COMPARISON OF MODEL PREDICTED AND OBSERVED LIGHT CURVES OF GEO SATELLITES

by

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Key Nomenclature

RSO       Resident Space Object
GEO       Geostationary Orbit
BRDF      Bidirectional Reflectance Distribution Function
SSA       Space Situation Awareness
EPIC      Efficient Photometry In-Frame Calibration
TLE       Two-line Element Set
SRP       Solar Radiation Pressure
EKF        Extended Kalman Filter
UKF        Unscented Kalman Filter
Abstract

Although the amount of light received by sensors on the ground from Resident Space Objects (RSOs) in geostationary orbit (GEO) is small, information can still be extracted in the form of light curves (temporal brightness or apparent magnitude). Previous research has shown promising results in determining RSO characteristics such as shape, size, reflectivity, and attitude by processing simulated light curve data with various estimation algorithms. These simulated light curves have been produced using one of several existing analytic Bidirectional Reflectance Distribution Function (BRDF) models. These BRDF models have generally come from researchers in computer graphics and machine vision and have not been shown to be realistic for telescope observations of RSOs in GEO. While BRDFs have been used for Space Situational Awareness (SSA) analysis and characterization, there is a lack of research on the validation of BRDFs with real data. This research is focused on comparing telescope data provided by Applied Defense Solutions, as processed by their Efficient Photometry In-Frame Calibration (EPIC) software, with predicted light curves based on the Ashikhmin-Premoze BRDF and two additional popular illumination models, Ashikhmin-Shirley and Cook-Torrance. I computed predicted light curves based on two line mean elements (TLEs), shape model, attitude profile, observing ground station location, observation time and BRDF. The selected BRDFs provided accurate apparent magnitude trends and behavior, but uncertainties due to lack of attitude information and deficiencies in our satellite model prevented us from obtaining a better match to the real data.
I. Introduction

1. Motivation

Light curves, the observed brightness of an object over time, have been used for estimating properties of space objects. The use of light curves to estimate the shape and state of asteroids has been studied in [1] and [2]. The use of light curves, can be applied to small and/or dim objects in high-altitude Earth orbits (e.g. geosynchronous orbit) as well. The apparent magnitude of an RSO is a function of its size, orientation, and surface material properties. Consequently, one should be able to estimate these characteristics using appropriate algorithms.

Attitude estimation using light curve data was demonstrated by Wetterer and Jah [3]. In Linares et al. [4], light curve data is used to estimate the shape of an RSO along with its rotational and translational states using a Multiple Model Adaptive Estimation (MMAE) algorithm. The MMAE contained a bank of filters with different hypothetical candidate RSO shape models. By computing the likelihood associated with each hypothesis, the MMAE could determine which of the candidate shape models is most probable given the observations [4]. These works used simulated light curves produced using one of several existing analytic Bidirectional Reflectance Distribution Function (BRDF) models. These BRDF models have generally come from researchers in computer graphics and machine vision and have not been shown to be realistic for telescope observations of RSOs in GEO. While BRDFs have been used for Space Situational Awareness (SSA) analysis and characterization, there is a lack of research on the validation of BRDFs with regards to real data.

No work has attempted to compare results from distinct BRDFs with the data obtained thru telescope observations of RSOs. This comparison allows us to further understand these illumination models and the scope of their ability to approximate reality. This research aims at estimating light curves by propagating a set of RSOs thru a specified time range as viewed from a particular ground station and
comparing them with observed data from that ground station. This research is not only aiming at studying the approximation of each BRDF model to the real data, but it also provides an opportunity to compare the accuracy of the three selected models with each other. Although no consensus has been reached on which of the three BRDF models used in this paper is the “best”, this research aims at revealing which model best resembles the real data collected for our particular RSO cases and propagation.

2. Theory

A. Simulation Overview

The simulation used in this work consists of a light curve generating model. The model propagates the translational and rotational motion of RSOs, including effects due to the non-uniformity of Earth’s gravity field, third-body perturbations, solar radiation pressure (SRP), gravity-gradient torques and atmospheric drag. In order to compute SRP and drag perturbations, each RSO configuration used in the simulation was modeled as a convex system of flat plates. The forces on each plate were computed individually and summed to compute the total acceleration and torque on the RSO. Several different shapes of RSOs can be simulated in the measurement generator, including flat plates, cuboids, and hexagonal prisms (which are meant to approximate cylinders) as shown in Fig. 1.
The RSO state is used to generate simulated measurements of an RSO, including apparent magnitude, right ascension, and declination. The right ascension and declination are angles that define the location of the object of interest on the celestial sphere, as defined at the J2000 epoch. The apparent magnitude of an RSO, as measured by an observer on the Earth, is a function of the amount of radiant flux received by the RSO from the Sun and of the fraction of light that is reflected in the direction of the observer. This fraction is computed by summing the amount of light reflected by each of the $n$ flat plates that form the body of the RSO model.

The simulation includes the capability to model three different possible attitude profiles for the RSOs modeled. These attitude profiles are major-axis spinning, nadir-pointing, and sun-pointing. These profiles are modeled by calculating a small moment on the spacecraft at each time step, which is then propagated by a 6 degree of freedom propagator to cause the RSO model to point in the direction indicated by the selected attitude profile. For this particular research, only nadir-pointing
satellites were studied. In addition, the estimation models use extended Kalman filters (EKFs) to process the simulated measurements. The fundamental concept behind an EKF states the notion that the true state is sufficiently close to the estimated one, and the error dynamics can be accurate enough to be represented by a linearized first-order Taylor series expansion [1].

**B. Apparent Magnitude Model**

The “brightness” of a celestial object is actually measured in terms of the radiant flux, denoted by $F$, received at the sensor from the object. The radiant flux is the total amount of light energy of all wavelengths that crosses a unit area oriented perpendicular to the direction of the light's travel in unit time. It can be shown that an object's apparent magnitude $m$ is related to the radiant flux $F$ received from the object by

$$m = m_{\text{sun}} - 2.5 \log_{10} \left( \frac{F}{F_{\text{sun}}} \right)$$

(1)

where the apparent magnitude and radiant flux of the Sun is $m_{\text{sun}} = -26.74$ and $F_{\text{sun}} = 1368 \text{ W/m}^2$ respectively.

Let a given RSO be represented as a series of $N$ flat plates, as depicted in Fig. 1, where the position vector of the $j^{th}$ plate $\mathbf{r}_j$, expressed in body coordinates, is defined as the distance of the plate's center-of-area relative to the RSO's center-of-mass. The orientation of the $j^{th}$ plate with respect to the body frame is represented by three orthogonal unit vectors: $\mathbf{u}_{n,j}$ represents the direction normal to the plate and $\mathbf{u}_{u,j}$, $\mathbf{u}_{v,j}$ represent the $u$ and $v$ directions, respectively, of the $uv$-plane, i.e., the plane formed by the plate.

In general, the radiant flux received from a RSO is due to the reflection of sunlight from the illuminated surfaces of the RSO. As illustrated by Fig. 2, the geometrical configuration between the
Sun, object, and observer will determine the amount of reflected radiant flux received by the sensor, which in turn determine if the RSO is observable. In Fig. 2, the unit vectors that describe the inertial orientation of the \( j \)th plate have been transformed from body to inertial coordinates via the standard transformation

\[
\mathbf{u}_{i,j} = \left[T(q^b_i)\right]^T \mathbf{u}_{i,j}^b, \quad k = n,u,v
\]

(2)

where \( T(q^b_i) \) is the inertial-to-body transformation matrix (using quaternion parameterization) and \( q^b_i \) is the RSO's inertial-to-body attitude quaternion. The direction to the Sun relative to the RSO is represented by the unit Sun direction vector expressed in inertial coordinates and defined as

\[
\mathbf{u}_{sun} = \frac{\mathbf{r}_{sun}^i - \mathbf{r}^i}{\left\| \mathbf{r}_{sun}^i - \mathbf{r}^i \right\|}
\]

(3)

where \( \mathbf{r}_{sun}^i \) and \( \mathbf{r}^i \) are the inertial position vectors of the Sun and RSO, respectively. The direction to the observer relative to the RSO is represented by the unit observer direction vector expressed in inertial coordinates and defined as

\[
\mathbf{u}_o = \frac{\mathbf{r}_o^i - \mathbf{r}^i}{\left\| \mathbf{r}_o^i - \mathbf{r}^i \right\|}
\]

(4)

where \( \mathbf{r}_o^i \) is the inertial position vector of the Earth-based observer. The unit Sun and observer direction vectors form the Sun-observer plane, as illustrated in Fig. 2. The unit half vector \( \mathbf{u}_{h} \) bisects the angle \( \theta \) between the unit Sun and observer direction vectors and is defined as
From Fig. 2, it follows that the $j^{th}$ surface is illuminated by the Sun and a reflection occurs only when the angle between the unit Sun direction vector $\mathbf{u}_{\text{sun}}'$ and the unit vector normal to the surface $\mathbf{u}_{n,j}'$ is less than 90 degrees or (equivalently) when the dot product of these two vectors is positive and greater than zero. Moreover, the reflection is observable by an Earth-based observer only when the dot product between the unit observer direction vector $\mathbf{u}_{o}'$ and the unit vector normal to the surface $\mathbf{u}_{n,j}'$ is also positive and greater than zero. If these two criteria are met for even one of the $N$ surfaces, then the reflection geometry is such that the RSO is observable and a measurement is obtained. Mathematically, this observability condition $\psi$ is represented by

$$
\psi(\xi_{1,j}, \xi_{2,j}) = \begin{cases} 
1 & \text{(observable)} \quad \text{if } \xi_{1,j} > 0 \text{ and } \xi_{2,j} > 0 \\
0 & \text{(not observable)} \quad \text{else} 
\end{cases}
$$

where $\xi_{1,j} = [\mathbf{u}_{\text{sun}}']^T \mathbf{u}_{n,j}'$ and $\xi_{2,j} = [\mathbf{u}_{o}']^T \mathbf{u}_{n,j}'$.

From (5)

$$
\mathbf{u}_{k}' = \frac{\mathbf{u}_{\text{sun}}' + \mathbf{u}_{o}'}{\|\mathbf{u}_{\text{sun}}' + \mathbf{u}_{o}'\|}
$$

Figure 2 Reflection Geometry
2.1 BRDF Models Summary

Bidirectional reflectance distribution functions are used to create RSO brightness models. A BRDF is defined as a ratio of reflected radiance to incident irradiance. The BRDF for an object models the amount of incident light which is reflected by an object and is the sum of the specular and diffuse reflections, which are both functions of its material properties, as well as the angles of incidence and reflection of incoming light. Thus, an RSO’s apparent magnitude is highly dependent on shape and attitude. When defining a BRDF’s reflected light from a surface, one must consider the direction of the incoming light source $\mathbf{L}$, the direction of the observer $\mathbf{V}$, the halfway vector $\mathbf{H}$ between the light source and the observer, and the surface normal direction $\mathbf{N}$. These vectors are specified in Fig. 1 along with their corresponding angles as defined from the surface.

![Figure 3 BRDF Reflection Vectors](image)

The BRDF models the amount of light reflected by each plate, $t$, as shown in Fig. 2. More specifically, BRDF is defined as the ratio between reflected (directional) radiance and incoming surface irradiance [10]. The amount of directional radiance reflected by an object is defined as the sum of the specular and diffuse reflections, which are both functions of its material properties, as well as the angles of...
incidence of the incoming irradiance. Thus, RSO apparent magnitude is highly dependent on shape and attitude.

The measured radiant flux $F_j$ of the $j^{th}$ Sun-illuminated plate is related to the intensity of light $L_{o,j}$ reflected in the direction of the observer and inversely proportional to the square of the distance $d_j$ from the plate to the observer,

$$F_j = \frac{L_{o,j}}{d_j^2} \quad (7)$$

The distance $d_j$ from the plate to the observer, neglecting the distance from the RSO center-of-mass to the plate's center-of-area, is simply

$$d_j = \| \mathbf{r}' - \mathbf{r} \| \quad (8)$$

The intensity of reflected light $L_{o,j}$ or radiance is proportional to the intensity of incident sunlight $L_{s,j}$ and the reflectance behavior of the plate,

$$L_{o,j} = \rho_j L_{s,j} \quad (9)$$

where $\rho_j$ is the BRDF. The intensity of sunlight light (irradiance) $L_{s,j}$ incident upon a plate of surface area $A_j$ is related to the solar radiant flux $F_{\text{sun}}$ and the cosine of the incident angle,

$$L_{s,j} = F_{\text{sun}} A_j \left[ \mathbf{u}'_{\text{sun}} \right]^T \mathbf{u}_{o,j} \quad (10)$$
Here $\left[u^i_{\text{sun}} \right]^T u^i_{n,j}$ has been used in place of the cosine of the incident angle. Finally, since multiple plates could conceivably be illuminated and observable at a given time, it then follows that the measured radiant flux $F$ of the RSO is given by

$$F = \sum_{j=1}^{N} F_j = \frac{F_{\text{sun}}}{\left\|r_o-r'_o\right\|} \sum_{j=1}^{N'} \rho_j A_j \left[u^i_{\text{sun}} \right]^T u^i_{n,j}$$

(11)

where $N' (N' < N)$ is the number of illuminated plates that meet the observability criteria defined in Eq. 11. It is important to note that this expression does not account for shadowing effects.

In the Ashikhmin and Shirley model [5], the apparent magnitude is computed for each surface of the RSO and the value corresponding to the brightest magnitude is accepted as the magnitude measurement. This is valid if and only if one surface of the RSO is illuminated and observable. However, if more than one side of the object is illuminated and observable, this model would be inaccurate. In order to make the measurement model truly applicable to a wide variety of objects, both resolved and unresolved, the model was modified to include the contributions of all illuminated and observable reflecting surfaces.

The highly nonlinear nature of the specular reflectance term contained in the Ashikhmin and Shirley BRDF made it less tractable for use in an EKF because the EKF requires the first order derivative or Jacobian of the measurement with respect to the estimated states. A modified version of the Ashikhmin and Shirley BRDF was found in an unpublished paper by Ashikhmin and Premoze [6], where it was shown that the modified BRDF produced a better overall match to real data. More importantly, it was observed that the spectral reflectance term in this modified BRDF
model is of a form that is more amenable to computation of the analytic Jacobian of the measurement. For these reasons, we used the Ashikhim and Premoze BRDF described in [6], and were able to derive, implement and validate the analytic Jacobian of the apparent magnitude measurement model in our algorithms.

The BRDF, denoted by $\rho$, which has units of inverse steradians (sr$^{-1}$), and is decomposed into a specular component $\rho_s$ and a diffuse component $\rho_d$.

$$\rho = \rho_s + \rho_d \quad (12)$$

Specular reflection occurs when light incident upon a surface appears to be focused in one direction. In other words, there is a bright spot, called a specular highlight, which is more readily apparent on shiny surfaces. For the ideal reflector, such as a mirror, the angle of incidence $\phi$ equals the angle of specular reflection, as shown in Fig. 4.

![Figure 4 Perfect or Ideal Reflection vs. Specular Reflection](image)
If \( R \) is the direction of the specular reflection and \( V \) is the direction of the viewer, then for an ideal reflector (i.e., perfect reflection) the specular reflection is visible only when \( R \) and \( V \) coincide. For real objects, however, the specular reflectance can be seen even if \( R \) and \( V \) do not coincide, i.e., it is visible over a range of values that form a cone about the \( R \) direction, as depicted in Fig. 4. In general, the shinier the surface is the smaller the range is for specular visibility. Consequently, a specular reflectance model must have a maximum intensity at \( R \), with an intensity that decreases as a function of \( \alpha \) the angle between \( R \) and \( V \). The anisotropic specular reflection component in Eq. 12 is defined as

\[
\rho_s = \frac{\sqrt{(n_u + 1)(n_v + 1)}}{8\pi} \frac{\left(\mathbf{u}_h^T \mathbf{u}_s^i\right)^z}{\sum_i \left(\mathbf{u}_h^T \mathbf{u}_s^i\right) + \left(\mathbf{u}_v^T \mathbf{u}_s^i\right) - \left(\mathbf{u}_w^T \mathbf{u}_s^i\right) \left(\mathbf{u}_h^T \mathbf{u}_s^i\right) - \left(\mathbf{u}_v^T \mathbf{u}_s^i\right) \left(\mathbf{u}_h^T \mathbf{u}_s^i\right) F_{ref}}
\]

where \( z \) is the Phong exponent given by

\[
z = \frac{n_s \left(\mathbf{u}_h^T \mathbf{u}_s^i\right)^2 + n_v \left(\mathbf{u}_v^T \mathbf{u}_s^i\right)^2}{1 - \left(\mathbf{u}_w^T \mathbf{u}_s^i\right)^2}
\]

and \( F_{ref} \) is the Fresnel fraction given by

\[
F_{ref} = R_i + (1 - R_i) \left(1 - \left[\mathbf{u}_w^T \mathbf{u}_s^i\right]^5\right)
\]

Moreover, \( n_u \) and \( n_v \) are parameters that represent the roughness of the reflecting material along the \( u \) and \( v \) directions, respectively, of the \( uv \)-plane. For small values \( (n_u = n_v = 10) \), the material is like
rough metal; whereas for large values \( n_s = n_v = 10^4 \), the material is like a perfect mirror. In Eq. 15, \( R_s \) represents the material’s specular reflectance for normal incidence, where \( 0 < R_s < 1 \).

Diffuse reflection occurs when light incident upon a surface scatters isotropically (i.e., the same in all directions) such that the apparent brightness of the surface to an observer is the same regardless of the viewing angle. In other words, diffuse reflection is the reflection of light from an uneven or granular surface such that the incident ray appears to be reflected in a number of directions simultaneously, as illustrated in Fig. 5. The non-Lambertian diffuse reflection component is defined as

\[
\rho_d = \frac{28R_d}{23\pi} (1 - R_s) \left[ 1 - \left( 1 - \frac{u_m^T u_n}{2} \right)^5 \right] \left[ 1 - \left( 1 - \frac{u_m^T u_n}{2} \right) \right] \]

(16)

where \( R_d \) is the diffuse albedo of the surface, where \( 0 < R_d < 1 \) and a value of 1 represents total reflectance (i.e., no absorption).

The simulation was originally set up with the Ashikhmin-Premoze BRDF, which is a modified, simplified version of the Ashikhmin-Shirley illumination model. The Ashikhmin-Premoze BRDF differs from the Ashikhmin-Shirley model mainly in that it simplifies the denominator of the specular term, and it allows for the use of an arbitrary normalized function \( p(h) \) instead of a specific anisotropic Phong function [1]. The Ashikhmin-Premoze BRDF was selected for implementation in the original simulation due to its spectral reflectance term being amenable to computation of the analytic Jacobian of the measurements.
The other two models included in the simulation for this light-curve analysis, Ashikhmin-Shirley and Cook-Torrance, are commonly used BRDF models in computer graphics that have been shown to provide accurate representations of their intended visual target. All three BRDF models incorporate diffuse and specular reflectivity terms. While the specular term consists of the light reflected in a specific direction, as displayed in Fig. 5, the diffuse term is composed of randomly scattered light, as shown in Fig. 6.

![Figure 5 Specular Reflection](image1)

![Figure 6 Diffuse Reflection](image2)

2.2 Ashikhmin-Shirley and Ashikhmin-Premoze BRDFs (standardize vector terms)

Ashikhmin-Shirley is an anisotropic BRDF model which provides good modeling of metals and plastics, along with general purpose surfaces, and utilizes a Fresnel term to evaluate specular reflectivity. However, unlike Cook-Torrance, Ashikhmin-Shirley’s model has been adapted in order to conserve energy through the implementation of a non-Lambertian diffuse term [3]. It is computationally expensive to evaluate, but due to its useful properties and intuitive parameters, the Ashikhmin-Shirley model was selected as one of the BRDF methods for this research. Equations 17 and 18 display the specular and diffuse components of the Ashikhmin-Shirley BRDF, respectively. The specular term consists of the two parameters $n_u$ and $n_v$, two Phong-like exponents which control
the shape of the specular lobe [5]. For this research, \( n_d \) is equal to \( n_v \), \( R_d \) and \( R_s \) are parameters which specify the “specular reflectance at normal incidence” and the “diffuse reflectance of the ‘substrate’ under the specular coating,” respectively [5]. Although there was experimentation regarding the value of these four parameters, their selection was based on the values used in the studied research [5]. The \( F \) term, as evaluated by Eq. 19, is Schlick's approximation to the Fresnel fraction [5]. In the Fresnel term, \( F_0 \) is material's reflectance at normal incidence [5]. As a note, while the Ashikhmin-Shirley uses Eq. 1 for its specular calculation, the Ashikmin-Premoze changes the denominator term \( (\hat{V} \cdot \hat{H}) \max \left((\hat{N} \cdot \hat{L}), (\hat{N} \cdot \hat{V})\right) \) to \( \hat{N} \cdot \hat{L} + \hat{N} \cdot \hat{V} - (\hat{N} \cdot \hat{L}), (\hat{N} \cdot \hat{V}) \) [6].

\[
\rho_s(\hat{L}, \hat{V}) = \frac{\sqrt{(n_u+1)(n_v+1)}}{8\pi} \frac{\left(n_u(\hat{H} \cdot \hat{u})^2 + n_v(\hat{H} \cdot \hat{v})^2\right)}{(1-(\hat{N} \cdot \hat{H})^2)} \frac{1}{(\hat{V} \cdot \hat{H}) \max\left((\hat{N} \cdot \hat{L}), (\hat{N} \cdot \hat{V})\right)} F(\hat{V} \cdot \hat{H})
\]

\[
\rho_d(\hat{L}, \hat{V}) = \frac{2BR_d}{23\pi} (1 - R_s)(1 - \left(1 - (\hat{N} \cdot \hat{L})^2\right)^5)(1 - \left(1 - (\hat{N} \cdot \hat{V})^2\right)^5)
\]

\[
F = F_0 + \left(1 - F_0\right)\left(1 - \hat{V} \cdot \hat{H}\right)^5
\]

2.3 Cook-Torrance BRDF

The Cook-Torrance lighting model is one that is acceptable for a wide range of objects, being a general model for handling rough material surfaces. Like the Ashikhmin-Shirley model, the Cook-Torrance method consists of diffuse and specular terms, as defined by Eq. 20 and 21, respectively. It is also computationally expensive to evaluate, although the complexity behind the Cook-Torrance method allows for a more accurate and physically-based specular reflection value through the use of a model specific Fresnel reflectance term. Looking at the diffuse term presented in Eq. 20, \( \rho \) is the diffuse reflectance, which is a value between 0 and 1 [7]. The specular term of the Cook-Torrance model consists of a microfacet distribution term (Eq. 22), a geometric term (Eq. 23), and a Fresnel term (Eq.
The distribution term provides information on the orientation of the microfacets, with similar orientations on smooth surfaces and more diverse orientations on rough surfaces. This research used the Beckmann distribution, seen in Eq. 22, as the microfacet distribution term, where $\alpha$ is the angle between $\hat{N}$ and $\hat{H}$. The geometric factor allows for the inclusion of shadowing and self-masking effects between the microfacets, and the Fresnel term, similar to the one observed with the Ashikhmin-Shirley model, provides the specular reflection value, telling us which portion of the light was reflected and which other was transmitted. [8] The value of parameters left to the user’s discretion were selected based on the studied research and their results and suggestions [8].

\[
R_d = \frac{\rho}{\pi} \tag{20}
\]

\[
R_s = \frac{FDG}{\pi(N \cdot L)(N \cdot V)} \tag{21}
\]

\[
D = \frac{e^{-[\tan\alpha m]^2}}{\pi m^2 (\cos \alpha)^4} \tag{22}
\]

\[
G = \min \left\{ 1, \frac{2(N \cdot H)(N \cdot V)}{(V \cdot H)} , \frac{2(N \cdot H)(N \cdot L)}{(V \cdot H)} \right\} \tag{23}
\]

\[
F = \frac{(g - \bar{V} \cdot \bar{H})^2}{2(g + \bar{V} \cdot \bar{H})^2} \left[ 1 + \frac{[(\bar{V} \cdot \bar{H})(g + \bar{V} \cdot \bar{H}) - 1]}{(\bar{V} \cdot \bar{H})(g - \bar{V} \cdot \bar{H}) + 1} \right] \tag{24}
\]

\[
g^2 = n^2 + (\bar{V} \cdot \bar{H})^2 - 1 \tag{25}
\]

\[
n = \frac{1 + \sqrt{F_0}}{1 - \sqrt{F_0}} \tag{26}
\]
2.4 Satellite Solar Panel Rotation

Aside from establishing the bus facets of each satellite, a close approximation of the solar panels was also included. Although some basic information regarding each satellite bus was found online, the solar panel dimensions in the simulations are educated selections based on each model. The idea behind the different facets of the shape model is to allow the bus to be nadir pointing while keeping the solar arrays orthogonal to the sun vector. The simulation was modified to match this attitude configuration.

Figure 7 Bus and Solar Panel Attitude
3. Provided Telescope Data

3.1 Summary of Data

The data provided for this research was obtained from the company Applied Defense Solutions, after the raw data observations was processed through their EPIC software. The raw data was collected from different amateur observers and ranged in quality. All the telescopes used to collect the data are 14”-17” class with Andor, Zyla, or Fairchild sensors. Exposures were around 5 seconds, with a Johnson R filter used primarily.

The EPIC software provides a .mat file containing a wide range of fields, including the following:

- sensor - the sensor ID
- site_lat - site latitude
- site_lon - site longitude
- site_alt - site altitude
- son - RSO object ID (matches the object ID in the filename)
- mat_time - matlab-formatted time
- mjd - modified Julian date
- jd - Julian date
- ets - elapsed time in seconds
- ra - right ascension (without aberration correction)
- dec - declination (without aberration correction)
- est_mag - estimated visual magnitude
- est_mag_uncert - estimated visual magnitude uncertainty
- eps - flux
- zero_point - estimated zero point
- zp_sigma - estimated zero point 1-sigma error
- zp_uncert - estimated zero point uncertainty
- is_frame_interpolated - true/false whether or not frame statistics were interpolated
- ra_stellar - right ascension (with stellar and diurnal aberration correction)
- dec_stellar - declination (with stellar and diurnal aberration correction)

For this research, the data of interest include the sensor ID, the observation site data, the Julian date of observation, and of course the estimated visual magnitude, which is the data which will be directly compared against the simulation results.
3.2 Satellites of Interest

This research focused on five selected GEO satellites: ANIK F1R (NORAD: 28868), AMC-15 (NORAD: 28446), Galaxy 15 (NORAD: 28884), and Galaxy 16 (NORAD: 29236).

### Table 1 RSO Basic Information and Dimensions

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Bus Model</th>
<th>Width x Height x Depth (m)</th>
<th>Launch Mass (kg)</th>
<th>Dry Mass (kg)</th>
<th>#SP's</th>
<th>SP Width x Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANIK F1R</td>
<td>Eurostar-3000S</td>
<td>40.4 x 9 x 9</td>
<td>4500</td>
<td>2135</td>
<td>2</td>
<td>15 x 10</td>
</tr>
<tr>
<td>AMC-15</td>
<td>A2100AXS</td>
<td>7.5 x 3.5 x 3.5</td>
<td>4021</td>
<td>2050</td>
<td>2</td>
<td>10 x 10</td>
</tr>
<tr>
<td>Galaxy 15</td>
<td>GEOStar-2</td>
<td>4.2 x 4.2 x 4.2</td>
<td>2033</td>
<td>885</td>
<td>2</td>
<td>15 x 10</td>
</tr>
<tr>
<td>Galaxy 16</td>
<td>LS-1300</td>
<td>7.5 x 2.9 x 3.4</td>
<td>4640</td>
<td>1859</td>
<td>2</td>
<td>10 x 10</td>
</tr>
</tbody>
</table>

Satellite ANIK F1R was launched on September 9, 2005 from the Baikonur Cosmodrome, with a launch mass of 4500 kg and a dry mass of 2135 kg. Satellite AMC 15 was launched on October 15, 2005 from the Baikonur Cosmodrome, with a launch mass of 4021 kg and a dry mass of 2050 kg. Satellite Galaxy 15 was launched on October 13, 2005 from the Guiana Space Center, with a launch mass of 2033 kg and a dry mass of 885 kg. Satellite Galaxy 16 was launched on August 18, 2006 from the Sea Launch (Odyssey platform), with a launch mass of 4640 kg and a dry mass of 1859 kg. All five selected satellites include a cuboid shaped bus model.

In order to model the satellite bus and solar panel, online research was conducted on all the satellites of interest. A rudimentary description of the satellite bus dimensions was obtained and included. Due to the lack of information of the solar panels, an approximate size was selected for each bus model. It must be stated that the shape models used in MATLAB are not created from confirmed engineering detail, therefore they are only basic approximations to the actual satellites.
3.3 Observation Locations

After reviewing and evaluating the EPIC results, the data obtained from these four satellites provided the clearest and most distinctive observed light curves. In addition, these light curves were obtained from a specific stations, MITRE_USAFA. Although telescope data was provided from four distinct observation stations (MITRE_USAFA, RME02, C14HYPDorne, and RCOS14Westeros) only MITRE_USAFA is used in this research due to its significantly better observation results on the selected satellites.

<table>
<thead>
<tr>
<th>Table 2 Observation Station Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>MITRE_USAFA</td>
</tr>
<tr>
<td>Latitude (degrees)</td>
</tr>
<tr>
<td>Longitude (degrees)</td>
</tr>
<tr>
<td>Altitude</td>
</tr>
</tbody>
</table>
4. Simulation Configuration

To test the BRDF models, information on each of the RSOs was included in a MATLAB script and processed by a measurement generating function. This function’s purpose is to simulate observations based on the specified object’s provided properties. In this case, the observation time, initial TLE, estimated shape model, expected attitude and size, among other parameters, were incorporated into the simulation for each RSO and processed by the measurement generating function. Assumptions were made based on basic knowledge or online research of each RSO in order to successfully incorporate it into the simulation. In order to find the correct nadir-pointing bus direction for each satellite, experimentation with the bus rotation was conducted using the Ashikhmin-Premoze BRDF, rotating the angle from 0 to 360 degrees until an approximately close fit of the values resembling the truth was found. Therefore, discrepancies were expected between the simulated and actual data due to the lack of detailed available information on features such as attitude, absolute size of the RSOs, and exact TLEs during the propagation time. This research assumed nadir-pointing orientation for all of the RSOs, but rotation of the RSO about the nadir was required to find the best fit with the real data. This orientation approximation as well as the limited accuracy of the TLEs are the driving factors for the uncertainty in our models.

By including each RSO’s information, propagating the data for a specific time, and then comparing the results with the actual observed data, we were able to determine how accurately the BRDF models approximate reality. The data was processed using the three described BRDF models, Ashikhmin-Premoze, Ashikhmin-Shirley, and Cook Torrance, providing a range of apparent magnitudes of each RSO for the selected time frame. The resulting apparent magnitude was then compared with the collected apparent magnitude values provided by ADS.
5. Results and Discussion

5.1 Satellite ANIK F1R

Simulations for ANIK F1R were created for one time interval from the MITRE-USAFA observation station. Figures 9 through 18 show that the three BRDFs are able to detect the general path of the apparent magnitude, but the Cook-Torrance illumination model proved to best match the data values and direction. Ashikhmin-Premoze results provided the next closest approximation while Ashikhmin-Shirley results deviated the most from the truth. It must clarified that while the individual BRDF plots show the minutes of data observation past the J2000 epoch, the all-inclusive BRDF plots show the range of minutes of the observation for a more accessible understanding of the actual observation time gap.

Figure 9 ANIK F1R Ashikhmin-Premoze vs. Real Data Set 1

Figure 10 ANIK F1R Ashikhmin-Shirley vs. Real Data Set 1

Figure 11 ANIK F1R Cook-Torrance vs. Real Data Set 1
Figure 17 ANIK F1R Predicted vs Real Data Set 2

Figure 18 ANIK F1R Predicted vs Real Data Set 2 (normalized)
5.2 Satellite AMC-15

Simulations for AMC-15 were created for one time interval from the MITRE-USAFA observation station. Figures 19 through 48 show that the three BRDFs are able to detect the general path of the apparent magnitude, but the Cook-Torrance illumination model proved to best match the data values and direction. Ashikhmin-Premoze results provided the next closest approximation but in general stay within the same values as the Ashikhmin-Shirley results, which deviated the most from the truth. It must clarified that while the individual BRDF plots show the minutes of data observation past the J2000 epoch, the all-inclusive BRDF plots show the range of minutes of the observation for a more accessible understanding of the actual observation time gap.
Figure 22 AMC-15 Predicted vs Real Data Set 1

Figure 23 AMC-15 Predicted vs Real Data Set 1 (normalized)

Figure 24 AMC-15 Ashikhmin-Premoze vs. Real Data Set 2

Figure 25 AMC-15 Ashikhmin-Shirley vs. Real Data Set 2

Figure 26 AMC-15 Cook-Torrance vs. Real Data Set 2
Figure 32 AMC-15 Predicted vs Real Data Set 3

Figure 33 AMC-15 Predicted vs Real Data Set 3 (normalized)

Figure 34 AMC-15 Ashikhmin-Premoze vs Real Data Set 4

Figure 35 AMC-15 Ashikhmin-Shirley vs Real Data Set 4

Figure 36 AMC-15 Cook-Torrance vs Real Data Set 4
Figure 37 AMC-15 Predicted vs Real Data Set 4

Figure 38 AMC-15 Predicted vs Real Data Set 4 (normalized)

Figure 39 AMC-15 Ashikhmin-Premoze vs. Real Data Set 5

Figure 40 AMC-15 Ashikhmin-Shirley vs. Real Data Set 5

Figure 41 AMC-15 Cook-Torrance vs. Real Data Set 5
Figure 42 AMC-15 Predicted vs Real Data Set 5

Figure 43 AMC-15 Predicted vs Real Data Set (normalized)

Figure 44 AMC-15 Ashikhmin-Premoze vs. Real Data Set 6

Figure 45 AMC-15 Ashikhmin-Shirley vs. Real Data Set 6

Figure 46 AMC-15 Cook-Torrance vs. Real Data Set 6
Figure 47 AMC-15 Predicted vs Real Data Set 6

Figure 48 AMC-15 Predicted vs Real Data Set 6 (normalized)
5.3 Satellite Galaxy 15

Simulations for Galaxy 15 were created for one time interval from the MITRE-USAFA observation station. Figures 49 through 63 show that the three BRDFs are able to detect the general path of the apparent magnitude, but the Cook-Torrance illumination model proved to best match the data values and direction. Ashikhmin-Premoze results provided the next closest approximation while Ashikhmin-Shirley results deviated the most from the truth. It must clarified that while the individual BRDF plots show the minutes of data observation past the J2000 epoch, the all-inclusive BRDF plots show the range of minutes of the observation for a more accessible understanding of the actual observation time gap.

Figure 49 Galaxy 15 Ashikhmin-Premoze vs. Real Data Set 1

Figure 50 Galaxy 15 Ashikhmin-Shirley vs. Real Data Set 1

Figure 51 Galaxy 15 Cook-Torrance vs. Real Data Set 1
Figure 52 Galaxy 15 Predicted vs Real Data Set 1

Figure 53 Galaxy 15 Predicted vs Real Data Set 1 (normalized)

Figure 54 Galaxy 15 Ashikhmin-Premoze vs. Real Data Set 2

Figure 55 Galaxy 15 Ashikhmin-Shirley vs. Real Data Set 2
Figure 56 Galaxy 15 Cook-Torrance vs. Real Data Set 2

Figure 57 Galaxy 15 Predicted vs Real Data Set 2

Figure 58 Galaxy 15 Predicted vs Real Data Set 2 (normalized)

Figure 59 Galaxy 15 Ashikhmin-Premoze vs. Real Data Set 3

Figure 60 Galaxy 15 Ashikhmin-Shirley vs. Real Data Set 3
Figure 61 Galaxy 15 Cook-Torrance vs. Real Data Set 3

Figure 62 Galaxy 15 Predicted vs Real Data Set 3

Figure 63 Galaxy 15 Predicted vs Real Data Set 3 (normalized)
5.4 Satellite Galaxy 16

Simulations for Galaxy 16 were created for one time interval from the MITRE-USAFA observation station. Figures 64 through 88 show that the three BRDFs are able to detect the general path of the apparent magnitude, but the Cook-Torrance illumination model proved to best match the data values and direction. Ashikhmin-Premoze results provided the next closest approximation while Ashikhmin-Shirley results deviated the most from the truth. It must clarified that while the individual BRDF plots show the minutes of data observation past the J2000 epoch, the all-inclusive BRDF plots show the range of minutes of the observation for a more accessible understanding of the actual observation time gap.

Figure 64 Galaxy 16 Ashikhmin-Premoze vs. Real Data Set 1
Figure 65 Galaxy 16 Ashikhmin-Shirley vs. Real Data Set 1
Figure 66 Galaxy 16 Cook-Torrance vs. Real Data Set 1
Figure 67 Galaxy 16 Predicted vs Real Data Set 1

Figure 68 Galaxy 16 Predicted vs Real Data Set 1 (normalized)

Figure 69 Galaxy 15 Ashikhmin-Premoze vs. Real Data Set 2

Figure 70 Galaxy 15 Ashikhmin-Shirley vs. Real Data Set 2
Figure 71 Galaxy 15 Cook-Torrance vs. Real Data Set 2

Figure 72 Galaxy 15 Predicted vs Real Data Set 2

Figure 73 Galaxy 15 Predicted vs Real Data Set 2 (normalized)

Figure 74 Galaxy 15 Ashikhmin-Premoze vs. Real Data Set 3

Figure 75 Galaxy 15 Ashikhmin-Shirley vs. Real Data Set 3
Figure 81 Galaxy 15 Cook-Torrance vs. Real Data Set 4

Figure 82 Galaxy 15 Predicted vs Real Data Set 4

Figure 83 Galaxy 15 Predicted vs Real Data Set 4 (normalized)

Figure 84 Galaxy 15 Ashikhmin-Premoze vs. Real Data Set 5

Figure 85 Galaxy 15 Ashikhmin-Shirley vs. Real Data Set 5
In addition to plotting the apparent magnitude versus time of observation, the phase curves, which display the brightness of the satellite as a function of the phase angle, where also plotted. In astronomy, the phase angle refers to the angle between two meeting vectors, where one vector consists of the incoming light source onto the satellite and the other vector is the light reflected from it and taken in by our observing source. This angle is visually explained in Fig. 72.
These phase curves (Figures 74 through 89) allow for a slightly different view of our data and illumination model comparison but fall right in line with our previous plot results.
Figure 91 ANIK F1R Predicted vs. Real Data Set 2 Phase Curve

Figure 92 AMC-15 Predicted vs. Real Data Set 1 Phase Curve
Figure 93 AMC-15 Predicted vs. Real Data Set 2 Phase Curve

Figure 94 AMC-15 Predicted vs. Real Data Set 3 Phase Curve
Figure 95 AMC-15 Predicted vs. Real Data Set 4 Phase Curve

Figure 96 AMC-15 Predicted vs. Real Data Set 5 Phase Curve
Figure 97 AMC-15 Predicted vs. Real Data Set 6 Phase Curve

Figure 98 Galaxy 15 Predicted vs. Real Data Set 1 Phase Curve
Figure 99 Galaxy 15 Predicted vs. Real Data Set 2 Phase Curve

Figure 100 AMC-15 Galaxy 15 Predicted vs. Real Data Set 3 Phase Curve
Figure 101 Galaxy 16 Predicted vs. Real Data Set 1 Phase Curve

Figure 102 AMC-15 Galaxy 16 Predicted vs. Real Data Set 2 Phase Curve
Figure 103 Galaxy 16 Predicted vs. Real Data Set 3 Phase Curve

Figure 104 Galaxy 16 Predicted vs. Real Data Set 4 Phase Curve
In order to assess the accuracy of the BRDF models in a numerical approach instead of only having the visual assistance of the above plots, the mean and standard deviation of the difference between the real and predicted apparent magnitude of the light curves was calculated. The more accurate the simulated results are to the real data, the closer we would expect the mean and standard deviation values to be to zero.

Table 3 displays these mean and standard deviation values for each satellite data set evaluated with the three distinct BRDFs. As can be observed, as we move to the right of the table, the values for both assessing standards decrease. We can see that the mean and standard deviation values for all satellites are highest with the Ashikhmin-Premoze BRDF and lowest with the Cook Torrance illumination model, which is exactly what we expected purely based on the presented comparison plots.
Table 3 Satellite Mean and ST of Difference in Apparent Magnitude Calculated with Distinct BRDFs

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Ashikhmin-Premoze</th>
<th>Ashikhmin-Shirley</th>
<th>Cook Torrance</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMC-15 (Data Set 1)</td>
<td>3.798884</td>
<td>0.18981</td>
<td>3.7976</td>
</tr>
<tr>
<td>AMC-15 (Data Set 2)</td>
<td>3.527111</td>
<td>0.069005</td>
<td>3.52346</td>
</tr>
<tr>
<td>AMC-15 (Data Set 3)</td>
<td>3.081096</td>
<td>0.124355</td>
<td>3.078487</td>
</tr>
<tr>
<td>AMC-15 (Data Set 4)</td>
<td>1.005482</td>
<td>0.058581</td>
<td>0.250757</td>
</tr>
<tr>
<td>AMC-15 (Data Set 5)</td>
<td>3.633568</td>
<td>0.131813</td>
<td>3.630133</td>
</tr>
<tr>
<td>AMC-15 (Data Set 6)</td>
<td>0.667753</td>
<td>0.136743</td>
<td>-0.33764</td>
</tr>
<tr>
<td>Galaxy 16 (Data Set 1)</td>
<td>2.239825</td>
<td>0.030494</td>
<td>0.799238</td>
</tr>
<tr>
<td>Galaxy 16 (Data Set 2)</td>
<td>2.284993</td>
<td>0.09396</td>
<td>1.303856</td>
</tr>
<tr>
<td>Galaxy 16 (Data Set 3)</td>
<td>3.243209</td>
<td>0.18076</td>
<td>3.221699</td>
</tr>
<tr>
<td>Galaxy 16 (Data Set 4)</td>
<td>1.428654</td>
<td>0.067288</td>
<td>0.310296</td>
</tr>
<tr>
<td>Galaxy 16 (Data Set 5)</td>
<td>3.311048</td>
<td>0.121998</td>
<td>3.296717</td>
</tr>
<tr>
<td>Galaxy 15 (Data Set 1)</td>
<td>0.690406</td>
<td>0.110989</td>
<td>-0.37897</td>
</tr>
<tr>
<td>Galaxy 15 (Data Set 2)</td>
<td>1.580143</td>
<td>0.502364</td>
<td>-0.07385</td>
</tr>
<tr>
<td>Galaxy 15 (Data Set 3)</td>
<td>1.598037</td>
<td>0.181613</td>
<td>1.414389</td>
</tr>
<tr>
<td>ANIK F1R (Data Set 1)</td>
<td>1.050888</td>
<td>0.28105</td>
<td>0.045047</td>
</tr>
<tr>
<td>ANIK F1R (Data Set 2)</td>
<td>1.576345</td>
<td>0.143256</td>
<td>1.033706</td>
</tr>
</tbody>
</table>
It can be observed from the above plots that most of the light curves present themselves in straight lines during the observed time periods. Since the apparent magnitude is approximately equal to the equation \( a + bt \), where \( a \) is the bias, \( b \) is the slope, and \( t \) is time, a linear least squares fit of the observed and model predicted data was conducted. An example of this linear approximation can be seen in Figure 89.

![Apparent Magnitude vs Time](image)

**Figure 106 Linear Least Squares Fit Example**

Table 4 below shows the \( a \) and \( b \) values for each BRDF-predicted model as well as for the observed data sets. This least squares fit was intended to simplify the real and predicted results in a linear fashion in order to observe whether differences could be more easily observed and characterized through comparison of the bias \( (a) \) and slope \( (b) \). In order to compare the difference, the bias and slope values of each telescope data set was divided by the bias and slope captured by each illumination model. Ideally, this division would lead to a value of 1, which is what we expect if the predicted values exactly matched the observed ones. However, the table shows there is a range of discrepancy between all models.
Although the Ashikhmin-Premoze and Ashikhmin-Shirley results are almost similar for some of the data sets and could therefore have the same correction to better fit the observation results, they do not show a consistent bias through all cases. Therefore, no general correction can be applied to the models.

Based on the plots above and Table 3, we would expect the bias and slope values of the Cook-Torrance model to best approximate the ones from the observed data. However, this is not the case. Out of our selected data sets, the Cook-Torrance model does approximate the value of 1 better than the other two illumination models in some cases (approximately 1/3 of the data sets), but due to the spread in apparent magnitude values, the least squares fit provided lines for the Ashikhmin-Premoze and Ashikhmin-Shirley models which sometimes approximated the observed bias and slope better.

Overall, this method was not conducive to finding a characteristic pattern or bias between the illumination models and the truth to potentially find a correcting factor.
<table>
<thead>
<tr>
<th>Data Set</th>
<th>Slope</th>
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<th>Slope</th>
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6. **Conclusion**

6.1 **Summary of Thesis Results**

In general, our results showed that out of the three BRDF models for this research, Cook-Torrance best approximated the apparent magnitude trends and values obtained from the telescope data. The Ashikhmin-Shirley BRDF was the next most accurate at correctly matching the real data while the Ashikhmin-Premoze model came last. However, many of the observed times the Ashikhmin-Premoze and Ashikhmin-Shirley BRDF models performed almost the same, which is not a surprising result considering the similitude of the models. Although the solar panels in our shape models were rotated to always be perpendicular to the solar vector, the predicted models did not match the solar panel glint displayed by the real data.

6.2 **Limitations of Research**

Due to the lack of knowledge of the exact attitude and size of the RSOs, the BRDF results were obtained by making multiple assumptions and are limited in their portrayal of reality. Therefore, although some of predictions closely matched the observed data, none of them were able to fit it perfectly. In addition, limitations in our shape model and established dynamics did not allow us to capture the solar panel glint effect that we observed in some of the data sets.

6.3 **Future Work**

Future work includes further modification of the MATLAB code to improve the representation of the shape model and its dynamics to better capture solar panel glints. This will require more complex and realistic shape models as well as a more exact knowledge of the satellite attitude. In addition, inclusion of distinct BRDF models would allow for further understand on the validity of these illumination models and the effect on apparent magnitude results as the BRDFs are changed. In
addition, this research focused on using EKFs, future work could see how the implementation of an Unscented Kalman filter (UKF) or another filter would alter results. Also, a quantitative study of the average error between the BRDF predictions and the real data for more and longer propagation windows will be conducted. Lastly, more data with longer periods of observation would allow for enhanced simulation runs.
7. Works Cited


