OPTIMUM PARAMETER COMBINATIONS FOR MULTI-H
FULL RESPONSE CONTINUOUS PHASE MODULATION

Ding Xingwen  Chang Hongyu  Chen Ming
Beijing Research Institute of Telemetry, China

ABSTRACT

According to IRIG 106-15, the ARTM CPM waveform, a kind of multi-h partial response continuous phase modulation (CPM), has almost three times the spectral efficiency of PCM/FM and approximately the same detection efficiency of PCM/FM. But the improved spectral efficiency of ARTM CPM comes at the price of computational complexity in the receiver. This paper focuses on multi-h full response CPM, which generally has less detection complexity than ARTM CPM, but also has good spectral efficiency and detection efficiency. Taking the minimum Euclidean distance, spectral efficiency and detection complexity as judgment criterions, optimum parameter combinations for multi-h full response CPM are presented.

KEY WORDS
Multi-h Full Response Continuous Phase Modulation, Optimum Parameter Combinations, Minimum Euclidean Distance, Spectral Efficiency, Detection Complexity.

INTRODUCTION

CPM is a non-linear modulation with memory, since the consecutive symbols are correlative with each other. CPM signal with the characteristic of constant envelope has a certain tolerance against the non-linear distortion caused by non-linear power amplifiers, so as to greatly simplify the design complexity of communication system. Also, the characteristic of continuous phase can reduce the spectrum sidelobe, so as to decrease the channel interference. It is known that the performance of CPM signals can be improved by combining with multi-h phase codes [1].

In general, CPM signals can be represented as
\[ s(t, \mathbf{a}) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \phi(t, \mathbf{a}) + \phi_0), \quad -\infty < t < \infty \] (1)

Where \( E \) denotes symbol energy, \( T \) denotes symbol duration, \( f_c \) denotes carrier frequency, \( \phi(t, \mathbf{a}) \) denotes the information bearing phase and \( \phi_0 \) is the initial phase.

For multi-\( h \) full response CPM signal, its modulation index changes cyclically at the end of every symbol interval over \( k \) values \( H_k = (h_0, h_1, \cdots, h_{k-1}) \). In equation (1), \( \phi(t, \mathbf{a}) \) can be expressed as:

\[
\phi(t, \mathbf{a}) = 2\pi a_n h_n q(t-nT) + \pi \sum_{i=-\infty}^{n-1} a_i h_i = \theta(t, a_n) + \theta_n, \quad nT \leq t \leq (n+1)T
\] (2)

where \( \theta(t, a_n) \) denotes the instantaneous phase, which means the changing portion of the phase in the current symbol interval; \( \theta_n \) denotes the cumulative phase before the current symbol; \( a_i \) denote information symbols in the \( M \)-ary alphabet: \( a_i \in \{\pm 1, \pm 3, \ldots, \pm M-1\} \); \( h_i \) are modulation indices, \( h_i = h_{i \mod k} \); \( q(t) \) is the phase function, which can be expressed as the integral of a certain frequency pulse \( g(t) \):

\[
q(t) = \int_0^t g(\tau)d\tau
\] (3)

Two commonly used frequency pulse shapes \( g(t) \) are listed in Table 1.

<table>
<thead>
<tr>
<th>REC</th>
<th>( g(t) )</th>
</tr>
</thead>
</table>
| REC | \[
\begin{cases}
\frac{1}{2T}, & 0 \leq t \leq T \\
0, & \text{otherwise}
\end{cases}
\] |
| RC  | \[
\begin{cases}
\frac{1}{2T}[1-\cos(\frac{2\pi t}{T})], & 0 \leq t \leq T \\
0, & \text{otherwise}
\end{cases}
\] |
ANALYSIS ON ERROR PERFORMANCE

The minimum Euclidean distance between all possible pairs of the transmitted signal sequences is the most useful criterion for evaluating the error performance of CPM schemes with MLSD. The squared Euclidean distance between any two signals \( s(t, \alpha) \) and \( s(t, \beta) \) corresponding to the two symbol sequences \( \alpha \) and \( \beta \), which split apart at time \( t = 0 \) and remerge later, is defined as [3]

\[
D^2 = \sum_{i=0}^{n-1} \int_{T}^{(i+1)T} \left[ s(t, \alpha) - s(t, \beta) \right]^2 dt = \frac{2E}{T} \sum_{i=0}^{n-1} \int_{T}^{(i+1)T} \left( 1 - \cos[\phi(t, \alpha) - \phi(t, \beta)] \right) dt
\]

(4)

where \( n \) is observation interval length, \( D^2 \) is additive from interval to interval and depends on the phase difference \( \phi(t, \alpha) - \phi(t, \beta) \).

The minimum Euclidean distance \( D_{\text{min}} \) is the minimum overall Euclidean distances corresponding to all possible merging events. For multi-\( h \) schemes, the minimization also is performed over all cycled shifts of \( h \) values. When \( n \) is sufficiently large, the largest minimum Euclidean distance (i.e. the free distance) is obtained. It can be shown that any pair of signal sequences achieves the free distance within a small number of symbol intervals [4]. Generally, the minimum squared Euclidean distance \( d_{\text{min}}^2 \) is normalized with respect to the bit energy \( E_b \) as [3]

\[
d_{\text{min}}^2 = \frac{D_{\text{min}}^2}{2E_b}
\]

(5)

where

\[
E_b = \frac{E}{\log_2 M}
\]

(6)

The error rate performance of any wideband CPM signal system is usually expressed in terms of the minimum squared Euclidean distance of signals. In fact, only at high SNR, the asymptotic error probability is approximately given by [2],

\[
P_e \approx Q\left( \sqrt{\frac{d_{\text{min}}^2 E_b}{N_0}} \right)
\]

(7)

where \( Q(\cdot) \) is the standard \( Q \) function.
It is seen from equation (7) that the performance of a CPM system can be improved by increasing the minimum Euclidean distance. Multi-h phase code is one such technique, which can be used to increase the minimum squared Euclidean distance [1][2].

An upper bound on the minimum squared Euclidean distance, called $d_n^2$, can be obtained by calculating the distance for all pairs of phase paths in the trellis giving a first inevitable merge, a merge which occurs independently of the values of modulation indices [1]. The basic idea behind the multi-h continuous phase modulation (MHCPM) concept is to delay the first inevitable merge, thus creating a possibly larger minimum squared Euclidean distance.

**ANALYSIS ON SPECTRAL PERFORMANCE**

The bandwidth that a signal occupies is an important aspect of its total performance, as important as its error performance. Many methods have been developed for calculating the power spectra of CPM [1,5-10]. Most of these methods fall into three categories: the direct approach [5-6], the Markov chain approach [7-8], and the autocorrelation function approach [9-10].

In the direct approach, the power spectrum is obtained by the Fourier transform of a finite-time segment and taking the expectation or average value with respect to the random phase and data sequence. This general method has the ability to provide exact spectral density calculations, via numerical integration. The power spectrum of $M$-ary multi-h full response CPM signals is obtained by this method in [5], where a $M$-ary multi-h full response CPM is treated as an $M^k$-ary signal set over $KT$. Since $M^k$ distinct MHCPM waveforms must be taken into account, computation of the spectrum is roughly proportional to $M^k$.

Since the MHCPM signal has a periodic phase trellis with $q$ phases at each level, time-varying with a period of $k$, then the MHCPM signal can be described as a Markov chain approach, numerical integration usually is not required. The major drawback of this approach is the need to pay careful attention to the state modeling of the multi-h phase code. The computational complexity grows exponentially with $M$. Furthermore, the state model changes for different modulation index sets $H_k = (h_0, h_1, \ldots, h_{k-1})$.

In the autocorrelation function approach, the power spectrum is computed by the numerical Fourier transform, so the numerical integration is required. The power spectrum of MHCPM is calculated by the efficient method applicable for correlated data symbols [9], where the computational complexity is polynomial in $k$. 
In general, the roll-off of spectral sidelobe of MHCPM is determined by the smoothness of the frequency pulse shapes \( g(t) \). It is known that the spectral envelope decays as \( f^{-2(m+1)} \), if the \( m \)-th order derivative of \( g(t) \) is the lowest order, which is not continuous everywhere [11]. The width of the spectral main lobe increases with increasing \( h \) and \( M \).

**NUMERICAL RESULTS**

Generally, multi-\( h \) full response CPM signals with \( REC \) frequency pulse shape can produce higher minimum squared Euclidean distances than multi-\( h \) full response CPM signals with \( RC \) frequency pulse shape. And for multi-\( h \) full response CPM signals, the spectral properties of \( RC \) frequency pulse shape almost have little superiority, compared with \( REC \) frequency pulse shape. Thus, we only focus on 2-ary multi-\( h \) full response CPM signals with \( REC \) frequency pulse shape.

Table 2  Optimum parameter combinations for 2-ary multi-\( h \) full response CPM with \( REC \) frequency pulse shape

<table>
<thead>
<tr>
<th>Multi-( h ) Code</th>
<th>( H_k )</th>
<th>Minimum Squared Euclidean Distances ( d_{min}^2 )</th>
<th>Normalized 99% Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/16 11/16 12/16</td>
<td></td>
<td>4.34</td>
<td>1.78</td>
</tr>
<tr>
<td>7/16 8/16</td>
<td></td>
<td>3.31</td>
<td>1.18</td>
</tr>
<tr>
<td>4/15 5/15</td>
<td></td>
<td>1.56</td>
<td>1.00</td>
</tr>
</tbody>
</table>

In this section, the minimum squared Euclidean distances and normalized 99% bandwidths of 2-ary multi-\( h \) full response CPM signals are presented. Table 2 lists three representative optimum parameter combinations for 2-ary multi-\( h \) full response CPM with \( REC \) frequency pulse shape. 

\( H_k = (10/16, 11/16, 12/16) \) is the representation of high coding gain. \( H_k = (4/15, 5/15) \) is the representation of high bandwidth efficiency. \( H_k = (7/16, 8/16) \) is the representation of a tradeoff between coding gain and bandwidth efficiency. As references, for CPFSK signal ( \( h = 0.7 \), pre-modulation filter with -3 dB point at 0.7 times the data rate), the minimum squared Euclidean distances \( d_{min}^2 \) is 2.43, the normalized 99% bandwidth is 1.17; for ARTM CPM signal, the minimum squared Euclidean distances \( d_{min}^2 \) is 1.29, the normalized 99% bandwidth is 0.56. Furthermore, All these 2-ary multi-\( h \) full response CPM signals have much less detection complexity than ARTM CPM, but have more detection complexity than CPFSK.
CONCLUSION

This paper analyzes the error performance and spectral performance of multi-h full response CPM signals, which generally have less detection complexity than ARTM CPM, but also has good spectral efficiency and detection efficiency. And three representative optimum parameter combinations for 2-ary multi-h full response CPM are proposed.

REFERENCES