PERFORMANCE OF DIRECTIONAL MODULATION SYSTEMS IN FADING CHANNELS

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ABSTRACT

Physical layer security techniques have received increasing interests in recent years due to their ability to provide additional security and their ability to be combined with conventional higher layer security measures. One such physical layer security technique is the directional modulation (DM), where the goal is to provide unhindered communication in an intended direction while hindering the communication in unintended directions. In this paper, we study the effect of Rician fading on a system employing a DM technique that minimizes the intersymbol distance in unintended directions. The performance of the system is investigated for varying ratios of the line-of-sight and scattered signal power, or Rician K-factors. The effect of Rician fading on the bit error rate (BER) performance in intended and unintended directions is studied.

INTRODUCTION

As the data transmitted using a wireless communication system is vulnerable to interception, security is an important aspect of any wireless communication system. Historically, wireless communication security efforts have been taken care of using effective encryption of the data [1]-[2]. Recently, physical layer security measures have received increasing attention, as they can be combined with existing encryption techniques, allowing for further increased security. There are many methods that can provide physical layer security [3]-[5], and they include the beamforming and directional modulation (DM) methods [6]-[8]. The beamforming approach reduces the signal-to-noise ratio (SNR) in unintended transmission directions while maintaining sufficient SNR in an intended direction. In contrast, the goal of the DM approach is to distort the received constellation along the directions of the eavesdroppers, while maintaining an undistorted received constellation to the intended receiver.

In [6], a DM transmit vector design method is proposed where the minimum inter-symbol distance is minimized in unintended directions and kept unaffected in an intended direction. This is accomplished by iterating between a quadratic minimization stage and a linear transportation problem stage. The proposed DM algorithm is shown to provide increased security in comparison to a traditional beamforming array. However, [6] does not consider the presence of fading. Thus, the results given in [6] correspond to the line of sight (LOS) channel component only.

In this paper, we investigate the effect of Rician fading on a DM system employing the DM algorithm proposed in [6]. We formulate an analytical expression for the average bit error rate (BER) of a DM system in the presence of Rician fading. As finding a closed-form expression for the average BER is difficult, we adopt a Monte Carlo integration approach. We study the BER performance of the DM system for various ratios of LOS and scattered signal power. The effect of increasing the number of transmit antennas is studied with regard to diminishing the BER degradation effect of fading as well as increasing security.

Notations: We use bold lower case letters for column vectors, and bold upper case letters for matrices. For a set of vectors, $\mathbf{b}_i$ denotes the i-th vector, and $b_{i,m}$ is the m-th element of vector $\mathbf{b}_i$. For a matrix $\mathbf{A}$, $A_{m,n}$ denotes the (m,n)-th element. Re$(\cdot)$ and Im$(\cdot)$ are the real and imaginary parts of a complex number. The notation $[\cdot]^T$ denotes transpose, $[\cdot]^H$ is the Hermitian operator, $||\cdot||_l$ is the l-norm, $\binom{n}{k}$ denotes $n!/(n-k)!k!$, $\mathbf{0}_{m,n}$ is a zero matrix of size $m \times n$, $\mathcal{CN}(m, \sigma^2)$ denotes the complex Gaussian distribution with mean m and variance σ^2, and $E[\cdot]$ denotes expectation.

SYSTEM MODEL

We consider an N-element uniform linear array (ULA) with inter-antenna spacing d_a located at a distance d from an intended receiver as shown in Figure 1. DM provides an undistorted M-ary

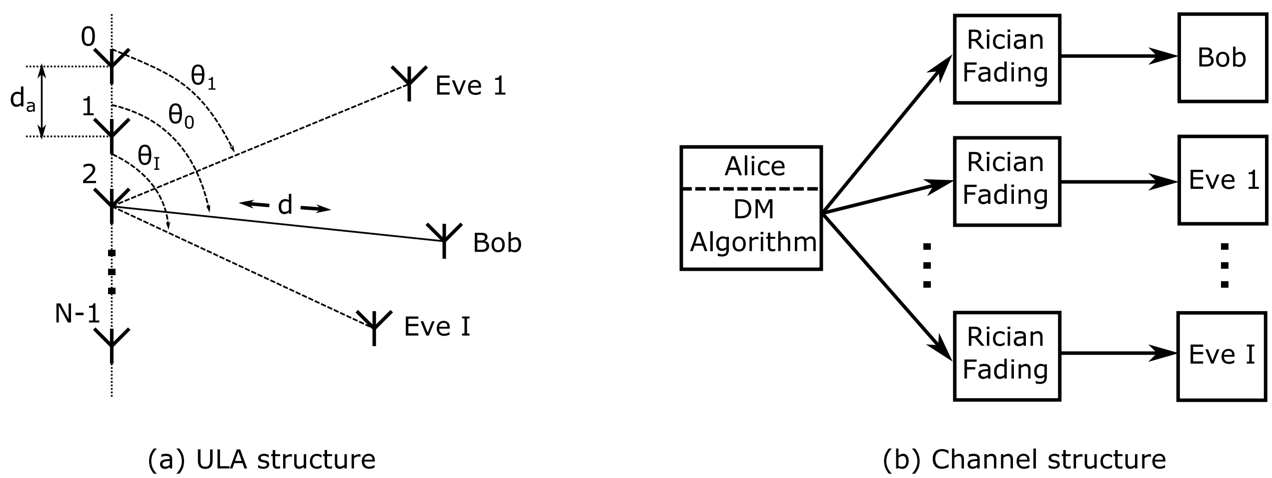

Figure 1: A ULA that transmits through a Rician fading channel to an intended receiver and I eavesdroppers.

constellation $\mathcal{U}$ in an intended direction θ_0, while distorting that constellation in multiple unintended directions θ_i, $i = 1, 2, \ldots, I$. The complex $N \times 1$ overall Rician channel vector is expressed as [9]

$$\mathbf{h}(\theta) = \sqrt{\frac{K_r}{1+K_r}}\mathbf{g}(\theta) + \sqrt{\frac{1}{1+K_r}}\mathbf{c}, \tag{1}$$

where K_r is the Rician K-factor, $\mathbf{c}$ is the scattered channel component with elements distributed as $\mathcal{CN}(0,1)$, and $\mathbf{g}(\theta)$ is the LOS channel component. The LOS channel component for a receiver located in the direction θ is calculated as

$$\mathbf{g}(\theta) = [1, \exp(j\phi_1(\theta)), \ldots, \exp(j\phi_{N-1}(\theta))]^T, \tag{2}$$

where $\phi_n(\theta) = (2\pi/\lambda)nd\cos\theta$, $n = 1, \ldots, N-1$, and λ is the wavelength. The transmission of symbol $u_m \in \mathcal{U}$ corresponds to an array excitement vector $\mathbf{x}_m = [x_{m,1}, x_{m,2}, \ldots, x_{m,N}]^T$. The noiseless received symbol in the direction θ due to u_m is

$$s_m(\theta) = \sqrt{P_r}\mathbf{h}^H(\theta)\mathbf{x}_m = \sqrt{\frac{K_r P_r}{1+K_r}}\mathbf{g}^H(\theta)\mathbf{x}_m + \sqrt{\frac{P_r}{1+K_r}}\mathbf{c}^H\mathbf{x}_m, \tag{3}$$

where $P_r = P_t d^{-\alpha}$ is the received power, P_t is the transmitted power, and α is the path loss exponent. For a given transmitted symbol $\mathbf{x}_m$ and transmission direction θ,

$$E[s_m(\theta)] = \sqrt{\frac{K_r P_r}{1+K_r}}\mathbf{g}^H(\theta)\mathbf{x}_m \quad \text{and} \quad \text{var}(s_m(\theta)) = \frac{P_r}{1+K_r}||\mathbf{x}_m||^2.$$

There are a total of $K = \binom{M}{2}$ symbol pairs. The symbol pair (m, p) is also referred to as the k-th symbol pair with intersymbol distance $D(k, \theta) = |s_m(\theta) - s_p(\theta)|$. The DM signal optimization is accomplished via minimization of $D_l(k, \theta_i) = |\mathbf{g}^H(\theta_i)\mathbf{x}_m - \mathbf{g}^H(\theta_i)\mathbf{x}_p|^2$ along the undesired directions θ_i, $i = 1, \ldots, I$. This requires: (1) the assignment of specific symbols pairs to given unintended directions, and (2) the design of array excitement vectors $\mathbf{x}_m$, $m = 1, \ldots M$ for the specific symbol pair assignments along the unintended directions.

THE DM ALGORITHM

Since the DM algorithm is based on the transmit directions θ_i, we consider only the LOS channel component while performing the DM design. The DM transmit vector design method proposed in [6] is used. The DM algorithm consists of three important parts: (1) Symbol pair assignment (SPA), (2) Transmit vector optimization (TVO), and (3) the iterations between the SPA and TVO stages.
The SPA stage is formulated as a transportation problem, where the K symbol pairs are assigned to the I unintended directions and a dummy direction θ_{I+1} is introduced to balance the supply and demand. The $K \times (I+1)$ decision matrix $\mathbf{Y}$ has elements $y_{k,i} \in \{0, 1\}$, where $y_{k,i} = 1$, $i = 1, 2, \ldots, I$, indicates the selection of symbol pair k for direction θ_i. Column $(I+1)$ of $\mathbf{Y}$ has elements $y_{k,I+1} \in \{0, 1, \ldots, S_{max}\}$ that absorb excess supply, where S_{max} is the maximum number of times a symbol pair can be assigned. The corresponding $K \times (I+1)$ cost matrix is $\tilde{\mathbf{C}}$ with elements $\tilde{C}_{k,i} = D_l(k, \theta_i)$, $i = 1, 2, \ldots, I$, and $\tilde{C}_{k,i} = 0$ for $i = I+1$. The i-th column of $\mathbf{Y}$ is denoted as $\mathbf{y}_i$, and the i-th column of $\tilde{\mathbf{C}}$ is denoted as $\tilde{\mathbf{c}}_i$. The decision vector, which gives symbol pair assignments, and cost vector are then $\mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \ldots, \mathbf{y}_I^T]^T$ and $\tilde{\mathbf{c}} = [\tilde{\mathbf{c}}_1^T, \tilde{\mathbf{c}}_1^T, \ldots, \tilde{\mathbf{c}}_I^T]^T$ respectively. The final transportation problem is expressed as

$$\min_{i,j} \ \mathbf{y}^T\tilde{\mathbf{c}} \tag{4}$$

$$\text{s.t.} \sum_{k=1}^{K} y_{k,i} = 1, i = 1, \ldots, I, \quad \sum_{k=1}^{K} y_{k,I+1} = KS_{max} - I, \quad \text{and} \quad \sum_{i=1}^{I+1} y_{k,i} = S_{max}, \ k = 1, \ldots, K.$$

The TVO stage seeks to minimize [6]

$$|D_l(k,\theta_i)|^2 = |\mathbf{g}^H(\theta_i)\mathbf{x}_m - \mathbf{g}^H(\theta_i)\mathbf{x}_p|^2 = (\mathbf{x}_m - \mathbf{x}_p)^H \mathbf{g}(\theta)\mathbf{g}^H(\theta)(\mathbf{x}_m - \mathbf{x}_p), \tag{5}$$

for $i = 1, \ldots, I$ given symbol pair assignments. Let **x** be $2NM \times 1$ vector defined as

$$\mathbf{x} = [\mathrm{Re}(\mathbf{x}_1^T),\ \mathrm{Im}(\mathbf{x}_1^T),\ \ldots, \mathrm{Re}(\mathbf{x}_M^T),\ \mathrm{Im}(\mathbf{x}_M^T)]^T. \tag{6}$$

Next, let **B** be the $2IN \times 2IN$ block diagonal matrix whose i-th block corresponds to unintended direction θ_i and is found as

$$\tilde{\mathbf{B}}(\theta_i) = \begin{bmatrix} \mathrm{Re}(\mathbf{g}(\theta_i)\mathbf{g}^H(\theta_i)), & -\mathrm{Im}(\mathbf{g}(\theta_i)\mathbf{g}^H(\theta_i)) \\ \mathrm{Im}(\mathbf{g}(\theta_i)\mathbf{g}^H(\theta_i)), & \mathrm{Re}(\mathbf{g}(\theta_i)\mathbf{g}^H(\theta_i)) \end{bmatrix}.$$

In order to control BER performance in the intended direction, a new constraint is introduced, and we denote η as the amplitude reduction in the intended direction θ_0 due to the DM algorithm. Define the $2M \times 2MN$ block diagonal matrix **H** whose i-th block is

$$\tilde{\mathbf{H}}_0 = \begin{bmatrix} \mathrm{Re}(\mathbf{g}^T(\theta_0)), & \mathrm{Im}(\mathbf{g}^T(\theta_0)) \\ -\mathrm{Im}(\mathbf{g}^T(\theta_0)), & \mathrm{Re}(\mathbf{g}^T(\theta_0)) \end{bmatrix}.$$

Lastly, define the vector

$$\mathbf{u} = [\mathrm{Re}(u_1),\ \mathrm{Im}(u_1),\ \ldots,\ \mathrm{Re}(u_M),\ \mathrm{Im}(u_M)]^T \tag{7}$$

Defining a matrix $\mathbf{C} = \mathbf{A}^T\mathbf{B}\mathbf{A}$, where the matrix **A** consists of 1, 0, and -1 as defined in [6], the TVO problem is expressed as

$$\begin{aligned} &\min_{\mathbf{x}} \frac{1}{2}\mathbf{x}^T(\mathbf{C}^T + \mathbf{C})\mathbf{x} \\ \text{s.t.} \quad &\mathbf{H}\mathbf{u} - N\eta\mathbf{u} = \mathbf{0}_{2M,1} \\ &-1 \leq \mathrm{Re}(x_{m,n}) \leq 1 \\ &-1 \leq \mathrm{Im}(x_{m,n}) \leq 1 \\ \text{for}\ \ &m = 1,\ \ldots, M,\ \ \text{and}\ \ n = 1,\ \ldots, N \end{aligned} \tag{8}$$

The DM algorithm is initialized with a random symbol pair-direction assignment. The TVO stage is performed and the solution is used to produce the cost matrix $\tilde{\mathbf{C}}$ for the SPA stage. The new assignments from the SPA stage are then passed to the TVO stage. This process continues until the symbol pair assignment does not change. Because the solution of the DM algorithm is not guranteed to be globally optimal, the DM algorithm is performed with multiple symbol pair assignments, and the solution that offers the minimum solution to (8) is finally selected.

AVERAGE BIT ERROR PROBABILITY

We consider received symbols in the presence of complex Gaussian noise with zero mean and variance N_o, where N_o is the noise spectral density. We then define the SNR as $\gamma = E_b/N_o$, where

$$E_b = \frac{1}{M\log_2 M}\sum_{m=1}^{M} E[|s_m(\theta_0)|^2], \tag{9}$$

and $E[|s_m(\theta_0)|^2]$ is calculated as

$$E[|s_m(\theta_0)|^2] = \frac{K_r P_r}{1+K_r}|\mathbf{g}^H(\theta)\mathbf{x}_m|^2 + \frac{P_r}{1+K_r}||\mathbf{x}_m||_2^2. \tag{10}$$

Due to the presence of fading, it is necessary to determine the average BER, which can expressed for Gray coded systems as

$$\bar{P}_b \approx \frac{1}{\log_2 M}\int_{\mathbb{C}} P_e(\mathbf{g},\mathbf{x})p_{\mathbf{c}}(\mathbf{x})d\mathbf{x}, \tag{11}$$

where $P_e(\mathbf{g},\mathbf{c})$ is the symbol error rate (SER) for a particular channel realization $\mathbf{h}$, and $p_{\mathbf{c}}(\mathbf{x})$ is the probability density function (PDF) of $\mathbf{c}$. Using the nearest neighbor approximation, $P_e(\mathbf{g},\mathbf{c})$ can be expressed as [10], [11]

$$P_e(\mathbf{g},\mathbf{c}) \approx \frac{1}{M}\sum_{m=1}^{M} N_m Q\left(\frac{d_{min}^{(m)}}{\sqrt{2N_0}}\right), \quad \text{with} \quad d_{min}^{(m)} = \min_{p=1,\ldots,M,p\neq m}|s_m(\theta) - s_p(\theta)|, \tag{12}$$

where N_m is the number of nearest neighbors for $s_m(\theta)$, and $Q(x) = 1/\sqrt{2\pi}\int_x^\infty \exp(-y^2/2)dy$. Using (12) in (11), along with the complex Gaussian PDF of $\mathbf{c}$, allows for an analytical evaluation for $\bar{P}_b$. However, obtaining a closed-form expression for the average BER in the presence of Rician fading is difficult. Therefore, we compute (11) numerically. We adopt a Monte Carlo integration approach. We randomly generate T channel vectors $\mathbf{c}_i$ according to (1) and find the SER $P_e(\mathbf{g},\mathbf{c}_i)$ using (12) for each channel vector. The average BER is then

$$\bar{P}_b = \frac{1}{T\log_2 M}\sum_{i=1}^{T} P_e(\mathbf{g},\mathbf{c}_i) \tag{13}$$

In order for (13) to give an accurate estimate of the average BER, it is necessary to choose T to be large. Increasing T decreases the variance of $\bar{P}_b$ but simultaneously increases the computational complexity of using (13).

RESULTS AND DISCUSSION

Our results consider quadrature phase shift keying (QPSK) modulation with $M = 4$ symbols and $K = 6$ symbol pairs. We fix $\eta = 0.5$ in (8), $S_{max} = 8$, and SNR $\gamma = 10$ dB. The considered ULA has $d_a = \lambda/2$ spacing, $\alpha = 2.2$, and is located at a distance of $d = 1000$ m from the intended receiver. In the DM algorithm, we consider $I = 20$ unintended directions with the θ_i's distributed evenly on either side of θ_0.

Figure 2 shows the received QPSK constellation for the DM system in the intended direction $\theta_0 = 90^o$ and unintended direction $\theta_i = 65^o$ with and without the presence of $K_r = 10$ Rician fading. The DM system maintains the QPSK constellation in the intended direction $\theta_0 = 90^o$. In the unintended direction $\theta_i = 65^o$, the distance between symbols is reduced significantly by the DM algorithm. In the presence of Rician fading, the QPSK constellation is randomly distorted for both the intended and unintended directions. We use a single realization of $\mathbf{h}$ with $\mathbf{c} = [-0.5594 + j0.1600,\ -1.0487 + j0.1396,\ -0.3061 + j1.2109,\ 0.1718 - j0.0177,\ 1.0188 + j1.0130]$. The

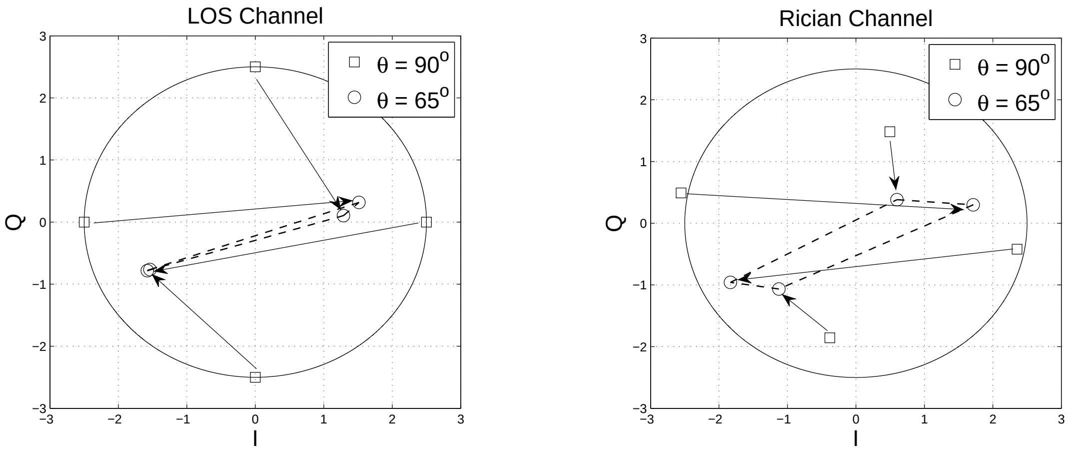

Figure 2: The received QPSK constellation for the intended direction $\theta_0 = 90^o$ and unintended direction $\theta_i = 65^o$. We use $K_r = 10$.

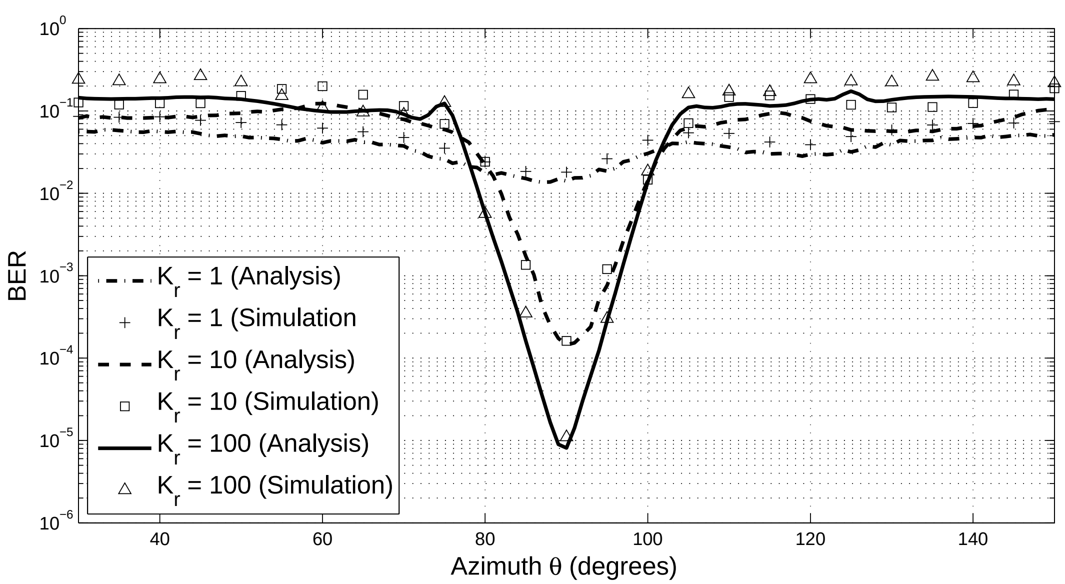

Figure 3: BER vs Azimuth angle for various K_r values with $N = 5$.

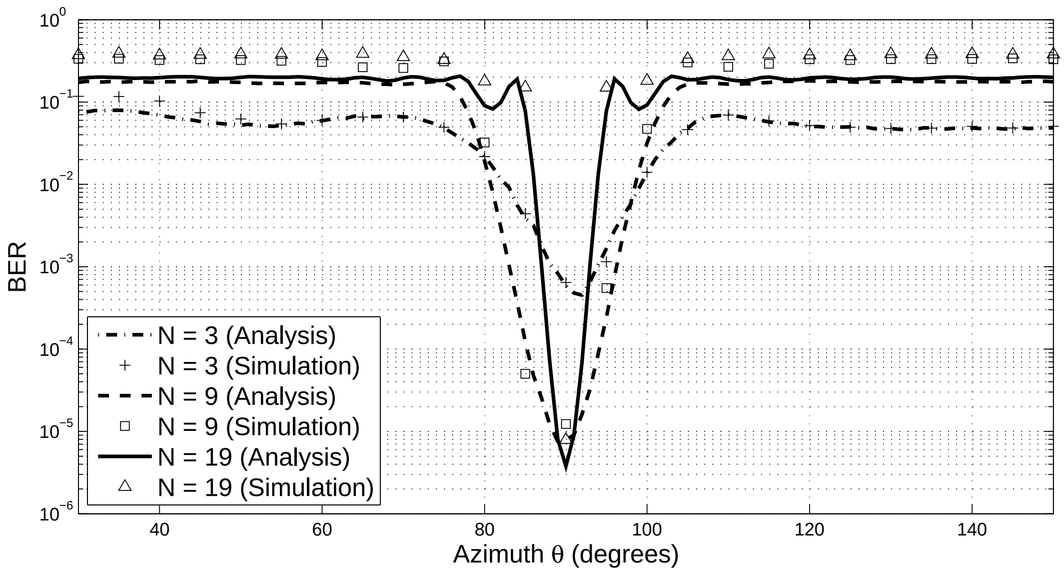

Figure 4: BER vs Azimuth angle for various N values with $K_r = 10$.

Rician channel results in the minimum distance among symbols in the intended direction being decreased, while also spreading symbols apart in the unintended direction, improving the BER performance for eavesdroppers for this specific realization of the channel.

In Figure 3, the BER performance of the DM system is shown for various K_r values with intended direction $\theta_0 = 90^o$. The Monte Carlo integration approach for obtaining $\bar{P}_b$ is shown to closely match with the simulation results. For low K_r values, the BER performance is degraded in the intended direction. Additionally, the BER is lower in unintended transmission directions because of symbol spreading due to fading. As one would expect, the results shown for $K_r = 100$ are similar to those presented in [6] as the LOS channel component is highly dominant.

Figure 4 presents the BER performance of the DM system for various values of N with $K_r = 10$ and intended direction $\theta_0 = 90^o$. The results indicate that there are dual benefits of increasing N. First, increasing N results in a narrow, low BER region with poor, angle-flat BER performance in unintended directions. Secondly, due to increased transmitter diversity, increasing N helps reduce the BER degradation due to fading. The figure suggests that a ULA with high N will result in high secrecy in unintended directions and low BER in the intended direction even in the presence of fading.

Figure 5 shows BER performance for various N values with $K_r = 5$ and intended direction $\theta_0 = 60^0$. For this figure only, we use 3 unintended directions for $\theta < \theta_0$ and 17 unintended directions for $\theta > \theta_0$. We first note that because K_r is lower than in Figure 4, the BER performance is worse in the intended direction. Secondly, the main lobe for the $N = 19$ case in Figure 4 has $\bar{P}_b \leq 10^{-1}$ for an angle range of about 10.5^o, whereas the main lobe for the $N = 19$ case in Figure 5 has $\bar{P}_b \leq 10^{-1}$ for an angle range of about 12^o.

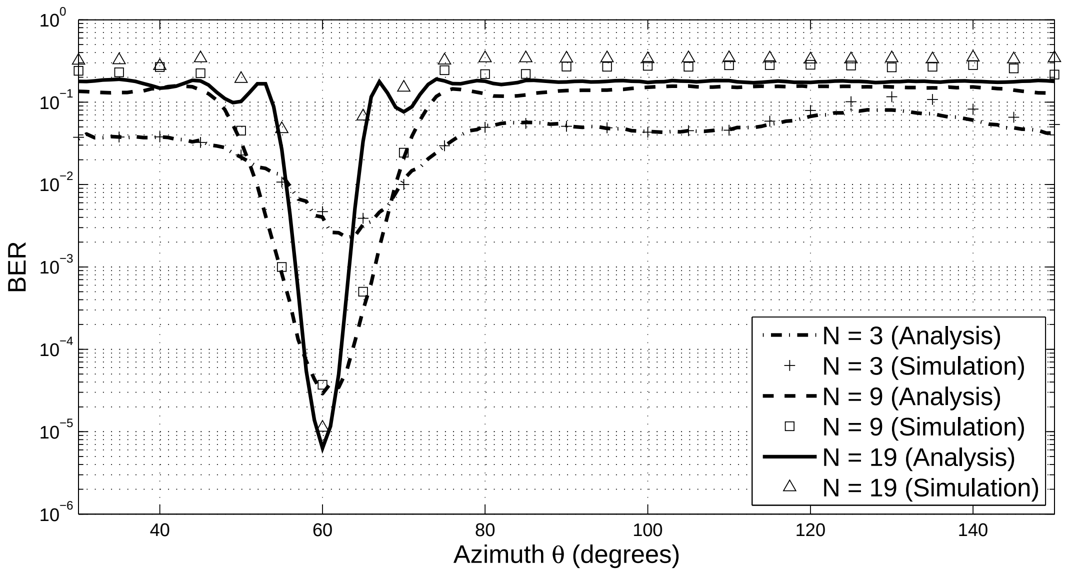

Figure 5: BER vs Azimuth angle for various N values with $K_r = 5$.

CONCLUSION

We consider a DM communication system as in [6] in the presence of Rician fading. An analytical average BER evaluation method is proposed using a Monte Carlo integration approach. Discussions and results are provided for a number of physical scenarios as well as Rician K-factors. It is shown that fading can degrade the BER in the intended direction while simultaneously improving BER in the unintended directions. The results presented suggest that a higher number of transmit antennas can reduce the effect of BER degradation as well as increase security in the presence of fading.

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