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ABSTRACT

The optical and X-ray properties of HZ Her are analyzed within the framework of a model in which the primary is illuminated by X-ray radiation from a degenerate secondary companion. System parameters are derived on the basis of a simplified model and yield a minimum distance to HZ Her of ~ 5 kpc. It is shown that the X-ray pulses and 35 day on-off characteristics as well as the shape and modulation of the optical light curve can be understood if (i) the primary is illuminated by X-ray emitted in a beam fixed in a rotating neutron star secondary undergoing forced precession and (ii) energy is supplied by mass transfer from primary to secondary resulting in the formation of a disc around the X-ray source which contributes significantly to the total light.

Key Words: X-ray source - binary system - neutron star - mass transfer - precession

1. Introduction

The X-ray source 2U1705 + 34 was discovered by Tananbaum *et al* (1972) to be an eclipsing binary system with orbital period ~ 1.7 days. In addition the source was found both to emit X-ray pulses with a period of 1.24 seconds and to exhibit long term (~35 day) cyclic "on-off" changes in intensity. Liller (1972a) suggested that the source be identified with the optical variable HZ Her, the light curve of which has subsequently been shown to exhibit 1.7 day eclipse characteristics in phase with the X-ray source (Bahcall and Bahcall 1972, Liller 1972b). Davidsen *et al* (1972) have also provided some evidence for a weak and transient component of periodicity 1.24^s in the optical radiation. The identification seems beyond question. In order to account for the observations Bahcall and Bahcall (1972) and Forman *et al* (1972) have proposed a binary model consisting of a primary evolving off the main sequence and a neutron star secondary which is the source of X-rays. A substantial proportion of the light observed from the primary is reradiated energy from the X-ray source. The purpose of the present paper is to examine in more detail the illumination hypothesis.

Theoretical light curves are calculated for various system parameters for a simplified model in which the X-ray emission is essentially isotropic and in which the optical radiation comes exclusively from the photosphere of the primary. The results are used to show how the light curve may, in principle, be used to solve for these parameters and to obtain approximate values appropriate to HZ Her. The detailed agreement between the observed light curve and that predicted on this simplified basis is, unfortunately, rather poor especially near minimum light. We show, however, that the latter discrepancy may be understood in terms of emission from a disc of material surrounding the X-ray source. This material is in the course of transfer from primary to secondary, a process which provides the basic energy supply to the X-ray source (Prendergast and Burbidge 1968). We also show that the 35 day X-ray "on-off" characteristics and the smaller modulation of the same period in the visual light (Kurochkin 1973) may be attributed to beaming of the X-ray radiation along directions fixed in a rotating neutron star undergoing forced precession.

Section 2 contains a description of our basic model together with a summary of the observational and theoretical constraints thereon. Results for spherically symmetric X-ray illumination are presented in Section 3 and used to derive approximate model parameters for HZ Her. Section 4 contains a discussion of material surrounding the X-ray source and is followed in Section 5 by a preliminary analysis of the precession and X-ray beaming. Our conclusions are summarized in Section 6, which also contains some possible tests of the model and a brief discussion of the evolutionary status of HZ Her.

2. The Model

§2a. Basic Model and Observational Constraints

Our basic model for HZ Her consists of a binary system in which the secondary, the source of X radiation, is a degenerate star. On this basis we will attempt to calculate various observable quantities of the system in order to test the hypothesis. Observation in both X-ray and optical regions of the spectrum have already furnished a substantial number of constraints on the system. Data relevant to the present discussion are summarized in Table 1. The notation is defined as follows: the eclipse, pulse, and long term modulation periods are denoted by P, P_P and P_M respectively; M_1 and M_2 are the primary and secondary masses; i is the angle between the normal to the orbital plane and the line of sight; ϕ_E the fractional X-ray eclipse duration; a_2 the distance from the centroid of the system to the X-ray source; and f_E the X-ray energy flux (ergs $cm^{-2}s^{-1}keV^{-1}$) at the earth. Two additional constraints which are essential to our analysis have been provided by optical observation. First the 35 day "on-off" X-ray modulation is not reproduced in the visual although there is evidence for comparatively small changes in the optical light curve as a function of phase in the 35 day cycle (Kurochkin 1973). Second, the optical spectrum does not consistently show Balmer emission lines even at maximum light (see, for example, Davidsen et al 1972).

§2b. Theoretical Considerations

In order to facilitate the computations of optical light curves under X-ray illumination of the primary we have made the following assumptions:

(i) The rotation period of the primary and the orbital period are more or less equal so that the standard Roche potentials may be used for this star.

(ii) The primary fills its Roche lobe so that mass transfer takes place.

(iii) A fraction η of the X-radiation incident on each part of the surface of the primary is absorbed and is reradiated locally. The latter assumption may be justified as follows. For the range of surface temperatures observed, the X-rays do not penetrate more than several (10-10^2) optical depths τ_v at the wavelength typical of the reradiated (visible) component (i.e. $\tau_x \sim \varepsilon\, \tau_v$, $10^{-2} \lesssim \varepsilon \lesssim 10^{-1}$). The radiation leak time from these depths is of order $t_\ell \sim 10^2$s, a time scale in which the (inevitably present) circulation currents could transfer energy only a small fraction of the stellar circumference of the star even if they attain the local sound speed.

(iv) The illumination may be treated as quasi-steady state since $t_\ell \sim 10^2$s greatly exceeds the X-ray pulse period 1.24^s.

(v) The intrinsic effective temperature distribution over the surface of the primary obeys von Zeipel's (1924) law. Thus the net local effective temperature T_{eff} is given by

$$\sigma T^4_{eff} = \left(L_* g / \int g \, dS\right) + \eta \mu F_x \, H(\mu) \qquad (1)$$

where L_* is the intrinsic luminosity of the primary, g is the local gravity, the integral is over the stellar surface, F_x is the incident X-ray flux and μ is the cosine of the angle between the normal to the surface and the direction to the secondary. $H(\mu)$ is the Heaviside function. (This approximation may break down in the presence of a deep outer convection zone [Lucy 1967]).

(vi) The specific intensity of the X-ray radiation is the same for all parts of the surface which can receive it (i.e. for which $\mu > 0$). This cannot be justified except on grounds of simplicity but will be adopted for the moment for computational convenience. More plausible alternatives will be discussed further in §5.

(vii) The temperature stratification is given by

$$T^4 = \frac{3}{4} T^4_{eff} \, (\tau_v + 2/3) \qquad (2)$$

the Eddington grey body solution of the same total flux. This is a reasonable approximation provided $\varepsilon << 1$ as would be the case if most of the X-ray emission occurred at energies above ~ 5 keV. The X-ray observations (Clark et al 1972, Ulmer et al 1972) indicate that this condition is indeed fulfilled. Electron scattering is the principal atmospheric source of opacity to hard X-rays and for photons of mean energy ~ 25 keV we would then expect that $\eta \sim 0.1 - 0.2$.

Within this framework we may compute the optical flux seen from the direction of the observer as a function of phase in the orbit. We have chosen to evaluate this flux at two wavelengths, 4200Å and 5500Å, corresponding approximately to the effective wavelengths of the B and V bands. Our calculations of specific intensity have been made using the grey body approximation and a Planckian source function because (a) the approximate temperature distribution hardly justifies greater accuracy and (b) a similar approach applied to rotating stars (Roxburgh and Strittmatter 1965) gave results for broad band photometry in good agreement with subsequent more detailed computations (Collins and Harrington 1966, Hardorp and Strittmatter 1968).

For computational convenience we set up a coordinate system 0 - XYZ centered on the primary. OX is in the direction of the secondary and OY is in the orbital plane. Spherical polar coordinates θ, measured from OX, and ϕ, measured from the plane OXY, were used to specify position on the Roche surface. In this frame the direction to the observer OE rotates. If OE' is the projection of OE in the orbital plane, we denote the angle E'OX by ψ, where $d\psi/dt = 2\pi/P$. The cosine of the angle between the normal $\underline{n} = (n_1, n_2, n_3)$ to the surface and the direction to the observer is given by

$$\mu_\circ = (n_1 \cos \psi \sin i + n_2 \sin \psi \sin i - n_3 \cos i) \qquad (3)$$

The flux $F_\nu(\lambda,\psi)$ in the direction of the observer is then given as a function of wavelength and orbital phase by

$$F_\nu\ (\lambda,\psi) = \int r^2(\theta,\phi)\mu_\circ\ I_\nu(\mu_\circ,\lambda)p^{-1}H(\mu_\circ)\ \sin\theta\ d\theta d\phi \tag{4}$$

where p is the direction cosine between the radial vector and the normal and $r(\theta,\phi)$ is the radius. The specific intensity $I_\nu\ (\mu_\circ,\lambda)$, (ergs/sec/ cm^2/Hz/ster) is given in this approximation, by

$$I_\nu\ (\mu_\circ,\lambda) = \int B_\nu\ (T,\lambda)e^{-\tau/\mu_\circ}\ d\tau \tag{5}$$

where $B_\nu\ (T,\lambda)$ is the Planck function.

For comparison with observation we have written

$$\begin{aligned} v(\psi) &= c_v - 2.5 \log F\ (5500,\psi\ ) \\ b(\psi) &= c_b - 2.5 \log F\ (4200,\psi\ ) \end{aligned} \tag{6}$$

where the constants c_v, c_b have been fixed so that a star with the parameters of Sirius (M_v = 1.44, T_{eff} = 10,380°K, R = 1.23 x 10^{11}cm, Hanbury-Brown et al 1967) should have v = b = 1.44 when calculated by means of equations (4) and (6). Our indices v, b thus correspond approximately to absolute V and B magnitudes. In practice the Johnson V band is very little affected by hydrogen or metal line blanketing and we therefore set our value of v equal to V. In order to obtain a more realistic estimate of B-V from our computations we have proceeded as follows. A series of b-v colors for spherical stars of different T_{eff} was computed yielding a (b-v) : T_{eff} relation. Similar calibrations for B-V vs. T_{eff} exist and for normal Population I stars we have adopted that of Johnson (1966). Elimination of T_{eff} thus yields a calibration of (B-V) in terms of (b-v). For Population II stars, metal deficiency results in a different (B-V) : T_{eff} relation for stars cooler than ~ 10000°K. In order to take this into account we have derived a calibration appropriate to extreme Population II by applying Melbourne's (1960) deblanketing corrections to the Johnson relation.

We have chosen as independent parameters T_o, the effective temperature at the point furthest from the X-ray source ($\theta = -\pi$), and ℓ_x the effective X-ray luminosity, defined as the luminosity of an equivalent spherically symmetric source the energy from which is absorbed entirely by the atmosphere if it impinges thereon. Thus for the spherically symmetric case $\ell_x = \eta L_x$ where L_x is the total X-ray luminosity. Clearly T_o and L_* are equivalent for a given Roche geometry. Calculations were originally made for two values of T_o (7000°, 8000°K) and three values of ℓ_x (2, 5, 8 x 10^{36} ergs/sec) respectively. These provide a coarse grid of light curves from which more accurate parameters may in principle be derived.

The presence and length of eclipse of the X-ray source may be estimated from the spherical star formula

$$\phi_E = \cos^{-1}([1 - r^2/a^2]^{1/2}/\sin i)/\pi \tag{7}$$

where r is the radius of the primary and a is the separation of centers. An appropriate mean value of r/a for the present case is (Plavec 1967)

$$r/a = 0.38 - 0.2 \log (M_2/M_1) \tag{8}$$

The quantity ϕ_E may, however, be obtained directly from our calculations without the approximation involved in (7) by noting that eclipse occurs during phases of the orbit for which the condition that $\mu_o < 0$ at all points at which $\mu = 0$ is satisfied.

It is clear that, given sin i, the observed values of the mass function, $a_2 \sin i$, and ϕ_E specify M_1, M_2 and a (cf. For man et al 1972). Conditions during eclipse, in particular spectral type or color, then allow estimates to be made of distance. We shall therefore attempt to use our computational results, together with the observational data of Table 1 to determine sin i and hence the remaining parameters of the system.

3. Results and Interpretation

In Table 2 we list results for the orbital parameters of HZ Her as a function of our independent variable i. The minima in M_1, r and a are well known. Also listed in Table 2 are the bolometric magnitudes for models with T_o = 7000°K, and the fraction x of the effective X-ray emission which impinges on the primary.

Results for $(B-V)^{min}$ are shown in Figure 1 as a function of i for two values of T_o and for both color calibrations; there is clearly no dependence on ℓ_x. The observed $(B-V)^{min}$ value is shown as a horizontal line and suggests that T_o is fairly close to 7000°K, the precise value depending on i and metal abundance. The temperatures required to give the observed $(B-V)^{min}$ have been derived for each inclination and both Populations and the results are listed in Table 2. The corresponding values of M_{bol} are plotted against M_1 in Figure 2 again for both Populations. Also shown in Figure 2 are theoretical mass luminosity relations. For Population I we have derived from Iben (1967) an approximate L, M relation for stars beginning their evolution across the Hertzsprung gap (i.e. at more or less constant luminosity). For Population II we have been unable to find model calculations in the correct mass range. To obtain an approximate M-L relation we have proceeded as follows. Faulkner (1967) gives main sequence quasi-homology relations for L in terms of mass and composition. From these we have computed the shift in M_{bol}, at constant mass and hydrogen to helium ratio, which would result from a reduction in metal abundance by a factor exceeding 10 from normal Population I values. We have further assumed that this luminosity shift is applicable to all evolutionary phases near the main sequence and that the slope of the M-L relation is essentially population independent. The approximate Population II sequence in Figure 2 is then obtained. It is quite clear from the diagram that there are no self-consistent solutions for M_1, M_{bol} and sin i in the case of Population I. For Population II, however, such solutions exist and suggest orbital inclinations i $\lesssim$ 75°. Unfortunately, the situation is less clear cut than appears at first sight, particularly if there has been a previous history of substantial mass transfer. For example, once a core envelope structure has been established with a chemical composition discontinuity, the luminosity is fairly insensitive to mass loss, at least initially. It is also, in principle, possible that the primary having previously lost mass is expanding at a later stage of stellar evolution when it would then be intrinsically brighter. A Population I composition cannot therefore be excluded. For the moment, however, we will confine our attention to the Population II case since (a) the present state of both the observational and theoretical data hardly warrants a more detailed discussion and (b) this assumption allows us to set a minimum distance to the system. The question of Population will be discussed further in Section 6.

In Figure 3 we plot values of $(B-V)^{max}$ against sin i for the six combinations of T_o and ℓ_x in our basic grid. As expected, $(B-V)^{max}$ is strongly dependent on ℓ_x but is only weakly influenced by T_o. We should, however, note the "cross over" effect in color as a function of i for the higher X-ray luminosities - an effect which is absent for ℓ_x = 2. This may be explained as follows: For low incident X-ray flux the color is dominated by the intrinsic radiation from the primary and accordingly the cooler star appears redder at maximum for all orbital inclinations. For higher X-ray fluxes the emission from the "pole" is predominantly reradiated energy and has a higher color temperature than that due to the intrinsic stellar luminosity. As i decreases, the net color becomes increasingly influenced by the redder stellar radiation and this clearly manifests itself more strongly (for the same ℓ_x) in the more luminous (and hotter) model. The intrinsically hotter star thus appears redder than its cooler counterpart since the reradiated X-ray energy is more important in the latter case.

As far as we are aware, no measure of $(B-V)^{max}$ is available for HZ Her. Davidsen et al (1972) have, however, shown that $(U-B)^{max} \sim -0.70$. From this we estimate a value of $(B-V)^{max}$ between -0.15 and -0.16. Our calculations can be refined as more accurate observations become available. The "observed" value of $(B-V)^{max}$ is represented by a horizontal line in Figure 3 from which it is clear that ℓ_x lies in the range $4 - 8 \times 10^{36}$ ergs/sec but that solutions for ℓ_x and T_o exist at all values of sin i. This multiplicity can be removed by use of a third distance independent observational parameter namely the amplitude, δm_B, in the B magnitude light curve.

Theoretical values of δm_B are plotted in Figure 4 and clearly depend strongly on both ℓ_x and T_o. The observed value of $\sim 1.8^m$ is also shown. From Figures 2, 3, 4 it is clear that a solution should exist in the region of $T_o \sim 7000°K$ and $\ell_x \sim 5 \times 10^{36}$ ergs/sec. A detailed investigation of solutions, error limits, etc. is hardly worthwhile given the present quality of both the observational and theoretical data. Our purpose here is merely to illustrate the potential of the method and to derive approximate results. In Table 2 we also list values of δm_B for different values of i under the restriction that T_o and ℓ_x have been chosen to yield the observed values of (B-V) at minimum and maximum light. A solution for i ~ 65° seems appropriate to HZ Her and is consistent, within the uncertainties with the theoretical mass luminosity relation shown in Figure 2. The distance modulus for this case is $\Delta m = 13.5^m$ corresponding to a distance of ~ 5.4 kpc. We note that if the metal abundance is higher than for Population II, higher values of T_o and ℓ_x are required. Even if i differs substantially from 65°

a maximum reduction of ~ 10 percent in radius could occur (cf. Table 2). A distance of ~ 5 kpc is probably a minimum estimate.

With the distance known, we may compute the intrinsic X-ray luminosity of HZ Her if we assume that the pulsed radiation is emitted isotropically. On this basis, and assuming a power law spectrum $F(E) \propto E^{-0.5}$ with a cut off at ~ 40 keV (Clark et al 1972, Ulmer et al 1973) the intrinsic luminosity is $L_x \sim 1.5 \times 10^{37}$ ergs/sec and the efficiency factor $\eta \sim 0.30$. The latter seems slightly high but is not inconsistent with our previous estimates.

A theoretical B-magnitude light curve for the i = 64° model is shown in Figure 5 together with the observed curve taken from Davidsen et al (1972). We note that while the agreement near maximum light on the ascending curve and in the amplitude is good, the correspondence is otherwise rather poor. In particular we note that the observed curve is both displaced from and falls to a sharper minimum than the predicted curve. A somewhat more satisfactory fit is obtained with the photographic light curve of Bahcall and Bahcall (1972) in that the observed curve is fairly symmetric around maximum light and the approach to minimum, while steep, is not as sharp as in the data of Davidsen et al. For the same color data, the δm_B observed by Bahcall and Bahcall requires that i < 60°. The difference between the two observed light curves is, however, almost certainly significant and would be consistent with Kurochkin's (1973) claim that the light curve depends on phase in the 35 day cycle. The results obtained so far should therefore only be considered as indicative of the true parameters, although the method illustrated above should still be applicable when a more adequate account of the light curve is given. It is to this question that we now turn.

4. Mass Transfer and Third Light

In the previous section the light curve of HZ Her was used to obtain an estimate of the total X-ray luminosity. With $L_x \gtrsim 10^{37}$ ergs/sec the question of energy supply becomes of crucial importance. For example, if by analogy with pulsars, the X-ray emission is produced by a rotating neutron star, a luminosity $L_x \sim 10^{37}$ ergs/sec would imply a rotational braking time scale $t_b \sim 1$ year unless energy is supplied more or less continuously to the system. The obvious source is gravitational energy released in transferring material from the primary to the secondary. The transfer rate will be determined essentially by the free fall time t_{ff} from the inner Lagrangian point. We therefore have

$$L_x \sim \frac{\Delta m}{t_{ff}} \frac{GM_2}{r_2} \sim \frac{\pi \ell^2 h n m_H}{t_{ff}} \frac{GM_2}{r_2} \tag{9}$$

where r_2 is the radius of the secondary, Δm, ℓ, h, n denote the instantaneous mass of material between the stars, the extent, thickness and proton density of the (assumed mainly ionized) gas distribution between the stars respectively, and m_H is the mass of the proton. Setting $\ell \sim 2 \times 10^{11}$ cm, $h \sim \ell/3$, $L_x \sim 2.5 \times 10^{37}$ ergs/sec, $M_2 \sim 0.2\ M_\odot$, $r_2 \sim 10^6$ cm, we obtain $n \sim 3 \times 10^{12}\ \mathrm{cm}^{-3}$. The optical depth to electron scattering is then $\tau_{es} \sim 1/3$ and satisfies the requirement $\tau_{es} \lesssim 1$, for X-ray heating of the primary to occur at all. Note that a white dwarf companion ($r \gtrsim 10^8$ cm) would not allow this condition to be fulfilled. The fact that τ_{es} is close to unity does, however, mean that occasional irregularities in n may lead to variations in the light curve or the X-ray eclipse; evidence for the latter has indeed been noted by Gursky (1972) and Ulmer et al (1973).

The hard X-ray flux alone implies a very high degree of ionization in the circumstellar disc. We should, however, point out that the ionization equilibrium in this material is also strongly influenced by the X-ray energy that is re-emitted in the Lyman continuum from the primary. This is because an incident ~ 10 keV photon will, on average, cause $\gtrsim 10^2$ Lyman continuum photons ($\eta \sim 10^{-1}$) to be re-emitted from the stellar surface of which a proportion p ($p \gtrsim 10^{-1}$) will reenter the gas. In addition, the photo ionization cross section is ~ 10^9 times greater for the UV photons. The ionization rate from these photons is approximately equal to the recombination rate calculated on the basis of the above gas parameters. While further investigation is required, there appears to be no inconsistency in this aspect of the model.

A further restriction is set by the absence of prominent Balmer emission lines. Since the dimensions and density of the material are comparable with those found in discs surrounding main sequence Be stars (Burbidge and Burbidge 1953) and since the stellar luminosities are also similar, we would expect to see a Balmer recombination spectrum unless the temperature in the gas considerably exceeds 10^4°K. The observations thus seem to require a kinetic temperature of $T_g > 10^5$°K. In these circumstances, the dominant emission mechanism will be free-free emission. The corresponding emission rate from volume V is given by

$$F_\nu^{ff} = 6.8 \times 10^{-38}\, n^2\, T_g^{-1/2} V \text{ ergs/sec/Hz} \qquad (10)$$

which, with the above cloud parameters and $T_g \sim 10^5$°K gives, at 4000A, $F_\lambda^{ff} = 4.5 \times 10^{30}$ ergs/sec/A. The continuum flux from the primary at 14th magnitude (the minimum brightness when the gas is unobscured) is equivalent to a flux $F_\lambda \sim 5 \times 10^{31}$ ergs/sec/A. The free-free emission would thus represent ~ 10 percent of the light observed at this phase and may therefore significantly influence the light curve.

The total free-free emission is given by

$$L_{ff} \sim 1.4 \times 10^{-27}\, n^2\, T_e^{1/2} V$$
$$\sim 2 \times 10^{35} \text{ ergs/sec} \tag{11}$$

or approximately 1 percent of the hard X-ray luminosity. To within the uncertainties this is consistent with the amount of energy transferred to the gas by electron scattering of the hard X-ray photons. Most of the free-free photons will be emitted at soft X-ray energies (~0.1 keV) and these will readily be absorbed in the upper atmosphere of the primary. However, since the energy involved is substantially less ($\sim 10^{-1}$) than that absorbed from hard photons no temperature reversal will occur in the photosphere and thus no photospheric emission lines should appear in the spectrum.

As noted above a significant distortion of the light curve could be caused by free-free emission from the gaseous disc which surrounds the X-ray source and ultimately provides the energy for the X-ray emission. With an electron scattering optical depth $\tau_{es} \sim 1/3$ this cloud may also become an effective source of optical radiation through scattering of photons emitted in the heated polar regions of the primary; the equivalent of a 15th-16th m source would seem entirely possible. We have therefore investigated the effect of emission from such a disc on the light curve assuming that the volume emissivity of the gas is uniform. Results for the i = 60° case are shown in Figure 6 together with the observed light curves and the theoretical results for the star alone. The disc dimensions were set at $\ell \sim 1.4 \times 10^{11}$ cm and $h = \ell/4$ and the volume emissivity was normalized so that the disc contributes 10 percent of the total light at maximum. It is clear from Figure 6 that the general character of the light curve at minimum may readily be reproduced in this manner. Similar results have been obtained for other inclination angles. A disk of expanded dimensions would result in a brighter and more rounded minimum while one substantially smaller would produce a light curve showing sharp eclipse characteristics near minimum as is the case for the X-ray emission. The adopted radius of the disc turns out, however, to be of the same order as the distance between the primary surface and the neutron star and a light curve of the observed type is therefore precisely what one would expect under the mass transfer hypothesis.

The third light could in principle alter our previous conclusions concerning distance, particularly if the color at minimum is strongly affected. In practice, however, this seems unlikely because (a) the sharp minimum implies a relatively small third light contribution at this phase and (b) (B-V) varies strongly with effective temperature in this range. Our distance estimates which depend mainly on the effective temperature at minimum are unlikely to be grossly distorted by the presence of third light. Values of δm_B could be more strongly influenced thus increasing the uncertainty in orbital inclination.

Finally, we note that while the theory can account fairly satisfactorily for the shape of the light curve at minimum, the problem of the asymmetry around maximum light in the data of Davidsen *et al* and the apparent changes in light curve still require explanation.

5. Precession

So far we have assumed that the X-ray emission is more or less spherically symmetric despite its pulsed nature. The continued apparent illumination of the primary even during the 35 day X-ray off period together with the observed variable asymmetries in the optical light curve (Davidsen et al 1972, Kurochkin 1973) are difficult, if not impossible to account for on this basis. A more plausible hypothesis is that the X-ray pulses are caused by beamed emission from a rotating, magnetic neutron star (Pringle and Rees 1972, Davidson and Ostriker 1973) and that the 35 day modulation is due to precession as proposed by Brecher (1972).

The introduction of such a beam clearly brings sufficient additional unknown parameters into the problem to preclude any detailed analysis on the basis of presently available observational data. We shall therefore confine our attention to the mechanism of precession and to the qualitative effects of a precessing beam on the observed X-ray and optical emission. We note, however, that since the X-ray source does not influence the star as seen at minimum light our distance estimates will not be seriously affected.

Simple geometrical considerations lead to plausible estimates of at least some of the beam characteristics. Since angular momentum directed along the normal to the orbital plane is continuously being added to the neutron star it seems likely that both the axis of precession and the X-ray source spin vector will be close to this normal. For illustrative purposes we will assume that the precession axis is perpendicular to the orbital plane and that the rotation axis is inclined at an angle $\Delta\alpha$ thereto. In order that the primary should be illuminated throughout the cycle <u>and</u> that the light curve should show a dependence on precession phase X, the following must be satisfied:

(a) The beam axis should be inclined at an angle $\beta \sim \pi/2$ to the spin direction.

(b) The beam halfwidth $\Delta\beta$ should be smaller than the angle $\theta_1 \sim \sin^{-1}(r_1/a)$ subtended by the primary.

(c) The precession angle $\Delta\alpha$ should satisfy the conditions $\Delta\alpha \gtrsim \Delta\beta$ and $\theta_1/10 \lesssim \Delta\alpha \lesssim \theta_1$. The lower bound is rather uncertain and depends on the degree of variation with precession phase in the light curve.

The X-ray data similarly require that

(d) $\Delta\alpha > \Delta\beta$

in order to achieve strong modulation (assuming that the beam intensity falls off with a characteristic angle $\sim \Delta\beta$) and that

(e) $\beta - i = \Delta\alpha + s\Delta\beta$ where $s \geq 0$ in order that the line of sight should come within $\sim s\Delta\beta$ of the beam center but not cross it.

The model proposed by Pringle and Rees (1972) in which the pulsed X-ray emission is emitted along the axis of a dipole type magnetic field and in which material can be accreted easily provided this axis lies near the orbital plane, would seem consistent with the observational requirements.

In order to illustrate the effect of such illumination we have assumed that the X-rays are emitted in a single axially symmetric beam with a Gaussian intensity profile.

$$I(\Delta\theta) = I_o \exp\left(-(\Delta\theta/\Delta\beta)^2\right) \qquad (12)$$

where $\Delta\theta$ is the angle between the direction of emission and the beam axis. The predicted light curves for phases $X_p = 0$, 0.25, 0.5 and 0.75 (phase zero refers to X-ray minimum) are shown in Figure 7 for the 60° model with beam parameters $\Delta\alpha = 9°$, $\Delta\beta = 9°$, $\beta = 84°$ and I_o corresponding to an effective (i.e., absorbed) intensity of 1.3×10^{36} ergs/sec/steradian. A third light contribution normalized to add 10 percent at maximum is also included. We note the considerable variation in shape and amplitude of the light curve as a function of precession phase and the asymmetry at phases 0.25 and 0.75. We should emphasize that no attempt has been made at this stage to fit the observed light curve but merely to illustrate the qualitative effect of beaming. In fact the data of Kurochkin (1973) contains indications that the precession axis is displaced from the normal to the orbital plane and that the source has two beams, only one of which approaches the line of sight to the Earth.

Kurochkin's data also suggests that the magnitude at minimum depends on X_p. This clearly cannot be ascribed to changes in direct illumination of the primary as the neutron star precesses. We would suggest, however, that the temperature and hence normal scale height of the gaseous disc surrounding the X-ray source depends on beam direction. The third light contribution would then also vary with X_p. When the beam cuts the orbital plane along the line of centers the relative illumination of the primary will be reduced by scattering from the disc. At the same time the heating of the disc material will be maximized causing expansion. The result would be a maximum third light contribution when the re-radiation from the stellar surface is least which is in qualitative agreement with Kurochkin's observations. A more detailed theoretical investigation is currently in progress together with an observational program to establish more accurately the precession phase and wavelength dependence of the light curve. The results should allow determination of beam and orbit parameters. It is clear, however, that the precessing beam hypothesis affords a reasonable qualitative understanding of the present observational evidence. The origin of the precession remains to be discussed.

Brecher (1972) suggested that the 35 day cycle is due to free precession of an oblate neutron star and this may indeed be the case. An alternative is, however, possible and may be checked for self-consistency. If the magnetic field axis is inclined at angle β (where $\beta \neq 0$, $\pi/2$, etc.) to the spin axis of the neutron star the resultant couple exerted on the star can have a component perpendicular to the spin and will then cause forced precession. The same consideration applies to the effective torque due to accretion. While the details of the motion cannot be specified without knowledge of the radiation mechanism and of the interaction between the neutron star and the surrounding material, an approximate self-consistency check is possible. If the torque G exerted on the neutron star is inclined at an angle γ to the neutron star spin direction, the precession rate Ω is given by

$$\Omega = \frac{2\pi}{P_M} = \frac{G \sin \gamma}{H \sin \Delta\alpha} \tag{13}$$

where the angular momentum H of the neutron star is

$$H \sim (1/15) M_2 r_2^2 \omega = (1/15) M_2 r_2^2 (2\pi/P_P) \tag{14}$$

and r_2 is the radius of the secondary. This precession rate need not be strictly periodic(nor indeed represent a pure precession) since the interaction may not be precisely constant in magnitude and relative direction over long periods. The rate at which the torque does work may be set approximately equal to the energy dissipated, that is to the X-ray luminosity L_x. We then have

$$L_x \sim G \omega \cos \gamma \tag{15}$$

From equations (13), (14) and (15) it then follows that

$$r_2^2 \sim 2.8 \times 10^5 \frac{\tan \gamma}{\sin \Delta\alpha} \left(\frac{L}{M}\right) \tag{16}$$

using the observed precession. For $L \sim 2.5 \times 10^{37}$ ergs/sec, $M \sim 0.1 M_\odot$ and $\Delta\alpha \sim 9°$ we then obtain $r_2 \sim 5 \times 10^5 (\tan \gamma)^{1/2}$ cm which is in reasonable accord with that expected for a neutron star. Within the uncertainties, the hypothesis of forced precession therefore seems self-consistent, although for the derived secondary mass, r_2 seems a little low.

The apparent stability of the spin rate may also be understood on the basis of the Pringle-Rees type of model provided the relative orientation of the magnetic field and angular momentum vectors satisfies $0 < \beta < \pi/2$ (or $\pi \lesssim \beta \leq 3\pi/2$). In these circumstances addition of angular momentum by accretion down the magnetic field lines will tend to orient the angular momentum vector more nearly perpendicular to the orbital plane thus tending to raise the magnetic field axis out of the plane. This will reduce the rate of accretion and thus slow down the process; an equilibrium orientation is clearly possible.

6. Summary and Discussion

We have shown that the light curve of HZ Her may be understood in terms of illumination by a beam of X-rays from a rotating companion neutron star undergoing forced precession. An additional third light contribution is predicted if the energy supply comes from mass transfer to the neutron star; it would account for the shape of the observed light curve near minimum. A method of determining orbital parameters has been applied to a highly simplified model to obtain approximate values for HZ Her. The orbital inclination probably satisfies $60° < i < 70°$ and corresponding system parameters are listed in Table 2; these should be viewed only as indicative until the full effects of beaming and third light are included in the calculations and the observational data are improved. The primary mass must, however, be close to 1.45 $M_{\odot}$ and the distance cannot be much less than 5 kpc.

A number of predictions may be made on the basis of the present model and may provide suitable tests thereof. They are,

(i) A substantial fraction of hard X-ray photons should be scattered either in the primary atmosphere or in the intervening gas. The flux of scattered photons in the direction of the observer should amount to $\sim 1 - 3 \times 10^{-2}$ of the maximum beam intensity and should vanish at eclipse. It should, however, be present even during the "off" part of the 35 day cycle.

(ii) If the "third light" is due to free-free emission its relative contribution should increase at longer wavelengths. An infrared excess and a veiling of the absorption lines which vanish at eclipse would be predicted. If, however, electron scattering is important, only veiling would occur. The electron temperature is too high and Doppler broadening thus too large to allow detection of the polar B star spectrum near minimum. A continuum contribution should, however, be readily detectable in the U band.

(iii) The amplitude of the true radial velocity variations of the primary should be $\lesssim$ 80 km.

(iv) The optical emission should be polarized at orbital phases ~ 0.25, 0.75 by amounts comparable with those found in Be stars (~1 percent).

Finally some comment is required on the origin and evolutionary status of HZ Her. At first sight the existence of this system 2.5 kpc above the galactic plane seems consistent with the result of §3 that a Population II metal abundance is required if the primary is evolving off the main sequence. An immediate problem, however, arises in the evolutionary time scale which for a ~ 1.4 $M_\odot$ star of this composition is ~ 10^9 years. Either such stars can form at comparatively recent epochs or the primary of HZ Her has a far more complicated evolutionary history. (A reduction in helium abundance would result in an evolutionary time scale of nearer 10^{10} years but only at the expense of greater disagreement than for Population I with the theoretical M-L relationship for the turn-off point).

The close proximity of the neutron star companion suggests that multiple mass exchange has indeed occurred. According to current theoretical ideas a neutron star can arise only from the collapse of a star whose initial mass exceeds the limiting value for a white dwarf, 1.2 $\pm$ 0.1 $M_\odot$ (Hamada and Salpeter 1961). The age of a Population II system, usually taken to be ~ 10^{10} years, is, however, close to that of a 0.7 - 0.8 $M_\odot$ star of normal initial helium abundance. If the binary separation has always been modest neither component would have much exceeded this mass initially. The present total mass ~ 1.6 $M_\odot$ thus implies that very little material has been lost from the system even during the formation of the neutron star. This rather surprising conclusion receives some support from the fact that the binding energy of a low mass (~0.10 $M_\odot$) neutron star is comparable with the energy necessary to convert nuclear processed material back to neutrons. The "explosion" of the outer layers of the pre-neutron star could therefore, in principle, have been a fairly quiet affair and have allowed most of the ejected material to be re-captured by the primary. The situation is, of course, relaxed if either (i) the initial Population II system was deficient in helium, or (ii) the initial separation was much greater than at present, or (iii) the age of the system is less than 10^{10} years.

Possibilities (ii) and (iii) above could, for example, be combined to provide another explanation of HZ Her, namely that the system is basically of Population I and owes its present position above the galactic plane to a "sling shot" effect of a supernova explosion involving a massive star. This type of mechanism has recently been invoked to account for the distribution of pulsars above the galactic plane (Gott et al 1970). In the present case it would require both that the binary remained bound during the explosion and the the orbital separation was subsequently greatly reduced possibly by tidal interactions if the neutron star initially moved in a highly eccentric orbit.

The two sample explanations offered here are clearly neither exclusive nor compelling. At this stage we can conclude only that the evolutionary status of HZ Her is far more complicated than the early hypothesis of simple mass exchange and that much further study is required to unravel its history.

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TABLE 1

	X-ray	Ref.	Optical	Ref.
P	1.70017 days	1	$m_B^{max} \sim 13.0$	2,3
$\frac{(M_1 \sin i)^3}{(M_1 + M_2)^2}$	1.69×10^{33} gms	1	$m_B^{min} \sim 14.8$	2,3
ϕ_E	0.14	1,2	$(U-B)^{max} \sim -0.70$	3
$a_2 \sin i$	3.95×10^{11} cm	1	$(U-B)^{min} \sim +0.30$	3
P_P	1.24 s	1	$(B-V)^{min} \sim 0.23$	4
P_M	~ 35 days	1		
$f_E(E)$	$\sim 4 \times 10^{-10}$ ergs $cm^{-2}s^{-1}$ kev^{-1} at 2-6 keV	1		

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2. Forman, *et al* 1972, Ap. J., 177, L103.

3. Davidsen *et al* 1972, Ap. J., 177, L97.

4. Serkowski 1972 (private communication: Result obtained during polarization measures)

TABLE 2

sin i	0.863	0.900	0.944	0.967	0.982	0.997
i	~ 60°	~ 64°	~ 71°	~ 75°	~ 79°	~ 85°
q	0.06	0.10	0.20	0.30	0.40	0.55
M_1 ($M_\odot$)	1.48	1.41	1.46	1.59	1.75	2.06
r (10^{11}cm)	3.19	2.96	2.77	2.74	2.75	2.82
a (10^{11}cm)	4.85	4.82	5.02	5.31	5.63	6.14
M_{bol}(7000° K)	0.38	0.57	0.74	0.77	0.76	0.70
x	0.11	0.09	0.07	0.06	0.05	0.045
T_I (°K)	7065	7230	7310	7355	7385	7410
T_{II} (°K)	6740	6820	6895	6915	6930	6935
$\delta m_{B,II}$	1.74	1.82	1.91	1.96	2.00	2.04
$M_{B,II}^{min}$	0.95	1.16	1.31	1.35	1.34	1.32

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FIGURE CAPTIONS

Figure 1 - $(B-V)^{min}$ as a function of sin i for T_o = 7000°K and 8000°K. The solid line refers to the Population II color-temperature calibration and the dashed line to the Population I calibration.

Figure 2 - M_{bol} as a function of primary mass M_1. The straight lines are theoretical mass-luminosity relations. The curved lines are the relationships which give the observed $(B-V)^{min}$ value. The notation "I" and "II" refers to stellar Population.

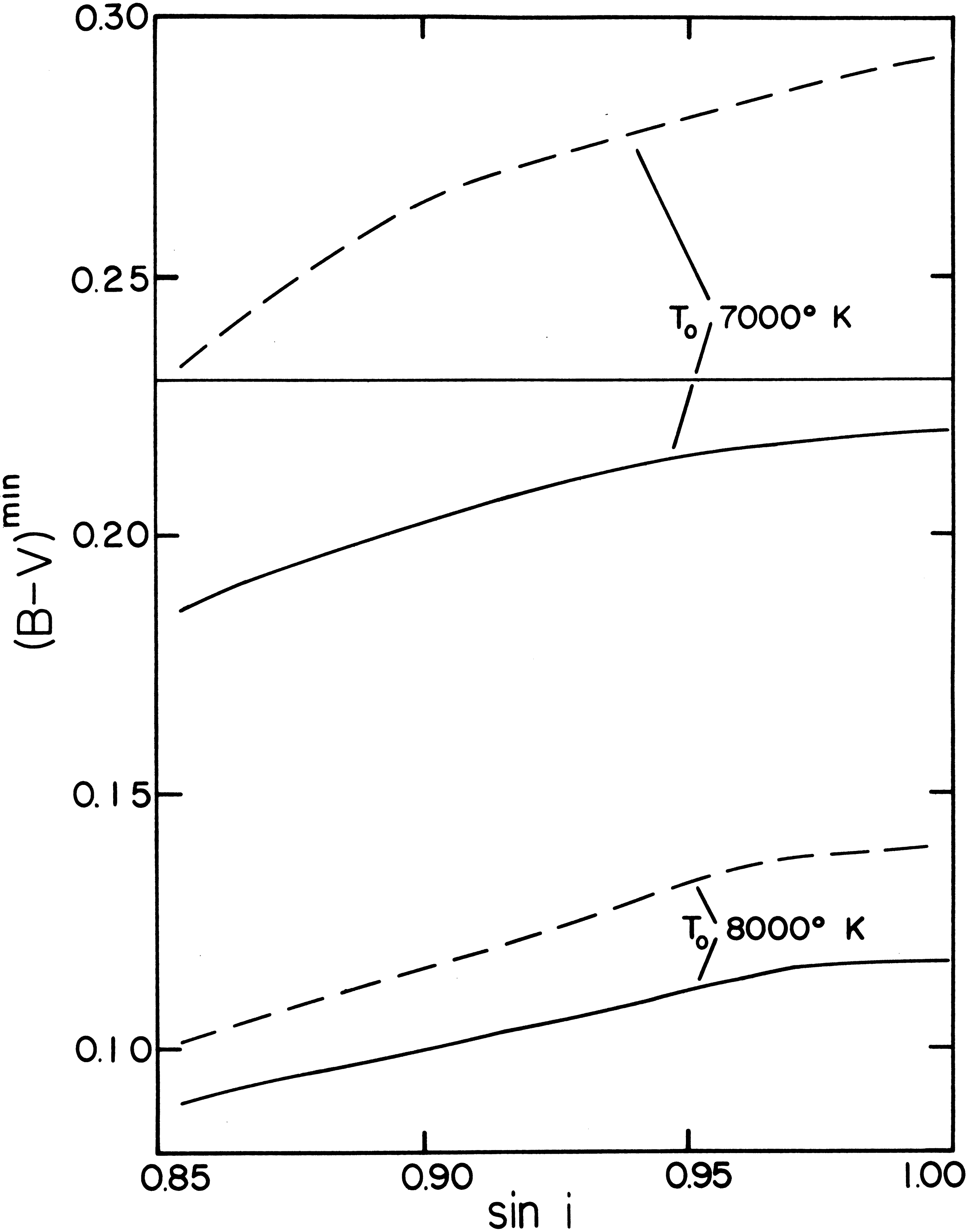
Figure 3 - $(B-V)^{max}$ as a function of sin i. For solid curves T_o = 7000°K and for dashed curves T_o = 8000°K. The numbers above each set of curves refer to the impinging effective X-ray luminosity, ℓ_x, in units of 10^{36} ergs s^{-1}.

Figure 4 - δm_B as a function of sin i for Population II. The solid curves refer to T_o = 7000°K and the dashed curves to T_o = 8000°K. The number appended to each curve is the value of the effective X-ray luminosity, ℓ_x, in units of 10^{36} ergs s^{-1}.

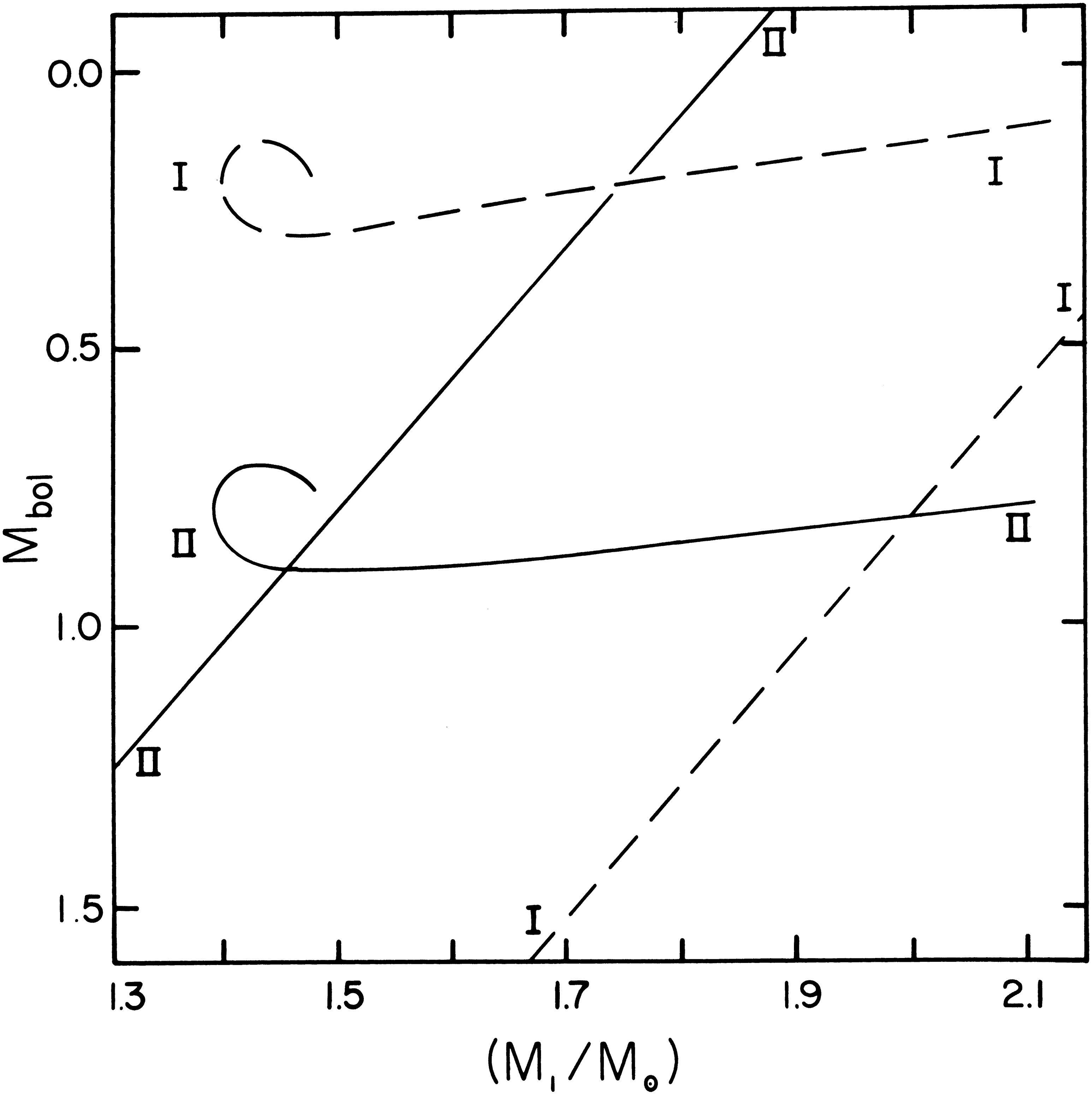
Figure 5 - The B light curve. The observations are represented by filled circles (Davidsen _et al_) and by open circles (Bahcall and Bahcall). The solid curve is the theoretical one for i = 64°.

Figure 6 - The B light curve. The observations are represented by filled circles (Davidsen _et al_) and by open circles (Bahcall and Bahcall). The thin solid curve is the theoretical one for i = 60° excluding light from the disc. The thick solid curve, also at i = 60°, includes light from the disc normalized to contribute 10 percent at maximum light.

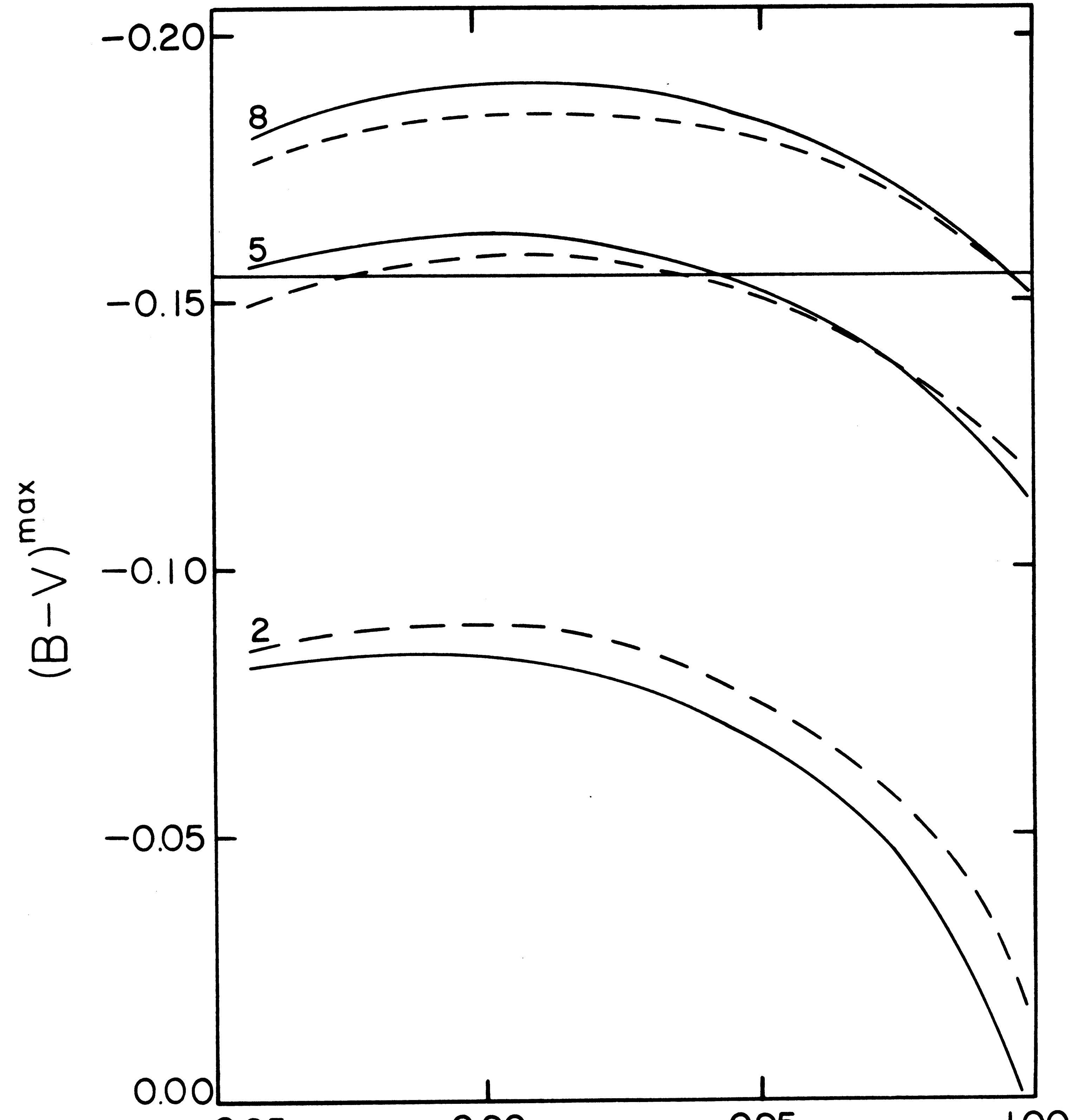
Figure 7 - Light curve variation during the 35 day X-ray cycle. The different curves refer to the following phases of the X-ray cycle (phase zero at X-ray minimum) - • - • = $X_P = 0$; ---: $X_P = 0.5$; thin solid line: $X_P = 0.25$; thick solid line: $X_P = 0.75$. For beam parameters see text.



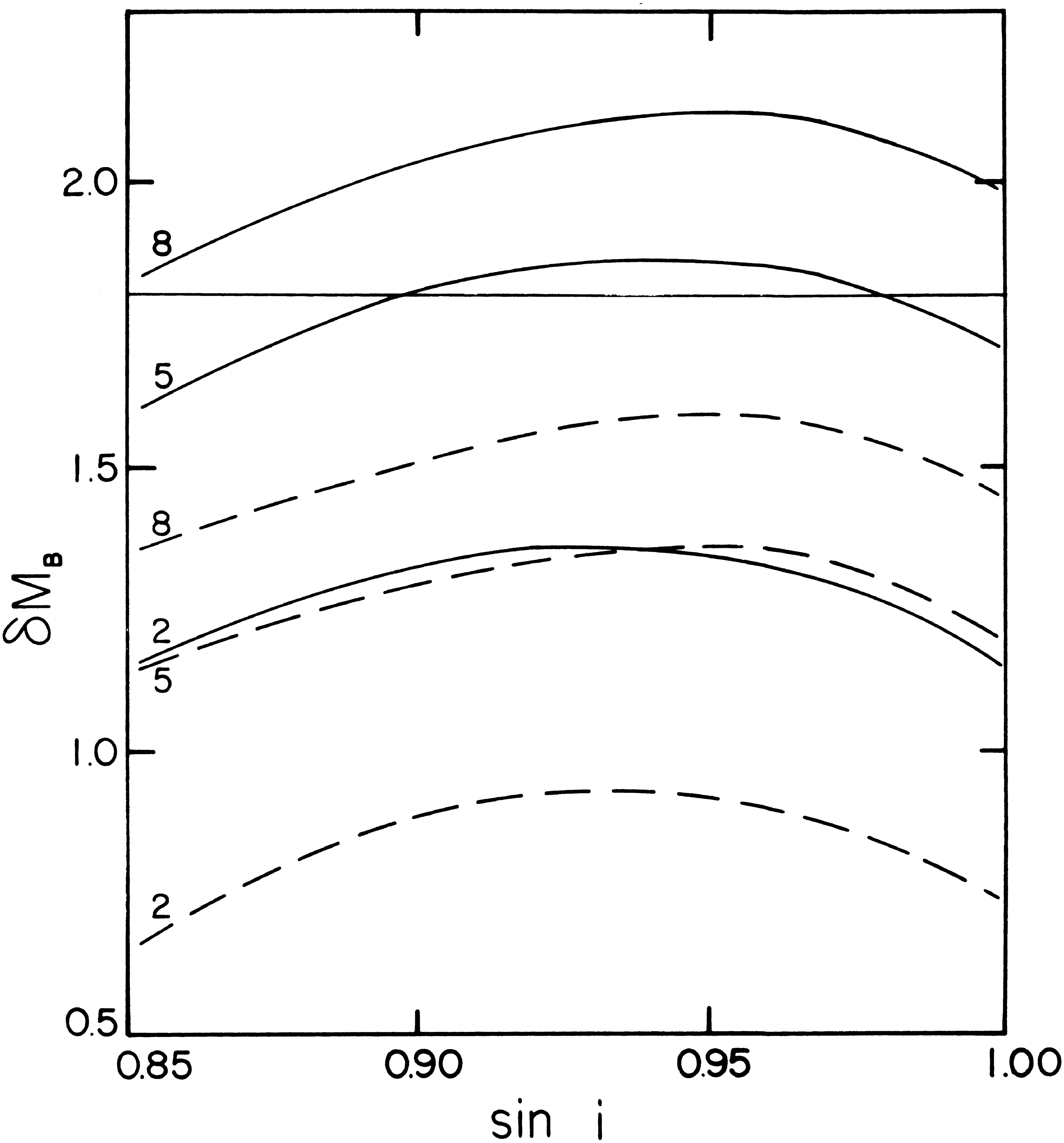
0.30
0.25
0.20
0.15
0.10
0.85
0.90
0.95
1.00
sin i
(B−V)^min
T0 7000° K
T0 8000° K



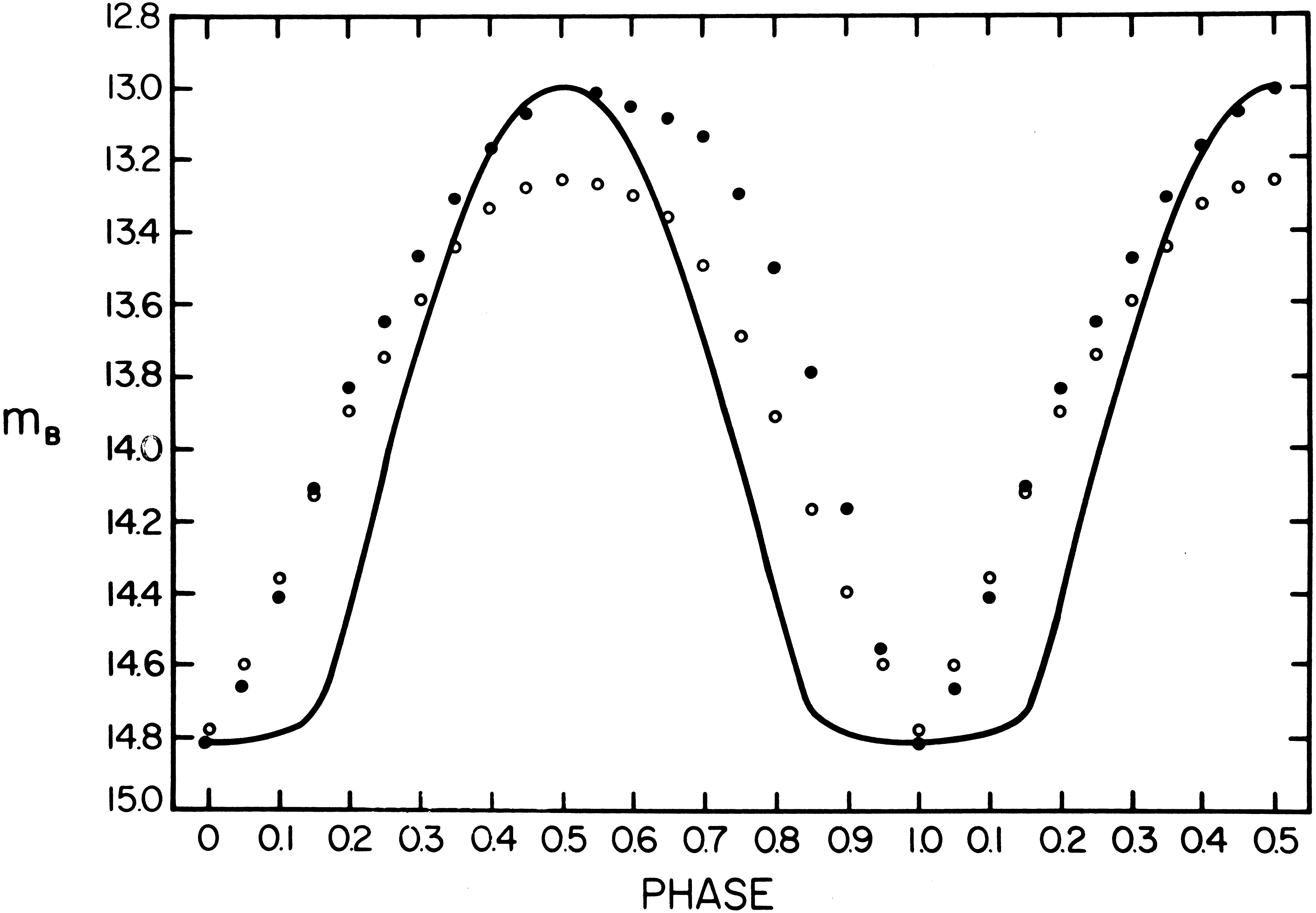
II
I
I
I
II
II
II
I
0.0
0.5
1.0
1.5
M_{bol}
1.3
1.5
1.7
1.9
2.1
$(M_1/M_\odot)$



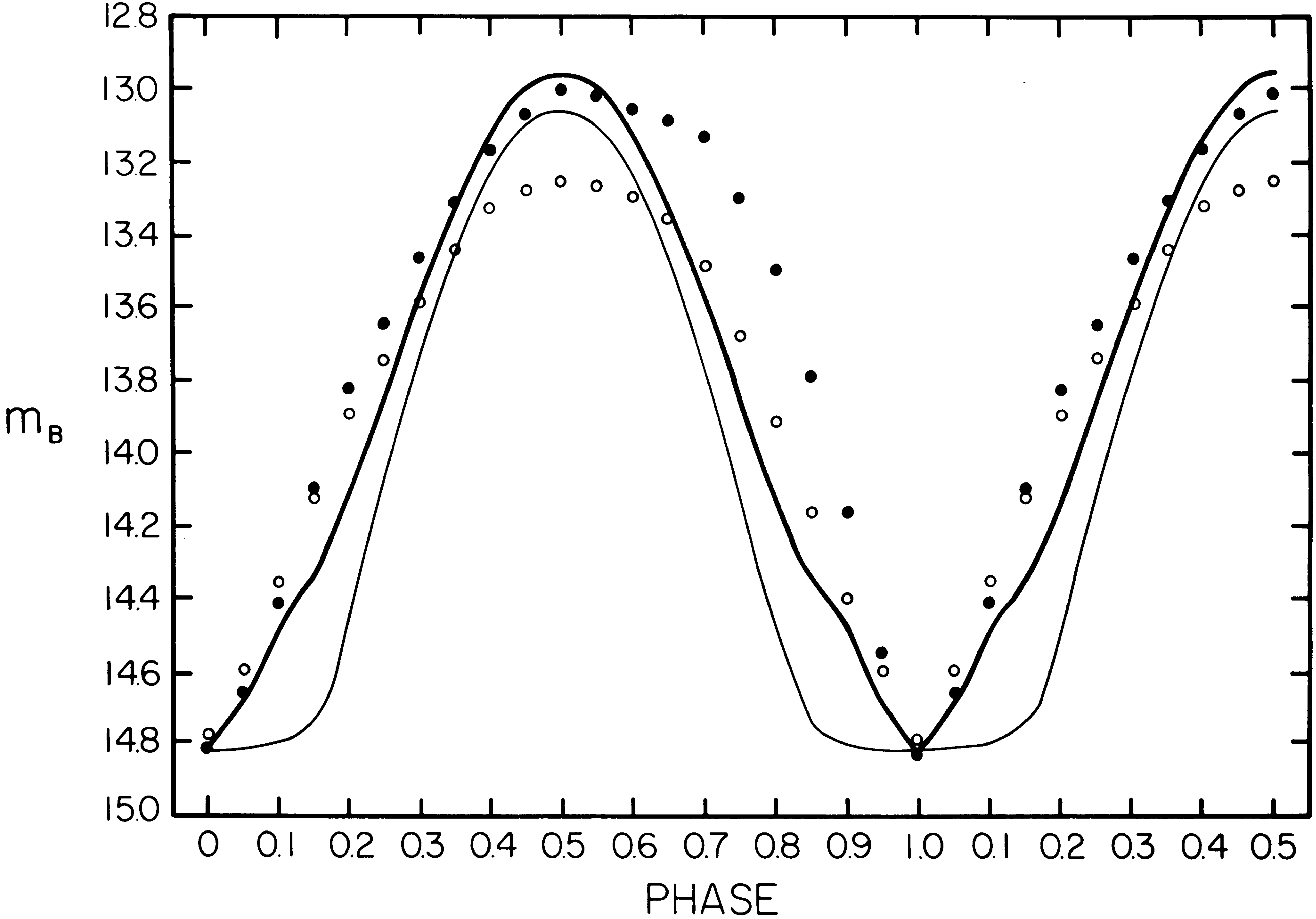
-0.20
-0.15
-0.10
-0.05
0.00
0.85
0.90
0.95
1.00
sin i
(B-V)^max
8
5
2



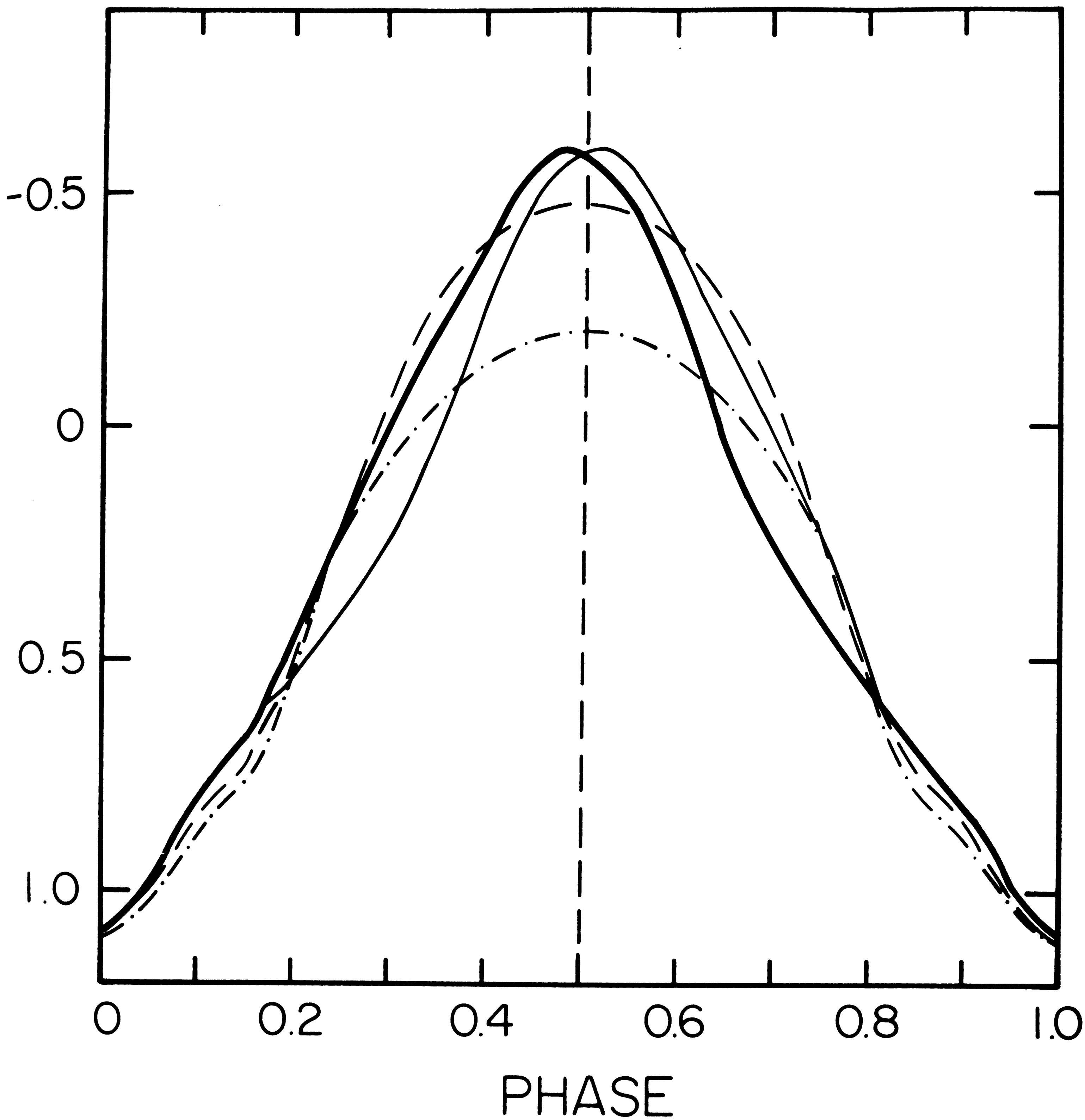
2.0
1.5
1.0
0.5
8
5
8
2
5
2
0.85
0.90
0.95
1.00
δM_B
sin i



m_B
12.8
13.0
13.2
13.4
13.6
13.8
14.0
14.2
14.4
14.6
14.8
15.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.1
0.2
0.3
0.4
0.5
PHASE



m_B
12.8
13.0
13.2
13.4
13.6
13.8
14.0
14.2
14.4
14.6
14.8
15.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.1
0.2
0.3
0.4
0.5
PHASE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0
0.2
0.4
0.6
0.8
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