# A CASE STUDY: A TASK-BASED INTERVIEW WITH A FIRST YEAR ALGEBRA LATINA STUDENT

By

BROOKE JUNDT

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A Thesis Submitted to The Honors College

In Partial Fulfillment of the Bachelor's degree
With Honors in

Mathematics

THE UNIVERSITY OF ARIZONA

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Approved by:

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Dr. Cynthia Anhalt
Department of Mathematics

**Abstract**

The purpose of this case study is to report on one Latina student's success in algebra by uncovering components of a support system for this student's learning and achievement in high school algebra. The focused algebraic activity is application of skills in a pattern sequence problem in a first year algebra course. This issue is important to the field of mathematics education because the National Assessment of Educational Progress (NAEP 2017) data has shown that Latino/a students' success rate in a first year algebra needs attention due to schools not meeting the academic needs of Latino students. This study examines one individual's thinking on an algebra task, her application of skills, and some personal thoughts on her self-perception as a mathematics learner. The student in this case study attends a school that provides structural support to students enrolled in first year algebra. In addition, the mathematics teacher of the course shared her insights on how she provides students opportunities to be successful in algebra. The researcher of this study was fulfilling a field practicum requirement for a senior-level course in a mathematics teacher preparation in the high school algebra class.

**Acknowledgements**

In the process of creating this honors thesis I have multiple people that deserve acknowledgement. First, Dr. Cynthia Anhalt, my thesis advisor, for going above and beyond supporting and believing in me every step of the way. I could have not had a better role model or advisor throughout this process. Second, Paloma should be acknowledged for her participation in my study. Without her my study would not exist, so I am grateful for her willingness to participate and the time I spent with her discussing mathematics topics. Third, I would like to acknowledge the Algebra Teacher for allowing me to interview one of her students as well as work in her classroom for a whole semester. Lastly, I'd like to acknowledge the school district that allowed me to interview one of their students for the purpose of my study. I could have not done this without the support of the people listed and my friends and family.

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## A Case Study: A Task-Based Interview with A First Year Algebra Latina Student

### Introduction

The issue that is being examined is the support system that allows for high school Latino/a students to be successful in mathematics. The goals of the study were to uncover a Latina student's algebraic thinking, her personal thoughts on her success in mathematics, and her academic support. This thesis will review historical information about Latino/a students and their successes in mathematics based on previous studies and literature. This study includes a description of the student's environment and academic support system and an analysis of a personal interview and a task-based interview utilizing an algebraic pattern sequence problem.

### Statement of Purpose and Relevance

The purpose of this study was to uncover components of a support system for a Latina student's learning and achievement in high school algebra through interviews and a focused algebraic thinking activity. The focused algebraic activity is based on a pattern sequence that requires application of first year algebra concepts. This issue is important to the field of mathematics education because the National Assessment of Educational Progress (NAEP 2017) data has shown that Latino/a students' success rate in a first year algebra needs attention due to schools not meeting the academic needs of Latino students. This study examines one individual's thinking on an algebra task, her application of skills, and some personal thoughts on her self-perception as a mathematics learner. In addition, the teacher of the course shared her insights on how she provides students opportunities to be successful in algebra and how she personally supports them academically.

## Methodology

### Participants

The participant of the study is one self-identified Latina student, Paloma that was invited to participate by the teacher in a freshman algebra class. The class is called Algebra with Algebra Support. The students meet with their teacher twice a day: once for algebra class and then again later in the day for the algebra support class. The Algebra Support class is apart of the school's Freshman Academy program to better support their students' academic success. Paloma was recommended by the teacher to participate in this study due to her success and positive work ethic in first year algebra class. She was invited to participate in a personal interview and a task-based interview with her permission as well as permission from her parents.

### Setting

The setting of this research study was at a public high school in the southwest region of the United States. The school student population was predominately Latino/a. The high school offered a range of courses for their students. Specifically, the school offered Algebra 1, Geometry, Algebra 2, and College Algebra (Pre-Calculus). Another highlight of the course offerings at the high school were the academies that they created to support their students. For example, they had an academy to promote real-world experiences for their students to prepare them with college and career skills. The Freshman Academy created opportunities and an environment for the students to be known by their teachers through more structured contact time be valued in the classroom, and to provide motivation to achieve academic success. The freshman student schedules also consisted of a homeroom class which focused on topics that help students focus on learning, develop positive study habits, and learn about frameworks for

developing a personal growth mindset specifically in mathematics learning (Boaler, 2016).

The particular class that was being studied was an algebra one course, titled Algebra with Algebra Support. The Algebra Support component is a separate class period where the students meet with their teacher for a second time during the school day to allow the class to spend more time on the content. The students receive an elective credit for this course and the teacher utilizes this time to review for assessments, have application activities, and it allows the students to have more time to process the content as a whole group with their teacher as an accessible resource.

The regular algebra classroom consisted of twenty students that were predominately Latino/a. The class covered topics like creating linear equations, solving for slope, slope-intercept form, standard form and point slope form of linear equations, solving systems of equations, and arithmetic sequences; these topics are typical topics for an algebra one course. From observations, the students demonstrated deep respect for their teacher and understood that their participation in class was vital to their success. Occasionally there were behavioral issues or students getting off task, but the teacher had set the classroom up in such a way where inappropriate behavior was not tolerated. Overall there seemed to be comradery between the students in the classroom. While they were shy sometimes participating in front of the whole group, they worked well together in small groups. The students understood the value of participating in class and how that would directly relate to their success in class. For example, the teacher encouraged them to work together in small groups before sharing out in front of the whole class to gain confidence in their mathematical understanding. If the students were asked to work on a problem they were expected to be working with the students around them, and then when it was time for a class discussion the teacher would call on representatives to share on

behalf of their group. The teacher encouraged classroom discussion as well as incorrect answers as they were instances where the students could learn from themselves as well as each other.

In addition to the algebra class as the physical setting, a community of learners was developed during the academic year due to the teacher's deliberate goals of establishing a community of learners from which all students learn from each other as they worked through problems in small groups and through listening to each other. The teacher was an important component of the support system established in the classroom. The teacher not only supported the students in the academics, but also provided emotional stability in their everyday school experience. It is important to point out that the teacher is a graduate of the same teacher preparation program as my program, which requires a degree in mathematics with an emphasis in mathematics teaching, thus preparing the candidate in both the mathematics content and developing mathematics pedagogy.

As the researcher and author of this study, I became part of this learning environment in the regular algebra class, and the students gradually began to view me as an additional academic resource, therefore, this study has components of ethnography due to the nature of the setting and my role in the classroom as a teacher-intern.

**Data Sources**

The information that is being gathered to answer the research questions stem from Paloma's work and thinking while she was engaged in the algebra task, a personal interview, classroom observations field notes, and a brief interview with the teacher. Paloma worked independently on the algebra pattern task and was asked to verbalize her thinking while working on the problem. I did not help her with the algebra task; however, I prompted the student to share

her thinking while she worked on the problem. I documented the student thinking as field notes, which provided insights later during analysis of the task-based interview.

**Research Questions**

- What are Paloma's perceptions about herself as a learner and her success in mathematics?
- What connections did Paloma make while engaging in the algebra pattern task between the skills she had been learning and the application of the skills?

**Student Interview Protocol**

The student interview protocol was designed in two parts; the first part is a list of questions with expected verbal responses, and the second part is an algebraic activity that requires the student to use mathematical skills and reasoning to solve a patterned based algebra problem.

The first part of the protocol was created to ask the student about her personal background, to get an understanding of who she is as a person, as well as to gather information about her attitude towards mathematics. The questions were:

1. Tell me about yourself and who you are.
2. What have you learned in math?
3. What has made you successful in math?
4. What math do you plan on taking in the future?
5. How do you see yourself in math?

The second part consisted of the algebraic task (see Figure 1). The student was given the patterned task and was asked to use color tiles to create the pattern and then create the following figures. In addition, the task required a table of data points, a description of observations of the

pattern, and a mathematical rule for the pattern. Once the rule was determined, the task required an explicit connection between the terms of the equation and the geometric representation, a prediction of the graph followed by the graph itself on a coordinate grid.

The nature of the relationship of the task is linear. The students had been working on recognizing and writing linear equations, specifically slope, y-intercept, the variable, and the form $y = mx + b$ as a linear equation. My goal was to uncover how the student would make connections between skills and application in working with a linear relationship. The algebra task was not explicit in giving the student the slope or the y-intercept, but rather it provided the student a pattern sequence that required application of the concepts. The student had to consider the variable as the nth-term or the figure number. Figure 1 includes the pattern figure that was posed to the student along with associated scaffolded tasks (see Appendix A).

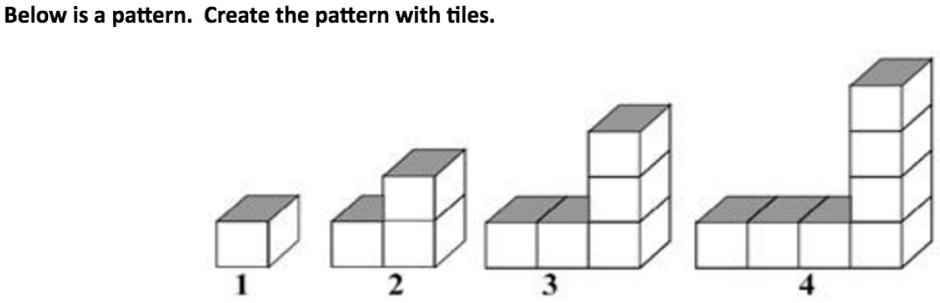



Figure 1. Algebra task.

## Review of Literature

This literature review focuses on areas that are relevant to Latino/a students' academic achievement. First, a brief discussion on the condition of Latino/a students in education helps place the current study in reference to the general Latino/a student population and statistics in addition to a summary of the literature that has focused on a deficit perspective of Latino/a students' academic success. Second, Culturally Relevant Teaching (CRT) tenets are discussed as a way to frame the various components that help students be successful in school. Third, a focus on access and opportunities allow for ways to raise issues of equity in mathematics education.

### Condition of Latinos in Education

It has been found that in the last ten years there has been a lot of improvement in education for the Latino/a community, but there is a lot more that still needs to be done (Excelencia in Education, 2015). Data has been collected and assessed throughout the United States from 2015 on Latino/a students in the educational system from kindergarten to graduate school. To set a background of Latinos in the educational system, first here are some facts on Latinos in the nation. Latinos are the second largest racial group in the United States, in 2012 they accounted for 17% of the population in comparison to the 63% of Whites (Excelencia in Education, 2015). Latino/a children are more likely to live in poverty than others, with 33% of Hispanic families with children under 18 below the poverty level (Excelencia in Education, 2015). Lastly, the Latino/a representation in K-12 education has grown to 24% of the public school system (Excelencia in Education, 2015). With percentages of Latino/a children attending public schools rising, there needs to be action taken within the schools to ensure that they are not failing school, dropping out, or being excluded against because of their background and culture.

Right now Latinos are the second largest group enrolled in secondary education behind Whites and things need to change to ensure that they are given the same opportunities and experiences as every other child in the school system (Excelencia in Education, 2015).

Gutiérrez says in her reviewed work of the book *Latino High School Graduation: Defying the Odds*, that there are successful students that exist, but they are normally not the focus of the research. Instead researchers focus on at-risk students and those studies perpetuate the negative stereotypes of the Latino/a student (1996). She says, "Because there are many people who are misinformed or ill-informed about Chicano/Latino students and their potential to learn or their ability to contribute to society at large, I am especially critical of research that has the potential to perpetuate stereotypes" (Gutiérrez, 1996). The Latino/a students in school do not all follow the negative stereotypes that have been given to them for decades, as shown by her research in the 90s. There are successful Latino/a students in mathematics that should have been given more attention to so that educators and researchers can start a new way of including aspects in the classrooms that allow for the Latino/a students to be even more successful.

**Culturally Relevant Teaching**

Children from minority backgrounds have a tendency to struggle in the public school system, and one of the reasons is because educators are trying to put culture into the student's education, versus trying to put education into their culture (Ladson-Billings, 1995). Students are being failed by their school systems simply because their culture is different than their educators' culture. A response to this is culturally responsive teaching or pedagogy. Culturally responsive teaching (CRT) defined by Gay (2002) is, "using the cultural characteristics, experiences, and perspectives of ethnically diverse students as conduits for teaching them more effectively." So

basically, allowing the students to learn about things or in an environment that is capitalizing on their culture, background, and experiences, so that the students have something to relate what they are learning to. Ladson-Billings (1995) defines culturally relevant pedagogy as "specifically committed to collective… empowerment… [that] rests on three criteria or propositions: (a) students must experience academic success; (b) students must develop and/or maintain cultural competence; and (c) students must develop a critical consciousness." This definition focuses on three aspects that the students should be able to accomplish through their education. One of them is cultural competence where the students are able to use their culture and the knowledge that they have of their culture in the classroom. "Culturally relevant teachers utilize students' culture as a vehicle for learning" (Ladson-Billings). So both of these definitions are focusing on the idea that the teachers, educators, schools, and anyone that has an impact on student learning will use the students' culture in the classroom as a guide to enable the students' learning.

**Equity: Access and Opportunities**

"An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential" (NCTM, 2014, p. 59). All students in mathematics education have the right to be given equal opportunities to ensure their success. Teachers, parents, administrators, and students need to steer their thinking from the unproductive beliefs about access and equity in mathematics to the productive beliefs about access and equity in mathematics so that all students are given the support they deserve to be successful in mathematics. Specifically, three unproductive beliefs are highlighted followed by the productive belief. First, the belief that all students have innate levels of ability in

mathematics needs to be changed to the belief that all students have the ability, opportunity, and effort to be successful in mathematics (NCTM, 2014). Second, the belief that students living in poverty, or low-income areas lack the behavioral characteristics to be successful in mathematics is countered by the productive belief that effective teaching practices, which includes engaging students in challenging mathematical task, allow for all students regardless of background or socioeconomic status to have the potential to open up greater opportunities for higher-order thinking and for raising the mathematics achievement for all students, including poor and low-income students (NCTM, 2014). Third, the belief that all students need to receive the same learning opportunities in order to have the same academic outcomes needs to be shifted to the belief that all students need to receive differentiated supports to ensure that they all are mathematically successful (NCTM, 2014).

Another important part of access and equity in the classroom to consider is how the teacher responds to the students' backgrounds and experiences when designing, implementing, and assessing instruction (NCTM, 2014). "Acknowledging and addressing factors that contribute to differential outcomes among groups of students is critical to ensure that all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content, and receive the support necessary to be successful" (NCTM, 2014, p. 60). When a teacher addresses the factors that can cause unbalance in the classroom it allows for the students to recognize that they are all different mathematical learners and thinkers, which is why mathematical instruction is not one size fits all, rather the teacher can meet students' needs by acknowledging their diversity and learning needs.

## Results

The results are reported systematically and include the interview questions and responses about the student's self perceptions as a learner of mathematics, the student's work and responses for the algebra task, and the teacher's reflection about her beliefs on teaching mathematics with a lens on equity and access for students.

### Personal Interview

The interview questions and Paloma's responses are below.

Question 1: Tell me about yourself and who you are.

> Paloma: I'm 15 years old. I'm short, 4'11''. I love to play basketball but I hurt my wrist. Same with softball. I like to join/experience new things. My favorite class is algebra, this class. I don't have problems with school, I like it.

Question 2: What have you learned in math?

> Paloma: Basically like how to write equations and how to solve them. I have problems with fractions but I'm getting better. Same with word problems. I do know how to, I learned more about exponents. I know how to add, subtract, divide, basic math.

Question 3: What has made you successful in math?

> Paloma: Well just not giving up. If I do a mistake it's not as bad as other people think. I know my teacher and classmates wouldn't make fun of me. Even if they did, it would be okay. I know my teacher and classmates are here to help me.

Question 4: What math do you plan on taking in the future?

> Paloma: Geometry, calculus – my brother is in it now and I see him struggling, so I would like to challenge myself.

Question 5: How do you see yourself in math?

Paloma: It depends on what [the teacher] is showing us. If I knew it from eighth grade then it's easier. Otherwise if it is new I do practice problems and it gets better. So it's not challenging. I know I can make mistakes and that's how I learn.

**Algebra Task**

The questions for the algebraic tasks are below with Paloma's responses and my field notes (FN).

1. Create the next figure in the pattern. How many tiles are in figure #5?

FN: Paloma wrote down how the figures changed by adding a plus two by the picture.

Interviewer: "Can you make the 5th figure with the tiles?"

Paloma: "It doesn't matter how it looks right?"

Interviewer: "You want to make it look like the others [same formation]."

Paloma: "There are 9 tiles in figure 5."

2. How many total tiles in figure 6?

Paloma: "There are 11 tiles in figure 6."

3. Write observations about the pattern – how is the pattern changing? How can you describe it?

Paloma: "For each figure it adds 2 more."

4. Make a table and collect data.

| Figure Number | Number of Tiles |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | 5 |
| 4 | 7 |

5. Describe a method for finding the total number of tiles in figure #10.

Paloma: “You could either keep adding 2 [four times], or since figure six has 11 tiles, you add 8 more. Because 2 that you’re adding [each time] plus the four [more figures] that you need to get to [figure] ten, it would equal 19 [tiles].”

6. Write a rule or an equation to predict the total number of tiles for any figure x.

FN: Paloma wrote something down and tried to check it. $y = x + 2$ then erased it. She then compared the row of the figures to the column of the figures.

Paloma: “Wait I got this. You multiply by two and subtract one.”

FN: Paloma checked her table and got excited that she had an insight into the pattern. Paloma wrote down $y = x \cdot 2 - 1$.

7. Explain how your rule is connected to the geometric figure.

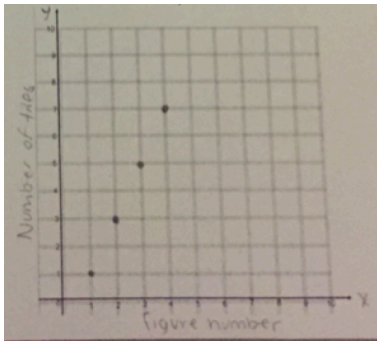
Interviewer: “Where do you see the x, the 2, and the -1?”

Paloma: “x would equal the number of tiles [in the row], and 2 is from multiplying [doubling] that you’re putting on the top [column on top of the row] and the 1 is from subtracting it from the top [column].”

8. Predict what the graph will look like.

Paloma: “I think the graph will be a straight line but when I label it, it would have some spaces in between.”

9. Show a graph of the data. Label the axes. Explain how to “see the slope.”



### The Teacher's Beliefs

From interactions and emails, the teacher has shared with me about her beliefs in teaching mathematics to underrepresented student populations, and strategies that she believes are essential to student learning. She mentioned:

> I certainly want all students to feel like they have the capability for success. As often as I can, I celebrate the successes embedded in mistakes. For example, my […] students will often complete an entire problem, find a solution and then realize they mistook their own handwriting and solved the problem with an 8 instead of a 5. [...] Yes, it may have led to an incorrect answer but they understood what was needed to solve the problem and that's what's more important. I also provide plenty of opportunity for students to work in pairs or groups so they can gain confidence in their work or feel more comfortable reaching out to a peer if they're stuck. And as often as I can I stress the fact that anyone can be good at math. Students have grown accustom to the idea that some people are simply 'math people' and it's part of my job to try and dispel that myth. (Algebra Teacher)

## Analysis and Discussion

### Personal Interview

From the student interview I was able to gain a better insight into Paloma's life at and outside of school as well as how she views herself in mathematics. The questions were designed to capture her current thinking about who she is and her relationship with mathematics and her future course choices in mathematics.

For the first question, "Tell me about yourself and who you are" Paloma gave responses

that show that she engages in activities outside the classroom that include sports, experiencing new things, and even that her favorite class is algebra. This allowed me to become more familiar with Paloma as a person versus limiting my knowledge of her background beyond the mathematics classroom, and it allowed for her to become more comfortable talking to me in preparation for the next questions.

For the second question, "What have you learned in math?" Paloma talked about how she has learned to write and solve equations, work with word problems, and become more comfortable with fractions. Her response was directly related to the algebra task that I gave her because she had to apply her knowledge of the components of linear equations to the pattern task that required her to write an equation that modeled the pattern.

For the third question, "What has made you successful in math?" the first thing Paloma mentioned is that she does not give up. She was familiar with the quest for perseverance and creating a growth mindset during mathematics learning as she had been studying about these topics during her homeroom. The exposure to these topics helped the student understand the expectations that the school had set for her in learning mathematics. It was very apparent that Paloma demonstrated an outlook for a personal growth-mindset. Paloma also talks about how she knows that her classmates and teacher are there to support her and help her when she needs it.

For the fourth question, "What math do you plan on taking in the future?" Paloma demonstrated that she would like to continue her studies in mathematics. She discussed how she would like to be challenged in mathematics so that she can learn new things.

For the fifth question, "How do you see yourself in math? " Paloma mentioned that she works at getting better at new concepts by doing practice problems and that she is not afraid of

making mistakes, and by making mistakes is how she learns. She has figured out a process that makes sense to her to allow for her to be successful when learning new material in mathematics.

**Algebra Task**

The algebra task was designed to help the student make connections with the skills she had been learning, such as slope, the use of a variable, and graphing. The problem required her to interpret and generalize an algebraic pattern to be able to write a linear equation. The task asked the student to create a continuation of a pattern, describe the pattern that she sees, form expressions that lead to the generalization of an equation, graph the equation, and verbalize how she sees the slope in her graph. Paloma was familiar with linear equations, as it was a topic that the class had spent time on, and the pattern task was meant to challenge the student to make connections with the knowledge and skills she had gained in class to a different representation of a linear relationship.

While observing Paloma work on the algebra pattern task I recorded everything she was doing and saying to make sure that I could understand her thought process as much as possible. When she first started working on the problem she wrote down how she saw each figure changing in the pattern to see if she could come up with a pattern that represented what was happening. She very quickly realized that each figure was adding two from the previous figure. I prompted her to create the fifth figure with the tiles and she asked me, "It doesn't matter what it looks like right?" to which I responded with, "You want it to look like the others." She created the fifth figure out of the tiles and I took a picture of it (see Figure 2) after she recorded how many tiles would be in figure five. I then prompted Paloma to continue on the next questions of the task. She answered the question about how many tiles were in figure six without hesitation,

and then in her own words verbalized how she saw the pattern changing. While she was creating a table of values, off to the right hand side of it, she marked how the number of tiles was adding two to the previous value each time.

For question six, Paloma needed to take time to really think about an equation that could represent the growth in the figures. She started by looking at the figures and thinking about how each was growing by two, so the first equation that she wrote down was, $y = x + 2$. After she wrote that down, she checked her equation against the figures and the table that she created. She wrote down $1 + 2 = 3$and $2 + 2 = 4$, which are true statements, but they do not accurately represent the growth of the figures. So she crossed those out, and started thinking again. She was looking at comparing the bottom row in the figure to the column of the number of tiles. Finally she said, "Wait, I think I got this. You multiply by two and subtract one." She then wrote $y = x \cdot 2 - 1$.As I observed her substitute values in her equation with her table of values, I could see her excitement because the equation that she came up with was accurately representing the pattern in the figures.

This kind of task prompted the student to apply the knowledge and skills that she had been learning in her classroom in a way that added conceptual understanding to algebra topics. Paloma was able to make connections to the meaning of the variable when she interpreted that the variable represented the figure number in the pattern sequence. She noticed that the figure number coincided with the number of tiles in the row of the corresponding figure.

It is important to note that Paloma's first equation that she wrote down to model this pattern was $y = x + 2$. She recognized that each figure in the pattern was adding two from the previous figure, so she believed that an accurate representation of that was the equation,

$y = x + 2$. Here was a critical point in her time with the task, she could have very easily decided that she came up with an equation that followed what she noticed was happening and stopped. But instead, Paloma checked her work against what she knew was true, the table of values that she created, and found that her equation did not model the pattern. Now Paloma was back at the beginning, she still needed an equation that modeled what was happening, and her first attempt was incorrect. Here she came to a crossroads, she could continue working and applying what she knows to persevere through the problem or she could give up and say that she tried it without success. As stated before, Paloma had learned about growth mindset in her homeroom and in her interview she talked about how she never gives up and that making mistakes allows for her to learn. By continuing this problem, Paloma showed through her actions that she truly has a growth mindset and she can preserve to be successful in mathematics.

After Paloma recognized that her original equation did not model the pattern, she continued to look at the pattern, her work, and the table she created in order to write an equation that models the pattern. Then the equation came to her, and again she checked her equation against her table to ensure it was an accurate equation. This is a milestone in her algebraic thinking. The whole time leading up to her correct equation, Paloma noticed how the figure was changing by two tiles each time. So, that is what her thinking was based on, which to her meant that the change from figure to figure could be represented as plus two. Her thinking had to make a jump from adding two each time, to the idea that the figures are built off the previous figure, therefore the equation will require more, and in this case that meant multiplication and subtraction (a slope and a y-intercept).

Paloma created two equations while she was working through the algebra task to model

the pattern, $y = x + 2$and $y = x \cdot 2 - 1$, and both were written in slope-intercept form of a linear equation. She applied what she knew from what she learned in class about slope and writing linear equations to this problem, without being prompted to write her equation in a certain way. This shows that Paloma has an understanding of a form of written linear equations.

At the very end of the task Paloma was asked to predict what the graph of the equation that represented the pattern would look like. She responded with, "I think the graph will be a straight line but when I label it, it would have some spaces in between." So then I asked her what she meant by that, and she began to explain to me that she would not be connecting the dots, coordinate points, because the graph would not exist at those places. What Paloma was getting at, without expressing with mathematical language, was the idea that the graph she created was discrete and not continuous. She recognized and made the connection that the tiles cannot be cut into pieces or have fractional representations, all of the tiles and figure numbers were going to be whole numbers, therefore it was inaccurate for her to connect the points on her graph because that would be a misrepresentation of the algebra pattern.

**Productive Beliefs**

The three Productive Beliefs (NCTM, 2014) that I examined with application to the case study of Paloma are: (1) all students have the ability, opportunity, and effort to be successful in mathematics; (2) effective teaching practices allow for students regardless of background or socioeconomic status to have the potential to open up greater opportunities for higher-order thinking and for raising the mathematics achievement for all students; and (3) all students need to receive differentiated supports to ensure that they all are mathematically successful.

The first Productive Belief, *all students have the ability, opportunity, and effort to be*

*successful in mathematics,* is evident in Paloma's case. Paloma had been learning concepts in her algebra class that are typical across most algebra classes, and the opportunity for application of the learned knowledge and skills through a variety of problems is a natural progression. The pattern sequence type of task provided the opportunity for Paloma to make mathematical connections between the various concepts in linear relations for which interdependent skills are sometimes taught in isolation and independent from one another. Paloma was able to demonstrate her ability to think mathematically through an abstract concept, that being of creating an equation to satisfy the pattern sequence, and make connections through a concrete experience (using color tiles) and verbalizing her thinking, thus materializing her effort to be successful in this algebraic task.

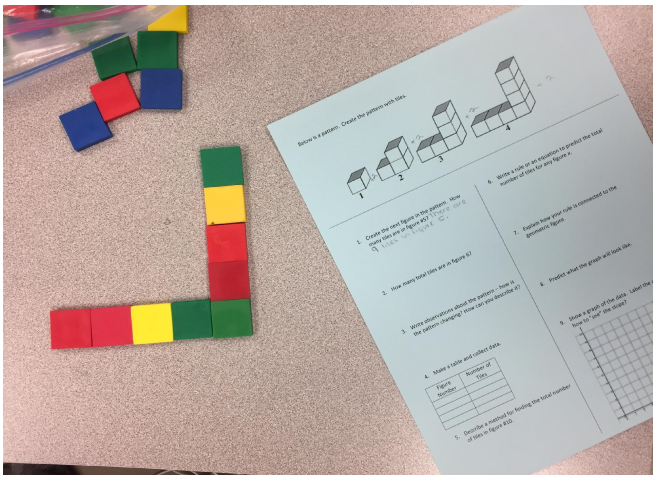


Figure 2. Paloma's concrete representation.

The second Productive Belief, *effective teaching practices allow for students regardless of background or socioeconomic status to have the potential to open up greater opportunities for higher-order thinking and for raising the mathematics achievement for all students,* is extremely important for Paloma because she is a self identifying Latina student who strives for academic

achievement. Paloma is encouraged and motivated in mathematics because of her success in learning. Because the school serves underprivileged families, the school has adopted strategic research-based programs to promote academic success for the students. All students were expected to treat each other with respect, and the teacher embraced the students' ethnicity and culture as part of their assets by which they live and learn.

The third Productive Belief, *all students need to receive differentiated supports to ensure that they all are mathematically successful,* was evident with the school structure support system of the Algebra Academy. She is at a high school that recognizes the need to support the first year students academically, and specifically in mathematics, Paloma is supported by having an algebra support class everyday as well as by teaching her about the growth mindset and how every student has the abilities to succeed in mathematics. The teacher has a seating chart that strategically placed students together in pairs and in groups of four to ensure that every student is supported by peers. During this time the teacher is able to work one-on-one or in small groups with students based on their learning needs. Everyday Paloma is given an opportunity to have specialized attention for her mathematical needs and in turn, have successful experiences in algebra class.

**Support Structure**

The support structure that is present in Paloma's life is a key component of her success in mathematics. The high school that she attends has immediate support for first year students created by having the Freshman Academy. Paloma is able to feel valued at school and that her teachers want her to be successful, and she has the opportunities to learn about topics such as the growth mindset and how this type of mindset impacts success in the mathematics classroom. In

the mathematics classroom she has a teacher who wants all of her students to "feel like they have the capability for success" and she celebrates the "success embedded in mistakes." Paloma is truly supported by her teacher in her mathematics endeavors, and her teacher believes that it is her job to dispel the myth that her students believe that only certain privileged students or people are "math people." Lastly, Paloma is supported intellectually in the environment of her mathematics classroom. Her teacher believes that it is important for the students to have the opportunities to work together in pairs or in groups to foster the students' confidence in their mathematical thinking. The school structure, the teacher, and the classroom environment are all key components in the support structure that Paloma has which has allowed her to become successful in developing algebraic thinking.

## Conclusion

Although this study is limited to one first year algebra student and her time spent in algebra classes, the findings shed light on her success in mathematics, and specifically her success in algebraic thinking. There are many factors as to why Paloma is successful in mathematics, including those we do not know that may be part of her support structure outside of school. At school, Paloma is in an environment that has supported her success in mathematics, which includes the school structure, the classroom, her teacher, and her peers, thereby fostering her self-determination. Latinos are the second largest racial group in the United States and it is important to recognize the impact that has on student populations in the classroom (Excelencia in Education, 2015). All students regardless of their background should be supported by their mathematics teachers so that all students have the opportunities to be successful in mathematics like Paloma.

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## Appendix A

1. Create the next figure in the pattern. How many tiles are in figure #5?

2. How many total tiles are in figure 6?

3. Write observations about the pattern – how is the pattern changing? How can you describe it?

4. Make a table and collect data.

| Figure Number | Number of Tiles |
|---|---|
| | |
| | |
| | |
| | |

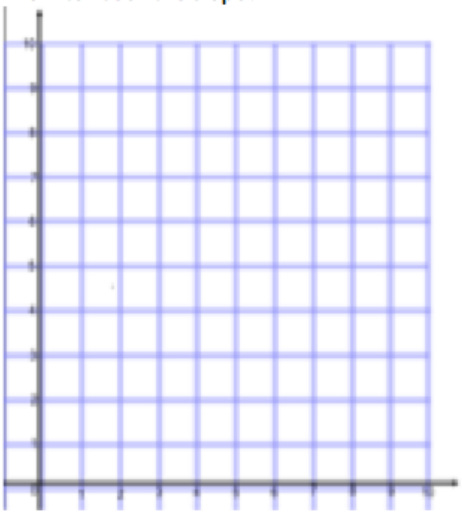
5. Describe a method for finding the total number of tiles in figure #10.

6. Write a rule or an equation to predict the total number of tiles for any figure *x*.

7. Explain how your rule is connected to the geometric figure.

8. Predict what the graph will look like.

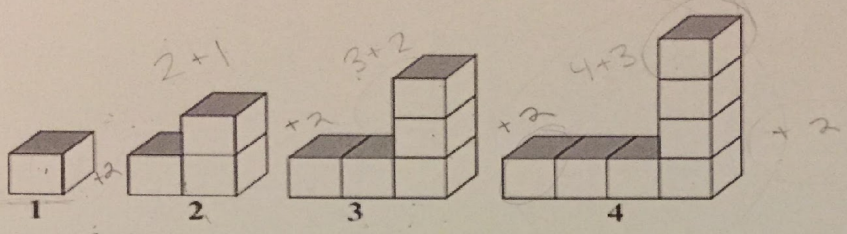
9. Show a graph of the data. Label the axes. Explain how to "see" the slope?



## Appendix B

Student #3

Below is a pattern. Create the pattern with tiles.




#5 = 9 +2 = #6 = 11

1. Create the next figure in the pattern. How many tiles are in figure #5? There are 9 tiles in figure 5.

2. How many total tiles are in figure 6? There are 11 tiles in figure 6.

3. Write observations about the pattern – how is the pattern changing? How can you describe it? For each figure it adds 2 more.

4. Make a table and collect data.

| Figure Number | Number of Tiles |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | 5 |
| 4 | 7 |

+2 +2 +2

5. Describe a method for finding the total number of tiles in figure #10. You could either keep adding 2 or since figure six has 11 tiles you 8 more because 2 that youre adding plus the four that you need to get to ten. it would equal 19.

6. Write a rule or an equation to predict the total number of tiles for any figure x.

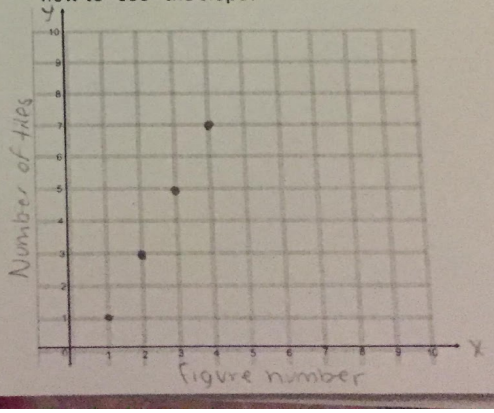
$y = x \cdot 2 - 1$

1+2=3
2+2=4

7. Explain how your rule is connected to the geometric figure. X would equal the number of tiles and 2 is from multiplying that youre putting on the top and the 1 is from subtracting it from the top.

8. Predict what the graph will look like. I think the graph will be a straight line but when I label it, it would have some spaces in between.

9. Show a graph of the data. Label the axes. Explain how to "see" the slope?

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