WHAT RESEARCH CAN TELL US ABOUT THE UNDERSTANDING OF ALGEBRA

By

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ABSTRACT
Algebra is a very important component of mathematics. For students, an adequate understanding of algebra serves as a foundation for the math to come. Concurrently, algebra is also a subject where many students struggle and form mathematical misconceptions. In order to take proactive steps in being able to more effectively teach students algebra, it is first important to identify the processes involved in understanding algebra. This paper discusses the different methods of thinking used when learning algebra as well as common misunderstandings formed by teachers and students. To obtain a better understanding of students’ processes when solving algebra tasks, five students were each given four math tasks to complete individually. These tasks all involved sketching the curve of a graph given the context of a situation. After analyzing the students’ responses it became clear that there are specific types of thinking that the students were misusing during the completion of the tasks. From this information, one can have a better understanding of student processes and be able to better adapt instruction to meet students' needs.
INTRODUCTION

From learning algebra throughout my schooling to receiving instruction on how to properly teach algebra, it seems like this topic is where many students and teachers struggle. Many people see algebra simply as numbers and procedures, which can make it difficult to give it meaning and ensure that students remember it. That being said, algebra has always been one of my favorite topics because working with numbers appeared to come naturally to me. While I was never able to put meaning to the work that I was doing, I was able to remember the procedures in order to complete algebra problems. Algebra was a subject that I succeeded in due to memorization, yet this can become problematic as more content and significance is attached to it.

While many students may struggle with the computation and procedural aspects of algebra, I found that I struggled most when looking at the “big picture” of algebra. That is, when connecting algebra to situations or graphs. Due to this, I became interested in how people interpreted algebra in these situations and if they were successful with their interpretations or not. As a former student of algebra and future teacher of algebra I felt this information would be valuable for my future teaching methods. Being able to recognize how people understand algebra through graphs and models can help me to better guide my instruction and ensure students truly comprehend.

In order to determine how people interpret graphs, procedures, and the connections between them I felt it would be beneficial to determine how students understand the algebra tasks given to them. By seeing how students interpreted and worked through such tasks, I felt I would be able to conclude the methods students use most and be able to cater my instruction to meet their needs.
LITERATURE REVIEW

When working through an algebra problem there are often many different ways to approach the situation. The various approaches often involve unique ways of interpreting or focusing on the mathematics involved. For example, two popular methods of examining an algebra question involving functions are point wise thinking and global thinking. These two ways of thinking can be thought of as opposites as point wise thinking involves examining specific points or individual pieces of a problem while global thinking suggests looking at the situation as a whole. In order to utilize these types of thinking, Kieran (2007) suggests that there are math activities which “include problem solving, modeling, working with generalizable patterns, justifying and proving, making predictions and conjectures, studying change in functional situations, looking for relationships or structure, and so on” (p. 714). To put these methods into context, when a student is asked to classify a given graph as linear or quadratic they can look at the difference between specific points to determine the pattern, which would be pointwise thinking, or examine the graph and its behavior as a whole, using global thinking. These two ways of thinking are very well known and widely used, which is why there are many different opinions about these approaches and their importance to students.

While there is “an agreement among mathematics educators today on the importance of different representations in the learning of mathematics, not much is known about the nature of the processes involved in working with different representations” (Even, 1998, p. 105). In an attempt to learn more about the two processes and how they are connected, researchers have done studies looking at the ways that students solve problems. In many cases involving understanding equations and their behavior, it was found that students were more successful when using the global approach. According to Even (1998), the students were able to answer
multiple questions of increasing difficulty which “may lead to the conclusion that the use of a
global approach to functions implies a better understanding” (p.111). However on the other hand,
it was also found in certain situations that surface characteristics of graphs were enough for some
students to come to incorrect conclusions. So this implied that 'the use of a global approach to
functions in itself is not always "better."' (Even, 1998, p. 112). In a similar study it was
discovered that “the participants were not able to flexibly use algebraic and graphical
representations to solve quadratic equations, and irrational equations” (Huang & Kulm, 2012, p.
427). It was deemed that the participants particularly had weaknesses in being flexible with the
different representations and knowing when to isolate the algebraic and graphic representations.
Specifically from this study, it was concluded that “the participants lacked the learning
experience in integrating graphic and algebraic representations to solve problems related to
quadratic functions highlights the necessity of creating this kind of learning opportunity in
curriculum and teaching” (Huang & Kulm, 2012, p. 428). It is apparent that students must be
able to utilize both of these skills; however it appears that switching between the two methods is
where students often struggle.

Another similar argument about methods can be found with the study of the object versus
process methods of solving algebra tasks. Even (1998) explains that “a common distinction today
is between an operational approach to a concept as a process, and a structural approach as an
object” (p. 109). Similarly to the explanation of global and point wise thinking, using the object
method would suggest looking more so at the item as a whole while the process method would
look more at specifics. Ideally, these two approaches would be used interchangeably and with
enough flexibility so that one could interpret when to use each method. It is common that one
person may become more comfortable with a specific method, which can then in turn limit their
perspectives on functions. While there is no “better” method, it does appear that there are situations where one way can display the correct standpoint more easily.

Moschkovich (1993) explains that “students must come to grips with connections across representations, and depending on context or interpretation, with different perspectives regarding the functions themselves” (p. 74). It is displayed that both the process and object perspectives are “an essential part of learning about functions and graphs”, however the student must first have a proper understanding of algebra in order to be comfortable with these methods interchangeably (Moschkovich, 1993, p. 98). This is where a lot of people are at fault. Often times, people don’t have a comprehensive understanding of the subject which is why they lack so much in one of the processes. It is explained that both of the perspectives “shed light on the behavior of functions, but the perspectives are differentially useful” (Moschkovich, 1993, p. 97). This is also another section where it appears a lot people struggle with the different approaches. Since throughout the learning of the subject one can become accustomed to a specific way of thinking, it can be very difficult to then switch to a different mindset in order to look at the task as a whole and choose a method to use. This constricted mindset can cause people to overlook simple structures within the task because they are either focusing too narrow or wide. While it is said that all problems can be done using either, Moschkovich (1993) explains that the students’ knowledge of functions and fluidity with the subject will play a big factor in which method they choose and the ease with which they complete the task.

In a different study done looking at teachers’ instruction of mathematics, various teachers were seen using these methods and incidentally teaching them. In this research done by Harel (2008), the confusion around object and process is seen directly. In this case, the mathematical task doesn’t necessarily focus on functions and looking at graphs, but instead defining an object
such as a variable versus a process such as some sort of transformation. Specifically, the teacher in the study shows ‘no distinction is made between an object, such as a “scalar,” and a process, such as “multiplication”’ which then leads to misunderstandings by the students (Harel, 2008, p. 124). This situation again emphasizes how difficult it can be for people to differentiate between the two approaches and fluidly utilize them. This is concerning, because as Harel (2008) mentions, when the instructor fails to distinguish between the methods “it is thus unlikely that students will pay careful attention to mathematical definitions, beyond memorizing what they need for testing” (p. 125). The research done here displays that this issue doesn’t stem simply from the student misunderstanding, but also from misguidance from teachers.

It is another common debate on how much schooling high school math teachers truly need to be effective teachers. Since secondary education is typically more specialized than that of elementary education, many believe that the instructor should have a more in depth background in the subject that they will be teaching. On the other hand though, some believe that sufficient pedagogical knowledge is satisfactory for teaching at the secondary level as long as there is some understanding of the subject. Many studies have been done on this topic and for the most part they all come to the same conclusion; that in order to sufficiently teach the students, an instructor needs to have a deep understanding of both content and pedagogical knowledge.

In Even’s (1998) study, prospective secondary math teachers were the ones focused on. They were in the last stage of their formal preservice preparation, and it was the goal to determine the extent of their knowledge of functions. Specifically, it was important to determine if they were flexible with the different representations of functions. Many of the subjects again struggled with the difference between global and point wise thinking in situations where “seeing a quadratic expression did not immediately bring to mind the graphic representation of a
quadratic function” (Even, 1998, p. 108). As discussed previously, it was found that the subjects struggled with freely using the different types of analysis. From this, Even (1998) determined that “procedural knowledge can help in monitoring naive conceptual knowledge” and that a combination of the two methods is necessary for teachers to utilize and pass onto their students (p. 120).

Another study was conducted by Chazan (2008) in which teachers’ pedagogical content knowledge is compared to their curriculum knowledge. Due to the introduction of a new curriculum, the teachers were asked to look over and discuss some of the new topics. From the interviews of high school teachers, it was seen that their views weren’t always consistent. While one teacher was able to successfully make sense of equations in two variables as comparisons of functions of two variables, others struggled with this concept. Even though the idea wasn’t grasped at first, the “consideration of this conception of an equation was an important resource that the teachers used to construct their understandings” (Chazan et al, 2008, p. 87). The teachers essentially had to relearn some aspects of what they knew before. From this, it became apparent that “this sort of mathematical thinking should be conceptualized as part of the work of teaching” (Chazan et al, 2008, p. 99). This then lead to the discussion of teacher educators and their future for educating teachers. As well as unit planning, the big picture that teachers have for the courses they teach need to be taken into consideration and strengthened.

Kieran on the other hand adds another factor into the mix, which is that knowledge of the ways students are thinking about particular aspects of algebra and functions, is an equally important component of pedagogical content knowledge. Research has shown that there are “discrepancies between teachers’ predictions of student’s difficulties and students’ actual difficulties” (Kieran, 2007, p. 740). This in turn can have a significant effect on teachers’
perceptions of their students and their learning. It is common for more teachers to underestimate students on conjecture tasks, which will then negatively affect the instruction presented as it will be less challenging than truly needed. It is seen that “limitations in both content knowledge and pedagogical knowledge may influence teacher’s beliefs and their instructional decisions and actions” (Kieran, 2007, p. 742). Essentially, teachers need to have an adequate amount of content and pedagogical knowledge in order to have a better understanding of their students’ performance. In order to eliminate the discrepancies between teachers’ perceived outcomes and actual outcomes, it is necessary that teachers completely comprehend what is being asked of the students and how to present that information to them in the best way. Then together with both of these pieces there should be no discrepancies.

In another study done with teachers, Harel (2008) observes how teachers treat mathematical meaning in the classroom. Before the study is conducted, the teachers attended a workshop where both content and pedagogical knowledge for high school algebra were addressed. These specific teachers were chosen because of their relatively limited math background. After looking at the teaching done by the participants it was concluded that there was no “intellectual purpose for what is being taught” and “a lack of differentiation between meanings of different terms” (Harel, 2008, p. 119). There appeared to be a disconnection between what the teachers knew and understood about the algebra and what was being shown to the students. The teachers all understood the terms and symbols used, however they weren’t always clearly communicated to students. In the end it was believed that “institutional constraints were a factor in these teachers’ treatment of meanings” such as their goals for their classrooms (Harel, 2008, p. 126). This, in addition to the fact that “the teachers’ knowledge base may be limiting their actions in the classroom”, leads us to believe that teachers need to be equipped
with the proper resources, support, and knowledge in order to successfully help their students (Harel, 2008, p. 126). While it is possible for the students to succeed without concrete understanding of math terms and symbols, it is critical that students are comfortable with them in order to make greater math connections in the future.

This issue of lack of teacher knowledge is addressed directly in research done by Huang and Kulm (2012). It is displayed that “equipping teachers with appropriate knowledge needed for teaching is the key to high-quality teaching that aims at achieving high quality student learning” (Huang & Kulm, 2012, p. 417). Through their work with high school teachers, many statements were made regarding the importance of adequate teacher preparation programs and deeper understanding of the content. In order for the students to excel in mathematics “it is not a simple issue of adding more mathematics and mathematics education courses” for teachers (Huang & Kulm, 2012, p. 428). This work all begins with prospective teachers who first must obtain a solid base needed for teaching. The argument is that first the curriculum must provide a core of coherent content areas, and then pre-service teachers need to have the opportunity to develop their own knowledge and skills needed for organizing a classroom and promoting students learning. A common finding in this research was that the participants lacked experience in integrating representations and implementing lessons.

Similarly, in a study done by Campbell (2014), it was found that student achievement is directly affected by the teachers’ knowledge and presentation of the subject. The findings from this study “provide evidence of the relevance of teacher knowledge and perceptions for teacher preparation and professional development programs” (Campbell et al, 2014, p. 425). Similarly to what was stated in Keiran’s (2007) work, this article explains how teachers’ beliefs on math teaching and learning as well as the teachers’ awareness of their students’ mathematical
disposition influences teaching practice. This in turn can negatively affect the instruction and classroom environment. When there is a lack of proper awareness, it is easy for teachers to depress student success. The math achievement of students in classrooms with teachers who held beliefs on mastery of procedural skill was far lower than that of teachers with strong math and pedagogical knowledge.

At this point it is clear that the common thought is teachers must have adequate knowledge and understanding in order to adequately aid their students in comprehension. Ball, Thames, and Phelps (2008) take a moment to figure what this knowledge truly entails for teachers. Essentially, it is suggested that teachers must be able to do the work that they present to students and more. They have to also have the knowledge to choose math representations effectively, justify ideas, anticipate what students will think, and when to highlight various mathematical points. Ball, Thames, and Phelps (2008) again emphasize the idea that “just knowing a subject well may not be sufficient for teaching” (p. 404). As stated many times throughout the previous studies, the teachers must have various avenues of knowledge aside from their content knowledge. All in all this idea can be expressed as “teachers need to know mathematics in ways useful for, among other things, making mathematical sense of student work and choosing powerful ways of representing the subject so that is it understandable to students” (Ball, Thames, & Phelps, 2008, p. 404).

In recent years, the presence of technology in the classroom has been increasingly popular. In addition, “over the past decade, school mathematics has moved to assimilate graphing technology into practices of teaching and learning” (Chazan et al, 2008, p. 87). While the implementation of more technology is fairly widespread across schools, there are still some instances where the use of technology is not necessarily welcomed. Some educators believe that
technology is simply a visual aid and reduces certain unnecessary steps, while on the other hand there are many who still think the calculators do too much work for the students and students will forget basic skills. As technology become more prevalent in society as a whole, it is important to understand its place in the classroom.

For most, calculators have been and continue to be used as a tool for various mathematical tasks. According to Chazan (2008), many educational reformists “suggest that the use of graphing technology has the potential to support making school algebra a more meaningful activity for students” (p. 88). Once students have mastered basic calculation, the idea is that they are able to use a calculator in order to speed up the process and not focus so much on the calculation aspects. After the calculation is taken care of, students have more time to look at the results and determine what they mean. However, the use of calculators is still controversial because of the “fear that reliance on graphing calculators will erode students” algebraic skills, or the idea that the sorts of techniques that are supported by calculator technology change remove the algebra from school algebra (Chazan et al, 2008, p. 88). Many people worry that the students will begin to depend on their graphing calculators instead of simply using it as a support. For educators, it is important to ensure that students are able to graph and equate both with and without the calculator as well as truly understand the meaning of the answer being given.

After various research studies, it has been displayed that many reformed teaching models actually encourage the student usage of calculators. As curriculum shifts to being more concept based, it has become more appropriate to use technology to illustrate connections and meaning. “Research has emphasized that the insertion of such technology into the algebra classroom does not remove the need for paper-and-pencil algebraic techniques” but actually does the opposite (Kieran, 2007, p. 739). Technology truly enables students to focus more on meaning and
understanding. It has been shown that “the use of this technology as a didactical tool can occasion mathematical discussions that do not normally occur in algebra classrooms” (Kieran, 2007, p. 744). So after understanding the algebra and processes, technology can actually allow students to spend more time exploring connections and making comparisons.

Technology in the form of calculators is also notoriously common for creating visuals and graphing. This technology “has the potential to give students visual feedback that emphasizes the various meanings of equivalence” (Chazan, Cognitive Complexity, p. 131). For data that is not easily graphed by hand, the use of technology can easily put the data into perspective for students. This also then helps the students make meaning of the data and find properties of its graphs. While this is beneficial in many cases, Chazan (Cognitive Complexity) introduces the idea that these graphing representations are not always transparent and can actually cause misinterpretations in some cases. In addition, occasionally the graphs are too complex and can make understanding harder to reach. When this occurs, “learning to reach such representations is complex and requires teaching and learning” (Chazan, Cognitive Complexity, p. 131). Chazan (Cognitive Complexity) also mentions how the use of calculators relies heavily on letters as variables and a certain form for equations. This could actually cause tension with the curriculum when students attempt to graph equations.

When it comes down to it, a lot of the proper use of technology is based on the teachers’ implementation of it. Carlson (2002) argues that when “properly grounded and coupled with sufficient teacher training, these technology offer valuable tools for students and learning to apply covariational reasoning to analyze and interpret dynamic function situations” (p. 372). There are many opportunities for technology to be an aid in graphing and studying dynamic events. Simultaneously though, it can easily take away from students basic skills if introduced
before such skills are mastered. Technology can be a great help and deepen mathematical understanding, but it can also hinder students if not used correctly.

Through all of these readings the different opinions on methods, teaching and technology become very apparent. While there is no correct or incorrect view, it is evident that global or process thinking is needed just as much as point wise or object thinking. These two categories of solving algebra problems are both useful in their own rights. Then it is also seen how important it is for teachers to understand more than just the content they are teaching. In order to successfully help students learn, teachers need to be skilled in their subject, adaptive, and understanding of the students’ needs. When teaching the students it is also appropriate to incorporate the use of new technologies as long as it doesn’t take away from what the students are learning. Technology in the classroom is an important tool that can help students make more connections and focus more on the meaning. With the different ways that students solve problems, the different teaching methods, and the uses of technology, it is clear how diverse the learning and understanding of mathematics can be.
METHODS

In order to conduct this research, four math tasks were used. The tasks are listed below (See Figure 1). These tasks were presented to students interview style, where the student worked individually on them and the interviewer asked questions. During the interview the students completed all tasks in whichever order they pleased and were asked to talk aloud as they worked through them. They were also told that there was no right or wrong answer. The interviews typically took about an hour to complete. During the interview I took notes on what the students were saying and writing on their papers. Once the students were done and had left the interview session I wrote down any trends I saw and what type of strategies they used.

Five students were interviewed and they were all chosen at random. They all opted in to do the survey by their own choice. All of the students went to the same high school in the southwestern part of the United States. They were all in algebra 1 with the same teacher.

Once the interviews were complete, I analyzed the data and compared the students’ solution methods. I looked to see if there were trends in the students thinking and any other things in common with their work. I analyzed the student work in conjunction with the methods and ideas found in the research I studied. The analysis was guided by the research literature on covariation and graphing in identifying themes. I then looked for these different themes in the data. The analysis of the data was guided by the following research questions:

1) What methods of quantitative reasoning do students use to solve contextualized graphing problems?

2) How do students interpret and use “time” when time is essential to the context of the problem, but not directly represented on the graph?
### Figure 1: Interview Tasks

Beginning at point A, a person riding on a Ferris wheel completes 3 clockwise revolutions at a constant speed. Sketch a graph showing the relationship between the person’s distance from the ground and the person’s distance from the power line. Assume that the scale is the same on both axes.

![Graph showing the relationship between distance from the ground and distance from the power line.](image)

Starting at a full stop, a person begins sprinting along the dotted line, speeding up as they run with constant acceleration. Sketch a graph showing the relationship between the person’s distance from the canal and the person’s distance from the sidewalk. Assume that the scale is the same on both axes.

![Graph showing the relationship between distance from the canal and distance from the sidewalk.](image)

Imagine you have a barrel nearly full with water as shown below. You open the spigot and water begins flowing out of the barrel at a constant rate. Sketch a graph of the relationship between the volume of the water in the barrel and the height of the water in the barrel, as the water is flowing out of the barrel.

![Graph showing the relationship between volume of water in the barrel and height of water in the barrel.](image)

Beginning at point P, a pace car completes 3 clockwise revolutions around a track at a constant speed. Race fans watch from stands A and B. Sketch a graph showing the relationship between the pace car’s distance from stand A and the pace car’s distance from stand B. Assume that the scale is the same on both axes.

![Graph showing the relationship between distance from stand A and distance from stand B.](image)
RESULTS

Below is a table summarizing the results that I found after interviewing five students.

<table>
<thead>
<tr>
<th></th>
<th>Pace car</th>
<th>Person</th>
<th>Ferris wheel</th>
<th>Barrel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>Increasing linear function</td>
<td>Decreasing linear function</td>
<td>Negative quadratic function</td>
<td>Decreasing linear function</td>
</tr>
<tr>
<td>Student 2</td>
<td>(Left Blank)</td>
<td>Increasing linear function</td>
<td>Increasing linear function</td>
<td>Decreasing linear function</td>
</tr>
<tr>
<td>Student 3</td>
<td>Constant function</td>
<td>Increasing linear function</td>
<td>Increasing exponential function</td>
<td>Decreasing exponential function</td>
</tr>
<tr>
<td>Student 4</td>
<td>Increasing periodic function</td>
<td>Decreasing linear function</td>
<td>Decreasing quadratic function</td>
<td>Decreasing linear function</td>
</tr>
<tr>
<td>Student 5</td>
<td>Discrete periodic function</td>
<td>Increasing linear function</td>
<td>Discrete periodic function</td>
<td>Decreasing linear function</td>
</tr>
<tr>
<td>Correct Solution</td>
<td>Decreasing linear function</td>
<td>Increasing linear function</td>
<td>Decreasing linear function</td>
<td>Increasing periodic function</td>
</tr>
</tbody>
</table>

After examining the work that the students did I found many common themes throughout. I will now discuss these themes in the following pages.

Global vs. Point wise Thinking

After interviewing the students and seeing their processes, it was interesting how they approached the tasks. There were trends with how they began the tasks and thought about the graphs. For the most part, it seemed like many of the students utilized global thinking to think of how the shape of the graph would look. I inferred this based on the way that the students focused more on the overall shape of the graph instead of specific points. Frequently, students made conjectures like "This is what the shape would be..." Or "It should look like this..." For these specific tasks, the global thinking was misleading for the students and caused them to draw
incorrect graphs. Their intuition for what the shape of the graph should be led them to make false
generalizations about the graphs or look over important details.

I noticed that the students often focused on the shape of the graph which lead them to
make linear graphs. Specifically in the Pace car and Ferris wheel problems, it seemed like the
students were inclined to make linear functions because the problem descriptions had the terms
"constant speed" or "constant rate". When the students were working through the problems, they
often noted and referred back to those terms when justifying why they drew their graph the way
they did. This generalization that the terms "constant speed" or "constant rate" applied to the
shape of the graph lead students to position their linear function in the incorrect place, as they
may have previously seen these functions beginning at the origin. In one case, the term
"constant" was interpreted to literally mean a constant function. It may have been the students'
global thinking and wide view of the task that led many of them to make this overgeneralization.

Their global outlook on these tasks also might have led the students to overlook many
details that could have been important when constructing their graphs. I noticed that the students
didn’t closely attend to the rate of change and varying rates of change when coming up with their
solutions. It seemed like the students got caught up on the term “constant rate” and this misled
them to make linear graphs. This was the case with the Ferris wheel task and the Car task. There
also appeared to be details overlooked when the students attempted the Barrel task. The fact that
the shape of the barrel is not cylindrical makes it so the water does not exit at a constant rate.
This was an instance where the task involved a varying rate of change which the students didn’t
pay attention to. There was also a common misconception with the Person task that the sidewalk
and canal in the picture was mirrored in the axes of the graph. It seemed like some students
interpreted the axes of the graph as the sidewalk and boardwalk from the visual. This led the
students to incorrectly position the line on their graphs. The lack of attention to detail while drawing the graphs may have been due to the students' more general approach of the tasks. If the students had taken more time to look at specific points this may have led them to see more of the important details.

One student seemed to have a habit of labeling the axes before beginning to draw a graph, which I thought would lead the student to more point wise thinking (See Figure 2). This was the case for the student when completing the Ferris wheel task as he indicated the points (1,1), (2,2), and (3,3) as the different times the Ferris wheel made a revolution. For the rest of the tasks though, it seemed like the student labeled the axes and then disregarded the labeling when creating the graph. I thought this was interesting because I thought labeling the axes would lead the student to plot points on the graphs to create the shapes, but he actually didn’t use the labeling to draw the graph. It might have been that this student had a habit of first labeling axes before thinking about what the graph should look like. It was almost as if the student had been trained to do this in some prior math schooling. So even though the student demonstrated point wise thinking when they first labeled heir axes, this didn’t encourage him to continue to use point wise thinking to create the graph. The student actually utilized global thinking to think about the overall shape of the graph, which seemed contradictory to the labeling.
Another thing that I noticed about a specific student was their tendency to make discrete graphs. This student made discrete graphs for the Pace car and Ferris wheel problems, almost as if the car and Ferris wheel were stopping periodically during their revolutions. It seemed like this misunderstanding stemmed from the student’s idea that the Pace car and Ferris wheel were not continuously moving. In this case the student’s overall view of the context led them to incorrectly draw the graph. While this may have been a misconception about the context that the student was making, it is also worth noting that these tasks are the only two that have points drawn in their visuals. There is a point P marked on the Pace car drawing and a point A labeled
on the Ferris wheel drawing. It may have been that these points on the visuals led the student to simply plot points for their graph instead of making a continuous function.

Attention to Detail

While completing the tasks, the students also failed to pay attention to a lot of the key details. For the majority of the tasks, attention to detail was essential to getting the task correct. It appeared as though the students noticed more of the unnecessary information within the task prompt which led them to easily miss the small details.

For example, for the Pace car and Ferris wheel tasks specifically, the students quickly noticed the terms “constant rate” or “constant speed”. In some cases, this may have skewed the students to think that the graph would have to be linear in some sense. While the students noticed this wording within the task, they failed to pay attention to the visual which shows that the Pace car track is actually closer to one stand and the Ferris wheel is further away from the ground than the power line. Due to this, the graphs should be closer to one of the axes in order to represent the shorter distance, to the power line in the case of the Ferris wheel, or to stand B in the case of the Pace car. So even though the visual clearly displays this important information, it seemed like the students overlooked this after deciding that the graph should be linear.

The same mistake was made while the students were completing the Barrel task. Within the task description the students are told that the water flows out of the barrel at a “constant speed” which the students took note of. This likely caused all but one of the students to draw the graph as a linear function. After reading the task description and determining that the graph would be some sort of straight line, the students failed to note how the shape of the barrel affects the volume of water within the barrel. The curvature of the barrel makes it so that the height of the water in the barrel and the volume of water in the barrel do not have a linear correlation. This
then changes the shape of the graph so that it is not linear in any sense. The students however did not pay attention to this important piece of information which led them to draw the incorrect graph. The students also may have missed this key detail due to lack of understanding or knowledge of barrels, yet it seemed like the students disregarded the shape of the barrel altogether.

In all the task prompts, the students are presented with information which includes the term “constant”. As stated previously, the students seemed to pay more attention to this wording than other key information. It appeared as though the students had previous experience which led them to make the assumption that this specific term was important. With the Pace car and Ferris wheel tasks, the students may have misinterpreted the terms “constant rate” or “constant speed” to imply that the graph would be linear. In these two tasks the terms refer to the speed of the car or Ferris wheel, which doesn’t apply to the graph in this case. So because the students focused on the terms “constant rate” or “constant speed”, it seemed like they felt it necessary to incorporate it into the graph even though it wasn’t necessary. The same situation occurred with the Barrel task where students identified the term “constant rate” and may have made the assumption that this applied to the graph. In both of these cases the students misinterpreted the importance of the term “constant”. It seemed like the students were accustomed to identifying this term and then creating a linear graph, which led them to wrongly draw the graph for these specific situations. In terms of paying attention to detail, the students were able to identify this specific vocabulary within the task prompts yet were unable to pay attention to the other important information which may have led them to realize that this vocabulary didn’t apply to the graphs.

The lack of attention to detail while completing these tasks appeared to be many of the students’ downfalls. I also found it interesting how the students’ inconsideration of details
frequently involved the students missing important cues from the visuals. It seemed as if the students didn’t pay as much attention to the visuals and relied more so on the wording of the task. This may be something the students have created a habit for since many times the visual is simply there to supplement the wording of a task and not to add further information. It is as if the students weren’t used to thinking of the visual as its own entity with its own information. The students’ lack of attention to the visuals though is where a lot of simple mistakes in the solutions were made.

**Visual Cues**

During the interviews, it was also apparent that the students may have misinterpreted some of the visual cues given within the task. As stated previously, it seemed like the students didn’t pay close attention to the information that the visuals presented. It appeared that the students failed to take time to interpret the visuals before drawing the graphs. In order to properly complete these tasks it is as though one needs to first interpret the prompt given, secondly analyze the visual, then translate the information into a more mathematical sense, and lastly draw the graph. While the students did a sufficient job of reading and understanding the prompts, they were unable to properly analyze the visual and think of what this information meant mathematically. In many cases this caused the students to incorrectly use the visual while drawing the graph.

For the Person task, it seemed as though a lot of students made a misconception of the graph due to the way that the visual was laid out. With the sidewalk and canal on the visual looking very similar to the x-axis and y-axis of a graph, it can be very easy to interpret the axes of the graph as so. This appeared to be the case for many students as they drew the graph the same direction as the person was walking in the visual. It is as if the visual of the sidewalk and
canal transferred onto the graph below and that is how students interpreted the shape of the graph. The visual is drawn this way in order to analyze the way in which participants interpret the visuals and carry it onto the graph. While completing the task it is important to recognize that the visual is separate from the graph and has to first be interpreted before drawing the solution. However when the students completed this task it appeared as though they were unable to make a distinction between the two visuals which led them to treat the graph as if it were the same as the figure.

While completing the Barrel task students also appeared to have trouble disconnecting from the visual of water flowing out of a barrel. All of the students drew decreasing functions for this task as if they were relating the shape of the graph to the water flowing out of the barrel. Even though this visual of water flowing out of a barrel is not explicitly shown, students are able to form this picture in their head given the context. Once the students thought of the water flowing out of the barrel and how that appears, it seems like the students were unable to disconnect from that idea while drawing the graph. The picture of the water level in the barrel going down as the water flows out may have led the students to draw decreasing functions. It seems as though the students were unable to interpret the water flowing out of the barrel in terms of the height of the barrel and volume of water in the barrel, and instead solely focused on the idea that the water level in the barrel was decreasing as the water flowed out.

There was also one student that misinterpreted the Pace car and Ferris wheel tasks which led the student to draw discrete graphs for both tasks. It may have been that the student misunderstood the concept that the Pace car and Ferris wheel are continuously moving, yet it is also interesting that these two tasks are the only ones with points in their visuals. In the Pace car task there is a point P labeled in the visual which represents where the Pace car begins. Then in
the Ferris wheel task there is a point A to note the person that is riding the Ferris wheel. The student that drew the discrete graphs for these tasks drew continuous graphs for the other tasks, which makes it seem like the student may have been triggered to draw discrete graphs for these two tasks because of the points on their visuals. The student was able to recognize that the Pace car and Ferris wheel were making cycles since the student drew periodic functions, yet the misconception that the graphs were discrete may have stemmed from the points on the visuals.
DISCUSSION

After completing this research I now better understand how students interpret and complete algebra tasks. It has become apparent the different outlooks students can have, such as global and point wise, and how the students use them while understanding and carrying out the task. From my findings, it seemed like the students mainly utilized global thinking while making sense of the tasks and figuring out a solution. In the case of this study, the students’ global outlook may have led them to overlook many key details which were important for reaching a correct solution. The students failed to realize many details within the visuals associated with the tasks, as well as what the task was specifically wanting from them.

In addition, while reading the tasks it appeared as though the students focused more on the task prompt than the visuals as they often misinterpreted which information was crucial for the task and which was unnecessary. For these specific tasks, the visual given along with the task prompt was imperative to creating a correct solution. The students however didn’t utilize the visuals given to them to their fullest ability and in some cases may have used them incorrectly. This appeared to be the case as the students mistakenly transferred cues from the visuals into the graphs.

With this information, it became apparent to me the misconceptions that students may have about graphing and specific algebra vocabulary. While completing the tasks the students appeared to pay too much attention to specific terms and not enough to details within the visuals. This seemed to be a habit that the students had formed from their prior schooling. As a future teacher this is helpful for ensuring that I don’t instill the same misleading information into students. Also I found that the students appeared to use a global approach to many of the tasks which may have led them to look over important information within the task prompt. This piece
of information is useful as it shows the importance of the interchangeability of global and point wise thinking. It is necessary for students to be able to use both methods and in this case, know when to use them together.

Overall, I feel like this research has aided my future teaching in the ways that it opened my perspective to the different ways students understand tasks and showed me common misconceptions that students may have learned in their schooling. With this information I will be able to better prepare myself to teach algebra and ensure students are learning the proper methods. I now see the importance of students' understanding of global and point wise thinking and how the coincide, as well as interpreting information given in task prompts and the visuals associated with them. From this study I can adapt my teaching and use these findings to teach students how to approach such tasks and analyze them. By teaching students these different skills, ideally they would be able utilize these abilities to complete any kind of similar task.
REFERENCES


