PARAMETRIC OPTO-MECHANICAL PERFORMANCE ANALYSIS OF MOUNTED LENSES UNDER THERMAL LOADING

by

Kal Kadlec

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This thesis has been approved on the date shown below:

____________________________  8/17/17
Dae Wook Kim  Date
Professor of Optical Sciences
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1 Abstract

Mounting of lenses in opto-mechanical assemblies can create surface figure errors and refractive index changes through thermal and pre-load stresses. As lenses and barrels change in size under temperature changes, the optical performance degrades due to stress and surface deformations [3]. Currently there is no way of determining the effect of these mechanical perturbations on the system wavefront without performing tedious finite element analysis. Most in-depth opto-mechanical analyses involve case-by-case studies with specific designs while previous general studies fail to take into account the complex geometries. The assumptions made by previous general studies ignore the effects of lens shape [4]. These omissions can have a large effect on the stiffness, stress and surface figure error. A parametric model can combine the best of both an in-depth and general study. By parametrizing the model, a simple analysis can be executed for approximating the environmental-mechanical effects on optical performance. This eliminates the time it takes for an opto-mechanical design to be iterated for an optical or mechanical engineer. This tool could be used for early opto-mechanical design or for finite element analysis verification. The parametric model allows the exploration of the broader design study without confining it to a local design space.

![Figure 1: Design Flow Chart: Current Design Process vs. Parametric Design Process](image)

2 Introduction

When a mechanical engineer receives an optical prescription, the system is at its optimal performance. The mechanical engineer cannot improve the system performance; he or she can only hope to influence it as little as possible. Unfortunately, the optical design must perform its function in some sort of real environment. This includes but is not limited to: humidity, radiation, vacuum, shock, vibration and thermal considerations. Many of these environmental effects can be accounted for in early design. When mounting a lens, stresses are induced through lens-barrel contact [5]. When there is a temperature change, these
stresses get much larger and can cause refractive index and surface figure errors. Unfortu-
nately, these stress-induced thermal effects are difficult to design for.

When designing an opto-mechanical lens system, there is no quick and accurate way for an engineer to approximate the affect of temperature induced stress on the optical performance beyond the first-order effects. The engineer must export the optical system into a computer-aided design (CAD) model, design the housing, perform finite element analysis, and then analyze the optical path difference (OPD) with another program. If the OPD error is too large, the mechanical engineer must iterate the design until the desired performance is achieved. This process can take hours or days to complete and requires multiple engineers with different disciplines. The opto-mechanical design process is long and complicated task. This struggle is more difficult during the initial design process, as the optical and mechanical design will change constantly during the design process.

By creating a parametric model, the optical or mechanical engineer have the ability to immediately start exploring the overall design range, and optimizing the system as shown in Figure 1. The current initial design process is a long iterative process while a parametric model can quickly arrive at design parameters. This means that extensive design work, design choices and finite element analysis (FEA) can be performed later. Not only is a parametric model easy to use, it allows the design parameters to be changed rapidly until the performance requirement is achieved. The intent is for this parametric model to synergize with FEA to create a more efficient design process. FEA will always be used for critical and final opto-mechanical designs, as parametric models must make assumptions that are not always sufficiently accurate.

The parametric model is created by performing a Design of Experiments (DOE) by running multiple finite element analyses with ANSYS Mechanical APDL. The FEA analyzes a lens with varying shapes under radial pressure and azimuthal moment. The surface displacements and stresses are exported to MATLAB for post-processing. The axial displacement of the surface as a function of radius is fit with a symmetric 4th-order polynomial. Along with the displacement, the mean stress and stiffness of each surface are also fit. This polynomial is regression fit to the varying input parameters to create a parametric model. This model can be scaled with the known relations of the other parameters to complete the parametric model. With this parametric model, optical performance can be analyzed as a function of various input parameters.

The model takes an input of shape, size and material parameters that are used for the opto-
mechanical design. This parametric model then outputs optical specifications for inputting into optical design software. The model also outputs a single lens OPD contribution which is useful for optical tolerancing.
3 Background

A comprehensive thermal and mechanical analysis of optical performance requires many considerations and careful setup. To understand the basis of the parametric model, the coordinate system will be defined, which is the reference frame for all the calculations. Next, the different thermal effects on optical performance will be examined. This includes opto-thermal effects which comprises size change and temperature dependent refractive index. The other category is the opto-mechanical thermal effects, which come from multiple sources of stress which cause surface deformation and stress birefringence. After establishing the load cases, the performance evaluation techniques will be discussed. Lastly, the regression fitting techniques are explained.

3.1 Coordinate System

The coordinate-system used for the Finite Element Analysis is chosen to be cylindrical because the analysis is axisymmetric. ANSYS Mechanical APDL uses a different sign convention than the optics convention. ANSYS Mechanical APDL’s cylindrical coordinates uses the x-axis as the radial direction, y-axis as the axial direction and the z-axis as azimuthal direction as shown in Figure 2.

![Coordinate System](image)

Figure 2: Coordinate System (Left - ANSYS Mechanical, Right - MATLAB)

The coordinate-system used for the postprocessing in MATLAB is cylindrical and matches the optical convention as shown in Figure 2. The $\rho$-axis is the radial axis, $z$-axis is the optical axis, and the $\theta$-axis is the azimuthal axis.
3.2 Opto-Thermal Effects

The first thermal perturbation to be discussed are the opto-thermal effects. These effects arise due to the tendency for matter to change size as a result of temperature variations. For optical and mechanical materials seen in opto-mechanical design, the material has a near linear response to temperature change[3]. Optical materials also have refractive indices which change under different temperatures [1]. These two effects are dependent on the material’s coefficient of thermal expansion (CTE), \( \alpha \), and temperature dependent refractive index, \( dn/dT \). These effects would apply to the lenses regardless of mounting configuration and can be analyzed using most optical design software. These thermal effects are the primary influence on optical performance. Fortunately, the changes due to temperature in refractive index and CTE are first-order effects which are easily understood and calculated. Opto-thermal effects can be minimized through proper axial alignment. Athermalizating the system is an easy way to compensate for these errors.

3.2.1 Size Change

One of the simplest effects of temperature changes is the expansion and contraction of the lens. For the sake of clarity and to separate this from stress-induced deformation, this expansion or contraction will be called “Size Change”. For small ranges of temperatures, the coefficient of thermal expansion (CTE), \( \alpha \), of most materials is assumed to be linear[5]. This means for a change in temperature \( \Delta T \), any dimension of a lens will vary linearly with temperature. For example, for a radius of curvature, \( R \), or center thickness, \( t_c \), the change in dimension is calculated [6]:

\[
\Delta R = \alpha R_0 \Delta T \quad (1)
\]
\[
\Delta t_c = \alpha t_{c0} \Delta T \quad (2)
\]

which means:

\[
R' = \Delta R + R_0 \quad (3)
\]
\[
t'_c = \Delta t_c + t_{c0} \quad (4)
\]

where \( \Delta R \) and \( \Delta t_{c0} \) are changes in their respective dimension and \( R' \) and \( t'_c \) are their respective final dimensions. These linear thermal effects not only perturb the lenses, but are also prominent in the lens housing. The lens housing will expand and contract linearly and radially. If there is a CTE mismatch between the lens and housing, the air separation between lens elements, \( L_0 \), can change at a different rate than the dimensions of the optical element.

\[
\Delta L = \alpha L_0 \Delta T \quad (5)
\]

where \( \Delta L \) is the change in air gap. This results in a change in optical path length and can cause defocus at the image plane. This problem can be alleviated by passively or actively athermalizing the system [5]. Fortunately, axial athermalization can be modeled through optical design software which means it can be corrected and predicted easily [7]. For the research on this particular model in this thesis, the OPD analysis also focuses on individual lens contribution, not system level OPD.
3.2.2 Temperature Dependent Refractive Index

Along with size change, the refractive index of material also changes with temperature. This is due to the approximately linear temperature dependence of the refractive index. These thermal effects are more pronounced in the infrared ranges; materials like Germanium and Silicon are extremely sensitive [8]. For a given optical material, there is a given $dn/dT$. This $dn/dT$ scales the refractive index linearly with change in temperature, $\Delta T$:

$$\Delta n = \frac{dn}{dT} \Delta T$$ (6)

where $\Delta n$ is the change in refractive index. The temperature difference not only affects the lens material but also the air space between the lenses. But, since this is just an analysis of individual lens performance, the air space refractive index will be disregarded.

3.3 Opto-Mechanical Thermal Effects

Eventually, every applied optical design needs to exist outside of an optical design program. A long-time design challenge in opto-mechanics comes from the necessity of mounting the lenses. More often than not, materials that make good optical elements do not make good mechanical materials. The most difficult to quantify opto-mechanical design challenges stem from the differences in CTE of the optical and mechanical materials. Although the contributions from these effects are small, they are difficult to account for in the initial design process. These effects need to be considered in critical and high-tolerance applications.

3.3.1 Sources of Stress

Mounting a lens in any way will cause some amount of stress in it. This means there are a lot of possible sources of induced stresses in optics. Unfortunately, it would be difficult to create a simple parametric model that had a comprehensive database of sources. For this particular parametric model, there will only be four sources included. The four sources are Radial Hard Mounting, Drop-In, Edge Bonding and Axial Mounting [9]. These load cases were chosen due to their axisymmetry with respect to the optical axis. Load cases which are non-axisymmetric complicate the already complex model by introducing a third degree of freedom.

**Radial Mounting - Hard Mounting** In this type of mounting, the lens is positioned using the Outside Diameter (OD) of the lens and the Inside Diameter (ID) of the lens barrel. This type of mounting requires high-precision manufacturing, as the optical performance depends on how tight and accurate the mounting surfaces are. When using this technique, the decenter and tilt of the lens’ optical axis is dependent on the manufacturing of the barrel and lens. The axial location of the lens is determined by the machined shoulder of the barrel. This mounting technique is useful if positioned correctly, as it has excellent performance under shock and vibration[9]. Unfortunately, this technique also requires heating
of the barrel sometimes to create the clearance for the lens to be inserted[9]. This configuration, seen in Figure 3, can also create stresses and deformations under large ranges of thermal loading which can degrade optical performance.

![Figure 3: Lens Barrel Hard Mount](image)

The major concern with this technique is the small to non-existent gap between the barrel and the optical element. If there is a change in temperature, the optical performance can degrade. A CTE mismatch between materials under thermal loading will create a radial pressure on the lens. This creates stresses within the lens that affect the optical performance.

**Radial Mounting - Drop-In Mounting** Drop-In mounting is similar to Hard Mounting, except instead of creating a high tolerance barrel to hold the optical element in place, the barrel is designed with a radial clearance. This radial clearance provides room to absorb the tolerated machining error. This method of mounting lenses is generally used for low-tolerance optical assemblies. With special considerations, this technique can also be successfully used with tight tolerances for higher-performance [9]. A spacer or retaining ring is used to locate the optical element axially. This technique, shown in Figure 4, is generally less expensive but comes at a cost of optical performance.

![Figure 4: Lens Barrel Drop-In Mount](image)
Radial Mounting - Edge Bonding  When bonding lenses into a barrel, there are a few common approaches. Most techniques involve either the three-point or ring bonds. A three-point bond involves placing three small points of cement on the radial edge of the lens with equal angular separation. A ring bond, as shown in Figure 5, involves applying cement to the entire radial edge of the lens [9]. The ring bond retains the lens with equal force in all radial directions.

Since the three-point bond is not axisymmetric, only the ring bond condition will be examined. For the purposes of simplicity and to eliminate the CTE effect of the bond, it is assumed that the bond gap is designed to add zero stress under thermal loading. The appropriate gap size can be determined with either the Bayar or Meunch equations [1]. With bonding, it is important to take care when the bond is in compression and has a high shape factor. The bond shape factor is the ratio of the cross-sectional area to the bond radial thickness [1]. This effect is extremely prevalent in lens bonding as the Poisson ratio and areas can be high while the bonds are commonly thin. This common bonding condition does not freely allow for volumetric expansion. Not allowing the bond to expand can cause incompressibility. This prevents the bond from absorbing the energy as intended [1].

The preference for bonding comes from the allowance for active alignment which allows for lens tilt and decenter compensation. Bonding doubles as a dampener when it comes to absorbing shock and vibration. The downside of bonding is that it takes longer to assemble and is more complicated [9]. Bonding is an order of magnitude more complex and requires extreme consideration of outgassing, adhesive CTE and curing shrinkage, among others.

Axial Mounting  Axial Mounting is the most accurate way to mount lenses without bonding. This technique typically references the optical surfaces when locating the lens. Using the optical surface as a reference surface eliminates the tolerance stack-up of using mechanical surfaces (edges or chamfers). This method requires a pre-load which needs to be large enough to withstand shock or temperature changes. This means that there is a force being applied to the lens before any thermal considerations are taken into account. Temperature changes can cause the lens assembly to tighten or relax. This requires calculating...
a preload that balances the induced stresses with the relaxing of the axial load.

Under an axial mount where the optical element is loaded in the same radial location ($D_{M1} = D_{M2}$), as shown in Figure 6, the effects on the optical surface are only local. Since the same load is distributed directly to the same line contact, the force does not induce any moment. The effects occur locally near where the force is applied. This local deformation is due to the Poisson effect, which is shown in Figure 6. The Poisson effect is the tendency for matter to expand in other dimensions when another dimension contracts. The local Poisson effect can be ignored, as most optical designs have a clear aperture to avoid the surface near the mount. If the optical element is loaded at different radial locations ($D_{M1} \neq D_{M2}$) as shown in Figure 7, the axial forces can induce an azimuthal moment. This effect is no longer local and affects the entire lens surface figure.

### 3.3.2 Stress Birefringence

When stresses are induced into a refractive optical element, the optical performance degrades. This is due to a phenomenon that occurs in all matter called stress-birefringence. Birefringence is an optical property of a material which has a different refractive index depending on the propagation and polarization of light [1]. Most refractive optical materials
are isotropic and in a stress-less configuration, only have one refractive index. Crystalline materials like sapphire and magnesium fluoride are inherently birefringent, even in a free-stress environment [10]. These materials have different refractive indices depending on the direction of propagation with respect to the direction of the crystal structure [1]. Applying a stress to any transmissive element will make it birefringent, including isotropic materials. Stresses in the plane orthogonal to the direction of light propagation, $\sigma_{11}$ and $\sigma_{22}$, and in the direction of propagation, $\sigma_{zz}$, produce refractive index changes, $\Delta n_1$ and $\Delta n_2$ [1]:

$$\Delta n_1 = k_{11} \sigma_{11} + k_{12} (\sigma_{22} + \sigma_{zz})$$

$$\Delta n_2 = k_{11} \sigma_{22} + k_{12} (\sigma_{11} + \sigma_{zz})$$

where $k_{11}$ and $k_{12}$ are the respective directional stress-optic coefficients. The difference in the optical path $\Delta OPD$ between two ray components is given by the difference in the index of refraction multiplied by the distance the ray traveled $L$: Refractive index change has a

\[ \Delta n_1 = \Delta n_{1_{A}} \]

\[ \Delta n_2 = \Delta n_{2_{A}} \]

\[ \Delta OPD = (\Delta n_2 - \Delta n_1) L \]

\[ \Delta OPD = k(\sigma_{11} - \sigma_{22}) L \]

where $k$ is $k_{11}$-$k_{12}$. The $\Delta OPD$ terms in equation 9 and 10 are the OPD errors between two polarization states. The OPD errors calculated in equation 11, 12 and 13 are the mean OPD error of the average polarization effect. To calculate the OPD effect, the refractive index change of the surfaces are averaged:

\[ \Delta \bar{n}_1 = \frac{\Delta n_{1_{A}} - S_{1} + \Delta n_{1_{B}} - S_{2}}{2} \]
\[ \Delta \bar{n}_2 = \frac{\Delta n_{2-S1} + \Delta n_{2-S2}}{2} \]  

(12)

The stress-induced wavefront error may be approximated by averaging the optical path of the two electric field components:

\[ OPD = \frac{\Delta \bar{n}_2 + \Delta \bar{n}_1}{2} L \]  

(13)

### 3.3.3 Surface Deformations

Not only does stress induce birefringence, it also induces surface deformations of the optical elements. The pressure or moment applied induces stress and strain which affects the surface figure. These surface deformations deform the surfaces in a non-linear way that cannot be easily compensated for.

**Normal Strain**  As matter is compressed or stretched it has a tendency to want to stay the same volume. In the radially loaded case, the lens is being compressed radially due to a pressure. This generally causes the lens to expand in the axial direction due to the Poisson effect [11]. This effect is relatively small due to the stress applied being a normal mode, as opposed to a bending mode. Since axial displacement is the direction of interest, the axial strain, \( \epsilon_{zz} \), is calculated. Using the generalized Hooke’s equation to calculate the axial strain [11]:

\[ \epsilon_{zz} = \frac{1}{E}[\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{\rho\rho})] \]  

(14)

where \( E \) is the elastic modulus, \( \nu \) is the Poisson ratio, and \( \sigma_{zz}, \sigma_{\theta\theta} \) and \( \sigma_{\rho\rho} \) are normal stresses in their respective directions. Due to the lens being unconstrained in the \( z \)-direction, there is no stress in this direction, therefore \( \sigma_{zz} = 0 \). Consequently, the only stress contributions are \( \sigma_{\theta\theta} \) and \( \sigma_{\rho\rho} \):

\[ \epsilon_{zz} = -\frac{\nu}{E}(\sigma_{\theta\theta} + \sigma_{\rho\rho}) \]  

(15)

This means that axial deflection and strain are linearly proportional to the Poisson ratio and inversely proportional to the elastic modulus.

**Bending Strain**  What affects the surface more than normal stresses are bending mode induced stresses. Bending mode induced stresses in the lens occur due to two loading conditions. One of which is a azimuthal moment, \( M_{\theta} \), due to axial mounting. The other occurs due to lenses radially loaded with large magnitude lens shape factors which cause a radial pressure, \( P_{\rho} \), due to radial mounting. Lens shape factor, \( q \), is defined as [12]:

\[ q = \frac{R_2 + R_1}{R_2 - R_1} \]  

(16)

Applying a moment, \( M \), results in an axial deflection, \( z \), which is expressed by [13]:

\[ \frac{M}{EI} = \frac{d^2 z}{d\rho^2} \]  

(17)
where \( I \) is the area moment of inertia and \( x \) is the distance “along the beam” which in this case is the radial distance, \( \rho \). Integrating both sides of the equation with respect to \( \rho \) results in the equation:

\[
z = \frac{1}{E} \int \int \frac{M}{I} d\rho
\]  

(18)

Due to the complex geometries of a lens, this equation is difficult to solve. Fortunately, FEA can solve the shape dependent portion of the equation.

To solve the parametric equation, the deflection output by the model scales by the material values. For pure radial forces, the normal mode scales by the term: \(-\nu/E\). Bending modes scales by the term: \(1/E\). One of the terms is dependent on the Poisson ratio while the other term is not. This makes determining the axial deflection due to material properties difficult as the normal and bending mode deflections are indistinguishable. To alleviate this Poisson ratio dependence, the Poisson ratio can be assumed to be constant. With the Poisson ratio constant, both terms are now dependent on only \(1/E\). This assumption can be made while only incurring a small error as most lenses have a Poisson ratio between 0.17-0.25, as shown in Figure 9 [2]. Also, as previously stated, the largest deformations occur due to bending stresses so the normal stresses will be assumed to be statistically less significant.

![Figure 9: Optical Glass Poisson Ratio as a Function of Composition [2]](image)

### 3.4 Optical Performance Evaluation

All of the opto-thermal and opto-mechanical thermal effects affect the optical performance through refractive index and surface figure changes. These effects need to be quantified
into metrics to be analyzed by the engineer. The parametric model outputs two metrics: newly updated values for the optical prescription and optical path difference (OPD).

### 3.4.1 Output Variables for Prescription Input

The program outputs worst case parameter values for the lenses under loading. These outputs are new variables like $R_1$, $R_2$, $t_c$ and $\Delta n$. The program also produces even polynomial terms.

**Polynomial Surface Fitting**  Since the surface is axisymmetric, the surface deformations are also axisymmetric. This means that the surface deformation, $\Delta z$, can be plotted as a function of distance from the optical axis, $\rho$.

$$\Delta z'(\rho) = a_1(\vec{P})\rho^4 + a_2(\vec{P})\rho^2 + a_3(\vec{P})$$  

(19)

where $a_n(\vec{P})$ is a constant that is a function of the parameters. Since this distribution is axisymmetric, the surface can be fit to even asphere terms. Even asphere terms are a common way of defining a lens surface. This means these terms can be input as optical surface data in optical designs to perform a comprehensive optical performance analysis.

### 3.4.2 Optical Path Difference

The root-mean-square (rms) wavefront error (WFE) or optical path difference (OPD) after passing through an optical element will be approximately the root-sum-square (RSS) of the contributions of each lens’ surfaces and index. RSS is used with an assumption that each OPD contribution (surface-1, surface-2 and refractive index) is uncorrelated. Each individual contribution for each surface or refractive index is added with one another. The WFE contribution of each lens is the RSS of the three contributions to the OPD of each lens:

$$OPD_{rms} = \sqrt{OPD_{n1-rms}^2 + OPD_{S1-rms}^2 + OPD_{S2-rms}^2}$$  

(20)

where each uncorrelated OPD contribution is:

$$OPD_{n-rms} = [OPD_{dn/dT} + OPD_{StressBirefringence}]_{rms}$$  

(21)

$$OPD_{S1-rms} = [OPD_{dR1/dT} + OPD_{StressDeformation1}]_{rms}$$  

(22)

$$OPD_{S2-rms} = [OPD_{dR2/dT} + OPD_{StressDeformation2}]_{rms}$$  

(23)

where the individual opto-thermal effects are:

$$OPD_{dn/dT}(\rho) = \frac{dn}{dT}(\Delta T)L(\rho)$$  

(24)

$$OPD_{dR1/dT}(\rho) = (n - 1)[z_{R1}'(\rho) - z_{R1}(\rho)]$$  

(25)

$$OPD_{dR2/dT}(\rho) = (n - 1)[z_{R2}'(\rho) - z_{R2}(\rho)]$$  

(26)
and the individual thermal opto-mechanical effects are:

\[ OPD_{\text{StressBirefringence}}(\rho) = \left( \frac{\Delta n_2 + \Delta n_1}{2} \right) L(\rho) \]  
\[ (27) \]

\[ OPD_{\text{StressDeformation}1}(\rho) = (n - 1)[\Delta z_1(\rho)] \]  
\[ (28) \]

\[ OPD_{\text{StressDeformation}2}(\rho) = (n - 1)[\Delta z_2(\rho)] \]  
\[ (29) \]

where \( z_R(\rho) \) is the surface sag, \( z \), of the respective radius, \( R \).

3.5 Least Squares Fitting - Polynomial

The scattered surface data extracted from the FEA model is defined by a line. Least squares is used to fit or regress to the scattered data. A least squares fit will create a solution that minimizes the sum of the squares of the residuals [14]. MATLAB will perform this regression analysis by adjusting constants in the base equation until convergence is reached. This technique is used in this analysis for fitting an equation to a discrete surface data (axial displacement as a function of radial distance). Least squares fitting in this analysis is also used when performing the parametric regression.

When using the least squares fit, the user needs to choose a base equation for the program, MATLAB in this case, to fit. For the surface fitting, an even-4th-order polynomial equation is used as discussed later on. This is a simple 2D fit and the polynomial type equation can be estimated by examination.

For the parametric fitting for the Design of Experiments (DOE), the equation has three independent variables (three parameters) and one dependent variable (value). This means that the regression involves a four-dimensional equation (three independent variables and one dependent variable). Again, a polynomial equation is chosen. Polynomial equations are useful for multiple reasons. Polynomial equations are customizable as they have the ability to fit to almost any data. Polynomials are also easily understood by most engineers and scientists.

4 Method

This analysis uses a combination of ANSYS Mechanical APDL, MATLAB and hand calculations. Before starting this problem, assumptions are made to create a simplified and accurate analysis. Next, the parameters which are used to create the DOE and parametric model are established. Many parameters and assumptions allow for the use of hand calculations which ease the the complexity of the parametric model.

To begin the parametric model, a finite element model is run with varying parameters as data points. These data points are then fit using the least squares method to polynomial equations. Using regression methods, these polynomials are least squares fit to the varying parameters to complete the DOE.
4.1 Assumptions

Creating a parametric analysis means simplifying the model to be accurate, yet still useful. This means making assumptions about the system. Without assumptions, there are no limits to the extent of the parametric model. These assumptions lay the groundwork for the construction of the parametric model. Some of these assumptions can be examined in the future to allow for a more comprehensive analysis.

**Axisymmetry**  Making the assumption of axisymmetry greatly simplifies this analysis. The system is assumed to be axisymmetric about the optical axis. This assumption allows for elimination of the azimuthal degree-of-freedom (DOF). This means that deformations or stresses of the optical surfaces are only a function of radius. Unfortunately, eliminating the azimuthal DOF removes the ability to perform any non-symmetric analysis. This would include methods like holding or bonding the lens in discrete points along the diameter, among others.

**Clear Aperture**  The clear aperture of the lens is limited to 95% of the lens. This means that the analysis only accounts for 95% of the OD. The other 5% is assumed to be covered with a shoulder, retaining ring or spacer. The main reason for this cropping is that it removes any non-linear edge deflections. These non-linearities often occur at very thin locations. Another reason for this is that it is a rule-of-thumb for opto-mechanical design.

When applying a clear aperture, real concave surfaces should have an edge chamfer. A chamfer is an operation in which the edge is beveled through machining or grinding. Thin surfaces can create high stress-concentrations which cause chipping and breakage of edges. The chamfer greatly reduces chipping and also provides a mechanical mounting surface. During FEA, each concave surface had a chamfer which allowed for a 95% of the clear aperture.

**Outside Diameter:Edge Thickness Ratio**  Like the clear aperture assumption, keeping the outside diameter to edge thickness ratio low is important. This assumption is also a general rule of thumb in optical and opto-mechanical design, as a thin edge can be chipped easily if care is not taken. This assumption requires the outside diameter of the lens should be at most 10-times larger than edge thickness when mounting. Thin-edge lenses are extremely sensitive to stresses and non-linear deformations as the edge will bend instead of the entire lens as discussed in Section 4.5.6. The main reason for making this assumption in the parametric model is to avoid the extreme non-linearities.

**Steady-State, Linear CTE & Uniform Temperature Distribution**  When it comes to quantifying the thermal loading of the model, the simplest is best. Analyzing the parameters as a function of time would add complexity while adding minimal functionality. Adding non-uniform temperature would be difficult to model and unnecessary for most applications. This means a uniform temperature distribution throughout the entire assembly.
The CTE is assumed to be linear for any amount of temperature change. If the CTE is non-linear, the value can be adjusted to accommodate this.

**Singlet Lenses**  This analysis is only performed for singlet lenses. This means that the assumptions must be made for doublet lenses, triplet lenses, diffractive elements, etc. This model did not analyze how cementing of optical surfaces affects the opto-thermal performance.

**Static Loading**  The analysis is not time dependent so the loading is static. As stated above, analyzing the parameters as a function of time would add complexity while adding minimal functionality for most applications.

**Bond CTE**  The bond CTE is assumed to be negligible due to the low elastic modulus of most elastomers compared to glass or metal. The bond thickness can also be optimized to match the expansion of the barrel and lens, since the effective CTE is based on the thickness of the bond. This is considered good design practices. If the bond is not athermalized, the bond CTE would be taken into account.

**Optical Path Integration**  For estimating OPD, the optical path analyzed for the clear aperture. The more accurate analysis would involve integrating along the ray path. This is difficult for a parametric model without knowing the optical prescription. The assumption is that the entrance area is approximately equal to the exit area.

**Surface Stress Distribution**  Ideally, the parametric model would be able to calculate the three orthogonal stresses as a function of radius. This becomes difficult due to the non-linearities that occur. There are large errors that occur that make this distribution difficult to fit. Although, the stress distribution values are not correct, the mean radial distribution of stresses is more accurate to calculate through the parametric model.

**Bond Diameter to Width Ratio**  The bond width, $d$, as shown in Figure 10, is assumed to be much larger than the bond thickness, $t$ ($d \gg t$). It is common for the diameter of the lens to be at least an order of magnitude larger than the bond thickness. This allows the assumption that the majority of the bond stiffness comes from the in-plane modulus of the bond.

**Poisson Ratio of Lens**  The lens is assumed to a Poisson ratio of 0.206. This assumption is made because the deflection due to bending and normal loading is indistinguishable. This assumption incurs little error since the bending mode is the dominant mode and the Poisson ratio of glass has a small variance as shown in Figure 9.

## 4.2 Parameters

The input parameters are:
• Environmental Parameters
  – dT - Change in Temperature [°C]*
  – Moment - Azimuthal Moment [N-m]

• Lens Parameters (Geometry and Material)
  – R1 - Radius of Curvature of Surface 1 [mm], (+) Convex (-) Concave
  – R2 - Radius of Curvature of Surface 2 [mm], (+) Convex (-) Concave
  – tC - Thickness at Center [mm]
  – OD_L - Outside Diameter [mm]
  – E_L - Elastic Modulus [MPa]
  – νL - Poisson Ratio
  – n - Refractive Index
  – k - Stress-Optic Coefficient [10^{-6} \text{ mm}^2/\text{N}]
  – k_{11} - Orthogonal 11-Direction Stress-Optic Coefficient [10^{-6} \text{ mm}^2/\text{N}]
  – k_{12} - Orthogonal 12-Direction Stress-Optic Coefficient [10^{-6} \text{ mm}^2/\text{N}]
  – dn/dT - Temperature Dependent Refractive Index [10^{-6} \text{ K}^{-1}]
  – CTE_L or α_L - Coefficient of Thermal Expansion [10^{-6} \text{ K}^{-1}]

• Barrel Parameters (Geometry and Material)
  – ID_B - Inside Diameter of Lens Barrel [mm]
  – OD_B - Outside Diameter of Lens Barrel [mm]
  – E_B - Elastic Modulus [MPa]
- $\nu_B$ - Poisson Ratio
- $CTE_B$ or $\alpha_B$ - Coefficient of Thermal Expansion $[10^{-6} \text{ K}^{-1}]$

- **Bond Parameters (Optional)**
  - $E_{\text{Bond}}$ - Elastic Modulus [MPa]
  - $\nu_{\text{Bond}}$ - Poisson Ratio

- **Toggles**
  - $\text{BondToggle}$ - Toggles Ring Bond in Gap ($\text{Bond} = '1'$, No Bond = '0'). If a bond is used, then the bond compliance term is applied to the pressure equation. If no bond is used, only the lens and barrel compliances are taken into account.
  - $\text{AthermToggle}$ - Athermalization Toggle ($\text{Athermized} = '0'$, Not Athermized = '1'). If there will be compensation for $dn/dT$ and size-change in the optical system, this toggle can be turned on to remove these OPD contributions. This will result in only the OPD calculation of only the thermal opto-mechanical contributions.

*Pressure calculation can be overridden by a user-input radial pressure if this value is already known*

The key for performing a Design of Experiments is to eliminate any variables from the study that can be calculated without the DOE. Fortunately, there are some in-depth materials analyses and material properties are well known. For example, there has been extensive analysis showing that the elastic modulus scales deflection linearly with respect to stress [11]. Therefore, material properties are able to be added to the parametric model later. On the other hand, behavior of complex axisymmetric shapes under radial pressure and azimuthal moments do not have extensive analyses. This requires that the parametric model will be a function of the geometry of the optical element.

An axisymmetric spherical lens shape can be fully defined by four parameters: $C_1$, $C_2$, $t_c$ and $OD$. The parameters of the model will be representend by $\vec{P}$:

$$\vec{P} = [C_1 \ C_2 \ OD]$$  \hspace{1cm} (30)

$\vec{P}$ appears to be missing a parameter, but $t_c$ is omitted on purpose. This is because $t_c$ is used to linearly scale the size of the lens. By keeping this term constant during the DOE, each $\vec{P}$ is a unique shape. The output values from the DOE are linearly scaled by $t_c$.

When creating the parametric model, curvature is used over the more commonly used optics term, radius of curvature. This is due to the geometry of the lens. When plotting curvature from negative infinity to positive infinity, as shown in Figure 11, the surface changes from large concavity to planarity then to large convex. This means plotting the physical behavior of a lens makes more sense for a plot crossing the zero curvature axis.
4.3 Calculations

4.3.1 Interference

When a lens barrel is assembled without bonding, stress is induced when there is a large enough temperature change to where the barrel and lens interfere. This interference occurs if the barrel and lens are made of materials with differing CTEs. In most cases, the CTE of the barrel is larger than the lens, which means as the temperature drops the diameter of the barrel gets smaller faster than the diameter of the lens. The change in radius, half the diameter, of a cylindrical part is expressed by the equation [6]:

$$\Delta r = \alpha r \Delta T$$

where $r$ is the radius of the part, $\alpha$ is the CTE, $\Delta r$ is the change in the radius of the part and $\Delta T$ is the change in temperature of the object. If the diameter of the lens ever exceeds the diameter of the barrel, this becomes an interference as shown by the equation:

$$\delta = r_{\text{clearance}} - (\Delta r_{\text{lens}} - \Delta r_{\text{barrel}})$$

where $r_{\text{clearance}}$ is the radial clearance between the lens and barrel at the null temperature, $\Delta r_{\text{lens}}$ and $\Delta r_{\text{barrel}}$ are the change in radius of the lens and barrel respectively. This interference is what induces stresses which induce the optical performance errors [4].

Figure 11: Lens Shape Regions (C1-Left and C2-Right)
4.3.2 Bond Stiffness

As previously stated, under high shape factors, polymers become highly incompressible [5]. Ring bonds as shown in Figure 10, are radially bonded lenses along the entire edge. When there is an interference between materials, a radial pressure is applied. Using the equation shown in Figure 12, the elastic modulus in the radial direction through the thickness of the bond:

\[ E_{11} = k_{11}M \]  \hspace{1cm} (33)

where \( E_{11} \) is the elastic modulus due to a radial stress, \( k_{11} \) is the correction factor in the in-plane radial direction and \( M \) is the maximum modulus. The maximum modulus, \( M \), is defined as [1]:

\[ M = \frac{(1 - \nu)E}{(1 + \nu)(1 - 2\nu)} \]  \hspace{1cm} (34)

where \( E \) is the elastic modulus of the bond material and \( \nu \) is the Poisson ratio of the bond material. Due to the pressure being applied normally: \( \sigma_{11} = P \). There is also stress induced in the hoop direction. Hoop stress is defined as:

\[ \sigma_{22} = \frac{Pd}{2t} = \frac{\sigma_{11}d}{2t} \]  \hspace{1cm} (35)

where \( \sigma_{22} \) is the hoop (azimuthal) stress, \( P \) is the applied radial pressure, \( d \) is the diameter and \( t \) is the thickness of the bond. So the contribution to the hoop stress from the radial normal strain is:

\[ \sigma_{22} = \frac{k_{12}k_{11}\nu M}{(1 - \nu)} \epsilon_{11} \]  \hspace{1cm} (36)

where \( k_{12} \) is the out of plane modulus correction factor and \( \epsilon_{11} \) is the in-plane radial strain. Substituting the hoop stress equation into the in-plane azimuthal stress equation:

\[ \frac{\sigma_{11}d}{2t} = \frac{k_{12}k_{11}\nu M}{(1 - \nu)} \epsilon_{11} \]  \hspace{1cm} (37)
Figure 13: Correction factors for ring bonds with various combinations of \( b/t \) ratio and Poisson ratio [1]

\[
\sigma_{11} = \frac{2t k_{12} k_{11} \nu M}{d} \epsilon_{11}
\]

(38)

\[
E_{12} = \frac{2t k_{12} k_{11} \nu M}{d} \frac{1}{(1 - \nu)}
\]

(39)

With the assumption of \( d \gg t \), the value of \( E_{12} \) is ignored. The rest of the matrix in Figure 12 is ignored as there are no other terms of importance. What is left is just the contributions from \( E_{11} \):

\[
E = k_{11} M
\]

(40)

Unfortunately \( k_{11} \) is not a constant. \( k_{11} \) is a function of \( \nu \) and the ratio between the bond width, \( b \), and thickness, \( t \). Using the correction factors lookup table from Figure 13, the value of \( k_{11} \) is interpolated. This elastic modulus is used for calculating the compliance of the bond in the interference fit shown in Equation 42.
4.3.3 Pressure

The interference-fit equation is used to derive the radial pressure exerted onto the optical element. This equation calculates the applied pressure, \( P \), for a radial interference, \( \delta \), and compliances which are derived from the radial geometry and material properties [13]:

\[
P = \frac{\delta}{\frac{d}{E_0}\left(\frac{d^2_0+d^2}{d^2_0-d^2} + \nu_0\right) + \frac{d}{E_i}(1 - \nu_i)}
\]  

(41)

where \( E_0 \) and \( E_i \) are the modulus of elasticity for the barrel and lens respectively, \( d \) is the nominal diameter of the interference location, \( d_0 \) is the outside diameter of the barrel and \( \nu_0 \) and \( \nu_i \) are the Poisson ratios of the barrel and lens respectively. This equation is derived from Hooke’s Law and the denominator can be broken down into a sum of compliances. This allows for manipulating the compliance terms and the addition of a third term \( C_{Bond} \):

\[
P = \frac{\delta}{C_{Lens} + C_{Barrel} + C_{Bond}}
\]  

(42)

where \( C \) variables are the radial compliance of each respective object. The compliance of each material must be calculated differently since each material has different material and geometric properties. Compliance is the inverse of stiffness, \( k \), and since it is linear is in units of meters per Newton [11].

\[
C = \frac{1}{k}
\]  

(43)

**Barrel Compliance**  The compliance of the barrel comes from the first term in the denominator of the interference-fit equation:

\[
C_{Barrel} = \frac{d}{E_0}\left(\frac{d^2_0+d^2}{d^2_0-d^2} + \nu_0\right)
\]  

(44)

**Lens Compliance**  The compliance of the lens is the most complicated since it has many shape variables. Since the interference fit equation only takes into account a cylinder, the compliance must be determined a different way. Fortunately, the FEA provides all the data required to find the radial compliance of the lens. Using Hooke’s Law [11]:

\[
F = k\Delta \rho_{max}
\]  

(45)

The FEA provides a maximum radial deflection, \( \Delta \rho_{max} \). Also, the pressure applied to the lens is known because the analysis is run for a unit load of 1 Pascal [11]:

\[
P = \frac{F}{A}
\]  

(46)

where \( P \) is the pressure applied, \( A \) is the area of the applied pressure and \( F \) is the force applied.

\[
A = t_{edge}(\pi D)
\]  

(47)

where \( t_{edge} \) is the thickness of the edge of the lens and \( D \) is the outside diameter of the lens. Combining the previous three equations for \( P = 1 \) Pascal:

\[
C_{Lens} = \frac{\Delta \rho_{max}}{PA} = \frac{\Delta \rho_{max}}{t_{edge}(\pi D)}
\]  

(48)
Bond Compliance  The compliance of the bond is derived from the axial stiffness equation [11]:

$$k_{axial} = \frac{AE}{L}$$

(49)

where $A$ is the cross-sectional area, $E$ is the elastic modulus and $L$ is the axial length. This equation can be converted into compliance easily to become:

$$C_{Bond} = \frac{t_{Bond} AE}{AE}$$

(50)

Although the cross-sectional area is not constant, the bond thickness is much less than the diameter. The elastic modulus is derived in Section 4.3.2. This means that the cross-sectional area difference between the outer diameter and inner diameter of the bond is negligible. Unfortunately, large shape factors (ratio of the loaded area to the force free area) can cause incompressibility. Large shape factors occur in thin bonds which can cause high stresses.

4.4 Finite Element Analysis

The finite element analysis program used for this Design of Experiments (DOE) is ANSYS Mechanical APDL. This program allows for an axisymmetric study and also a probabilistic design study (PDS). The program is also chosen for its availability for student use at the University of Arizona.

The analysis is run through two text file types, a model file and a PDS file. The PDS file creates random uniform variables for the model file to run. The model file provides the material, geometry, loading, constraints and export file for the post-processing. The FEA is usually the primary design tool for opto-mechanical analysis. By creating a DOE using FEA, the important design-work required is already done.

Four of the model files are split into the different combinations of lens curvature: biconvex (both positive curvature), biconcave (both negative curvature) and both meniscus (positive-negative and negative-positive) as shown in Figure 11. These four shape combinations all have two load-cases each: azimuthal moment and radial pressure. These are the eight different model files used for analysis. By themselves, these model files can analyze any lens for azimuthal moment and radial pressure. These models files are the data points for the parametric model.

4.4.1 Probabilistic Design Study

To perform the DOE, an experiment is performed by inputting various parameters (independent variables) and recording the output (dependent variable). The PDS file defines a range of values to define the variables. For this particular analysis, the only variables that change are the curvature of the first surface, $C1$, curvature of the second surface $C2$ and lens semi-diameter $LensSemiD$. The PDS files define each variable as a random variable with uniform distribution between a maximum and minimum as shown in Table 1. The
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 [mm(^{-1})]</td>
<td>.1</td>
<td>10000</td>
</tr>
<tr>
<td>C2 [mm(^{-1})]</td>
<td>-10000</td>
<td>.1</td>
</tr>
<tr>
<td>OD[mm]</td>
<td>50</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 1: Valid Parameter Range

center-thickness, CT, the thickness of the lens along the axisymmetric axis (optical axis), is constant at 0.1 meters (100 mm). The values shown in Table 1 are analyzed with the constant center thickness. To analyze a lens with a different center thickness, these ranges are scaled by the new center thickness divided by the PDS center thickness, 0.1 meters.

Each model file is run 1000 times with three new variables to analyze. When the PDS file is run in conjunction with the model file, the ANSYS can run all 1000 iterations in about 5-10 minutes.

Unfortunately, there is no way to condition the variables to make geometrically accurate lenses every time. Each model is a collection of areas of shapes which are subtracted or are the intersection of areas. There are times when creating lenses that the curvature is too sharp to reach the outside diameter as shown in Figure 14. This effectively means the lens has no physical edge thickness to apply a load. This means this particular condition must be removed in post-processing. This results in around 150-300 usable cases out of the 1000 for each model. This edge thickness will be explained more in depth in section 4.5.2.

![Figure 14: Lens Geometry Analysis](image)

4.4.2 Model Analysis

As previously stated, the model file is what takes the PDS file and converts it into material, geometry, loading, constraints and export file. The axisymmetry of the model means that
the lens is already constrained in the azimuthal direction. The lens is constrained axisymmetrically, with the y-axis being the optical-axis. This line is constrained in the x-direction (radial). The final constraint is the axial direction. This constraint is necessary to constrain the final degree of freedom. This axial constraint is placed at the edge of the optical surface. For a convex surface, this is at the full \( \text{LensSemiD} \). For a concave surface, this constraint is located at 97.5\% of the full \( \text{LensSemiD} \). This is where the edge chamfer surface and the lens surface coincide. The constraint is placed at the edge of the surface because it is assumed that this is where the axial constraint is applied.

Each lens has two loading cases. The first load case is a radial pressure applied at the \( \text{LensSemiD} \) with a unit magnitude of 1 Pascal towards the optical-axis as shown in Figure 15a and Figure 17a. The second load case is an azimuthal moment applied at the mid-point of the lens edge with a unit magnitude of 1 N-m as shown in Figure 15b and Figure 17b. The direction of positive moment is shown in Figure 16.
To apply the moment in an axisymmetric case, the process will not allow a directly applied moment. This requires creativity with the loading. To apply a moment, the nodes on the edge line are selected. From these nodes, the nodes that are located on the corners are removed. The leftover nodes are in some numerical order. The median node is selected then equal and opposite forces are applied to the node above and the node below. This creates a force couple which is effectively a moment at this location. To create a unit load, the distance between the two nodes is calculated and the force applied is scaled to equal 1 N-m.

Each iteration of the variables exports FEA information for every run. The model file exports the variables:

- Radius of Curvature of Surface 1 - $R_1$
- Radius of Curvature of Surface 2 - $R_2$
- Lens Semi-Diameter - $LensSemiD$

and nodal information for only the nodes along the first surface:

- Radial Location - $\rho_0$
- Axial Location - $z_z$
- Radial Displacement - $\Delta \rho$
- Axial Displacement - $\Delta z$
- Radial Stress - $\sigma_{\rho\rho}$
- Axial Stress - $\sigma_{zz}$
- Azimuthal Stress - $\sigma_{\theta\theta}$
• Shear Stress (In plane perpendicular to the optical axis) - $\tau_{\rho\theta}$

The information on S1 is the only nodal information necessary. S2 data can be calculated by performing an analysis for the same lens by replacing S2 values for S1 and vice versa. Due to the axisymmetry of the lens, shear stress in the plane perpendicular to the optical axis, $\tau_{\rho\theta}$, must be equal to zero. This means there needs to be no calculations to find the principal stresses in the plane perpendicular to the optical axis. These principal stresses, $\sigma_{11}$ and $\sigma_{22}$, are equal to the $\sigma_{\rho\rho}$ and $\sigma_{\theta\theta}$ stresses that are output by the FEA model. A 3D view of the an analyzed axisymmetric model is shown in Figure 18.

### 4.5 Post-Processing

To produce the parametric model, the FEA data is analyzed using MATLAB. The surface and parameter data of each iteration is a data point for the creation of the parametric model.

#### 4.5.1 New Surface Sag

When a lens undergoes radial pressure or azimuthal moment loading, the surface not only displaces in the axial direction but also in the radial direction, as shown in Figure 19. The surface data is used to plot a new surface as a function of radius, $\rho'$, and axial displacement, $\Delta z'$:

\[
\rho' = \Delta \rho + \rho_0 \tag{51}
\]
\[
z'(\rho) = \Delta z + z_0 \tag{52}
\]

The new axial deflection, $\Delta z'$, for the new radial location, $\rho'$, is calculated by using the sag equation:

\[
\Delta z'(\rho) = R - \sqrt{R^2 - \rho'^2} \tag{53}
\]

where $R$ is the radius of curvature of the surface.
4.5.2 Edge Thickness Condition

After creating the new surface values, each parameter iteration is sorted for viability seen in Figure 14. The edge thickness, $t_{\text{edge}}$, is calculated for each parameter set ($C1$, $C2$, $LensSemiD$). This condition is related to the surface sag of both the first and second surfaces:

$$t_{\text{edge}} = t_c - sag_1 - sag_2 \quad (54)$$

where $sag_1$ and $sag_2$ are the surface sag at the edge of the first and second surfaces respectively. If $t_{\text{edge}}$ is greater than zero, then the lens has an edge for mounting. If the edge thickness is less than zero, then that particular parameter iteration is discarded.

The lens also is required due to the $Outside Diameter:Edge Thickness Ratio$ assumption to have a edge thickness, $t_{\text{edge}}$, greater than 1/5th of $LensSemiD$, which is 10% of the lens outside diameter. Any lens iterations with an edge thickness less than this is discarded.

4.5.3 Surface Fitting

Each lens iteration now has a $\Delta z'$ as a function of $\rho'$ for its new surface. To create a parametric model, the surface needs to be a simpler characterization. The best and simplest fit is a least squares polynomial fit. Due to the symmetry, this has to be an even function. The best balance of simplicity (lower-order) and accuracy (higher-order) is an even fourth order polynomial:

$$\Delta z'(\rho) = a_1(\vec{P})\rho^4 + a_2(\vec{P})\rho^2 + a_3(\vec{P}) \quad (55)$$

where $a_1(\vec{P})$, $a_2(\vec{P})$, and $a_3(\vec{P})$ are constants where $\vec{P}$ is a vector consisting of the values of $C1$, $C2$ and $LensSemiD$. A quadratic polynomial did not fit the distribution very well, while
a 6th-order polynomial and higher were more complicated and had higher errors when fitting the parameters. The errors in the higher-order polynomials occur because each term comes with its own error during the parametric regression. These higher-order term errors are scaled to the same power as the term. This creates large errors as the terms become higher-order. An example of the surface fitting is shown in Figure 20.

![Figure 20: Polynomial Least Squares Fitting](image)

4.5.4 Stress Fitting

The stress distribution, $\sigma_{\rho\rho}(\rho)$, $\sigma_{\theta\theta}(\rho)$, and $\sigma_{zz}(\rho)$ of the lens is dependent on its radial distance from the optical axis. The stress distribution can be fit just like the surface, but unfortunately, the regression has problems with fitting to the fitted polynomial. The difficulty in regression comes from the very low variance of some of the stress distributions. This makes creating a good parametric fit difficult due to the equation requiring three separate variables (the three coefficients for the polynomial equation). To approximate the stress distribution and still create an accurate fit, the mean of the stress values with respect to radius is used.

$$\sigma_{\rho\rho} = a_4(\vec{P})$$

$$\sigma_{\theta\theta} = a_5(\vec{P})$$

$$\sigma_{zz} = a_6(\vec{P})$$

(56)

(57)

(58)

Next, it is necessary to calculate the $\Delta OPD_{max}$ due to the stress birefringence. To calculate the maximum $\Delta OPD$ between the orthogonal polarization states of the lens material, the difference between $\sigma_{11}$ and $\sigma_{22}$ are calculated:

$$\Delta \sigma_{max} = |\sigma_{11}(\rho) - \sigma_{22}(\rho)|_{max} = |\sigma_{\rho\rho}(\rho) - \sigma_{\theta\theta}(\rho)|_{max}$$

(59)

$$\Delta \sigma_{max} = a_7(\vec{P})$$

(60)
4.5.5 Stiffness Fitting

To find the radial stiffness of each lens iteration, the only data required is the max radial deflection of the lens, $\Delta \rho_{\text{max}}$:

$$\Delta \rho_{\text{max}} = a_8(\vec{P})$$  \hspace{1cm} (61)

4.5.6 Parametric Regression

To create a parametric solution for each constant term, $a_n(\vec{P})$, must have its own regression fit for the independent parameters, $\vec{P}$. Since there are three variables in the parameter vector, the fit cannot be visualized in a 2D plot or even a 3D surface plot. Plotting with three independent variables requires four dimensional spatial awareness which is difficult for humans. Fortunately, computers can perform least square fits without needing to visualize the surface.

Initially, the hope was to combine all four lens types (biconvex, biconcave and both meniscus) into one plot and have one single regression fit for the entire model. This model would give insight to the behavior of each parameter and its individual contribution to optical performance. This hope was found to be largely unrealistic, as the model is more non-linear than expected. This forced the parameters to be broken up into linear regions and become a step-wise model. To break up the non-linearity, the model is split into the four lens types shown in Figure 11.

Most general cases researching lens deflection have assumed a plane parallel plate for a lens. This is oversimplified but produces a predictable distribution. The stiffness and displacement can be calculated using simple mechanics of materials equations. The larger the lens curvature, the more the behavior deviates from that of a plane parallel plate. Unfortunately there are a few sources of non-linearity.

Due to the large non-linearities in the model, the best polynomial fit for the parametric equation is of very high order. The base equation for the parametric model is a 'polyijk' equation where the largest exponential term of each parameter is $i, j$ and $k$ respectively. For the biconvex lenses, the base equation used is a 'poly777' equation. For all other lenses, the base equation is a 'poly999' equation.

For the CX (S1 is Con’C’ave, S2 is Conve’X’) meniscus lens with a azimuthal moment loading, the model is broken down again into a step-wise function. The model is split at $C1=-4$ meters. This occurs because the lens is split into two distinct linear regimes, one for $C1>-4$ meters and one for $C1<-4$ meters.

4.5.7 Sources of Non-Linearities

There are different mechanical behaviors that cause the estimation of the surface behavior difficult. These three observed mechanisms: thin edge, high curvature and meniscus are discussed below. These unique bending modes made parametric regression fitting difficult due to their non-linearities.
**Thin Edge**  One source of non-linearity is when the lens has a thin edge. A thin edge would be when the edge thickness, \( t_{\text{edge}} \), is much smaller than the diameter of the lens. Normally with a biconvex lens, there is almost no bending effects that occurs. When the edge is thin, a biconvex lens will have a local bending effect which creates local surface non-linearities. Not only does this occur in biconvex lenses, it happens with all of the lens shapes. These cases are taken into account and removed in the assumptions. This effect is shown in Figure 21.

![Figure 21: Example of Lens Non-Linearity](image)

**Large Curvature**  When there is a concave surface, there is a bending mode that appears as shown in Figure 22. This effect becomes more prominent as the curvature increases. In Figure 22b, the edge of the lens has material that extends axial past the material at the
radial center. This material acts like a cantilever beam with a uniform pressure distributed along edge. As the curvature becomes larger, this local effect becomes more prominent.

(a) Deflection for Low Curvature Meniscus Lens  
(b) Deflection for High Curvature Meniscus Lens

Figure 23: Non-Linearities Due to Meniscus Curvature

Meniscus With meniscus lenses, as the surface sag at the edge of the lens becomes much larger than the center thickness, both surfaces of the lens start to create a more prominent global bending mode as shown in Figure 23. This occurs because the pressure is not applied straight through the center of the lens. The pressure creates a moment which causes the entire lens to severely bend.

5 Parametric Model Solution

Applying these methods in MATLAB results in a very easy to use tool. Analysis of the parametric model is solved using a MATLAB suite called SAGUARO. The model is able to analyze not only single lenses, but also multi-lens designs. The parametric model is also tested for goodness of fit.

5.1 SAGUARO Solver

The code uses SAGUARO as a graphical user-interface (GUI) for the user. SAGUARO is a modular open-source platform for processing MATLAB code created by the LOFT group at the University of Arizona. This opto-mechanical parametric module is called ‘OMPS’ (Opto-Mechanical Parametric Study) and is available through the LOFT website (http://www.loft.optics.arizona.edu/saguaro/).
To run the module, all of the OMPS files are to be downloaded and placed into the ‘User-Modules’ folder located in the program files. Once SAGUARO is running, the OMPS module is selected. The desired variables are chosen, as shown in Figure 24a, and the module is executed. The outputs are shown in the properties, as shown in Figure 24b. The SAGUARO module also displays the lens with some important values as shown in Figure 25.

(a) OMPS Module Inputs

(b) OMPS Module Outputs

Figure 24: OMPS Module Tables

Figure 25: OMPS Plot
5.2 Parametric Model

The parametric model takes the input design parameters and outputs the newly updated design parameters. Once the parametric model is created, the new parameters are input into the parametric model. A plot, as shown in Figure 26, input parameters and OPD Error are output. The first surface is shown on the bottom (blue) and the second surface is shown on the top (red). The original surfaces are shown as a solid line while the surface after deflection is shown with a dotted line with a “Deflection Scale”. Only the deflection due to thermal mounting stress is plotted. Plotting size change would just be showing a linear change in size. Below the title there is a deflection scale which shows how much the deflection was scaled when plotted. This can be adjusted within the program to show the deflection more effectively. On the left of the lens is a list of a few of the important input parameters: $R_1$, $R_2$, $t_C$, OD, Radial Pressure and Moment. On the right are the OPD contributions of the six different contributors for each lens along with a RSS OPD error. There is also a $\Delta$OPD polarization error which is the difference between orthogonal directions in the plane normal to the direction of propagation. This is useful if interested in polarization.

The parametric model is applied to a real system to find the true effect on the optical performance using Zemax. The sample objective from Zemax “Cooke 40 Degree Field” optical design is chosen for analysis with a 10x scale. Each lens barrel is designed to have a 0.05 mm diametric clearance at max material condition (MMC) and 2.5 mm thick barrel. The model is evaluated for a -40 °C uniform temperature change. The lens data is shown in Figure 27 and the lens layout in Figure 28. The lenses are individually analyzed using the parametric solver. For this particular analysis, it is assumed that the optical system is athermalized, which means OPD effects of size and dn/dT are ignored. The output parameters for the Cooke lens design are shown in Table 2. The first three parameters, $R_1$, $R_2$ and $t_C$ are dependent on the size change which is independent of mounting configuration. The polynomial terms are used as even asphere terms. The quadratic and quartic terms are plugged into the lens design as an ‘even asphere’ surface. The stress-induced birefringence influence on refractive index is negligible.
To perform some verification on the effect of the stress perturbations on the lens, the wavefront error from the parametric analysis is compared with Zemax. To analyze the performance in Zemax, the lenses are changed from ‘Standard’ to ‘Even Asphere’ types. Each surface is given respective asphere coefficients shown in Table 2. To calculate the thermal-induced wavefront error for the Zemax model, the wavefront error for the original optical design is subtracted from the optical design under thermal loading. The wavefront error for the thermally loaded on-axis field and 550 nm wavelength is 1.792 waves rms (985.7 nm rms). The wavefront error for the thermally loaded off-axis field and 550 nm wavelength is 4.186 waves rms (2302.0 nm rms). The wavefront error for the original on-axis field and 550 nm wavelength is 1.829 waves rms (1005.9 nm rms). The wavefront error for the original off-axis field and 550 nm wavelength is 4.136 waves rms (2274.7 nm rms). This results in an added off-axis wavefront error of 27.3 nm rms when
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lens 1</th>
<th>Lens 2</th>
<th>Lens 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1 [mm]</td>
<td>220.0787</td>
<td>-222.0555</td>
<td>796.6288</td>
</tr>
<tr>
<td>R2 [mm]</td>
<td>4356.5</td>
<td>-202.8486</td>
<td>183.9055</td>
</tr>
<tr>
<td>tC [mm]</td>
<td>32.5811</td>
<td>9.9965</td>
<td>29.5131</td>
</tr>
<tr>
<td>S1 - Piston Term [mm]</td>
<td>-5.53471e-06</td>
<td>7.58203e-08</td>
<td>1.27875e-06</td>
</tr>
<tr>
<td>S1 - Quadratic Term [mm]</td>
<td>-3.91729e-10</td>
<td>-5.62654e-11</td>
<td>-2.42885e-10</td>
</tr>
<tr>
<td>S1 - Quartic Term [mm]</td>
<td>1.94900e-12</td>
<td>-1.40721e-11</td>
<td>9.77696e-14</td>
</tr>
<tr>
<td>S2 - Piston Term [mm]</td>
<td>5.60116e-06</td>
<td>1.97955e-07</td>
<td>-1.32084e-06</td>
</tr>
<tr>
<td>S2 - Quadratic Term [mm]</td>
<td>-7.75632e-10</td>
<td>1.36615e-09</td>
<td>3.46263e-11</td>
</tr>
<tr>
<td>S2 - Quartic Term [mm]</td>
<td>6.58677e-13</td>
<td>-2.82904e-11</td>
<td>3.83120e-13</td>
</tr>
<tr>
<td>Δn (dT)</td>
<td>-5.2000e-05</td>
<td>-0.0002</td>
<td>-5.2000e-05</td>
</tr>
<tr>
<td>Δn (Stress Biref.)</td>
<td>-1.43083e-09</td>
<td>-1.4195e-11</td>
<td>6.04065e-10</td>
</tr>
</tbody>
</table>

Table 2: Output Parameter Values vs. Cooke Lenses

under thermal loading. Surprisingly, the on-axis wavefront error is 20.19 nm rms less when under thermal loading. Calculating the root sum square of each surface OPD contribution, as shown in Figure 29, Figure 30 and Figure 31 from each surface where \( n = 1.62041 \):

\[
WFE = \sqrt{OPD_{S1L1}^2 + OPD_{S2L1}^2 + OPD_{S1L2}^2 + OPD_{S2L2}^2 + OPD_{S1L3}^2 + OPD_{S2L3}^2} \tag{62}
\]

where each OPD term is a surface contribution from a surface of the lens.

\[
WFE = \sqrt{(30.56)^2 + (11.74)^2 + (18.51)^2 + (36.20)^2 + (0.94)^2 + (2.21)^2} = 66.34 \text{ nm rms} \tag{63}
\]

When it comes to performance significance, the effect of opto-mechanical thermal stresses on this Cooke lens only accounts for about 2% of the OPD error. For this particular lens, these effects are probably insignificant and not considered during opto-mechanical design. For a design with a smaller required wavefront error or more opto-mechanically sensitive lenses (meniscus lenses), this effect must be accounted for. These rapid design studies are possible thanks to the parametric model.

### 5.3 Parametric Model Performance

When analyzing using fits, it is important to evaluate the goodness of the fit. There are many metrics which could be used for evaluating the error. For the fits used in this analysis, the two metrics used are root mean square error (RMSE) and mean absolute error (MAE). The
first metric, RMSE, is defined as [15]:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} e_i^2} \quad (65)$$

where $n$ is the number of samples and $e_i$ is the difference between the actual (FEA) and experimental (Parametric Model). The second metric, MAE is expressed by the equation [15]:

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |e_i| \quad (66)$$

These error metrics will be used to verify multiple analyses. It is valuable to have both metrics to give a better understanding of the error. RMSE is convenient because it is a relative value [15]. Unfortunately, RMSE gives larger errors more weight since it is analyzing the mean of the square of the error before square-rooting. MAE on the other hand just takes the mean of the absolute error. For this particular fitting, there are occasions where the RMSE error is high but the MAE is low. This occurrence is common with low curvature...
### Parameter Lens Analysis

**Deflection Scale = 10^5**

- **Parameters**
  - $R1 = 796.836$ mm
  - $R2 = 183.933$ mm
  - $CT = 29.528$ mm
  - $OD = 150$ mm
  - Radial Pressure = $1.1983$ MPa
  - Moment = $0$ N-mm

### Output Surface Metrics

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Mean (St.Dev.) [m RMSE]</th>
<th>Mean (St.Dev.) [m MAE]</th>
<th>Mean Surface Deflection Magnitude [m]</th>
<th># Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>XX Pressure</td>
<td>6.89E-15(6.75E-15)</td>
<td>5.88E-15(5.67E-15)</td>
<td>2.52E-13</td>
<td>158</td>
</tr>
<tr>
<td>XC Pressure</td>
<td>6.16E-14(1.94E-13)</td>
<td>5.72E-14(1.61E-13)</td>
<td>3.04E-12</td>
<td>267</td>
</tr>
<tr>
<td>CX Pressure</td>
<td>4.81E-13(1.95E-12)</td>
<td>4.01E-13(1.57E-12)</td>
<td>3.22E-12</td>
<td>199</td>
</tr>
<tr>
<td>CC Pressure</td>
<td>6.04E-13(1.70E-12)</td>
<td>5.02E-13(1.41E-12)</td>
<td>2.04E-12</td>
<td>280</td>
</tr>
<tr>
<td>XX Moment</td>
<td>2.81E-11(5.49E-11)</td>
<td>2.19E-11(3.71E-11)</td>
<td>6.31E-10</td>
<td>158</td>
</tr>
<tr>
<td>XC Moment</td>
<td>1.58E-10(5.41E-10)</td>
<td>1.22E-10(4.04E-10)</td>
<td>4.98E-10</td>
<td>263</td>
</tr>
<tr>
<td>CX Moment(C1 &lt; -4)</td>
<td>8.75E-11(1.17E-10)</td>
<td>7.38E-11(9.9E-11)</td>
<td>1.37E-10</td>
<td>93</td>
</tr>
<tr>
<td>CX Moment(C1 &gt; -4)</td>
<td>3.62E-11(3.60E-11)</td>
<td>2.99E-11(3.06E-11)</td>
<td>6.62E-10</td>
<td>107</td>
</tr>
<tr>
<td>CC Moment</td>
<td>2.75E-10(6.67E-10)</td>
<td>2.24E-10(5.42E-10)</td>
<td>2.72E-10</td>
<td>282</td>
</tr>
</tbody>
</table>

**Table 3: Surface Fitting Residual Error**

The first analysis to verify will be the fitting of the surface to the even polynomial equation. Next, the regression fitting will be verified for accuracy. The relative error will be evaluated as a percentage of the magnitude of the mean actual value (FEA):

\[
\%Error = 100 \times \frac{Error}{Magnitude of Mean FEA Value} \tag{67}
\]

### 5.3.1 Parametric Model vs. FEA Result

There are 8 fits that are performed. There are the four shapes: biconvex (XX), meniscus (XC and CX) and biconcave (CC). Each of the four shapes have two different loading types, Moment and Radial Pressure.

**Surface Deflection Fitting** Each surface is least squares fit to an even 4th-order polynomial as shown in equation 55. Each constant, $a_n$, is fit based on the parameters. Each surface fit case is evaluated for RMSE and MAE. Then, the mean and standard deviation with respect to all the samples is taken and displayed in Table 3.
Unlike evaluating mean stress and stiffness, fitting a surface requires more accuracy. Mean stress and stiffness are a single value while fitting an axisymmetric surface requires fitting a non-linear curve. Errors that occur in the quadratic constant grow quadratically with $\rho$ and errors in the quartic constant grow quartically with $\rho$. The highest relative error of the moment and radial pressure analyses are the biconcave surfaces. The biconcave lens with radial pressure fit has a RMS percentage error of 29.6%. The biconcave lens with azimuthal moment fit has a RMS percentage error of 101.1%. Looking at this 101.1% more in depth in Figure 32: There is a slight trend for larger errors tending towards the smaller magnitude curvature but this is a weak correlation. There are no other trends that would indicate a way of breaking up the information into a step-wise function.

**Stiffness/Mean Stress Fitting** Next, the parametric model is evaluated for mean stress and stiffness vs the actual values. These values are only singular values and not distributed surface values. This means that there is no need to take the mean of all the fit samples. The mean of the magnitude of the actual values are still calculated for relative comparison. The only error metrics involved are the RMSE and MAE between the actual and theoretical.

For the stiffness regression, only the analysis for the pressure is performed. The errors are shown in Table 4. This is because there is only a need for the linear-radial stiffness and not the azimuthal-angular stiffness. This stiffness is used in the interference fit equation to calculate the applied radial pressure. The highest relative error is the biconcave analysis with a RMS percentage error of 64%.

The error analysis for the stress values is the same as the stiffness. For radial stress, the largest error occurs in the XC meniscus case under radial pressure loading with a RMS
### Output Stiffness Metrics

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Stiffness Error [N/m RMSE]</th>
<th>Stiffness Error [N/m MAE]</th>
<th>Mean Stiffness [N/m]</th>
<th>#Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>XX Pressure</td>
<td>1.81E6</td>
<td>1.32E6</td>
<td>2.86E9</td>
<td>158</td>
</tr>
<tr>
<td>XC Pressure</td>
<td>3.84E8</td>
<td>2.41E8</td>
<td>8.53E9</td>
<td>268</td>
</tr>
<tr>
<td>CX Pressure</td>
<td>4.89E6</td>
<td>3.43E6</td>
<td>1.61E9</td>
<td>199</td>
</tr>
<tr>
<td>CC Pressure</td>
<td>5.42E9</td>
<td>3.29E9</td>
<td>8.44E9</td>
<td>280</td>
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</tbody>
</table>

Table 4: Stiffness Fitting Error

### Output Mean Radial Stress Metrics

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>XX Pressure</td>
<td>5.19E-4</td>
<td>3.73E-4</td>
<td>.5583</td>
<td>158</td>
</tr>
<tr>
<td>XC Pressure</td>
<td>.0061</td>
<td>.0043</td>
<td>.0450</td>
<td>268</td>
</tr>
<tr>
<td>CX Pressure</td>
<td>.0058</td>
<td>.0041</td>
<td>1.7556</td>
<td>199</td>
</tr>
<tr>
<td>CC Pressure</td>
<td>.0068</td>
<td>.0044</td>
<td>1.5471</td>
<td>280</td>
</tr>
<tr>
<td>XX Moment</td>
<td>9.579</td>
<td>6.968</td>
<td>765.7</td>
<td>158</td>
</tr>
<tr>
<td>XC Moment</td>
<td>4.501</td>
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<td>295.0</td>
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<tr>
<td>CX Moment(C1 &lt; -4)</td>
<td>2.24E-7</td>
<td>1.36E-7</td>
<td>470.2</td>
<td>93</td>
</tr>
<tr>
<td>CX Moment(C1 &gt; -4)</td>
<td>1.08E-8</td>
<td>6.87E09</td>
<td>611.5</td>
<td>107</td>
</tr>
<tr>
<td>CC Moment</td>
<td>9.398</td>
<td>6.665</td>
<td>230.0</td>
<td>282</td>
</tr>
</tbody>
</table>

Table 5: Radial Stress Fitting Error

percentage error of 13.6% as shown in Table 5. For axial stress, the largest error occurs in the XC meniscus case under azimuthal moment loading with a RMS percentage error of 11.7% as shown in Table 6. For azimuthal stress, the largest error occurs in the XC meniscus case under radial pressure loading with a RMS percentage error of 12.1% as shown in Table 7.

The maximum stress difference in the orthogonal plane is used to calculate the maximum $\Delta \text{OPD}$ due to birefringence. The largest percentage error occurs in the XC meniscus under radial pressure loading analysis with an RMS percentage error of 10.5%.

### 6 Discussion

A parametric model that can express thermal effects on opto-mechanical performance is handy design tool for engineers. The parametric model is able to evaluate a thermal opto-mechanical design to, at worst, an order of magnitude. This provides a useful tool for the preliminary design process while the optical and mechanical designs are still fluid. This model suffers from a high amount of non-linearities which make regression fitting difficult. This results in errors that are only solved by breaking the model into more linear piece-wise functions. With this in mind, FEA modeling should always be performed on critical and final opto-mechanical parts.
### Output Mean Axial Stress Metrics

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>XX Pressure</td>
<td>4.60E-4</td>
<td>2.67E-4</td>
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<td>158</td>
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<tr>
<td>XC Pressure</td>
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</tr>
<tr>
<td>CX Pressure</td>
<td>0.0025</td>
<td>0.0018</td>
<td>0.2286</td>
<td>199</td>
</tr>
<tr>
<td>CC Pressure</td>
<td>0.0070</td>
<td>0.0046</td>
<td>0.2511</td>
<td>280</td>
</tr>
<tr>
<td>XX Moment</td>
<td>1.334</td>
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</tr>
<tr>
<td>XC Moment</td>
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</tr>
<tr>
<td>CX Moment(C1&lt;-4)</td>
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<td>93</td>
</tr>
<tr>
<td>CX Moment(C1&gt;-4)</td>
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</tr>
<tr>
<td>CC Moment</td>
<td>0.259</td>
<td>0.194</td>
<td>7.05</td>
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</tr>
</tbody>
</table>

Table 6: Axial Stress Fitting Error

### Output Mean Azimuthal Stress Metrics

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>XX Pressure</td>
<td>5.57E-4</td>
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</tr>
<tr>
<td>XC Pressure</td>
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<td>.0044</td>
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</tr>
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<td>CX Pressure</td>
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<tr>
<td>CC Pressure</td>
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</tr>
<tr>
<td>XX Moment</td>
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<td>158</td>
</tr>
<tr>
<td>XC Moment</td>
<td>7.153</td>
<td>5.355</td>
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<td>263</td>
</tr>
<tr>
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<td>4.78E-7</td>
<td>2.78E-7</td>
<td>802.8</td>
<td>93</td>
</tr>
<tr>
<td>CX Moment(C1&gt;-4)</td>
<td>1.53E-8</td>
<td>9.15E-9</td>
<td>744.8</td>
<td>107</td>
</tr>
<tr>
<td>CC Moment</td>
<td>16.78</td>
<td>11.94</td>
<td>434.8</td>
<td>282</td>
</tr>
</tbody>
</table>

Table 7: Azimuthal Stress Fitting Error

### Output Max (Orthogonal Plane) Stress Difference Metrics

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>XX Pressure</td>
<td>.0032</td>
<td>.0023</td>
<td>.103</td>
<td>158</td>
</tr>
<tr>
<td>XC Pressure</td>
<td>.0238</td>
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<td>.226</td>
<td>268</td>
</tr>
<tr>
<td>CX Pressure</td>
<td>.0351</td>
<td>.0231</td>
<td>1.789</td>
<td>199</td>
</tr>
<tr>
<td>CC Pressure</td>
<td>.0683</td>
<td>.0449</td>
<td>1.357</td>
<td>280</td>
</tr>
<tr>
<td>XX Moment</td>
<td>21.320</td>
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</tr>
<tr>
<td>XC Moment</td>
<td>7.985</td>
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</tr>
<tr>
<td>CX Moment(C1&lt;-4)</td>
<td>2.56E-8</td>
<td>1.55E-8</td>
<td>1.32E3</td>
<td>93</td>
</tr>
<tr>
<td>CX Moment(C1&gt;-4)</td>
<td>1.55E-8</td>
<td>9.60E-9</td>
<td>837.9</td>
<td>107</td>
</tr>
<tr>
<td>CC Moment</td>
<td>10.39</td>
<td>7.66</td>
<td>440.9</td>
<td>282</td>
</tr>
</tbody>
</table>

Table 8: Max Stress Difference Fitting Error
Not only did this research result in a parametric model, there are many lessons that can be learned from this process. The first of which is that there are many assumptions that are made in this model which eliminates all but the simplest geometries and thermal loading. Even with these simplifications, the model is extremely complicated. These complexities come from non-linearities that occur when there are any combinations of thin edges, high curvature or meniscus lenses. These non-linearities arise from an increasing global and local bending compliance as mentioned in section 4.5.7.

Another important lesson is the importance of athermalization of the optical system. Most optical-performance degradation due to temperature results from the non-stress effects like dn/dT and size change. The axial size change of the lens barrel, the change in radius of the lenses and the change in refractive index with respect to temperature are the largest contributors to thermo-optical errors. However, these contributions can be corrected with passive or active athermalization. Opto-mechanical errors are more transient and higher-order, therefore unable to be corrected easily.

Although useful, this parametric model is limited in its scope. At this point, the model is unable to analyze cemented lenses (i.e. doublets or triplets), non-uniform temperatures or non-axisymmetric mounting. The model requires making a large number of assumptions due to the nature of regression fitting. The model is also limited to linear regimes when it comes to CTE and dn/dT. For example, cryogenic analysis would require performing a more in depth material analysis before use of the parametric model.

The creation of this parametric model provides a foundation for future opto-mechanical parametric models. This model allows for much more complex fitting. A more complex parametric fitting model like Kriging or a Gaussian process regression would be able to interpolate the more difficult parts of the fit. This analysis, along with an optical analysis, can work in conjunction to perform optimization. The optimization would allow for calculating the most effective geometry to satisfy the optical, mechanical and environmental parameters. Even the ANSYS model by itself is a useful tool for mechanical engineers which can analyze lenses without the parametric model to produce a more accurate output. By keeping the model as simple as possible, new parameters are easily added to make the model more customizable.

7 Conclusion

This parametric model is the first to be able to quantify a mounted lens performance based on mounting configuration without case-by-case FEA simulations. This simple parametric model has built the foundation for a more complex model. Initial opto-mechanical design time can now be significantly cut down by having an estimate on the thermal-mechanical induced optical performance errors. Preliminary thermal opto-mechanical error budgets can now be created with more than just a gut feeling. What used to be complicated opto-mechanical design can now begin to be viewed as a simpler process.
8 Future Work

If there were accurate parametric equations for every calculation, engineering would be a lot easier. Unfortunately, parametrizing models creates a tradeoff between a broad scope and accuracy. Using regression methods to fit the parameters to a surface is difficult and there is no insight on the behavior when using 4D 9th-order polynomials. Although it would be difficult, calculating the surface deformations and stresses analytically would be very useful. There would be more insight into the model, there would be more flexibility with loading cases and possibly less assumptions.

There are many possible add-ons to this parametric model. A somewhat simple feature would be a lens mechanical failure analysis. Due to the brittleness of glass, lenses usually fail at the point of highest tensile stress. Using parameters like surface quality to quantify the stress concentrations can also be added to determine the probability of failure.

What would make this analysis easier is to connect it to a lens design software like Zemax. Zemax has the capability of importing and exporting lens information. Currently, all parameters have to be typed in by hand. Ideally, only the mechanical parameters would need to be input into the parametric model. The parametric model would automatically export the new lens information directly into the lens design software. This type of analysis would allow for multiple lens analyses at a time and provide system level optical performance.

A more complicated add-on would be incorporating thermal gradients based on the light absorbed by the lenses. As light energy passes through lenses, any light absorbed by the lenses adds heat. This heat creates thermal gradients in the lenses as most optical glasses have low thermal conductivity, which does not allow for uniform temperature distributions.

A thermal analysis on cementing of lenses would add a missing piece to this parametric analysis. Currently there is no way to parametrically calculate the effect of lens cementing on optical performance. This would be extremely useful for lens manufacturers and also any engineers designing opto-mechanical systems.

References


