LDPC CODED APSK FOR AERONAUTICAL TELEMETRY

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ABSTRACT

This paper presents the performance of capacity-approaching low-density parity check (LDPC) coded amplitude phase shift keying (APSK) over aeronautical telemetry channels. We show the bit-error rate results for code-rates of 1/2, 2/3, 3/4, and 4/5 with 16 and 32 point constellations. Results are presented and compared between an optimal and sub-optimal reduced-complexity demodulating system. We also compare the results with SOQPSK-TG under similar channel conditions and provide an estimate of backoff needed for implemention with power amplifiers.

INTRODUCTION

Forward-error correction (FEC) codes have become nearly ubiquitous in wireless communication systems and provide excellent power efficiency in terms of coding gain [1]. However, this efficiency comes at the cost of reduced spectral efficiency and increased implementation complexity. The decreased spectral efficiency can be mitigated by combining the FEC code with a more spectrally efficient modulation format [2]. Implementation complexity can be reduced by using sub-optimal complexity reduction techniques by sacrificing part of the gained power efficiency.

The FEC codes used in this paper are low-density parity check (LDPC) codes. LDPC were proposed by Gallager in 1960 [3]. The codes were ignored until the work of Mackay [4] brought them back into consideration. LDPC codes have the advantage of high coding gains along with a parallelizable decoding algorithm. The LDPC codes used in this paper belong to the Accumulate, Repeat-by-4, and Jagged Accumulate (AR4JA) family of codes developed at the Jet Propulsion Laboratory (JPL) for deep space applications [5]. These codes were later adopted in the IRIG 106 standard [6].

Amplitude-phase shift keying is a linear modulation that is well suited for telemetry systems. APSK consists of constellation points arranged in concentric circles. This provides better resilience to distortion from non-linear power amplifiers when compared to other linear modulations with similar spectral efficiencies such as QAM [7]. The APSK constellations used in this paper have been standardized in the Digital Video Broadcasting - Satellite - Second generation (DVB-S2) [8] standard for applications in digital television broadcasting.

Combinations of FEC codes and different modulations (coded modulations) is not new [9]. Coded modulations have also been previously studied for aeronautical telemetry applications. SOQPSK-TG [10] has been combined with serially concatenated convolutional codes (SCCC), parralel concatenated convolutional codes (PCCC), and LDPC codes. APSK has also been considered with PCCC (also known as Turbo Codes) [11]. Turbo-coded APSK was also studied with power amplifier models used in telemetry systems and it was found that Turbo-coded APSK with backoff can match the performance of SOQPSK-TG with improved spectral efficiency [12]. The AR4JA LDPC codes have also been coupled with different linear modulation formats including 16 and 32 APSK in [13].

This paper describes a system with LDPC codes paired with 16 and 32 point APSK constellations. We simulate the performance of the system over an Additive White Gaussian Noise (AWGN) channel. An overview of the system model is shown in Figure 1. The flow of this paper will follow this figure. Initial sections will describe each block in the system in detail, followed by a presentation of our simulation results and conclusions.

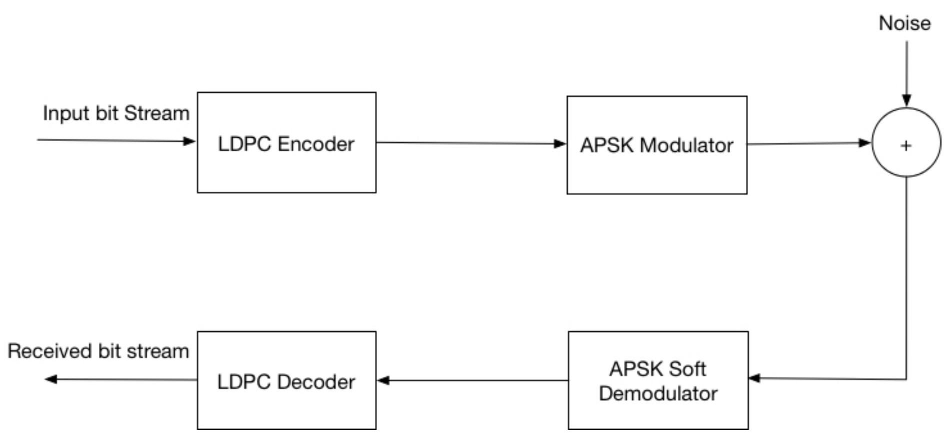

Figure 1: System overview

LDPC ENCODER

Our proposed system uses the AR4JA LDPC codes developed at JPL [5] and adopted into the IRIG 106 standard for aeronautical telemetry [6]. These codes are linear and binary and a codeword can be generated by multiplication of the message vector with a $k \times n$ generator matrix over GF(2). These codes are also systematic, which means each n length codeword vector conatins the k length message vector followed by $n - k$ redundant bits. This means the encoder need only store the last $n - k$ columns of the generator matrix, since the intitial k columns will be an identity matrix.

Also, the AR4JA codes are quasi-cyclic, which means the generator matrix is constructed of circulant sub-matrices. The encoder need only store the first column of each submatrix, and can subsequently generate the entire matrix by appropriately shifting these vectors. To generate a codeword, the encoder instead of brute-force matrix multiplication with the entire generator matrix need only generate the redundant parity bits and concatenate them with the message vector. An efficient algorithm specfied in [14] describes an encoding process to generate the parity bits by exploiting the systematic and quasi-cyclic properties of the code.

APSK MODULATOR

APSK is a linear modulation format in which constellation points are arranged in concentric circles. This enables APSK to have a lower peak-to-average power ratio (PAPR) than other linear modulation formats with similar spectral efficiencies. This makes APSK ideal for telemetry systems since the lower PAPR reduces distortions by non-linear power amplifiers operating in the saturation region [15]. The modulator can use either a 16 or 32 point constellation. The 16 point option will have a better PAPR at the cost of spectral efficiency. We use the constellations specified in the DVB-S2 standard for digital broadcast television which specifies the phase offsets and bit mapping. The ratio of radii of the rings of the constellation depends on the rate of the LDPC code used and is also specficied in [15] and [13]. It has been shown that these constellations provide a good compromise between minimizing PAPR and maximizing mutual information [7].

The modulator in our system receives the codeword vector from the LDPC encoder and divides it into blocks of 4 (for 16-APSK) or 5 bits (for 32-APSK). These blocks are then mapped to a symbol from the selected constellation. The modulator then applies a transmit pulse shape and upconverts the resulting complex baseband signal to a passband signal and sends it over the AWGN channel. Unlike Continuous Phase Modulation (CPM) systems, this system will not be able to drive the power amplifier at full saturation since the APSK constellation is not immune to non-linear distortions. The system would then need to incorporate backoff to ensure the signal stays in the linear region of the power amplifier. We discuss the backoff the system can operate under in the results section.

SOFT DEMODULATOR

The soft demodulator is the first block in the receiver subsystem. The term ‘soft’ implies that the demodulator does not make a ‘hard’ decision on the received noisy signals. Instead, the receiver returns the likelihood of each bit in the block of bits the symbol represents. The likelihoods are represented by log-likelihood ratios (LLRs). For a received signal r, the LLR for the ith bit is

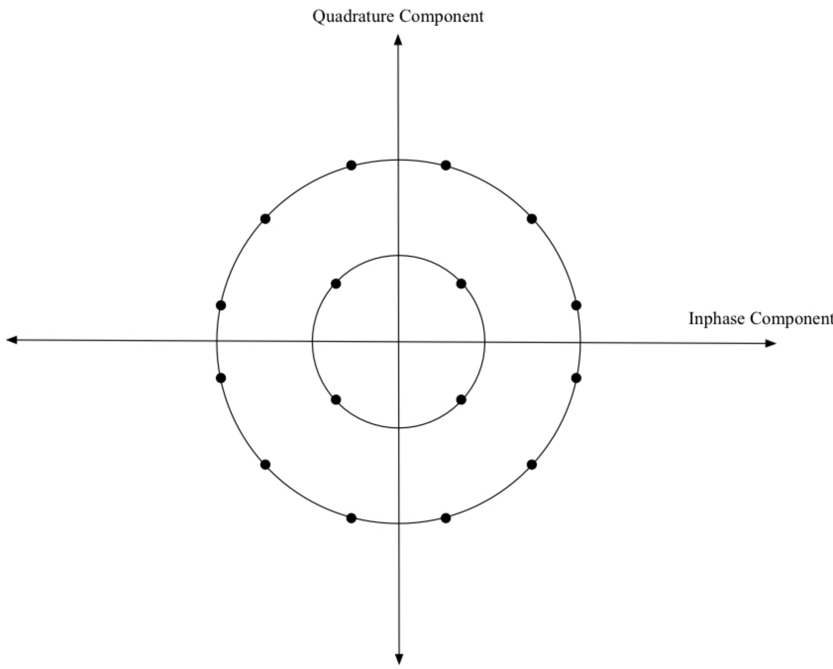

Figure 2: 16-APSK constellation described in [8]

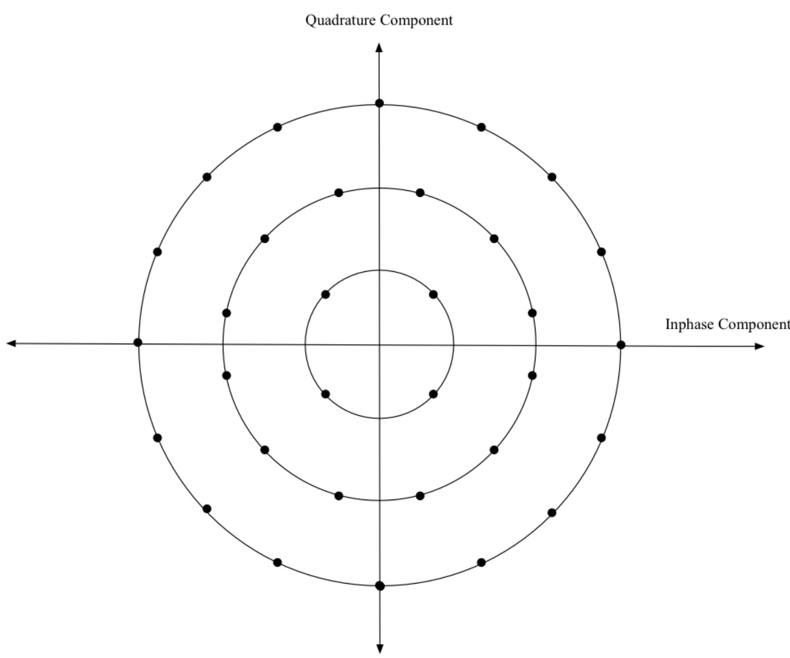

Figure 3: 32-APSK constellation described in [8]

$$\lambda_i = \ln \left(\frac{Pr(b_i = 0|r)}{Pr(b_i = 1|r)} \right). \tag{1}$$

We assume the bits are *a priori* equiprobable, thus λ_i becomes

$$\lambda_i = \ln \left(\frac{Pr(r|b_i = 0)}{Pr(r|b_i = 1)} \right) \tag{2}$$

when Baye's rule is applied. For an AWGN channel, the exact LLR is shown to be [13]

$$\lambda_i = \ln \left[\frac{\sum_{\mathbf{s}:b_i=0} \exp\left(\frac{< r, \mathbf{s} >}{2\sigma^2} \right)}{\sum_{\mathbf{s}:b_i=1} \exp\left(\frac{< r, \mathbf{s} >}{2\sigma^2} \right)} \right] \tag{3}$$

where **s** is the APSK symbol, σ^2 is the noise variance, and $< r, \mathbf{s} >$ represents the inner product between r and **s**.

The above method calculates the exact LLR at the expense of finding multiple inner products, a division, and a logarithm. A more computationally efficient technique [13] considers only the inner product with the constellation point with the least euclidean distance from the received noisy symbol. This reduces Eq. 3 to

$$\lambda_i = \frac{1}{2\sigma^2} \left(2 < r, \mathbf{s}^0_{\text{min}} - \mathbf{s}^1_{\text{min}} > + ||\mathbf{s}^0_{\text{min}}||^2 - ||\mathbf{s}^1_{\text{min}}||^2 \right) \tag{4}$$

where $\mathbf{s}^j_{\text{min}} = \arg \min_{\mathbf{s}:b_i} (||r - \mathbf{s}||^2)$. This closest point can be found in the 16-APSK constellation without any multiplications by dividing it into Voronoi regions [13] and finding the closest point through a combination of comparisons with $\text{Re}\{r\}, \text{Im}\{r\}, \text{Re}\{r\} \pm \text{Im}\{r\}$ and $\arg(r)$. The closest point in the 32-APSK constellation can be found by finding the closest three points on each ring and calculating the Euclidean distance. The LLRs for all bits in a codeword are sent to the LDPC decoder.

LDPC DECODER

Each LDPC code is associated with a (n, m) parity check matrix H. This parity check matrix can be represented graphically by a Tanner graph which is a bipartite graph [1]. The graph contains two types of nodes; bit nodes and check nodes. The decoding algorithm iteratively updates the check nodes with its associated bit nodes, and then updates the bit nodes with its associated check nodes. The optimal algorithm we use is described in [1]. The number of iterations can be varied, and the variation in coding gain with number of iterations for APSK have been presented in [13]. The complexity in this algorithm comes from the check node update metric which is given by

$$\eta_{m,n}^{[l]} = -2\tanh^{-1}\left(\prod_{i\in N_{m,n}} \tanh\left(-\frac{\lambda_i^{[l-1]} - \eta_{m,i}^{[l-1]}}{2}\right)\right) \tag{5}$$

where $N_{m,n}$ represents the bits associated with the mth check except bit n. It is obvious that the compution of the hyperbolic and inverse hyperbolic tangents is computationally expesive.

We also present results using a more computationally efficient [16] but sub-optimal "scaled-min" algorithm. This algorithm is identical to the optimal algorithm but with the check node update metric given by

$$\eta_{m,n}^{[l]} = -K\left(\prod_{i\in N_{m,n}} \text{sign}(-\lambda_i^{[l-1]} + \eta_{m,i}^{[l-1]}) \times \min_{i\in N_{m,n}}\{|-\lambda_i^{[l-1]} + \eta_{m,i}^{[l-1]}|\}\right) \tag{6}$$

where K is a constant. For our simulations we use $K = 0.75$. This greatly reduces complexity since the hyperbolic tangent functions have been replaced with multiplications, additions, and comparisons. This comes at a cost of coding gain which we show in our results varies from 0.1 to 0.5 dB.

RESULTS

In our simulations we considered code rates of 1/2, 2/3, 3/4, and 4/5 with information bit block sizes of 1024, 4096 (for rates 1/2, 2/3, and 4/5), 768, and 1536 (for rate 3/4). The BER performance of rates 1/2, 2/3, and 4/5 with APSK have been presented in [13]. Here we show the loss in coding gain with the reduced complexity algorithm. Table 1 shows the loss in performance for each combination of code rate, block size, and constellation. Figures 4 and 5 show the BER performance for rate 3/4 codes with the optimal and the reduced complexity algorithm.

We also compare the performance of our system with SOQPSK-TG. This comparison shows the capability of the system with backoff while still performing at least on par with an SOQPSK-TG system. For this comparison we use the sub-optimal decoder with 50 iterations. Also, we make this comparison at a bit error rate of 10^{-10}. To do this we extrapolate our results to 10^{-10} BER and compare it to the theoretical probability of error bound for SOQPSK-TG [17]. The results for rate 3/4 codes are shown in Figure 6. The backoff capability for all code rates, block-lengths, and constellations is summarized in Table 2

The backoff capability in Table 2 represents the power amplifier backoff the system can bear before the system performs worse than an SOQPSK-TG system. It can be seen that a lower code rate and higher block size can provide a higher backoff capability, however this comes at a cost of lower spectral efficiency and higher latency times.

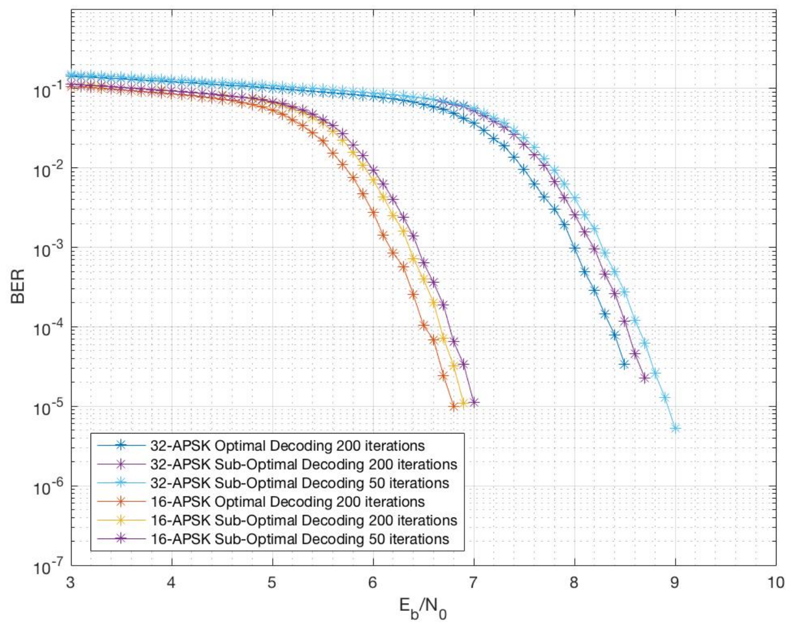

Figure 4: Bit Error Rate of Rate 3/4 codes with block size of 768 bits.

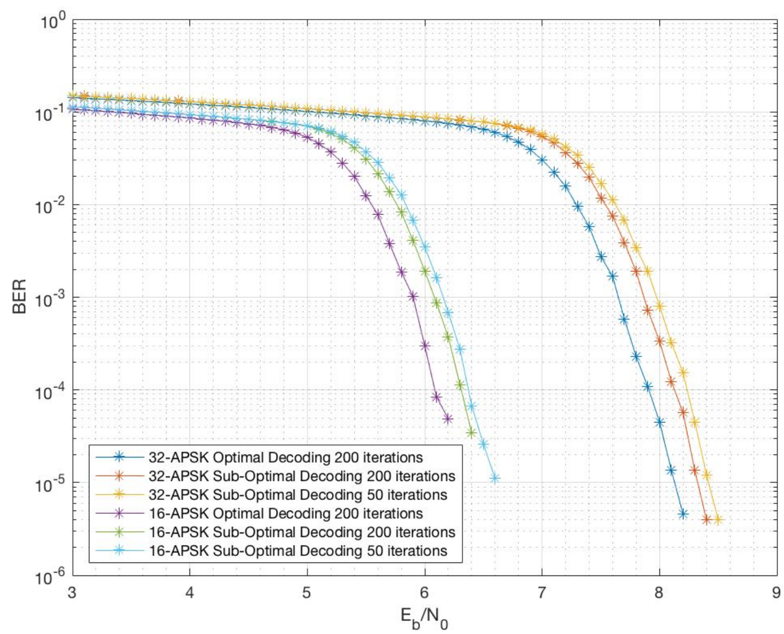

Figure 5: Bit Error Rate of Rate 3/4 codes with block size of 1536 bits.

Constellation	Block Size	Code Rate	Loss in Coding Gain
16-APSK	768 bits	3/4	0.25 dB
	1024 bits	1/2	0.38 dB
		2/3	0.37 dB
		4/5	0.20 dB
	1536 bits	3/4	0.29 dB
	4096 bits	1/2	0.50 dB
		2/3	0.40 dB
		4/5	0.10 dB
32-APSK	768 bits	3/4	0.30 dB
	1024 bits	1/2	0.48 dB
		2/3	0.40 dB
		4/5	0.20 dB
	1536 bits	3/4	0.25 dB
	4096 bits	1/2	0.11 dB
		2/3	0.20 dB
		4/5	0.22 dB

Table 1: Loss in coding gain for different code rate, blocksize, and constellation combinations.

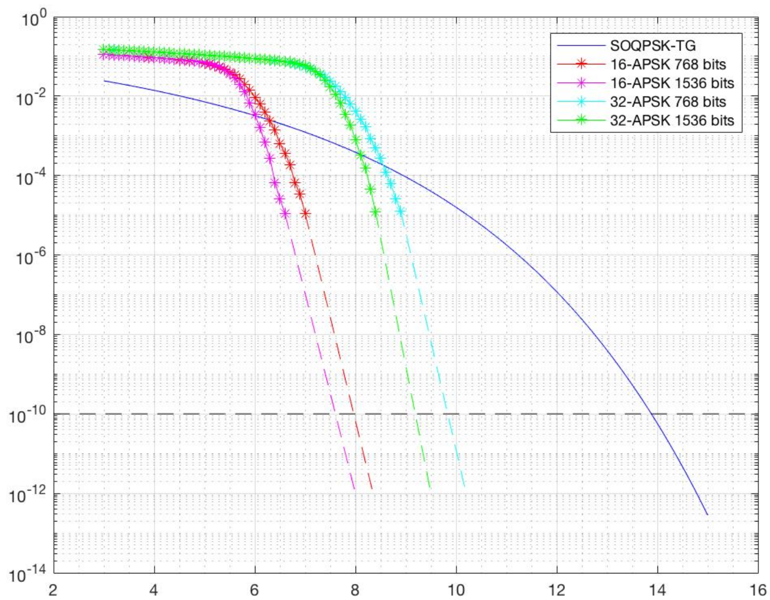

Figure 6: Comparison of Rate 3/4 codes with SOQPSK. The dashed lines represents extrapolation.

Constellation	Block Size	Code Rate	Backoff Capability
16-APSK	768 bits	3/4	5.14 dB
	1024 bits	1/2	5.51 dB
		2/3	6.09 dB
		4/5	5.56 dB
	1536 bits	3/4	6.28 dB
	4096 bits	1/2	9.38 dB
		2/3	8.17 dB
		4/5	6.60 dB
32-APSK	768 bits	3/4	4.00 dB
	1024 bits	1/2	6.50 dB
		2/3	5.16 dB
		4/5	3.60 dB
	1536 bits	3/4	4.69 dB
	4096 bits	1/2	7.47 dB
		2/3	6.17 dB
		4/5	5.06 dB

Table 2: Backoff capibility for different code rate, blocksize, and constellation combinations.

CONCLUSION

Coded-APSK systems provide an alternative to traditional CPM based telemetry systems with improved spectral efficiency. APSK, unlike CPM, is suseptible to non-linear distortions when used with power amplifiers. We have shown an LDPC coded APSK system which can be used with backoff to mitigate these distortions. The loss in performance due to complexity reduction techniques is documented and the backoff capbility of the system is shown. Future work includes analyzing the performance of this system when used with different power amplifier models and backoff required to match or better existing SOQPSK-TG systems while increasing spectral efficiency.

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