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ABSTRACT

The submicron-aperture fiber point-diffraction interferometer (SFPDI) can be applied to realize the measurement of three-dimensional absolute displacement within large range, in which the performance of point-diffraction wavefront and numerical iterative algorithm for displacement reconstruction determines the achievable measurement accuracy, reliability and efficiency of the system. A method based on fast searching particle swarm optimization (FS-PSO) algorithm is proposed to realize the rapid measurement of three-dimensional absolute displacement. Based on the SFPDI with two submicron-aperture fiber pairs, FS-PSO method and the corresponding model of the SFPDI, the measurement accuracy, reliability and efficiency of the SFPDI system are significantly improved, making it more feasible for practical application. The effect of point-diffraction wavefront error on the measurement is analyzed. The error of point-diffraction wavefront obtained in the experiment is in the order of $1 \times 10^{-4}$λ (the wavelength $\lambda$ is 532 nm), and the corresponding displacement measurement error is smaller than 0.03 μm. Both the numerical simulation and comparison experiments have been carried out to demonstrate the accuracy and feasibility of the proposed SFPDI system, high measurement accuracy in the order of 0.1 μm, convergence rate (~90.0%) and efficiency have been realized with the proposed method, providing a feasible way to measure three-dimensional absolute displacement in the case of no guide rail.

Keywords: Absolute displacement measurement; point-diffraction interferometer; spherical wavefront error; fast searching particle swarm algorithm

1. INTRODUCTION

Various techniques for high-precision displacement measurement, such as the coordinate measuring machine (CMM) and laser tracker, have been widely applied in modern fabrication and testing [1, 2], including the position location and size measurement, etc. Due to the limitation of heavy guide rails and huge platform, the CMM is not feasible for in-situ measurement. Besides, the achievable accuracy of the existing measurement technique is limited by the machining error of standard parts in the system. The method based on point-diffraction interferometer (PDI) [3-7] provides a feasible way for the in-situ high-precision measurement of three-dimensional absolute displacement, in which two optical fibers are applied to generate the lateral shearing spherical wavefronts. The point-diffraction spherical wavefronts from two fibers, which are installed on the target to be measured, interfere with each other, and the absolute displacement of the target can be retrieved from the interference field. Thus, the fiber point-diffraction spherical wavefront [8-14] plays the role as ideal measurement reference, and it determines the achievable measurement accuracy of the system.

The numerical aperture (NA) of traditional fiber PDI is smaller than 0.20 [4, 5, 15-17]. To extend the measurement range with optical fiber, a submicron-aperture fiber [8, 18] with cone-shaped exit end has been proposed to get both high-NA and high-intensity spherical wave. The submicron-aperture fiber point-diffraction interferometer (SFPDI) can be applied to realize the measurement of three-dimensional absolute displacement within large range, in which the performance of

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point-diffraction wavefront and numerical iterative algorithm for displacement reconstruction determines the achievable measurement accuracy, reliability and efficiency of the system. The PDI-based absolute displacement measurement uses a fairly complicated mathematical model, which needs to be solved by numerical iterative algorithms such as Newton iteration method and evolutionary algorithms [19-21]. In the Newton-type methods [22, 23], the derivative and even partial derivative need to be solved during iterations, and they are not feasible for the overload equation calculation due to the complex derivation. Generally, only a small part of volume data points are selected for analysis and it could lead to a great waste of the information on detecting plane and poor anti-disturbance ability. Besides, quite a long time is required to obtain an accurate solution with the traditional evolutionary algorithms, making it not suitable for real-time measurement.

This paper presents a fast searching method with a modified mathematical model based on SFPDI to realize the rapid measurement of absolute displacement within large range. The modified SFPDI with two submicron-aperture fiber pairs in x and y directions is applied to improve the measurement accuracy. Besides, a fast searching method based on particle swarm optimization algorithm (FS-PSO) and the corresponding modified model of SFPDI is proposed to determine the absolute displacement under measurement. The principle of the proposed method for absolute displacement measurement, including the system configuration, mathematical model and the theory of the FS-PSO, are introduced in detail, and both the numerical simulation and experiments are carried out to demonstrate the feasibility of the proposed method.

2. PRINCIPLE OF SFPDI

2.1 System configuration of SFPDI

Figure 1 shows the system configuration of SFPDI for absolute displacement measurement. The circularly polarized beams from a linearly-polarized laser operating at 532 nm are coupled into the single-mode fibers SF1, SF2, SF3 and SF4, respectively. All the exit ends of the four fibers are integrated into the measurement target with certain lateral offset in x and y directions, respectively. The point-diffraction interference field and phase distribution can be obtained with phase-shifting method, from which the corresponding three-dimensional absolute displacement of the target can be measured. To increase the NA of point-diffraction wavefront with single-mode fiber, the cone-shaped exit end, in which the taper surface is coated with Cr film and exit aperture formed from polished tip, is fabricated to obtain submicron aperture with the size being about 0.5 μm, as is shown in Fig. 1. To improve the measurement accuracy both in x and y directions, two submicron-aperture fiber pairs (SF1-SF2 and SF3-SF4 with coplanar exit ends, lateral offsets \(d=125\) μm in x and y directions) are integrated in a metal tube.

![Fig. 1. System configuration of SFPDI.](image)

2.2 Principle of SFPDI for absolute displacement measurement

To illustrate the principle of SFPDI for absolute displacement measurement, the lateral offset in x direction between submicron-aperture fibers is taken as the mathematical model of the SFPDI, as is shown in Fig. 2. The three-dimensional absolute displacement between the target and CCD detector can be measured according to the optical path difference (OPD) between two point-diffraction sources SF1 and SF2. The CCD plane is defined as \(xy\) plane, in which the central position \(O\) is taken as the origin of coordinate. As is shown in Fig. 2(b), the midpoint \(O'(x_0, y_0, z_0)\) of two sources SF1 and SF2 on the target with the distance \(d\) is defined as both the position of target (Fig. 2(a)) and the origin of the auxiliary target spherical coordinate system (Fig. 2(b)). The coordinates of SF1 and SF2 can be expressed as \((d/2, \theta, \sigma)\)
and \((d/2, \theta + \pi, \sigma + \pi)\), and we have the distances \(r_1\) (\(r_2\)) between SF1 (SF2) and a known point \(P(x, y, z)\) on CCD plane in Fig. 2(a),

\[
\begin{align*}
   r_1 &= \left[ (x-x_0 - d/2 \cdot \cos \sigma \sin \theta)^2 + (y-y_0 - d/2 \cdot \sin \sigma \sin \theta)^2 + (z-z_0 - d/2 \cdot \cos \theta)^2 \right]^{1/2}, \\
   r_2 &= \left[ (x-x_0 + d/2 \cdot \cos \sigma \sin \theta)^2 + (y-y_0 + d/2 \cdot \sin \sigma \sin \theta)^2 + (z-z_0 + d/2 \cdot \cos \theta)^2 \right]^{1/2},
\end{align*}
\]

(1)

where \(\theta\) and \(\sigma\) are the azimuthal angle and polar angle, respectively. Thus, the corresponding phase difference \(\phi(x, y, z)\) at the point \(P\) can be obtained as

\[
\phi(x, y, z) = \frac{2\pi}{\lambda}(r_1 - r_2).
\]

(2)

Fig. 2. Model of SFPDI for absolute displacement measurement. (a) Measurement model, (b) auxiliary spherical coordinate system.

The nonlinear equation \(f(\Omega)\) of the phase difference \(\phi\) at the point \(P(x, y, z)\) can be written as,

\[
f(\Omega; x, y, z) = [\phi(x, y, z) - \phi_0] - [\phi_{CCD}(x, y, z) - \zeta],
\]

(3)

where \(\Omega = (x_0, y_0, z_0, \theta, \sigma, d)\) indicates the three-dimensional absolute displacement between \(O\) and \(O'\). \(\phi_0\) is the calculated phase difference at the origin \(O\) according to Eq. (2), \(\phi_{CCD}(x, y, z)\) and \(\zeta\) are the measured phase difference values at the point \(P\) and the origin \(O\), respectively. Then, the overdetermined nonlinear equation \(F(\Omega)\) can be defined as

\[
F(\Omega) = \{f(\Omega; x_i, y_i, z_i)\}_{i=1,...,m} = \{(\phi(x_i, y_i, z_i) - \phi_0) - [\phi_{CCD}(x_i, y_i, z_i) - \zeta]\}_{i=1,...,m},
\]

(4)

where \(m (m \geq 6)\) is the number of the pixel points on CCD plane selected for absolute displacement measurement. Thus, the coordinates \(\Omega\) of two sources SF1 and SF2 can be obtained from the quadratic function according to the nonlinear least square method,

\[
\psi(\Omega) = \frac{1}{2} F(\Omega)^T F(\Omega) = \frac{1}{2} \sum_{i=1}^{m} f_i^2(\Omega; x_i, y_i, z_i),
\]

(5)

where the vector \(\Omega\) can be got from the global minimum value of the objective function \(\psi(\Omega)\), and the three-dimensional absolute displacement of target under measurement can be determined with numerical iterative reconstruction algorithm.

2.3 Fast searching particle swarm optimization algorithm for displacement reconstruction

To realize the efficient measurement of absolute displacement according to the highly nonlinear equation in Eq. (5), a modified FS-PSO method with fast searching methods adopted in iteration process can be applied to solve the equations, in which millions of pixel points on the detector can be applied to improve the measurement accuracy and reliability. To improve the computational efficiency and convergence speed of PSO, the pixel number \(m\) in Eq. (5) increases with the iteration steps in FS-PSO, and the acceleration function \(k(t)\) is defined to determine the change of equation number \(m\),

\[
\psi(\Omega) = \frac{1}{2} F(\Omega)^T F(\Omega) = \frac{1}{2} \sum_{i=1}^{m} f_i^2(\Omega; x_i, y_i, z_i).
\]
where \( t \) \((t \leq t_{\text{max}})\) is the number of iterations. As is shown in Fig. 1, the searching region is defined in the square region \( T \) with side length \( l \) on CCD, and the iteration number \( t_{\text{sup}} \) between two steps is

\[
t_{\text{sup}} = \frac{t_{\text{max}}}{q},
\]

where \( q \) is the step number in the retrieve process.

According to PSO algorithm process, the position of the \( i_{th} \) particle and the corresponding velocity of position change can be denoted as \( \Omega_j = \{\Omega_{ji}\}_{j=1,2,...,6} = (x_{i0}, y_{i0}, z_{i0}, \theta_{i1}, \sigma_i, d_i) \) and \( V_i = \{v_{ji}\}_{j=1,2,...,6} = (v_{i1}, v_{i2}, ..., v_{i6}) \), respectively. In the searching process, the best previous position of the \( i_{th} \) particle is defined as the best personal position \( P_i = \{P_{ji}\}_{j=1,2,...,6} = (P_{i1}, P_{i2}, ..., P_{i6}) \) and the best particle among all the particles as the global best position \( G_i = \{G_{ji}\}_{j=1,2,...,6} = (G_{i1}, G_{i2}, ..., G_{i6}) \). The velocity \( v_{ji}^{(t+1)} \) of the \( j_{th} \) unknown \( \Omega_j \) in \((t+1)_{th}\) iteration is updated at discrete intervals according to the following equation,

\[
v_{ji}^{(t+1)} = v_{ji}^{(t)} + c_1 \cdot b_1 \left[ P_{ji}^{(t)} - \Omega_{ji}^{(t)} \right] + c_2 \cdot b_2 \left[ G_{ji}^{(t)} - \Omega_{ji}^{(t)} \right],
\]

where \( c_1 \) and \( c_2 \) are two positive constants named as learning factors, \( b_1 \) and \( b_2 \) are uniform random variables in the range of \((0, 1)\). According to Eq. (5), it is a six-dimensional variable in the objective function \( \psi(\Omega) \), and the particles for each dimension move in the direction specified by the velocity matrix according to a simple relationship given by

\[
\Omega_{ji}^{(t+1)} = \Omega_{ji}^{(t)} + v_{ji}^{(t+1)}.
\]

In the FS-PSO, the nonlinear growth of equation number \( m \) for the objective function \( \psi(\Omega) \) can greatly improve computational efficiency and convergence speed of PSO. With the full use of a large amount of pixel information on detector, the random noise can be well suppressed and high measurement accuracy and reliability be obtained.

### 3. NUMERICAL SIMULATION

#### 3.1 Effect of point-diffraction wavefront error

In the SFPDI, the point-diffraction spherical wavefront from submicron-aperture fiber determines the achievable measurement accuracy of the system. The major parameters including exit aperture size, cone angle and NA of the submicron-aperture fiber determine the sphericity of point-diffraction wavefront, the effect of which could be estimated by the numerical analysis based on the FDTD method [14]. With the ray tracing method, the simulation is carried out to analyze the measurement error of absolute displacement introduced by point-diffraction wavefront error. Figure 3 shows the spherical error of point-diffraction wavefront and the corresponding absolute displacement measurement errors for different fiber exit apertures and taper angles of submicron-aperture fiber under various NAs.

**Fig. 3.** Spherical error of point-diffraction wavefront and the corresponding absolute displacement measurement error. Errors for different fiber (a) exit apertures with 35° taper angle, (b) taper angles with 0.5 μm exit aperture.
According to Fig. 3, both the point-diffraction wavefront error and displacement measurement error grow with the exit aperture, taper angle and NA, they are smaller than \( \lambda/1000 \) (the wavelength \( \lambda \) is 532 nm) and 1 nm, respectively. Thus, the effect of submicron-aperture fiber point-diffraction wavefront error with RMS value less than \( \lambda/1000 \) on displacement measurement can be negligible.

### 3.2 Feasibility of FS-PSO

The numerical simulation is carried out to demonstrate the feasibility of the proposed absolute displacement measurement method in Section 2. The pixel number on detector plane is set to 1920×1080 and pixel size is 4.65 \( \mu m \times 4.65 \mu m \). The maximum number of iterations \( t_{\text{max}} \) is set to 100, the whole retrieve process is divided into \( q=5 \) steps, and the side length / of searching region is 500 pixels. The measurement error of target position in the simulation is about (0.91 nm, 0.80 nm, 2.33 nm). Figure 4 shows the measurement results of 22 positions with the target moving from (0 mm, 5 mm, 150 mm) to (100 mm, 5 mm, 150 mm) in \( x \) axis and from (0 mm, 5 mm, 20 mm) to (0 mm, 5 mm, 320 mm) to (0 mm, 5 mm, 320 mm) in \( z \) axis. The RMS values of measurement error for the \( x \), \( y \) and \( z \) axes in Fig. 4(a) are 0.067 \( \mu m \), 0.071 \( \mu m \) and 0.052 \( \mu m \), respectively, and those in Fig. 4(b) are 0.070 \( \mu m \), 0.062 \( \mu m \) and 0.054 \( \mu m \), confirming the feasibility of the proposed measurement method with FS-PSO.

![Fig. 4. Absolute displacement measurement error in simulation. Errors in (a) \( x \) axis and (b) \( z \) axis.](image)

### 4. EXPERIMENTAL MEASUREMENT

An experimental SFPDI system has been set up for the absolute displacement measurement, in which the pixel number of CCD detector is 1920×1080 and pixel size is 4.65 \( \mu m \times 4.65 \mu m \). The aperture size of the submicron-aperture fiber with 35° taper angle is 0.5 \( \mu m \). To test the submicron-aperture fiber point-diffraction wavefront, the high-precision measurement based on shearing interferometry and differential Zernike polynomials fitting method has been carried out [24]. The RMS value of measured point-diffraction wavefront error is about 1.54×10^4\( \lambda \), and the corresponding absolute displacement measurement error is smaller than 0.03 \( \mu m \) according to Fig. 3.

To evaluate the accuracy of the proposed method, a control experiment with a high-precision CMM (HEXAB global classical) is performed, in which the target was moved from (0 mm, 5 mm, 150 mm) to (100 mm, 5 mm, 150 mm) along \( x \) axis and from (0 mm, 5 mm, 20 mm) to (0 mm, 5 mm, 320 mm) along \( z \) axis. Figure 5 shows the measured three-dimensional displacement deviations corresponding to the movement in \( x \) and \( z \) directions, in which the displacements obtained with CMM are taken as nominal value. According to Fig. 5, a good agreement can be observed between the CMM results and those from the proposed method. The RMS values of measurement errors in \( x \), \( y \) and \( z \) axes in Fig. 5(a) are 0.71 \( \mu m \), 0.51 \( \mu m \) and 0.80 \( \mu m \), respectively, and those in Fig. 5(b) are 0.67 \( \mu m \), 0.69 \( \mu m \) and 0.79 \( \mu m \).

To evaluate the repeatability of the proposed method, repeated measurements at (12.3 \( \mu m \), 4.3 \( \mu m \), 109.5 \( \mu m \)) are made for 30 times at the time interval of 5 min. Figure 6 shows the measurement errors of the FS-PSO and the L-M algorithm (a kind of Gauss-Newton algorithm) [8], in which the RMS values of FS-PSO and L-M algorithm are 0.87 \( \mu m \) and 0.84 \( \mu m \), respectively. As is shown in Fig. 6, the 8th, 14th and 29th measurements fail to get a convergent result with the proposed FS-PSO method, and the corresponding convergence rate is about 90.0%. Comparing with FS-PSO method, 20 measurements with L-M algorithm fail to converge, and the convergence rate is only 33.3%, illustrating the low anti-interference due to the small amount pixel points used in the iteration. Besides, the rapid measurement is realized with
the proposed measurement method, and the consuming time is only 1.61 s to finish the reconstruction of three-dimensional displacement.

Fig. 5. Absolute displacement measurement error in experiment. Measurement errors for the movement along (a) x axis and (b) z axis.

Fig. 6. Repeatability of absolute displacement measurement.

Table 1 shows the comparison measurement results between L-M algorithm and the proposed FS-PSO method, including the measurement error, convergence rate and time-consuming. According to Table 1, similar measurement accuracy is realized in repeatability experiment with both the methods; besides, great improvement in the convergence rate and efficiency is realized with the proposed FS-PSO method, and the large amount of sample points applied in the measurement enables the improvement of its anti-disturbance ability.

Table 1. Comparison of L-M algorithm and FS-PSO method.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Measurement error</th>
<th>Convergence rate</th>
<th>Time-consuming</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-M algorithm</td>
<td>0.84 µm</td>
<td>33.3%</td>
<td>2.57 s</td>
</tr>
<tr>
<td>FS-PSO method</td>
<td>0.87 µm</td>
<td>90.0%</td>
<td>1.61 s</td>
</tr>
</tbody>
</table>

5. CONCLUSION

In this paper, a fast searching method, which is based on modified model of SFPDI system and FS-PSO algorithm, is proposed to realize the accurate and rapid measurement of three-dimensional absolute displacement. The ideal point-diffraction spherical wave with both high NA and large light intensity is obtained from submicron-aperture fiber, with which the measurement range can be greatly extended. A modified FS-PSO method with a fast searching method is proposed to improve the measurement accuracy, reliability and efficiency performance of the SFPDI system.
computer simulation and experiments have been carried out to validate the feasibility of the proposed method. With the proposed method, high measurement accuracy, reliability and efficiency are realized, and it provides a feasible way to measure three-dimensional absolute displacement in the case of no guide rail.

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REFERENCES


