Polarization and Structure of the Crab Nebula

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ABSTRACT

Present knowledge of the optical, radio and X-ray polarization of the Crab is reviewed and discussed as it bears on the structure of the magnetic field, time scales in the nebula, and relations between nebula and pulsar. Not as much high-resolution polarimetry has been done on the Crab as might have been expected. Loops of field and a large-scale structure can be recognized, but it is not known whether the fields generally are smooth or chaotic on a small scale. Field lines tend to curl around the filaments. The large angular size of the X-ray source poses a difficulty to conventional theory. The form of the nebula does not single out the pulsar as its source, and the exact relation between pulsar and nebula is uncertain. The wave-field or "synchro-Compton" interpretation of the continuum emission is erroneous but has led to interesting observations of circular polarization. Circular polarization of the ordinary synchrotron radiation might be observable in the
radio band. Magnetic flux may have been generated in the nebula by winding of lines around the rotating pulsar. Polaroid photographs then suggest that the pulsar rotation axis is roughly NW-SE, but confirmation is lacking.

1. Introduction: Polarization of the Nebula

The prediction by Soviet theorists that the optical continuum of the Crab supernova remnant should be strongly polarized, and the detection of this polarization by Soviet observers in 1953,\(^1\) can be said to mark the birth of "high-energy astrophysics". It is the most dramatic and one of the most important contributions yet made by polarimetry to astronomy and deserves commemoration in any book on polarimetry.

In this age of pulsarology we tend to imagine that the form and polarization of the Crab nebula itself are old hat and well understood, but this is far from the truth. In the sixteen years since the high-resolution polarimetric map by Woltjer (1957), based on Baade's photographs, and the photoelectric map by Hiltner (1957), remarkably little observational or theoretical advance has been made. Woltjer mapped the linear polarization of the optical continuum over most of the nebula with a diaphragm 5.25" in diameter and a grid spacing of 5.59". Figure 1 (Woltjer 1958) shows this map, superposed on a photograph taken in a band including several nebular emission lines, so that the network of gaseous "filaments" (Trimble 1968) is visible. Within this network (in projection) lies the roughly elliptical (4' x 2') mass of continuum emission, seen most clearly when the lines are suppressed by an appropriate
filter. Figures 2 and 3 show two photographs of this continuum taken through Polaroid by Baade (1956); the arrows indicate the E-vector transmission directions of the filter. These are two of Baade's first series of eight, only five of which have been published (Baade 1956, Scargle 1969a). Additional photos through Polaroid by Münch, Baade and Arp, showing time changes, have been presented by Scargle (1969a). A positive print of the continuum without 2 Unfortunately the captions of these photographs are scrambled. The correct caption for the figure on page 198 is found on page 206; that for 199 is on 199; 202 is on 212, 203 on 215, 204 on 204, 205 on 210, 206 on 198, 207 on 208, 208 on 213, 209 on 203, 210 on 207, 211 on 211, 212 on 205, 213 on 214, 214 on 202, 215 on 209 (J. D. Scargle, private communication). The figures on pp. 198 and 199 are printed upside down with respect to the others. Position angle 45° should read 135°, and 67.5° should be 157.5°. Polaroid is shown in Figure 4, again with Woltjer's map superposed. In general, these photos and map support strongly the nearly universal belief that the continuum is synchrotron radiation (Ginzburg and Syrovatsky 1965) from a more or less tangled magnetic field which fills the nebula.

The net optical polarization of the entire Crab obviously lies roughly in a NW-SE direction. It is about 9.2% in position angle 159.6°, according to Oort and Walraven (1956). The polarization of the continuum alone must be somewhat larger, because their band included some filamentary radiation. A better figure could be derived by a summation over Woltjer's map, but apparently this has not been done. Burn (1966) quotes figures of 12% and 14%, but the source of these is not made clear. Woltjer first pointed out that the interstellar polarization toward the Crab is fortuitously in nearly the same position angle, so that about 2% must be subtracted from the observed polarization to get the intrinsic synchrotron polarization (Trimble 1971; cf. Cocke et al. 1970). The net intrinsic optical polarization of the synchrotron
continuum from the whole nebula is thus probably \( p \approx 7\% \) to 10\%. At short centimeter wavelengths the corresponding quantity is \( p \approx 8\% \), (Wilson 1972a) though this refers to a different and larger spatial distribution of emission. At longer wavelengths \( p \) is lower; this is probably due mainly to Faraday depolarization, especially in the filaments (Burn 1966, Wilson 1972a). There is also X-ray polarization, to which I shall return shortly.

2. **Structure of the Magnetic Field**

Despite many discussions, the details which these photos and map show have probably not been exploited fully by theorists. Threadlike features in the continuum have long been reported by optical observers (Scargle 1969b), and the Polaroid photos seem to show fibrous or ropy structure elongated perpendicular to the E-vector transmission direction (i.e., the axis of the Polaroid filter). Note, for example, the bright isolated loops at the north-east margin of the nebula in Figure 3. Since the Polaroid at a given orientation tends to pick out regions where the local projected B-field is perpendicular to the transmission direction, it appears that these fibers trace magnetic field lines. This impression is strengthened by the fact that, as the Polaroid is rotated through successive photos, some of the loops can be traced through large angles (Baade 1956). A sketch of some of the field lines is given by Oort and Walraven (1956).

How chaotic is the nebular field? In addition to these sizable and well-defined loops visible on the Polaroid plates, we have, as the Woltjer map shows, a rather uniform polarization pattern over the central part of the nebula; the magnitude and direction of polarization change, but not on a scale much smaller than the nebular diameter. Of course we see a projected summation: Along any one line of sight we see from the front of the nebula to the back. Nevertheless the polarization orientation pattern suggests that
the field is rather smooth and simple—comparable, perhaps, to a dipole with a few big twists and loops added. The percentage polarizations revealed by Woltjer's 5.25" diaphragm can be examined further for a rough check on this. Typically the values in the central region of the nebula are \( p \approx 20\% \). On the standard theory (Ginzburg and Syrovatsky 1965) the value expected for a uniform field is

\[
p = \frac{\gamma + 1}{\gamma + \frac{7}{3}},
\]

where \( \gamma \) is the usual spectral index of the electrons. Baldwin's (1971) review suggests that the effective \( \gamma \) in the optical band may be about 2.8, implying \( p \approx 74\% \); recent work on the reddening by Miller (1973) indicates \( \gamma \approx 2.1 \) and \( p \approx 70\% \). We naturally interpret the difference between 70% and the observed 20% as due to projection averaging (plus a smaller contribution from finite resolution). From this we can understand why values \( p > 70\% \) are in fact measured at a few places on the periphery of the nebula where we perhaps see only the last outward-looping line of force, while \( p \) is much lower near the center where several loops are seen crossing in projection along the line of sight.

Let us examine this idea semiquantitatively. If there are \( n \) independent, randomly oriented regions of uniform field along a line of sight, each gives rise to a linearly polarized beam with \( p \approx 70\% \). By adding Stokes parameters (Shurcliff 1962) and considering the statistics, we readily see that if \( n \) such beams of roughly equal strength are superposed, the expected linear polarization is \( p_n = p/\sqrt{2n} \). (We can see a posteriori and perhaps even a priori that this is just an example of a random walk.) Setting \( p_n = 20\% \) as the observations suggest, we find \( n \approx 6 \) for the number of independent elements in a column along a typical line of sight. On the other hand, looking at the polarization patterns transverse to the line of sight, as revealed by
Figure 4, we might judge the correlation length to be such that \( n \approx 3 \), certainly no larger than 4, across the bright part of the nebula. This small discrepancy in \( n \) is probably not significant, and no studies of this sort have been done; the model of superposed independent regions is probably a bad one. But we certainly cannot exclude the possibility that there is small-scale structure in the field, causing some depolarization, even within regions where the mean field on a scale of 5" appears to have a constant direction. It is not difficult to imagine substantial polarization of synchrotron radiation even from a region where the magnetic lines execute random walks, provided there is a preferred axis (cf. Burn 1966). For example, a gas cloud containing a tangled, chaotic, initially isotropic magnetic field, which is then allowed to expand anisotropically (say, along one axis only) with the fields frozen in (not at all implausible in an astrophysical context), can generate a substantial \( p \) even though the field remains quite tangled and the propagation of cosmic rays through the field quite slow.

It would be interesting to know how smooth the field is in the bright northeasterly loops mentioned earlier. Is \( p \) in these loops anywhere near the theoretical 70\% for a uniform field? Of course projection effects must be allowed for (Scargle 1971), but in isolated features like these they should not be large. Woltjer's map suggests that \( p \) is much smaller, 10 - 30\%. Maximum-resolution studies of a few selected regions of the nebula such as this would be of interest. Although a lot of work is involved, it is surprising that such studies have not been undertaken.

Another interesting point about these bright loops is this: The reason for their brightness is probably not solely that the field lines are properly aligned. Continuity would suggest that neighboring lines which do not show up brightly are nevertheless similarly aligned. Probably the visible loops represent regions of enhanced field or relativistic-electron density, or both. Possibly
these particular lines are connected to sources of the particles, while neighboring lines are not. In this case the bright loops would illustrate that cosmic rays diffuse (or stream) faster along the field than across it.

The general polarization pattern in Figure 1 is obviously related to the positions and orientations of the gaseous filaments. Woltjer (1958) suggested the most successful interpretation: that the magnetic lines tend to circle around the filaments. This view is supported to some extent by recent radio polarization maps (Wilson 1972a, c; cf. Conway 1971). It is not obvious, however, that the great radial "fans" of polarization vectors at the margins of the nebula can be explained in this way. To my eye the fan at the eastern edge seems to be centered on the dark bay in the continuum rather than on the nearby filament. Certain features like this relate more closely to the continuum than to the filamentary network.

3. Angular Sizes and Time Scales

Recent Cerenkov-light observations of gamma rays (Fazio et al. 1972) have pinned down the mean magnetic field in the bright parts of the Crab fairly well as $B \approx 1 \times 10^{-3}$ gauss. In such a field, an electron whose critical synchrotron frequency is optical ($\nu_c \approx 10^{15}$ Hz) will lose a large fraction of its energy in a characteristic time $t_s \approx 40$ years. Thus the optical electrons we see in the Crab cannot be much older than this; a more elaborate treatment (Oort and Walraven 1956) does not change the order of magnitude of the result. It follows that the optical electrons cannot be remnants of the supernova explosion 919 years ago. They must be generated continuously or at least recently, probably at or near the pulsar. At the distance of the Crab (1.5 to 2 kpc) one minute of arc is about two light years; then if the optical electrons are to fill the 4' x 2' continuum region in 40 years, they must stream outward fairly efficiently along field lines. There is not much time
for slow diffusive motion in a chaotic field. This argument becomes progressively more restrictive as higher frequencies are considered, since $t_s \propto \nu_c^{-1/2}$.

Indeed there is evidence that the continuum region is smaller at blue frequencies than at red (Scargle 1969b).

In view of this it is surprising that the Crab is also a synchrotron X-ray source of large size. It is at least $1.1'$ in diameter (Kellogg 1971). The measured polarization of $(15.4 \pm 5.2)\%$ at position angle $156^\circ \pm 10^\circ$ (Novick et al. 1972) is consistent with that of the optical continuum radiation from a central circular region of the same size. This is a strong argument for the synchrotron character of the radiation. While granting this, we should note that the location and spatial extent of the X radiation are not well known either along or across the line of sight. A line source along the major axis of the nebula would be compatible with the data. I believe the data also still allow the hypothesis that the spatial distribution of X-ray emission is the same as that of the optical synchrotron radiation.

The X-ray source is large despite the fact that $t_s$ is only about 1 year at $\nu_c = 10^{18}$ Hz. There is always the possibility that particle acceleration is widespread in the nebula. It is difficult, however, to construct a theory of particle circulation which will produce a size dependence on color in the optical, and yet permit a large size in the X-ray band. In a recent attempt, Wilson (1972b) is much vexed by the X-ray size, and his summary of the X-ray results appears to squeeze the data uncomfortably into his mold. Improved knowledge of the X-ray structure, especially as a function of frequency, is essential. The "knee" near the infrared band in the power-law spectrum of the Crab (Baldwin 1971) is obviously related to these lifetime considerations, but I will not discuss it here. The recent reddening observations by Miller (1973) pose difficulties with joining the optical and X-ray spectra smoothly, and suggest that there may be additional structure in the spectrum which will force us to more complicated models.
4. **Pulsar and Nebula**

Most people believe that the cosmic rays are produced at or near the pulsar; nearly everyone believes that the present field in the nebula was generated somehow by the pulsar or by its supernova predecessor. The details of this are a mystery. Wilson (1972a) asserts that the optical polarization pattern over the face of the nebula is symmetric about the pulsar (shown by a white arrow in Fig. 4), but I do not find this symmetry very striking. There is little in the gross features of the nebula to single out this star as its prime mover. The star is reasonably bright (V ≈ 16.5; Kristian et al. 1970) and does lie near the center of gravity of the continuum emission and, perhaps more important, near the center of expansion of the filaments (Trimble 1968). The kinematical and spectroscopic evidence was, however, inconclusive (Minkowski 1968, Trimble 1968) until after the discovery of the optical pulses. Piddington (1957), who as we shall see was quite prescient, nevertheless did not identify this star (nor any other) as the supernova remnant. Even the "wisps" very near the pulsar, which show changes correlated with pulsar activity (Scargle and Harlan 1970), are not strictly symmetric with respect to the pulsar position. The usual theories of relativistic-particle production by the pulsar are in conflict with the observed accelerations of the filaments, as shown by Trimble and Rees (1970). Altogether it is safe to say that if theorists had had to predict the optical appearance of the nebula, given only what is known about the pulsar and supernova, they would have been way off the mark.

It has long been realized that, if the usual synchrotron-radiation theory of the nebular continuum is correct, the magnetic energy of the nebula is so large at present that the nebula cannot have expanded isomorphically, with fields frozen in, from an initial star or other state stabilized by gravitation. The ratio of magnetic to gravitational energy is preserved in such an expansion,
and since it is presently $- 10^6$, the magnetic flux could not have been contained in a stable object (Woltjer 1971). This difficulty created a favorable climate for an alternative suggestion (Gunn and Ostriker 1971) that the "classical" theory of the radiation, imputing it to fast electrons in a static B-field, was completely erroneous, and that the nebular continuum was, instead, "synchro-Compton" radiation from fast electrons in a huge cavity filled with 30-Hz electromagnetic waves generated by the rotating pulsar. Indeed a favorable climate was essential for this idea, because it receives almost no support from Figures 1 - 4. But since no one had understood the form of the nebula in any detail anyway, the theorists were not deterred. Rees (1971a, b) carried the suggestion further and, laudably, offered a prediction: that the "synchro-Compton" optical continuum from the nebula should show a few per cent circular polarization at source points near the rotation axis of the pulsar—which, in this theory, lies (at least in projection) parallel to the general linear polarization in the central region of the nebula, i.e., roughly along the NW-SE (major) axis. The theory of the radiation has been developed quantitatively by Arons (1972) and Blandford (1972), who confirm Rees's approximate results.

This prediction led to the most interesting polarimetry that has recently been done on the Crab. Landstreet and Angel (1971) looked for optical circular polarization and found none at the level predicted. Martin, Illing, and Angel (1972), in a definitive study, have detected circular polarization $q$ of order $0.2\%$ - too small to fit the "synchro-Compton" theory. All are now agreed that this theory of the nebula must be abandoned. The variation of $q$ with wavelength and with position in the nebula indicates that it is due to birefringence of the interstellar grains, investigated theoretically by Martin (1972). Thus circular polarization becomes a powerful new tool for study of the inter-
stellar grains. Rees (1971b) remarked that "If it were found that no parts of the Crab nebula displayed even -1% circular polarization ... it would ... suggest that the popular "oblique rotator" magnetic dipole model for NP 0532 would require some reappraisal." This reappraisal apparently is now in progress (Gunn and Rees, in preparation).

It is possible that there is a small cavity around the pulsar which does fit the wave-field description. Scargle (1971) observed that there is an elliptical region of reduced surface brightness near the center of the nebula which might be identified with such a cavity. Perhaps circular polarization of order 1% could indeed be manifested here. The measurements by Martin, Illing, and Angel (1972) do not encourage this belief, but perhaps they do not yet eliminate the possibility.

To explain the gross features of the nebula we are forced back onto the conventional synchrotron theory. It is worth recalling that synchrotron radiation too produces circular polarization (Sciama and Rees 1967, Legg and Westfold 1968, Pacholczyk and Swihart 1971). The expected q goes as $\nu^{-1/2}$, so that it is very small in the optical band, but at radio frequencies, where the Crab has $\gamma \approx 1.5$ (Baldwin 1971), it would be about

$$q \sim 2 \left(\frac{B}{\nu}\right)^{1/2}$$

if the field were uniform. Here B is in gauss and $\nu$ in MHz. With B as large as $10^{-3}$ gauss (Fazio et al. 1972), we would have q - 1% at 40 MHz and ½% at 610 MHz. This should be observable (cf. Andrew, Purton, and Terzian 1967, Gilbert and Conway 1970, Conway et al. 1971, Berge and Seielstad 1972).

Depolarization due to field nonuniformity could be large; recall that the net linear polarization of the whole Crab, even in the optical band where Faraday depolarization can be neglected, is only 1/8 of the theoretical
value for a uniform field. But the circular depolarization can be smaller, and in any case the remaining polarization might be detectable. Parts of the nebula with high linear polarization should particularly be examined. Observation of the pattern of $q$ over the face of the nebula would give valuable new information on the field structure, since its sense and degree depend on the magnitude and orientation of $B$ (Legg and Westfold 1968).

In a remarkable paper, Piddington (1957), aware of the difficulty in accounting for the large magnetic flux of the nebula, suggested that what was needed, in addition to general expansion of the nebula, was amplification of the field by stretching of the lines due to winding around a rotating supernova remnant. He estimated the rotational period of the star to be as short as 5 min -- this in 1957! Other authors have advanced the same idea, especially since the discovery of the 30-Hz pulsar (Kardashev 1965, 1970; Sturrock 1970). It may well be that theorists will soon settle upon a model of the nebula not very different from Piddington's. If the lines of force, continually being stretched by wrapping, are also carried outward by a kind of stellar wind, the wind will wrap them around the windward sides of the denser and more resistive filaments -- in gross agreement with Woltjer's (1958) observation (M. J. Rees, private communication; cf. Wilson 1972c). We may hope to learn more about this by detailed polarimetry.

Can we tell the orientation of the pulsar rotation axis by looking at pictures of the nebula? After looking at Figures 2 and 3, I feel that the lines in the nebula tend to wind more around the NW-SE (major) axis, and it seems that the rotation axis should then lie in that general direction. Kardashev (1970), in an argument I do not understand, concludes that the lines should, and do, wind around the NW-SE axis, even though he apparently believes the pulsar rotation axis to be NE-SW! Perhaps some theorist can explain this paper to me; possibly it involves a misinterpretation of the conclusions of Wampler, Scargle, and Miller (1969).
There have been attempts to deduce the orientation of the rotation axis from "rotating-vector" models of the pulsar polarization; they are reviewed in this volume. The results are model-dependent. In particular there is a model-determined ambiguity of 90°, plus additional ambiguity in fitting the data. In general the analyses seem to indicate that the pulsar rotation axis is nearly in the plane of the sky, and roughly parallel or perpendicular to the major axis of the nebula. But there is an additional disagreement of 40° among three groups, so this does not mean much. We must conclude for now that the orientation of the rotation axis is unknown. There is a surprising disagreement between the Arizona and Caltech groups about a simpler matter: namely a disagreement of 20° in the position angle of the nebular polarization in the central region, in the sense that Arizona agrees with Woltjer's original result and Caltech disagrees. More data are obviously needed.

In closing this review I should mention two recent pieces of high-resolution polarimetric work on the Crab. Forman and Visvanathan (1971) made observations in the central region and found an "island" in the linear polarization pattern near "Wisp 2", which they associate with a hydromagnetic wave in the wisp. Scargle (1971) emphasized that such work is difficult because suitable corrections must be made for projection effects. Attempting such corrections himself, he measured linear polarizations in the wisps and concluded that they are high enough to be consistent with a uniform field. The nature of these wisps, and their relation to the pulsar and to the nebula at large, is not understood. We hope that high-resolution polarimetry, applied more extensively, will resolve some of these questions.

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confirming me in some of my prejudices about the Crab. Professor Woltjer and the North-Holland Publishing Company graciously permitted the use of the photographs.

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FIGURE CAPTIONS

**Figure 1.** Optical polarizations in the Crab nebula measured photographically and projected against a 200-inch photograph of the nebula taken by Baade in the light of the [SII] lines (Woltjer 1958). North is at top, east at left. Scale: 2.24"/mm. (These figures, and the journal enlargements from which they were taken, are not free of distortion. Persons desiring high precision should seek the use of the original plates.) The length of each line segment shown is proportional to the percentage polarization at the point centered on the segment; the segment shown at upper left to establish the scale represents 50% polarization. The small circle shows the diaphragm size.

**Figure 2.** A 200-inch photograph of the Crab nebula taken in the continuum (λ5400 to 6400) through a Polaroid filter (Baade 1956). Printed to the same scale as Fig. 1. The arrow indicates the E-vector transmission direction.

**Figure 3.** Same as Fig. 2, but with the Polaroid rotated through 90°. Printed to the same scale as Figs. 1 and 2.

**Figure 4.** The polarizations of Fig. 1 projected against a continuum photograph of the nebula without Polaroid (Woltjer 1957). Printed to the same scale as Figs. 1, 2, and 3. The white arrow indicates the 33 msec optical pulsar NP 0532.