HIGH-PRECISION MOTION ESTIMATION SYSTEMS FOR UUV NAVIGATION

Hua Lee and Vincent Radzicki
Department of Electrical and Computer Engineering
University of California, Santa Barbara

ABSTRACT

This paper is the summary of a sequence of research tasks in the area of 3D bearing-angle estimation for UUV homing and docking exercises. The main focus is to simplify the concept as well as computation efficiency of the homing and docking tasks, by elevating the estimation modality from the conventional twin-receiver configuration to the 2D circular arrays. The objective is to utilize the multi-element receiver array for the entire navigation procedure, including bearing-angle estimation, optimal path planning, and high-precision docking.

INTRODUCTION

The homing-docking procedures in UUV navigation involve three key components [1-4]. The first is the estimation of the bearing angle in the far field for the homing process. The second component is dynamic path planning for the optimal approach. As the UUV reaches the near field, the docking procedure is then activated, as the third and final task.

The early prototype for homing-docking operations was a twin-transmitter single-receiver system, designed for the estimation of the bearing angle in the far field. This new system extends the capability to full-scale multi-dimensional motion estimation for precision homing and docking.

The receiver arrays of the new system are in the form of multi-element circular arrays. This array configuration enables high-precision *3D* bearing-angle estimation, which leads to the accuracy improvement of dynamic path planning.

The mathematical analysis in the paper is formulated based on the concept of single-frequency coherent waveform for simplicity. It should be point out that the accuracy of the estimation process improves significantly with waveforms with increased frequency bandwidth.

In addition, to maintain the focus of the paper, the configuration of the systems is limited to the single-transmitter mode. Extension to multiple-element transmitter arrays for improved accuracy can be achieved with minor modifications.

BEARING-ANGLE ESTIMATION WITH CIRCULAR ARRAYS

One of the most well-known applications with circular arrays is the bearing-angle estimation of infrasound propagation. Figure (1) shows the physical implementation of infrasound arrays.

Figure (1): Circular infrasound arrays.

For applications in this category, the propagation of wavefront from a distant source can be modeled as the far-field case. Here we first denote the unknown bearing angle as θ_o. For the two-dimensional case, then the directional bearing-angle vector can be written in the complex form $exp(j\theta_o)$, or in the vector form,

$$[\ \alpha, \beta\ ] = [\ cos(\theta_o),\ sin(\theta_o)]$$

The phase offset due to the time delay corresponding to a coherent source in the far field at the incident angle $\theta = \theta_o$ can be written in the form

$$exp(j\omega\tau) = exp(j2\pi f(R/v)\ cos(\theta - \theta_o))$$

$$= exp(j2\pi(R/\lambda)\ cos(\theta - \theta_o))$$

where τ is the time delay, R is the radius of the circular array, λ is the wavelength corresponding to the coherent frequency f, and v is the propagation speed. If the radius of the array is smaller than the wavelength, the phase profile can be fully determined without phase unwrapping as

$$\Phi(\theta) = 2\pi(R/\lambda)\cos(\theta - \theta_o)$$

$$= \pi(R/\lambda)\exp(-j\theta_o)\exp(j\theta) + \pi(R/\lambda)\exp(j\theta_o)\exp(-j\theta)$$

As it can be seen, the phase profile $\Phi(\theta)$ is a single-mode periodic function with magnitude $2\pi(R/\lambda)$ and period 2π. This periodic function consists of a pair of complex-conjugate components. The Fourier coefficient of the first term is $\pi(R/\lambda)\exp(-j\theta_o)$, of which the phase can fully define the bearing angle numerically.

This formulation is based on the single-frequency model. As we superimpose the estimates from all frequencies within the available frequency bandwidth, the estimation of the bearing angle becomes more accurate. The implementation for wideband waveforms can be further improved with linear frequency weighting.

This approach consists of several important features. The first property is that this algorithm is capable and applicable to cases with bearing angles corresponding to multiple sources. The second feature is that the accuracy of the algorithm does not introduce degradation when implemented with *FFT* procedures. In addition, the computation efficiency of this technique is high due to the simplicity of the model.

FOURIER ANALYSIS

The second approach to bearing-angle estimation is the direct Fourier transform method. The formulation of this method is based on the property that the spatial-frequency spectrum of a plane-wave component consists of one single peak along a circle. The radius of the circle is $1/\lambda$, and the angular position of the peak matches the bearing angle of the incident plane wave exactly.

If we apply the two-dimensional Fourier transform to the wavefield pattern over the circular array, the kernel of the transformation is in the form

$$\exp(-j2\pi < \boldsymbol{f}, \boldsymbol{x} > = \exp(-j2\pi(f_x x + f_y y))$$

where $< \boldsymbol{f}, \boldsymbol{x} >$ denotes the inner product of the frequency vector

$$\boldsymbol{f} = [\, f_x ,\ f_y \,]$$

and the spatial-position vector

$$\boldsymbol{x} = [\, x ,\ y \,]$$

Because of the use of circular array, the position vector is in the form

$$\mathbf{x} = [\, x, y \,] = [\, R\cos(\theta), R\sin(\theta) \,]$$

And, because the wavefield pattern is coherent plane wave, for the two-dimensional case, the spatial-frequency spectrum is located along a circle of radius $1/\lambda$,

$$\boldsymbol{f} = [\, f_x\, ,\; f_y \,] = 1/\lambda\, [\, \cos(\varphi),\; \sin(\varphi) \,]$$

As a result, the transform kernel becomes

$$\exp(-j2\pi(f_x x + f_y y)) = \exp(-j2\pi(R/\lambda)\,(\cos(\theta)\cos(\varphi) + \sin(\theta)\sin(\varphi))$$

$$= \exp(-j2\pi(R/\lambda)\,(\cos(\theta - \varphi))$$

Then the Fourier transform of the phase variation of wavefield pattern translates into the form

$$\int_0^{2\pi} \exp(j2\pi(R/\lambda)\cos(\theta - \theta_o))\, \exp(-j2\pi(R/\lambda)(\cos(\theta - \varphi))\, d\theta = Q(\varphi - \theta_o)$$

where $Q(\varphi)$ is the autocorrelation of the periodic function $\exp(j2\pi(R/\lambda)\cos(\theta))$. Thus the Fourier spectrum of the coherent wavefield pattern detected over the circular aperture has one single peak at $\varphi = \theta_o$. Figure (2) shows the autocorrelation of one sample coherent component. The superposition of the full collection of the spectra will further improve the accuracy of the estimate.

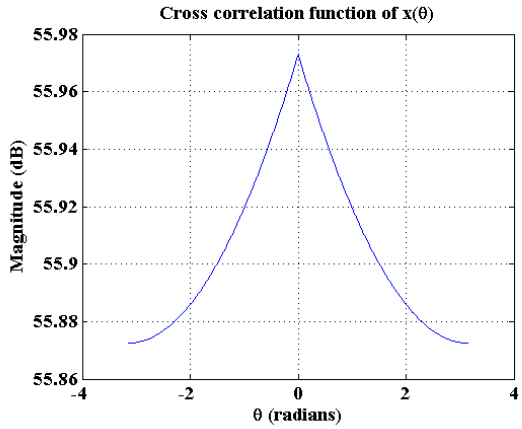

Figure (2): Autocorrelation of one coherent component

THREE-DIMENSIONAL CASE

Three-dimensional bearing-angle estimation is involved in a wide range of applications. More recently, it has become one of the critical components of homing-docking procedures for UUV's in underwater operations.

Consider a planar data-acquisition configuration, where the source plane is located at $z = 0$, and the planar receiving aperture is at $z = z_0$.

$$
\begin{aligned}
s(x, y, z_0) &= s(x, y, 0) * h(x, y, z_0) \\
&= s(x, y, 0) * \frac{1}{j\lambda r} \exp(j2\pi r/\lambda) \\
&= \iint s(x', y', 0)\, h(x - x', y - y', z_0)\, dx' dy'
\end{aligned}
$$

For a centered transmitter at $(x, y, z) = (0, 0, 0)$, the wavefield pattern over the aperture is

$$h(x, y, z_0) = \frac{1}{j\lambda r} \exp(j2\pi r/\lambda)$$

where r is

$$r = [\, x^2 + y^2 + z_0^2 \,]^{1/2}$$

For a small translational spatial offset of $(\Delta x, \Delta y)$ on the x-y plane, the received wavefield changes slightly because

$$
\begin{aligned}
r &= [\, (x - \Delta x)^2 + (y - \Delta y)^2 + (z_0)^2 \,]^{1/2} \\
&= z_0 \left[\, 1 + \left(\frac{x - \Delta x}{z_o}\right)^2 + \left(\frac{y - \Delta y}{z_o}\right)^2 \,\right]^{1/2}
\end{aligned}
$$

If the propagation distance z_0 is relatively large, we can approximate the term by taking the low-order terms of the Taylor's series

$$
\begin{aligned}
r &\approx z_0 \left[\, 1 + \frac{1}{2} \left(\frac{x - \Delta x}{z_o}\right)^2 + \frac{1}{2} \left(\frac{y - \Delta y}{z_o}\right)^2 \,\right] \\
&= z_0 + \frac{1}{2z_0} (x^2 - 2\Delta x\, x + \Delta x^2) + \frac{1}{2z_0} (y^2 - 2\Delta y\, y + \Delta y^2)
\end{aligned}
$$

To simply the procedure, we regroup them into four terms,

$$R = z_0 + \frac{1}{2z_0}(x^2 + y^2) + \frac{1}{2z_0}(\Delta x^2 + \Delta y^2) - \frac{1}{z_0}(\Delta x\, x + \Delta y\, y)$$

Then we place the terms of parameter r back into the convolution kernel. Subsequently, it can be partitioned into five terms

$$s(x, y, z_0) = h(x - \Delta x, y - \Delta y, z_0) = \frac{1}{j\lambda r} exp(j2\pi r/\lambda)$$

$$\approx \frac{1}{j\lambda z_0} exp(j2\pi z_0/\lambda)\, exp(j\pi(x^2+y^2)/\lambda z_0)$$

$$exp(j\pi(\Delta x^2+\Delta y^2)/\lambda z_0)\, exp(-j2\pi(\Delta x\, x+\Delta y\, y)/\lambda z_0)$$

We should note that the complex amplitude of the first term $1/j\lambda r$ is approximated as $1/j\lambda z_o$. This approximation is feasible, because amplitude of the integration kernel is less sensitive to approximation errors.

Removing the term through demodulation,

$$A(x, y) = \frac{1}{j\lambda z_0} exp(j2\pi z_0/\lambda)\, exp(j\pi(x^2+y^2)/\lambda z_0),$$

the modified wavefield at the aperture becomes $s'(x, y, z_0)$

$$s'(x, y, z_0) = exp(j\pi(\Delta x^2+\Delta y^2)/\lambda z_0)\, exp(-j2\pi(\Delta x\, x+\Delta y\, y)/\lambda z_0)$$

Now we denote

$$cos\theta_o = \Delta x/(\Delta x^2 + \Delta y^2)^{1/2}$$

$$sin\theta_o = \Delta y/(\Delta x^2 + \Delta y^2)^{1/2}$$

and

$$sin\varphi_o = (\Delta x^2 + \Delta y^2)^{1/2}/z_0$$

For a circular aperture of radius R, the receiver position can be rewritten in the form

$$x = R\, cos\theta \quad \text{and} \quad y = R\, sin\theta$$

Thus, the wavefield is now in the form

$$s'(x, y, z_0) = exp(j\pi(\Delta x^2+\Delta y^2)/\lambda z_0)\ exp(-j2\pi(\Delta x\ x+\Delta y\ y)/\lambda z_0)$$

$$= S\ exp(-j2\pi(\Delta x\ x+\Delta y\ y)/\lambda z_0)$$

$$= S\ exp(-j2\pi(R/\lambda)\ sin\varphi_o\ (cos\theta_o\ cos\theta\ +\ sin\theta_o\ sin\theta))$$

$$= S\ exp(-j2\pi(R/\lambda)\ sin\varphi_o\ cos(\theta - \theta_o\ ))$$

where S denotes the complex amplitude

$$S = exp(j\pi(\Delta x^2+\Delta y^2)/\lambda z_0)$$

Upon close examination, the phase term is similar to that of the two-dimensional case

$$\Omega(\theta) = 2\pi(R/\lambda)\ sin\varphi_o\ cos(\theta - \theta_o)$$

with the additional scaling factor $sin\varphi_o$. Physically, the *3D* bearing-angle vector can be written as

$$[\ \alpha, \beta, \gamma\ ] = [\ sin(\varphi_o)cos(\theta_o),\ sin(\varphi_o)sin(\theta_o),\ cos(\varphi)\ ]$$

During the homing-docking process, the objective is to incrementally adjust the approaching angle for the convergence of the *3D* bearing-angle vector toward

$$[\ \alpha, \beta, \gamma\ ] = [\ 0, 0, 1\ ]$$

The most unique and important feature of this approach is to formulate that the relationship between the bearing-angle estimation and translational displacement in the docking process. This enables the dynamic path planning for high-precision homing-docking procedures. The mathematical analysis in this section is limited to one coherent frequency for simplicity. The accuracy of the estimation improves when substantial frequency bandwidth is available.

CONCLUSION

This paper is the summary of a sequence of research tasks in the area of 3D bearing-angle estimation for UUV homing and docking exercises. The main focus is to elevate the estimation modality from the conventional twin-receiver configuration to the 2D circular arrays. The objective is to utilize the multi-element receiver array for the complete task, including bearing-angle estimation, optimal path planning, and high-precision docking.

The utilization of circular arrays as the format of the signal processing techniques in the paper is because of the direct application to UUV navigation. In many other applications, the configuration of the 2D arrays can be rectangular arrays or, for better data-acquisition efficiency, hexagonal arrays [5-6]. Because the mathematical formula is relatively independent of the array configurations, the use of non-uniform arrays is also feasible.

REFERENCES

1. Hua Lee, “Signal Processing Algorithms for UUV Navigation and Collision Avoidance,” *Journal of the Acoustical Society of America*, 125(4), p. 2537, April 2009.

2. Hua Lee, “Underwater Acoustical Imaging and Sensing Systems for Homing and Docking, Navigation, and Collision Avoidance,” *Acoustical Imaging*, Vol. 30, pp. 295-304, 2011.

3. Michael Lee, Daniel Doonan, Michael Liebling, and Hua Lee, “Resolution Analysis and System Integration of a Dynamically Reconfigurable FMCW Medical Ultrasound Imaging System,” *Proceedings of International Telemetering Conference*, 2012.

4. Melissa Johnson and Hua Lee, “Estimation of Angle of Arrival with Two-Dimensional Circular and Non-Uniform Arrays,” *Proceedings of International Telemetering Conference*, pp. 457-466, 2015.

5. Hua Lee, “Estimation of Angle of Arrival with Non-Uniform Reconfigurable Receiver Arrays,” *Journal of Acoustical Society of America*, 139(4), 2016.

6. Hua Lee, “Optimization of Computation Efficiency in Underwater Acoustic Navigation System,” *Journal of Acoustical Society of America*, 139(4), 2016.