WAVEFIELD-BASED MOTION ESTIMATION FOR RADAR AND SONAR ARRAY SYSTEMS

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ABSTRACT

This paper introduces a generalized and computationally efficient approach for the estimation of target motion parameters from received wavefield data collected from coherent sensing systems such as radar and sonar measurement arrays. The mathematical content of the algorithm is described, along with the general processing procedure to perform on recorded data. The algorithm presents a solution to the joint estimation of translational motion and periodic motion of targets, which has many practical applications for sensing and detection tasks. Experimental and simulation results are included supporting the effectiveness of the method.

1. INTRODUCTION

Motion estimation and target tracking has been a topic of great importance in the field of advanced sensing and imaging. Conventional approaches developed are based mainly on video image sequences for optical or infrared devices. Research efforts have more recently been devoted to motion estimation in the microwave and acoustic modes for applications to radar and sonar systems[1]. Advantages of such systems include situations with limited visibility, immunity to weather conditions, and robust tracking at far distances. Also, recent advances in the size and cost of sensors have led to their increased usage in a variety of upcoming technological areas such as autonomous navigation, portable computing devices, healthcare monitoring, and other exciting new fields. Given the developments of sensing systems in these areas, it is important that innovative approachs for the signal processing be explored that can yield useful and informative analysis.

Accurate motion estimation is required for many applications including trajectory estimation and route planning, collision avoidance, and target detecting and tracking. For real-time applications, one important objective is to perform motion estimation directly with measured wavefield patterns accurately, independent and parallel to the image formation procedure. Real-time processing is often critical for successful system operation, as analysis of events must often be presented to system operators immediatley.

Conventional motion estimation algorithms focus on the estimation of a singular motion parameter such as translational shift, angle of rotation, or frequency of vibration. This approach may be limited for certain tracking tasks that requrie joint estimation of multiple motion parameters. Specific examples of this include tracking flight trajectories of Unmanned Aerial Vehicles (UAV) while also estimating the frequency of rotation of the propeller blades to potentially identify the UAV model type. Another application of interest, is through-the-wall radar systems detection of people during search-and-rescue operations. This scenario may require the tracking of a walking person and simultaneously estimating biometrics of the person including breathing and heartbeat rates [2].

This paper introduces a motion estimation algorithm using the wavefield statistics acquired from coherent sensing and imaging systems. The approach presented here enables the motion estimation task with high computation efficiency as it can be done in parallel with image formation. Two steps are described for estimating tranlsational motion and periodic motion parameters indepenetly, and then a hybrid method comprised of both steps is outlined to address the joint estimation problem discussed earlier.

The first section describes the data acquisition procedure of measuring two-dimensional (2D) wavefield statistics from Frequency Modulated Continuous Wave (FMCW) sensing systems. The mathematical details of the algorithm are then discussed in Section 2 along with a systematic procedure of how it may be implemented. Lastly, numerical and experimental results are presented from data collected with a protoype FMCW microwave imaging system. The experiments focus on the estimation of translational motion parameters, and also the viability of measuring human biometrics such as heartbeat and breathing rates using radar sensing technology. A simulation study is also included exploring the effectiveness of the approach for the joint estimation of breathing and walking speeds of people.

2. FMCW MICROWAVE IMAGING OF MOVING TARGETS

FMCW sensing systems are widely used sensor systems for detecting and ranging of targets. A single FMCW sensing node transmits a probing single and receives the reflections from a target which can then be used to estimate the range of the reflecting target. It is usually more desirable to be able to localize the target accross two or three dimensions by mapping a target profile or image. This requires multiple FMCW transmitting and receiving nodes performing measurements on the entire target scene. The configuration of the transmitter and receiver sensor arrays determine the overall image formation process from the received data tracks.

Given the scenario, where the known target under investigation is moving instead of producing a single image, the imaging system needs to produce a video sequence of images. By characterizing the movement of the target over video frames, an estimate of the target's motion parameters can be performed. For each image frame m, an entire data collection cycle comprises the illumination and reception of data for each transceiver pair in the array. Before outline the image formation process, the range estimation of a target is first presented. With the range estimation procedure established, the image formation process for a single frame is described which can be repeated for a full video sequence of images.

For an imaging system confiugred in the Stepped-FMCW operating mode, the probing waveform is defined as

$$e_T(t) = Ee^{j2\pi f_k t} \tag{1}$$

where the seqeunces of sinuisodal frequencies are transmitted as $f_k = f_0 + k\Delta f$ for $k = 0, 1, ..., N-1$. If the target of interest is a single ideal point reflector, the transmit signal will reflect off and be detected at the receiver as

$$e_r(t) = \alpha E^2 e^{j2\pi f_k(t-\tau)} \tag{2}$$

where $\tau = \frac{R}{c_p}$ is the round trip flight time, R is the propagation distance from the transmitter to the receiver, and α represents the range attenuation and target reflectivity product.

To obtain a measurement of the delay time, the received signal will be demodulated by the transmit signal

$$\begin{aligned} e[k] &= e_r(t) \cdot e_T^*(t) \\ &= \alpha E^2 e^{-j2\pi f_0 \frac{R}{c_p}} \cdot e^{-j2\pi k\left(\Delta f \frac{R}{c_p}\right)} \end{aligned} \tag{3}$$

As can be seen, the second exponential in (3) is the only term dependent on k. It has been shown, the target location can be found through a DFT relationship over k with the demodulated sequence[3]. Assuming a scene that is the superposition of multiple targets, the relative location and reflectivity of multiple targets can also be found through this DFT relationship

$$p[n] \xleftrightarrow{DFT} e[k] \tag{4}$$

where the sequence $p[n]$ is referred to as the range profile and the mapping of the index n to the range of any target in the scene can be found through

$$R_n = \frac{c_P}{B} n_0 = \Delta_R n_0 \tag{5}$$

where $B = N\Delta_f$ is the total bandwidth of the system $\Delta_R = \frac{c_P}{B}$ is the range resolution of the system.

Next to estimate the 2D target profile $S(x, y)$ multilple range-profile estimates from the transceiver pairs are required. Let the transceiver sensor array be comprised of N_T transmitters and N_R receivers and the locations of the transmittting and receiving elements be $(x_T[i], y_T[i])$ for $i = 1, ..., N_T$ and $(x_R[j], y_R[j])$ for $j = 1, ..., N_R$ respectively. From the estimated range profiles of each transceiver pair $p_{ij}[n]$, the image profile can be formed by back projecting the appropriate range value to the correct location in the image profile scene.

This back-projection requires first computing the bi-static distance from each transceiver pair location to every location in the scene. That is for a particular pixel location (x, y), the distance the probing signal travelled from the ith transmitter and the subsequent distance the reflected signal travelled to reach the jth reciever is calculated as

$$R_{BS}(x, y) = \sqrt{(x - x_{T_i})^2 + (y - y_{T_i})^2} + \sqrt{(x - x_{R_j})^2 + (y - y_{R_j})^2} \tag{6}$$

The calculated range profile however may not have fine enough resolution, so the mapping procedure of bi-static range to image pixel value requires the linearly interpolated range profile for the appropriate transceiver pair $\hat{p}_{ij}(R)$. The interpolated range profiles are then mapped to the reconstructed image

$$\hat{S}_{ij}(x,y) = e^{-j2\pi f_0 R_{BS}(x,y)/c_p}\hat{p}_{ij}\left(R_{BS}(x,y)\right) \tag{7}$$

where the phase term serves provides a focusing effect on the final image.

The estimate of the target profile at the mth frame of data will be the superposition of each sub-image formed during one data acquisition cycle:

$$\hat{S}(x,y,m) = \sum_{i=1}^{N_T}\sum_{j=1}^{N_R}\hat{S}_{ij}(x,y,m) \tag{8}$$

The magnitude of the estimated profile $|\hat{S}(x,y,m)|$ can then be displayed as the final image to determine the relative location of reflecting targets present in the scene.

Any information present in the spatial profile $\hat{S}(x,y,m)$ will also be present in its spectrum. The wavefield spectrum of the reconstructed image is defined as

$$\hat{G}(f_x,f_y,m) = \mathcal{F}_{xy}\{\hat{S}(x,y,m)\} \tag{9}$$

The FMCW imaging modality can provide these wavefield statistics easily with a simple 2D DFT operation. For any coherent imaging system, the same wavefiled statistics may be computed and undergo subsequent processing that will be addressed in the remainder of this paper. The coherent wavefiled measurements can be processed in more computationally efficient methods that will be exploited to estimate target motion in the next section.

3. MOTION ESTIMATION FROM FMCW IMAGE SEQUENCES

Depending on the type of motion, different motion parameters require estimation to properly track a target's trajectory. Two main categories of motion will be explored in this section: translational motion and periodic motion. Once the respective approaches for parameter estimation in these contexts have been explained, the joint scenario will be explored of simultaneous translational and periodic motion. A hybrid approach combing the first two methods will be presented to estimate both parameteres to successfully complete the estimation task.

A. TRANSLATIONAL MOTION

The case of translational motion is the more common scenario encountered in sensing systems. For this case, the estimation process can be restricted to determining the 2-D shift parameters $[d_x,d_y]$ of the target between two sequential frames $\hat{S}(x,y,m+1)$ and $\hat{S}(x,y,m)$. Upon estimating the shift parameters from each image pair a history of the target's trajectory can be mapped over the entire video sequence.

The shift relationship can be expressed in the space domain and in the spatial freqeuncy domain as

$$\hat{S}(x,y;m+1) = S(x-d_x,\hat{y}-d_y;m) \tag{10}$$

$$\hat{G}(f_x,f_y;m+1) = \hat{G}(f_x,f_y;m)e^{-j2\pi f_x d_x}e^{-j2\pi f_y d_y} \tag{11}$$

where the shift property of the Fourier transform has be utilizied. This property will be the basis of the estimation routine.

The Smoothed Coherent Transform (SCOT) is a technique developed to aid in the estimation of shift parameters in the present of narrowband noise[4]. Microwave imaging systems typically encouter varying of amounts of background noise due to superfluous clutter in the scene and interference. The SCOT transform at a given frame $m+1$ is defined as

$$Q(x,y) = \int\int W(f_x, f_y) P_{m+1,m}(f_x, f_y) e^{j2\pi(f_x x + f_y y)} df_x df_y \tag{12}$$

where the first term in the integral is a normalization term that is defined as

$$W(f_x, f_y) = [P_{m+1}(f_x, f_y) P_m(f_x, f_y)]^{\frac{1}{2}} \tag{13}$$

Here the $P_{m+1}(f_x, f_y)$ is the power spectral density of the wavefield statistics at frame $m+1$ and $P_{m+1,m}(f_x, f_y)$ is the cross-spectral density between the wavefield statistics at frame m and $m+1$. Examing the product of these two terms is

$$W(f_x, f_y) P_{m+1,m}(f_x, f_y) \approx e^{-j2\pi(f_x d_x + f_y d_y)} \tag{14}$$

Here the phase history of the shift is the remaining term left after nomalization. It can be seen that the SCOT transform $Q(x,y)$ is the inverse Fourier Transform of this approximation. Therefore the maximum of the spatial function $Q(x,y)$ will be the location of the estimated shift parameter

$$[\hat{d}_x,\ \hat{d}_y] = \arg\max_{x,y}\ Q(x,y) \tag{15}$$

For the sequential video pairs from the wavefield measurements, one SCOT transform and maximum search can be applied. The result of the max search yields the estimate of the parameter of interest $[\hat{d}_x,\ \hat{d}_y]$. With these shifts stored over the entire video sequence the target trajectory can be then mapped and investigated for further analysis.

B. PERIODIC MOTION

If the target is moving in range periodically, the shift property is less useful as the wavefield measurements will contain a sinusoidal phase modulation component. To simplify the analysis, the model presented here will only describe a target moving in range with respect to the sensing system. The estimation procedure can be easily generalized to any direction, if the wavefield measurements are coherent over time, although the accuracy may degrade depending on array geometry and target position.

The change of target position in range at frame m can be modelled as

$$\Delta r_m = r_A \sin(\omega_o m) \tag{16}$$

where ω_0 is the normalized frequency of oscillation. If the target is illuminated by the FMCW probing signal and the target's motion is slow compared to the sweep rate the demodulated received signal will be of the form

$$
\begin{aligned}
e(k,m) = e(k)e^{-j2\pi f_k \Delta r_m / c_p} &= e^{-j2\pi f_k r_A \sin(\omega_o m)/c_p} \\
&\approx e(k)[1 - j2\pi r_A \sin(\omega_0 m)] \qquad (17)
\end{aligned}
$$

In this case $e(k,m)$ is the FMCW wavefield data measured by the sensing system at frame m. A taylor series approximation has been utilized to yield the second equation, which is valid for small amplitudes of vibration r_A.

The goal is to then to estimate the frequency of oscillation ω_0 from the wavefield data collected over time. Assuming the approximation in (18) is valid, it can be observed for each wavefield sample at frequency k the only time varying component is the oscillating signal $2\pi r_A \sin(\omega_0 m)$. Therefore to estimate the oscilallation frequency ω_0, for each wavefield component k a DFT may be computed over frame m yielding

$$
\begin{aligned}
e(k,l) &= DFT_m\{e(k,m)\} && (18) \\
&= e(k)\left(\delta(l) - \delta(l - M\frac{\omega_o}{2\pi}) + \delta(l + M\frac{\omega_o}{2\pi})\right) && (19)
\end{aligned}
$$

where M is the total number of data sequences.

The estimate can be performed by finding the location of the peak of $e(k,l)$ which should be constant accross all k and providing the appropriate scaling factor

$$
\hat{\omega}_0 = \frac{2\pi}{M} l_{max} \qquad (20)
$$

This technique then allows to produce a range profile from only the periodic target wavefield data $e(k, l_{max})$, and subsequent imaging and localization could follow.

C. JOINT ESTIMATION OF MULTIPLE MOTION PARAMETERS

If multiple motion signatures are present in the received data, this increases the difficulty of the parameter estimation procedure presented so far. The analysis here will again focus on a target moving only in the range direction while also vibrating sinusoidally simultaneously. If the target is moving at a constant velocity v_r, and vibrating sinusoidally at frequency ω_o then range position change over time m is

$$
\Delta r_m = v_r \cdot m + r_A \sin(\omega_o m) \qquad (21)
$$

Becuase the linear motion term and sinusoidal term are superimposed, separting the two in the phase history of the received data is more difficult. The wavefield data will be

$$
e(k,m) = e(k)e^{-j2\pi f_k \Delta r_m / c_p} = e^{-j2\pi f_k (v_r \cdot m - r_A \sin(\omega_o m))/c_p} \qquad (22)
$$

The goal is then to estimate only the linear shift first, and remove that component to isolate the periodically varying component.

Using the SCOT transform again we see on the wavefield data from track $m+1$ and m yields

$$Q(r;m+1) = \int W(f)P(f)e^{j2\pi f}df \tag{23}$$

Examing the product of the cross-spectral density and normalized spectral density

$$W(f)P(f) = e^{-j2\pi(v_r + r_A \sin(\omega_o(m+1)) - r_A \sin(\omega_o m))} \approx e^{-j2\pi v_r}$$

where the second approximation is true if $v_r >> r_A\left(\sin(\omega_o(m+1)) - \sin(\omega_o m)\right)$. That is the linear displacement is greater than the vibrational displacement. Therefore the estimate of the translational velocity will also be the maxima of the SCOT transform estimated

$$\hat{v_r} = \arg\max_r \; Q(r;m) \tag{24}$$

Now demodulation of the received data with the linear trajectory yields only the periodic component

$$e_p(k,m) = e(k,m)\cdot e^{-j2\pi f_k \hat{v_r} m/c_p} \approx e(k)e^{-j2\pi f_k r_A \sin(\omega_o m)/c_p} \tag{25}$$

The perioidic estimation technique outlined before can be used to estimate the freqeuncy of oscillation $\hat{\omega}_0$ from the technique outlined earlier. This technique can be gerenalized for targets moving with any translational trajectory and direction of oscillation. The general algorithm is summarized here

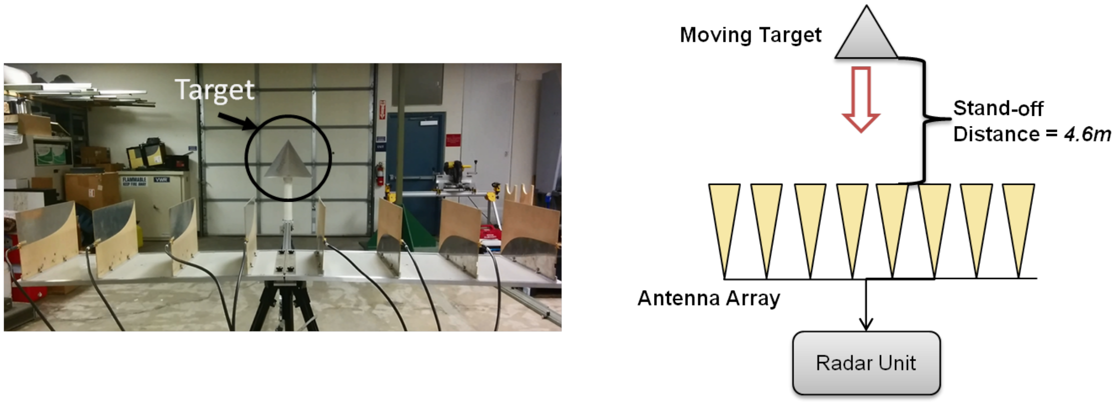
1. For the wavefiled samples $G(f_x, f_y)$ at frame $m+1$ and m use the 2D SCOT transform to estimate the translation shift $\hat{\mathbf{d}} = [\hat{d}_x \hat{d}_y]^T$.
2. Estimate the shift parameters over the entire video sequence of M frames and store them in the time-indexed parameter vector $\hat{\mathbf{d}}[m]$
3. Demdoulate the wavefield data with the shift parameters over the entire video seqeunce leaving only the periodically varying wavefield sample

   $\hat{G}_p(f_x, f_y) = \hat{G}(f_x, f_y; m)\cdot e^{-j2\pi(f_x\cdot\hat{d}_x(m) + f_y\cdot\hat{d}_y(m))}$
4. Estimate the frequency of oscillation $\hat{\omega}_0$ from the wavefield samples $\hat{G}_p(f_x, f_y)$ in either the range (x) or cross-range (y) direction.

It can be seen from the short summary that given the wavefieled samples collected from the sensing system, the relative design complexity of implementing the algorithm is low.

4. RESULTS

An analysis exploring the theory presented here was performed consisiting of both experiemental and simulated data. For the experimental work, a prototype S-FMCW radar system was utilizied as the data acquisiton system. The final hardware system included a smaller antenna array for portability. The hardware comprised of an 8-port AKELA vector signal generator and measurement unit (AVSMU) , 8 ultra-wide band Vivaldi antennas, and a PC programming interface to control



(a.) (b.)

Figure 1: (a) Experimental setup (b) Experimental diagram

the radar and collect data. The antennas were configured into a linear array of equally spaced antennas with spacing of $4.5cm$. All signal processing was computed in the MATLAB programming environment, which produced the graphical figures presented here.

To verify the translational motion algorithm, a simple experiment was designed. A corner reflector was situated approximately $4.6m$ down-range from the antenna array and mechanically moved along a motorizied rail directly towards the antenna array as seen in Fig.1. The radar unit collected data on the moving target over time while the actual position of the target was being recorded from the rail's mechanical tracking system, and the motion estimation algorithm presented here was utilized to estimate the target's trajectory from the raw radar data. In Figure 2, a reconstructed image of the target is displayed, and an estimate of the target's trajectory has also been plotted. As can be seen from the results the estimated trajectory tracked the actual target position closely over the collection cycle, and the average estimation error of the algorithm compared to the rail's mechanical tracking system was computed to be $0.59\frac{cm}{frame}$.

To test the frequency estimation procedure for periodic signals, a more realsitic scenario was constructued and a second experiment performed. For the measurement of biometric periodic signals of a human target, a careful experimental setup was configured in a controlled indoor environment as shown in Fig 3a. The person target was placed behind an 8-inch thick concrete wall approximatley $1.75m$ from the antenna array, and remained still for the duration of the experiment. The radar system was configured in CW mode with an illumination frequency of $3.941GHz$, and a only a single transceiver pair recorded data for 120 seconds from the breathing target. The vibration frequency estimation procedure described in Section 3B, was performed on the recorded data and the results can be seen in Fig. 3b. From the results, the estimation routine was able to succesfully identify the breathing freqeuncy of a person at $\hat{f}_B = 0.25Hz$ and the heartbeat rate of $\hat{f}_h = 1.5Hz$. The plot also contains a plot of the background signature with no target present to illustrate the strong signal level of the estimate routine relative to the background noise[5].

Lastly, to explore the joint motion scenario of a person walking a numerical simulation was performed to determine to validity of the numerical approximations. For the simulation, a simple

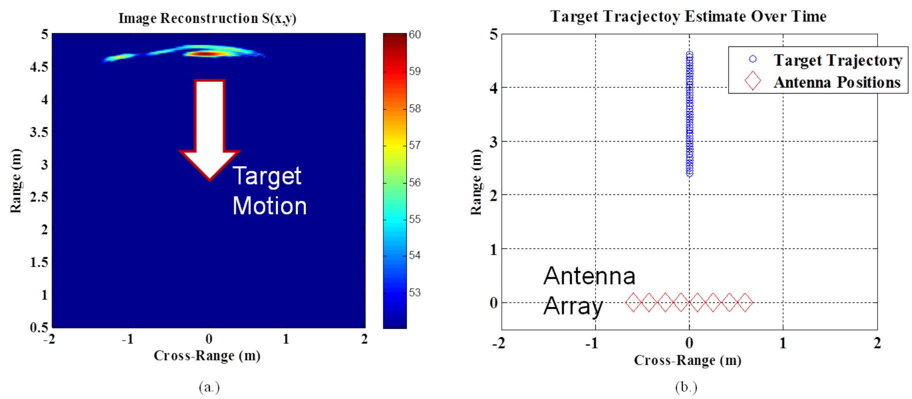


Figure 2: (a) Target image reconstruction and (b) Target trajectory estimate

scenario with a single person walking down-range towards a bi-static S-FMCW radar data collection system was constructed similar to the experimental hardware. To simpfy the scenario the breathing cavity of the person was set as the only reflecting material in the scene, and the probing waveform theory described in Section 2 determine the wave propagation and reflection physics in the simulation. The person was set to an offset distance from the radar of $50m$, the breathing speed was determine to be $12cycles/sec = 0.2Hz$, and their speed was set to $25cm/sec$. The FMCW frequency sweep transmitted a probing signal from $1GHz$-$4GHz$ with a sweep rate of $5sweeps/sec$. Figure 4a illustrates the simulation scenario described, and Fig. 4b is a plot of the target's range position over time. It is clear that the walking displacement and breathing displacement are both contributing to the time varying range shift.

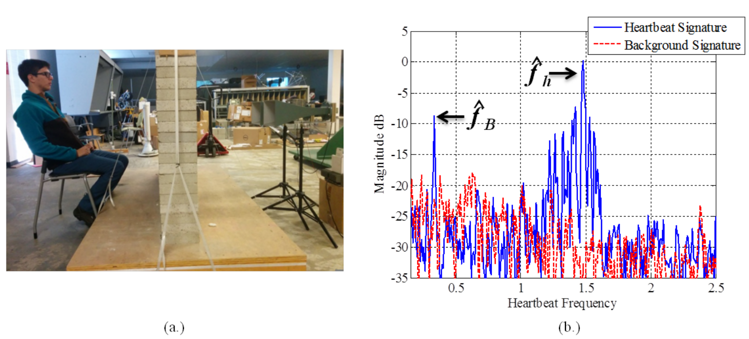


Figure 3: (a) Biometric detection experiment and (b) Period estimation results

The simulated system illuminates the target for around $25s$, and the simulated data is run through the joint motion estimation algorithm. The estimated breathing spectrum from the algorithm $e_p(f)$ can be seen in Fig. 4a, and the un-compensated breathing spectrum is displayed in Fig. 4b. As can be seen, when the linear motion of the target is uncompensated for, the relatively small peak of the breathing signal is not visible. However, properly estimating and compensating for the walking velocity accurately preserves the breathing content in the received data. The estimated breathing frequency $\hat{f}_B = 0.212Hz$ is very close to the true breathing rate known as a simulation parameter.

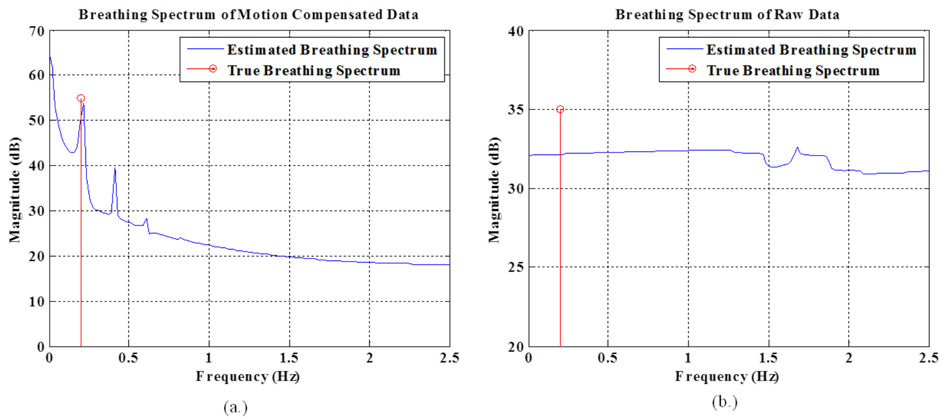


Figure 4: (a) Motion Compensated Estimation Results (b) Raw Data Estimation Results

CONCLUSIONS

A generalized approach for motion estimation for coherent imaging systems is presented here. The estimation alrogithm is capable of accurately estimating multiple estimation parameters succesfuly. The feasibility of the algortihm for the joint tracking of person's walking patterns and biometric signs has been verified with experimental data. This computationally efficient method for estimating mutliple motion parameters with relatively low-complexity design also has the potential to operate in real time situations for other applications. By simultaneously estimating both translational and periodic motion parameters this algorithm can be utilized to both track target trajectories while jointly performing classification tasks based on the frequency estimates. Future work will look to collecting experimental data on the multiple motion scenario in support of the simulation results described here.

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