ELASTIC NET FOR CHANNEL ESTIMATION IN MASSIVE MIMO

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ABSTRACT

Next generation wireless systems will support higher data rates, improved spectral efficiency, and less latency. Massive multiple-input multiple-output (MIMO) is proposed to satisfy these demands. In massive MIMO, many benefits come from employing hundreds of antennas at the base station (BS) and serving dozens of user terminals (UTs) per cell. As the number of antennas increases at the BS, the channel becomes sparse. By exploiting sparse channel in massive MIMO, compressive sensing (CS) methods can be implemented to estimate the channel. In CS methods, the length of pilot sequences can be shortened compared to pilot-based methods. In this paper, a novel channel estimation algorithm based on a CS method called elastic net is proposed. Channel estimation accuracy of pilot-based, lasso, and elastic-net based methods in massive MIMO are compared. It is shown that the elastic-net based method gives the best performance in terms of error for the less pilot symbols and SNR values.

Keywords - Elastic net, channel estimation, massive MIMO, compressive sensing

INTRODUCTION

Current wireless communication systems like IEEE 802.16M and 3GPP LTE/LTE-Advanced widely use MIMO technology [1]. MIMO technology in which multiple antennas serve more than one user terminal (UT) is called multi-user MIMO (MU-MIMO). MU-MIMO brings significant benefits such as higher data rate, improved reliability, and energy efficiency to the wireless systems of the future [2]. It is anticipated that the fifth generation (5G) wireless networks will be deployed by 2020. These future wireless networks are expected to support even higher spectral and energy efficiency, higher data rates, lower latency, and improved reliability. Massive MIMO is considered to be one of the fundamental technologies of 5G due to its potential for spectrum and energy efficiency, secure, and robust networks [2]. In massive MIMO, the BS employs a few hundreds of antennas to serve dozens of UTs in the same radio channel. Massive MIMO improves spectral efficiency and throughput by focusing the energy into much smaller regions with hundreds of antennas [2]. Energy efficiency can be achieved by adding the signals transmitted from the antennas of the BS constructively at the intended UTs and destructively at the other UTs. The spectral efficiency in massive MIMO is better than in conventional MIMO since dozens of UTs can be served simultaneously in the same frequency-time resource [2]. Consequently, massive MIMO is a very promising technology to build the wireless systems of the future.

In order to recover received signals accurately, channel state information (CSI) which is represented with a complex matrix needs to be estimated with minimum error. Channel estimation methods can be classified as pilot-based, blind, or semi-blind methods. In pilot-based methods, the complex channel matrix is evaluated by sending a set of training signal sequences, called pilot sequences, from each UT to the BS. Pilot sequences need to be orthogonal to prevent a specific type of inter-cellular interference called pilot contamination. The channel coherence time imposes an upper-bound on the number of symbols of each orthogonal pilot sequence [2]. Therefore, UTs in different cells reuse the same pilot sequences in a cellular network. Pilot contamination is caused by the interference between a desired UT and another UT which share common pilot sequences [2].

Blind channel estimation methods estimate the channel by using statistics of the received symbols after having been transformed by the channel without using any pilot sequences. Pilot contamination can be mitigated by using blind channel estimation methods, but they have higher computational complexity than pilot-based methods since they require advanced signal processing techniques. To compromise between the computational complexities of pilot-based and blind methods, semi-blind channel estimation methods can be used. Semi-blind methods use shorter pilot sequences and additional signal processing techniques to estimate the channel.

Pilot contamination is one of the biggest challenges in massive MIMO since it limits the gain that can be achieved. Using shorter pilot signal sequences to estimate the CSI gives promising solutions to the pilot contamination problem in massive MIMO. The channel response between the BS and the UT becomes sparse as the number of antennas of the BS increases [3]. In [4], they exploited the sparse nature of signals to reduce the pilot overhead for massive MIMO OFDM systems by using a CS-based technique. Different researchers have proposed compressive channel sensing (CCS) methods to estimate sparse channels. CS methods estimate the strong channel responses which correspond to desired channels while they treat the interfering channels as noise. Therefore, they provide very promising solutions to the pilot contamination problem. Channel estimation by using compressive sensing methods have been studied in the context of massive MIMO [5]-[6]. These compressive methods are based on different regularization and variable selection methods. One of the most popular compressive methods is least absolute shrinkage and selection operator (lasso) which is a L_1-penalized least squares (LS) method [7]. Since it shrinks some coefficients and sets others to 0, it provides a good estimation for the sparse channels. However, the lasso method does not perform well when the pairwise correlations among the group of variables are high and there are high correlations between predictors. Most of the CS based techniques for sparse channel estimation are based on L_1 penalization. Adaptive-lasso has been used to estimate the channel in [3]. Recently, the CS based techniques, which use Bayesian framework, have been studied. In [8], a Bayesian based CS method is proposed for pilot reduction in massive

MIMO systems. In [9], a new regularization technique is proposed that generally gives better performance than lasso called elastic net. With the additional regularizer (L2), the elastic net does automatic variable selection and continuous shrinkage at the same time. Therefore, it can select groups of correlated variables which improve its prediction accuracy compared to lasso. Since the correlations among the predictors are high in massive MIMO, we propose to use elastic net based channel estimation. To the best of our knowledge, elastic net method for channel estimation in massive MIMO has not been studied in the literature.

In this paper, we implement a massive MIMO system in MATLAB. Then, we compare the performance of the LS pilot-based channel estimation method, lasso channel estimation method and elastic net based channel estimation method in this massive MIMO system. We analyze the relative accuracy and computational complexity between these methods, with and without pilot contamination. We compare their accuracy for different number of pilots and values of SNR. Computational complexity is observed when the number of pilots changes. We show that elastic net outperforms LS and lasso with and without pilot contamination when the number of pilots and SNR decrease. Lasso has the worst performance among these methods. Moreover, we show that elastic net continues to estimate the channel successfully with less pilots. Therefore, we show that elastic net based channel estimation gives a promising solution to the pilot contamination problem. However, computational complexity of elastic net is higher than LS and lasso according to the simulation results.

SYSTEM MODEL

A massive MIMO uplink system in which the BS has M antennas and each UT has one antenna is considered in this paper. There are N UTs where $M \gg N$ in this system. We consider P number of pilot symbols per frame. The L received symbols at the M antennas of the BS can be represented as:

$$Y = SH + W, \tag{1}$$

where $Y \in \mathbb{C}^{L\times M}$ is a matrix of the L received symbols at the M receive antennas of the BS. $S \in \mathbb{C}^{L\times N}$ is a matrix of the transmitted L symbols. $H \in \mathbb{C}^{N\times M}$ is the CSI between N UTs and M antennas of the BS, and $W \in \mathbb{C}^{L\times M}$ is the additive white Gaussian noise (AWGN) matrix.

In massive MIMO, the radio propagation channel is composed of a superposition of different multipath components (MPCs). MPCs are caused by the interaction between the radio waves and the objects (or scatterers) in the environment. We can reduce the number of channel modeling parameters significantly by grouping MPCs with the same delay and directions into clusters. We extend the cluster-based COST 2100 channel model [10], which captures important massive MIMO channel characteristics.

PILOT BASED CHANNEL ESTIMATION

In pilot-based channel estimation methods, the BS estimates the complex channel matrix by using known pilot sequences transmitted from the UTs. They are preferable due to their low computational complexity. However, bandwidth is not used efficiently in pilot-based channel estimation methods since a predetermined section of the bandwidth is used for transmitting pilot sequences. Pilot contamination is another issue which occurs when pilot sequences are reused among different cells. One of the most common pilot-based channel estimation methods is the LS method. Let's assume the same pilot sequences are reused among K cells, i.e., $S_{p_1} = ... = S_{p_K}$. Here S_{p_i} stands for the pilot sequence of the ith cell. The LS estimate of the complex channel matrix at the BS of the cell i is defined as:

$$\hat{H}_i = (S_{p_i}^H S_{p_i})^{-1} S_{p_i} Y = H_i + \sum_{k \neq i, k=1}^{K} H_k + (S_{p_i}^H S_{p_i})^{-1} S_{p_i} W, \tag{2}$$

In (2), the interfering channels H_k, $k = 1, ..., K$, $k \neq i$ will contaminate the desired channel estimate H_i, which causes pilot contamination [11].

CHANNEL ESTIMATION VIA LASSO

Channel estimation based on lasso applies a L_1-penalized least squares (LS) method on each column vector of the complex channel matrix. By using lasso algorithm, the jth column vector of H, which is the complex channel matrix between N UTs and jth antenna of the BS, is given as:

$$\hat{h}_j = \arg\min_{h_j} ||y_j - S_p h_j||_2^2 + \lambda ||h_j||_1, \tag{3}$$

where S_p are the pilot sequences such that $S_p = (s_1|s_2|...|s_N)$ and $s_i = (s_{i_1}, s_{i_2}, ..., s_{i_P})$, $i = 1, ..., N$ are the predictors. y_j is the vector of received symbols from N users at the jth antenna of the BS. λ is a fixed non-negative tuning parameter.

CHANNEL ESTIMATION VIA ELASTIC NET

A novel method for channel estimation based on elastic net is proposed in this paper. The jth column vector of H which corresponds the complex channel coefficients between N UTs and jth antenna of the BS is estimated by using elastic net:

$$\hat{h}_j = \left(1 + \frac{\lambda_2}{n}\right) \arg\min_{h_j} ||y_j - S_p h_j||_2^2 + \lambda_2 ||h_j||_2^2 + \lambda_1 ||h_j||_1, \tag{4}$$

λ_1 and λ_2 are any fixed non-negative tuning parameters.

An algorithm called LARS-EN is proposed to solve the entire elastic net path in [9]. This algorithm is based on the LARS algorithm which is proposed for solving the lasso problem in [12]. Given a fixed λ_2, the elastic net problem is equivalent to the lasso problem on the augmented data set. In this algorithm, all coefficients are set to zero initially. Then, the predictor which is most correlated with response y_j is found. Let's assume this predictor is s_{j_1}. The largest possible step is taken in the direction of s_{j_1} until another predictor s_{j_2} has the same correlation with the current residual. The algorithm steps in the direction equiangular between s_{j_1} and s_{j_2} until another predictor s_{j_3} has the most correlation with the current residual. The algorithm continues along the least angle direction between s_{j_1}, s_{j_2}, and s_{j_3} until another predictor earns its way to the active set which is the most correlated set. It continues so on.

The predictors are simply the P pilot symbols transmitted from N UTs. Pilot symbols are known by the receiver. By applying an elastic net estimator to the P received pilot symbols at the jth antenna, the complex channel coefficients between N UTs and jth antenna can be estimated. Elastic net estimator is applied to each antenna of the BS separately.

SIMULATION RESULTS

A model of a multi-cellular massive MIMO system is constructed in MATLAB to test the relative performance of LS channel estimation and the proposed channel estimation via elastic net method. In the simulation, there are nine cells and each cell has a 10,000 square meter area. Each BS is equipped with 100 ideal dipole-type antennas and serves different number of UTs. There are 90 UTs in total. Each UT is equipped with one antenna and has a velocity less than 1 m/s. The channel is modeled as COST 2100 with Rayleigh fading and non-line of sight (NLOS) situation. The corresponding pseudo-color plot of the strength of complex channel coefficient matrix $H \in \mathbb{C}^{N\times M}$ is depicted in Figure 1. As expected, the channel matrix is approximately sparse. The cells are assumed to be synchronized during the time pilot symbols are transmitted. This is the worst-case scenario for pilot contamination. A carrier frequency of 2.4 GHz is assumed for this massive MIMO system.

First, RMSE between real and estimated complex channel coefficients matrices of LS, lasso, and elastic net methods for different number of pilots are compared when there is no pilot contamination. Figure 2 shows RMSE between real and estimated complex channel coefficients matrices of LS, lasso, and elastic net methods when the number of pilots increases from 18 to 30. The number of data symbols are decreased from 142 to 130 simultaneously. In this scenario, the SNR is 10 dB. Figure 2 shows that same RMSE is obtained with decreasing number of pilots for elastic net while performance of LS significantly decreases. It is seen that lasso gives the worst performance in terms of RMSE.

Then, RMSE between real and estimated complex channel coefficients matrices of LS, lasso, and elastic net methods for different number of pilots are compared when there is pilot contamination. Figure 3 shows that elastic net has nearly the same RMSE with LS for higher number of pilots because the performance of LS method gets worse with pilot contamination.

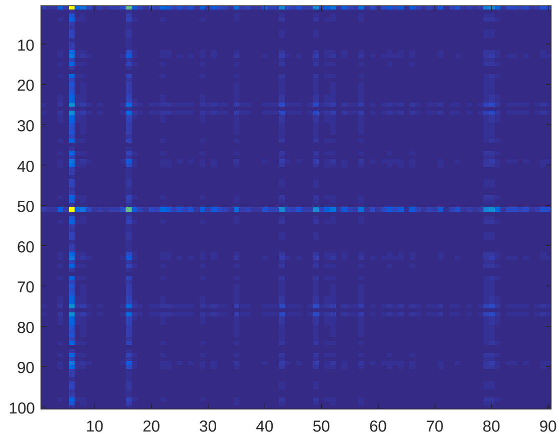

Figure 1: A pseudo-color strength of complex channel coefficient matrix

However, lasso performs worse than both LS and elastic net method.

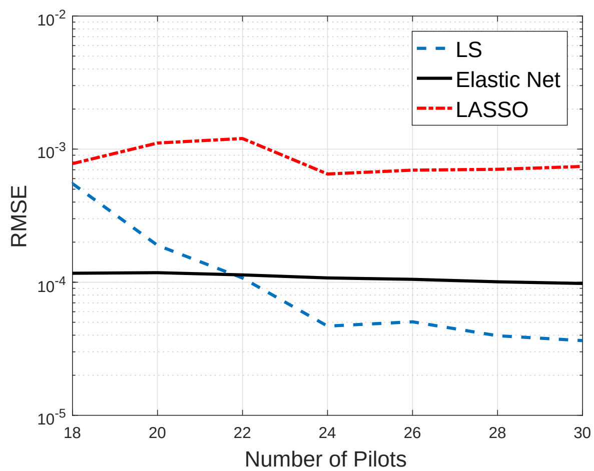

Figure 2: RMSE between real and estimated complex channel matrices of LS, lasso, and elastic net methods without pilot contamination for different number of pilots and data symbols

Figure 4 shows RMSE between real and estimated complex channel coefficients matrices of LS, lasso, and elastic net methods when SNR changes from 0 dB to 20 dB. The results in Figure 4 demonstrate that elastic net outperforms LS and lasso for decreasing values of SNR. Lasso has the worst performance for the all SNR values. Then, the pilot contamination effect on LS, lasso, and elastic net based channel estimation methods are compared in terms of RMSE when the SNR changes from 0 to 20 dB. It is shown in Figure 5 that elastic net gives less RMSE than LS. Lasso again has a greater RMSE than LS and elastic net for all the SNR values. It is shown in [13] that signals become less sparse while per element signal strength increases. Elastic net is affected adversely by the reduction in sparsity of signals.

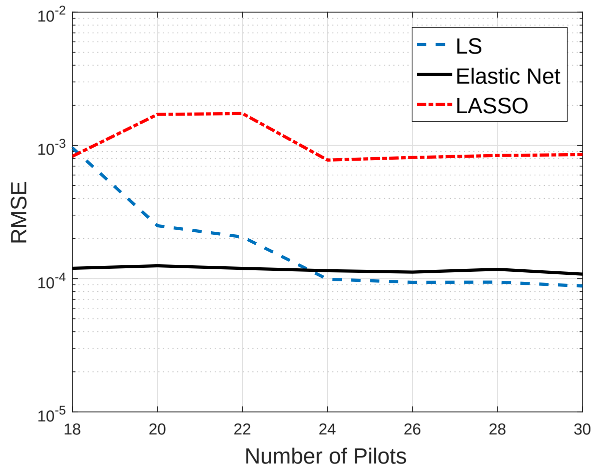

Figure 3: RMSE between real and estimated complex channel matrices of LS, lasso, and elastic net methods with pilot contamination for different number of pilots and data symbols

Even though the signal quality gets better with increasing values of SNR, less sparsity of signals causes a bottleneck. Therefore, elastic net does not perform better with increasing values of SNR.

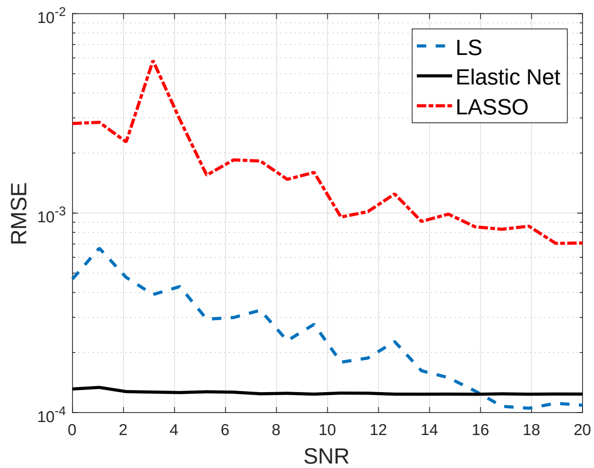

Figure 4: RMSE between real and estimated complex channel matrices of LS and elastic net methods without pilot contamination for different SNR values in dB

Then, we investigate the minimum number of pilots that is required to estimate the channel with lasso and elastic net methods when there is pilot contamination. Figure 6 shows RMSE between real and estimated complex channel coefficients matrices of lasso and elastic net methods when the number of pilots increases from 2 to 30. The number of data symbols are decreased from 158 to 130 in the meantime and the SNR is selected as 10 dB. It is seen that lasso and elastic net start to fail at estimating the channel when the number of pilots

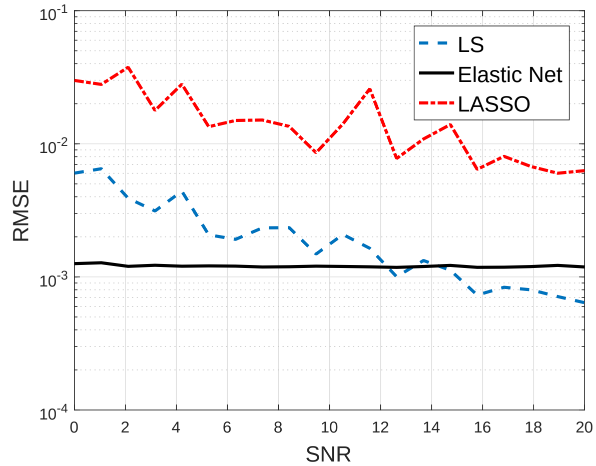

Figure 5: RMSE between real and estimated complex channel matrices of LS and elastic net methods with pilot contamination for different SNR values in dB

is below 10 and 8, respectively. According to these results, better performance than LS can be achieved by using elastic net even with a significantly less number of pilots compared to data symbols.

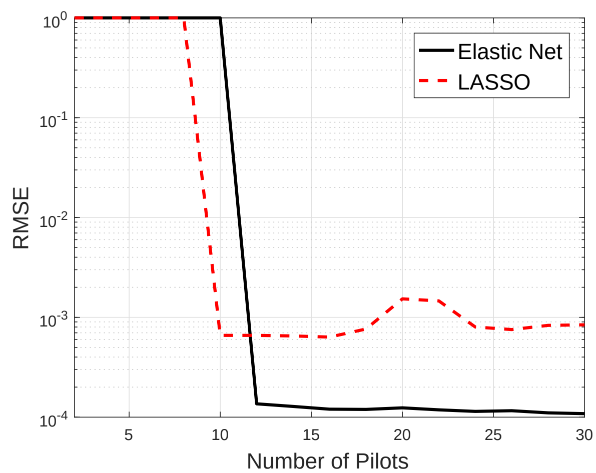

Figure 6: RMSE between real and estimated complex channel matrices of lasso and elastic net methods with pilot contamination for different number of pilots

Finally, computational complexity of LS, lasso, and elastic net are compared. We show the average run time of one frame for LS, lasso, and elastic net based channel estimation methods in Table 1. According to the results, lasso method has a higher runtime than LS. Elastic net has the highest runtime among these methods.

Table 1: Average run time of LS, lasso, and elastic net methods

Channel Estimation	Run Time
LS	0.0004 s
Lasso	0.16 s
Elastic Net	0.87 s

CONCLUSIONS

Massive MIMO is expected to be one of the key technologies in 5G wireless systems. Massive MIMO, in which significantly more BS antennas are used, has been proposed to improve spectral efficiency and data rates. Effective channel estimation for massive MIMO is one of the most important challenges in addressing pilot contamination. By exploiting sparse channel in massive MIMO, CS methods can be used to estimate the channel. Communication resources such as latency, bandwidth and energy can be used more efficiently by using CS methods. Moreover, pilot contamination can be overcome by reducing pilot signal sequences. In this paper, we propose a novel channel estimation method based on elastic net regularization and variable selection method. The pilot contamination effects on LS, lasso, and elastic net based channel estimation methods are compared by implementing a MATLAB simulation of massive MIMO. Both of the scenarios with and without pilot contamination are compared. Simulation results show that elastic net gives better performance than LS and lasso in terms of error rate for low values of SNR and less number of pilots. Therefore, we can reduce pilot contamination significantly with elastic net method. Moreover, communication resources can be used more efficiently in the elastic net method while achieving a better performance than the LS method with a decent number of pilot signal sequences. Computational complexity of these channel estimation methods are also compared by using MATLAB simulation. It was observed that LS has the least computational complexity. In the future, we plan to improve the elastic net based channel estimation algorithm in order to achieve less computational complexity.

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