Noise Predictive Information Rate Estimation for TDMR Channels

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ABSTRACT

In this paper, we use the forward recursion BCJR algorithm to estimate the symmetric information rate for Two Dimensional Magnetic Recording (TDMR) channels. In particular, we consider a TDMR read/write channel whose all components, including recording medium, write and readback processes are modeled in software. Since the primary source of noise in TDMR arises from irregularities in the recording medium and leads to highly colored and data-dependent jitter, the pattern dependent noise predictive (PDNP) algorithm is implemented to improve the accuracy and performance of SIR estimation. Furthermore, we study the performance gain of using the PDNP algorithm in SIR estimation through simulations over the Voronoi based media model for different TDMR channel configurations.

Index Terms

Symmetric Information Rate, Pattern Dependent Noise Predictive algorithm, Two Dimensional Magnetic Recording, and Voronoi Channel.

I. INTRODUCTION

Two Dimensional Magnetic Recording (TDMR) [1] is as an emerging technology that can achieve densities of 10 Tb/in^2 and higher. A pursuit of extremely high storage densities necessitates using only a small number of magnetized grains (5-10) to store one bit of information. The reduction in the number of grains per bit in TDMR results in variations of bit boundaries which consequently lead to data dependent and colored media noise [2]-[3]. These artifacts have to be handled using sophisticated signal processing and coding algorithms.

In this paper, we estimate the symmetric information rate (SIR) for Voronoi based TDMR channels. The SIR can be computed as a measure of the achievable storage densities. In [4], the SIR for binary-input channels with memory is computed by standard forward sum-product (BCJR) processing of the simulated channel output. Here, we implement the Pattern dependent noise prediction (PDNP) algorithm [5] to effectively handle the jitter noise observed in the TDMR channels as the noise is highly colored and data dependent. We embedded the PDNP algorithm in the forward recursion BCJR to compute more accurate approximation of the SIR for TDMR systems. By inserting a noise prediction/whitening process into the branch metric computation of the BCJR algorithm, PDNP is constructed. The PDNP-based SIR estimator uses different noise prediction filters for different input patterns in the trellis of the forward recursion BCJR

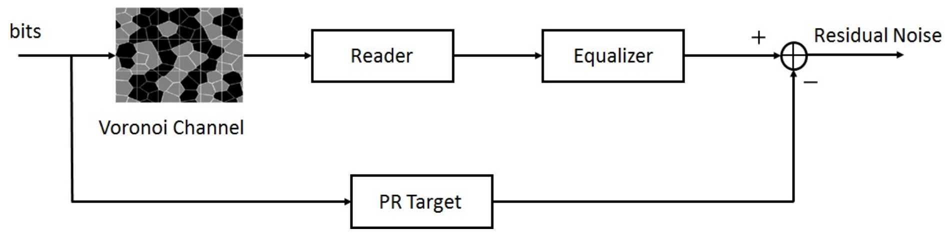


Fig. 1. The block diagram of TDMR system including read channel and equalizer is given. A simplified read channel architecture where the readback signal from the medium is equalized to achieve a controlled ISI called partial target response.

algorithm in order to take advantage of the data dependency of the media noise. Furthermore, throughout the simulations, we demonstrate the performance gain of using the PDNP algorithm in SIR computation for different TDMR systems.

The organization of the paper is as follows. In Section II, we describe a Voronoi-based media read-channel model and explain the techniques to design partial targets and equalizers. In Section III, we present the noise predictive SIR estimation algorithm. In Section IV, we incorporate the noise predictive SIR estimation algorithm to compute the SIR for TDMR channels and show the performance gain obtained of using the PDNP algorithm in SIR estimation. Finally, Section V concludes the paper.

II. TDMR READ CHANNEL MODEL AND EQUALIZER

The block diagram of a TDMR system is given in Figure 1. Let $x_{i,j} \in \{0, 1\}$ represent bits written onto the medium. Let $y_{i,j} \in \mathbb{R}$ represent the read-back samples obtained from the channel model. Let $\mathbf{x} = [x_{i,j}] \in \{0, 1\}^{n \times m}$ represent an $n \times m$ array of bits written on the medium and $\mathbf{y} = [y_{i,j}] \in \mathbb{R}^{n \times m}$ represent the corresponding array of read-back samples. Let $\mathbf{x}_{i,j}$ represent the set of bits that contribute to the read-back sample $y_{i,j}$. For the 3×3 read-head span case, $\mathbf{x}_{i,j}$ contains 9 bits $\{x_{i+k,j+l} \mid k, l \in \{-1, 0, 1\}\}$. The readback signal from the medium is equalized to achieve a desired target response, optimizing based on the MMSE criterion.

For TDMR systems, several channel models are proposed to aid simulations in studying the coding and signal processing algorithms. These models vary in complexity and accuracy in capturing the behavior of a magnetic medium. TDMR channel models can be classified into a) discrete grain models, b) Voronoi media models and c) micro magnetic media models. Voronoi based media models give a good trade-off between implementation complexity and the accuracy of representing a magnetic medium. In the sequel, the Voronoi based TDMR read channel model ([2]-[3]) utilized in this paper is introduced. The model consists of a media model, write process model and a readback model.

A. Voronoi based channel model

The grain centers are distributed on a medium according to the Poisson disk distribution with boundary sampling [6]. This distribution is characterized by the grain size parameter of the center-to-center (CTC) distance. In this model, the grain centers are separated by at least a distance of CTC, the minimum permissible distance between any two grain centers, and each grain has at least one neighbor.

B. Write Procedure

The magnetic medium is split into a rectangular grid, where each rectangular cell of size BL×TW represents a channel bit, where

- Bit Length (BL): the length of each bit in the down-track direction.
- Track Width (TW): the length of each bit in the cross-track direction.

The channel input signal $x(t_1, t_2)$ is defined by

$$x(t_1, t_2) = \sum_i \sum_j x_{i,j} \Pi_{TW}(t_1 - i \times TW) \Pi_{BL}(t_2 - j \times BL),$$

where $x_{i,j} \in \{-1, +1\}$ is the symbol which will be written on the $(i, j)^{\text{th}}$ bit cell and

$$\Pi_T(t) = \begin{cases} 1, & 0 \leq t < T, \\ 0, & \text{otherwise.} \end{cases}$$

When a bit is written into the medium, all grains whose centers lie within the bit region are polarized according to the bit value.

C. Read-back Procedure

The read back signal depends on the grain distribution, grain magnetization, and read-head design. We model the read-head response to be a 2-D Gaussian pulse. The 2D Gaussian pulse characterized by PW$_{50}(CT)$ and PW$_{50}(DT)$ at half-amplitude in the cross-track and down-track directions, respectively.

The media noise arises due to the irregular boundaries of grains and the random distribution of grain centers [3]. If two neighboring bits have the same polarization, there is no media noise caused from that border. The media noise is correlated in both down-track and cross-track directions. As we model the read head response as a 2D Gaussian pulse, the pattern dependent media noise can be approximated by a data dependent Gaussian distribution [7].

D. Equalizer and Partial Response Target

The equalizer used in the simulations for generating PR targets is the generalized partial response (GPR) equalizer which is proposed in [8]. The GPR objective is to find the target and the equalizer filter simultaneously by minimizing the mean squared error between the equalizer output and the desired signal, subject to a minimum-phase constraint on the target (monic constraint target). Here, we formulate the method for the 1D case. Let G be a $L_1 \times L_2$ matrix representing the GPR target of size $L_1 \times L_2$. The target is called 2D where $L_1 > 1$, otherwise, the target is 1D ($L_1 = 1$ and $G = [g_0, g_1, \ldots, g_{L_2-1}]^T$). Let $F = [f_0, f_1, \ldots, f_{N-1}]^T$ be a $1 \times N$ matrix representing the equalizer filter coefficients where N denotes the equalizer filter taps. The output at the equalizer is

$$z(i) = \sum_{l=0}^{N-1} f(l) y(i + l), \tag{1}$$

and the desired output at the equalizer is

$$\hat{z}(i) = \sum_{k=0}^{L_2-1} g(k)x(i+k). \tag{2}$$

where $x(n)$ is the input sequence written on the medium and $y(n)$ is the corresponding read-back samples. The error at the position (i) is defined as

$$e(i) = \hat{z}(i) - z(i). \tag{3}$$

The goal is to minimize the error vector e achieved by the equalizer as compared to the target chosen. In [8], the Minimum Mean Squared Error (MMSE) criterion is used to jointly design the partial response target and the equalizer such that

$$\begin{aligned} F &= R_{yy}^{-1} R_{yx} G, \\ G &= \frac{(R_{xx} - R_{yx}^T R_{yy}^{-1} R_{yx})^{-1} u}{u^T (R_{xx} - R_{yx}^T R_{yy}^{-1} R_{yx})^{-1} u}, \end{aligned} \tag{4}$$

where R_{xx} is a $L_2 \times L_2$ matrix with $(i,j)^{\text{th}}$ element given by $R_{xx}(i,j) = \mathbb{E}\{x_{k-i}x_{k-j}\}$, R_{yy} is a $N \times N$ matrix with $(i,j)^{\text{th}}$ element given by $R_{yy}(i,j) = \mathbb{E}\{y_{k-i}y_{k-j}\}$, R_{xy} is a $N \times L_2$ matrix with $(i,j)^{\text{th}}$ element given by $R_{yx}(i,j) = \mathbb{E}\{x_{k-i}y_{k-j}\}$, and u is a vector that has one element as 1 and all others zeros such that $u^T G = 1$.

III. Noise Predictive symmetric Information rate

The bit error rate is a useful performance metric for optimizing parameters of data storage systems, but it can not provide information about achievable user storage density. Here, we introduce a more comprehensive metric that can be used to simultaneously optimize all parameters and provide information about achievable densities. For this purpose, we use the symmetric information rate (SIR) between the input sequence $\mathbf{x}$ written on the medium and the equalizer output sequence $\mathbf{y}$. The SIR as a lower bound on the capacity of system can provide information about achievable storage density. The SIR is defined as

$$SIR = I(X;Y) = H(Y) - H(Y|X), \quad \text{when } p(x) \text{ is uniform.} \tag{5}$$

According to Shannon-McMillan-Breimann theorem [4], for a stationary ergodic finite state Markov process Y, we have

$$-\frac{1}{n}\log p(\mathbf{y}) \to H(Y), \tag{6}$$

as $n \to \infty$. As is well known, for any given block length n and any given channel output $\mathbf{y} = (y_1, y_2, \ldots, y_n)$, the probability $p(\mathbf{y})$ can be computed using the forward recursion BCJR algorithm [4], which operates on the trellis of the partial response channel. An ISI channel's trellis is described by the binary input alphabet X, output alphabet Y, finite set of states S, and by the conditional probability density function, $p(y_k, s|s')$ where $y_k \in Y$, s-the current state, and s'-the previous state. The first term $H(Y)$ in eq. (5), the entropy of the equalizer output, can

TABLE I
RS_{CT} (RS_{DT}) DENOTES THE READER RESPONSE SPAN IN CROSS-TRACK (DOWN-TRACK) DIMENSION. ALL THE PARAMETERS IN THE TABLE ARE SPECIFIED IN NANOMETERS. $\star$ INDICATES THAT THE PARAMETER IS VARIED IN THE SIMULATIONS. CTC= 7 NM.

	TW	BL	RS_{CT}	RS_{DT}	$PW_{50}(CT)$	$PW_{50}(DT)$
TDMR(1)	40	10	80	70	$\star$	34
TDMR(2)	$\star$	9	60	40	30	20

be computed using the forward recursion of the BCJR algorithm, as explained in [4]. For this computation, each trellis branch b at time k is assigned the metric $\mu(b_k)$ such that

$$\mu(b_k) = p(y_k, s_k|s'_k) = p(y_k|s'_k)p_{s'_k s_k}, \tag{7}$$

where $p_{s'_k s_k}$ is the probability of transition from state s'_k to s_k. The term $p(y_k|s'_k)$ is the distribution of residual noise of the equalizer. The noise is data dependent and colored. In order to mitigate these harmful effects, we incorporate noise-prediction filters which use the information from the neighborhood noise samples.

The PDNP algorithm uses different noise prediction filters for different input patterns in the trellis of the BCJR detector in order to take advantage of the data dependency of the media noise. The computation of the coefficients of the noise predictors and pattern-dependent variances are performed by a linear MMSE predictor [9]. Applying the residual noise n_k to a time varying linear predictor of the form

$$\hat{n}_k = \sum_{i=1}^{L_f} p_{i,k}(x_k)n_{k-i}, \tag{8}$$

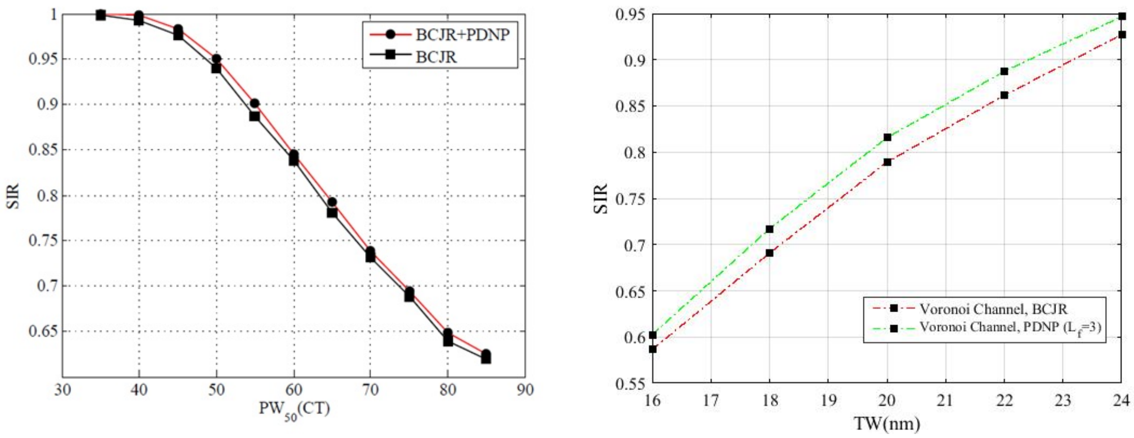
would then lead to an uncorrelated prediction error sequence $e_k = n_k - \hat{n}_k$. By employing noise prediction/whitening filters, we estimate the residual noise by an AWGN with variance σ_e^2 where σ_e^2 is the variance of prediction error sequence $e = (e_1, e_2, \ldots, e_n)$ which is dependent on the input data $\mathbf{x}$. Therefore, $p(y_k|s')$, the residual noise distribution, can be approximated by the AWGN noise with the same variance. The trellis is then processed from left (initial states) to right (final states) computing $H(Y)$. The data-dependent noise prediction is used to estimate the conditional entropy $H(Y|X)$ in eq. (5). According to the approximation, $H(Y|X)$ is also given by

$$H(Y|X) = H(\mathbf{e}) = \frac{1}{2}\log(2\pi e\sigma_e^2). \tag{9}$$

Finally, the SIR can be computed using the eq. (5).

IV. Simulation Results

All the simulations are done on TDMR systems with the parameters given in Table I. For both TDMR systems, the number of bit cells in the cross track and down-track directions is 5×4000. The center track (track 3) is the track of interest and the reader is located at the middle of



(a) (b)

Fig. 2. Investigating the performance gain obtained of using the PDNP in computing the SIR using the forward recursion BCJR. (a) shows the variations of SIR vs. ITI. By increasing PW_{50}(CT), ITI is increased since the reader senses more from two neighboring tracks. (b) shows SIR vs. TW.

this track. In the simulations, we have equalized the output of reader to a 1D PR target with L_{DT} taps in the down-track direction ($1 \times L_{DT}$ target) which is then followed by the forward recursion BCJR algorithm to compute the SIR for different TDMR channel configurations. We also incorporate the PDNP algorithm in the calculation of branch metrics in the BCJR algorithm to handle the data dependent and colored residual noise and have more accurate branch metrics. By inserting a noise prediction/whitening process into the branch metric computation of the BCJR algorithm, noise predictive information rate estimator is constructed which operates on a $2^{L_{DT}+L_f+1}$-state trellis where L_{DT} is the number of equalizer taps in the down-track direction and L_f is the number of whitening filter taps.

Figure 2(a) shows the performance gain obtained of using the PDNP algorithm in SIR calculation as a function of $PW_{50}(CT)$ for TDMR(1) system. We change the reader-span in the cross-track direction to have different ITI's. It is clear that for larger $PW_{50}(CT)$, ITI is increased, but still we have gain of using whitening filters in the branch metric calculation. Figure 2(b) demonstrates the performance gain of using PDNP algorithm in the SIR calculation as a function of TW for TDMR(2) system. In both experiments, the PR target size is $1 \times L_{DT} = 1 \times 3$ and the number of whitening filter taps is $L_f = 3$.

V. Conclusion

We have computed the symmetric information rate (SIR) using the forward recursion BCJR algorithm for TDMR channels to use as a new criterion to analyze the performance of TDMR systems. The SIR provides important information about the achievable storage density. We have modeled and studied the jitter noise of Voronoi based magnetic medium and its dependence on various data patterns written onto the medium. The media noise is colored and data dependent due to the following: 1) the PR equalizer makes the noise correlated, and 2) the irregular boundaries of grains and the lack of knowledge of these boundaries during the read back process makes the noise data dependent. In order to take advantage of the data dependency of media noise, we have used different noise prediction filters for different input patterns in the trellis of the forward recursion BCJR detector. Finally, we have demonstrated the performance gain of using the pattern dependent noise prediction (PDNP) algorithm in SIR calculation throughout simulations on different TDMR channel configurations.

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