Antiwindup Terminal Sliding Mode Control for Mars Entry Using Super-twisting Sliding Mode Disturbance Observer

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Abstract: Uncertainty and external disturbance during Mars entry process inevitably degrade the performance of Mars entry guidance and control algorithms. Traditional approaches focus on suppressing disturbances and compensating uncertainties, which usually results in larger control responses beyond the limited control capability of Mars entry vehicle. This paper further takes the limited actuator ability into consideration, and proposes the Mars entry robust attitude control strategy using the terminal sliding mode control (TSMC) with anti-windup (AW) and super-twisting sliding mode disturbance observer (SMDO). First, terminal sliding mode control with anti-windup is developed to robustly track the nominal attitude command under uncertainty and limited control capability. Then, the super-twisting sliding mode disturbance observer is introduced to online estimate the disturbances and further enhance the attitude control accuracy and robustness. Finally, the comparison simulation results illustrate that the proposed control strategy not only performs well in tracking the reference commands even in the presence of uncertain disturbance but also avoids control saturation issue.

Keywords: Mars entry; attitude tracking; super-twisting sliding mode disturbance observer; terminal sliding mode control; anti-windup

Introduction

As one sub-phase of the entry, descent and landing (EDL) process, Mars entry plays a vital role in the entire Mars landing exploration mission-cycle. It commences when the entry vehicle reaches the Mars atmospheric boundary and ends at the deployment of supersonic parachute, which covers the vast majority of the flight range from entry interface to touchdown (Li et al. 2014). The terminal state error of this phase finally evolves to be the majority component of landing error (Li et al. 2014; Zheng et al. 2015). To date, all Mars landers except 2012 Mars science laboratory (MSL) rely on the unguided entry technologies developed for the Viking missions in the 1970s, which results in a larger landing error (up to tens to hundreds kilometers) (Li et al. 2014; Paul et al. 2008). In order to improve the entry accuracy and then the final landing accuracy, an active closed-loop guidance and control (G&C) approach inherited from Apollo

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reentry mission was firstly adopted in the 2012 MSL mission, in which a reaction control system (RCS) was utilized (Paul et al. 2008). The final landing error of MSL was unprecedentedly reduced to ~10km, which is about one order of magnitude larger than that of the Apollo reentry missions (Li et al. 2014; Paul et al. 2008). Additionally, a similar compound attitude control configuration using RCS and flap was developed for the Mars Smart Lander and the Mars Surveyor Lander missions (Lockwood et al. 2006; Horvath et al. 2006), and an internal moving mass actuator was also proposed to support future high-mass Mars entry G&C operations (Atkins 2014).

The typical entry vehicle adopted a low-lifting configuration (achieved through the center of mass offsetting from the center of pressure) and controlled the orientation of the lift vector by modulating bank angle (Li et al. 2014; Zheng et al. 2015; Paul et al. 2008). Therefore, the Mars entry G&C operations are finally implemented through attitude tracking control. As Martian atmosphere is approximately 100 times thinner than that of the Earth, the low-lifting entry vehicle can only generate a very small amount of aerodynamic forces to correct trajectory errors. The low control capacity of entry vehicles inevitably leads to larger entry and landing errors in the presence of uncertainties (Li et al. 2014; Zheng et al. 2015; Paul et al. 2008). In order to rapidly correct the trajectory error, larger attitude maneuvers are often needed under uncertainties and disturbances, but the attitude maneuver capability of the entry vehicle is always limited in practice. Therefore, control saturation problem must be carefully taken into account for the attitude tracking control system of Mars entry vehicle (Li et al. 2014; Zheng et al. 2015; Paul et al. 2008).

It is believed that the dynamics uncertainty and external disturbance are the biggest obstacle to further improve the precision of Mars entry G&C (Li et al. 2014; Zheng et al. 2015). Preceding studies show that high-precision robust and adaptive G&C is one of the most essential approaches to suppress the adverse effect of uncertainties and disturbances during Mars entry, and achieve the pin-point landing on the surface of Mars (Li et al. 2014). For the attitude tracking control of Mars entry vehicle, Wang et al. (2014) applied the fuzzy sampled-data control and dynamic inversion method to achieve Mars entry vehicle's attitude tracking. Jiang et al. (2014) introduced T-S fuzzy model and H_∞ fuzzy control to robust tracking reference attitude command under external disturbance. Lei et al. (2016) developed a hybrid robust control algorithm for Mars entry attitude tracking with time-varying input delay using dynamic inversion method, T-S fuzzy model, and H_∞ control. Unfortunately, control saturation problem has not been taken into account in the studies above. An anti-windup compensator is often adopted to cope with the saturation constraints, after the nominal controller is designed. The anti-windup compensator is usually used to ensure that stability is maintained (at least in some region near the origin) and that less performance degradation occurs than when anti-windup is absent (Jeng et al. 2009). It has been widely adopted in the hypersonic flight vehicle and reusable launch vehicle (Chen et al. 2015). Recently, sliding mode controller and disturbance observer have been used to solve the Mars entry guidance and control problem with uncertainty and disturbance (Li et al. 2014; Zheng et al. 2015). The boundary of disturbance is needed for traditional sliding mode control. The disturbance observer can online approximate the composite uncertain disturbance item, which is beneficial to design a compensation controller, and easy to achieve high-precision and robust G&C (Wen et al. 2011). Dai et al. (2017) applied terminal sliding mode control (TSMC) and extended state observer to Mars entry guidance. Zhao et al. (2016) proposed finite-time disturbance observer based super-twisting sliding mode control for Mars entry. Su et al. (2015) presented a comprehensive design of disturbance observer and non-singular terminal sliding mode control. Hybrid robust attitude control algorithms are also widely developed for reentry vehicles (e.g. hypersonic flight vehicle, near-space vehicle, and reusable launch vehicle) via sliding mode control, disturbance observer, and backstepping control approaches (Wang et al. 2015; Dong et al. 2016; Zhang et al. 2013; Zhang et al. 2015; Hall et al. 2006; Tian et al. 2015). Compared with the attitude tracking control problem for Mars entry studied in this paper, a) those reentry vehicles mentioned above are significantly different from the typical low-lift Mars entry vehicle, so their dynamic uncertain factors and control capacity are different accordingly; b) the existing hybrid control approaches (e.g. combining sliding mode disturbance observer and sliding mode controller) developed for reentry vehicles have not consider limited control capacity and potential saturation issue, so they can not be directly used for the attitude tracking control problem of Mars entry that studied in this paper. In theory, high order sliding mode control not only can commendably overcome the chattering problem of one order sliding mode, but also retain the high precision and robustness (Sagliano et al. 2017; Huang et al. 2015). Furfaro et al. (2012) applied multiple sliding surface control to Mars entry guidance issue. Li et al. (2015) developed neural network based second-order sliding mode guidance for Mars entry. As one of useful second order sliding mode control methods, super-twisting algorithm is the unique that doesn't need any differential information of sliding mode variable respect to time in advance, and it has less adaptive learning parameters to be propitious to real-time control (Pico et al. 2013). Jiang et al. (2015) developed super-twisting sliding mode disturbance observer (SMDO) based adaptive compensation control for Mars entry attitude tracking. Since the super-twisting algorithm contains a discontinuous function under the integral, chattering is not eliminated but attenuated.

To date, there are some articles that address robust G&C for Mars entry under uncertainties and/or disturbances. However, few simultaneously consider the entry uncertainties and the control saturation issue due to limited actuator capability. The control saturation inevitably degrades the robustness and accuracy of the G&C. The aim of this paper is to extend the preceding work (Li et al. 2015; Jiang et al. 2015) and develop a new hybrid robust attitude control strategy to further enhance the robustness and accuracy of Mars entry in the presence of uncertain disturbances and limited actuator capability. In this paper, we propose the super-twisting sliding mode disturbance observer based anti-windup terminal sliding mode controller for Mars entry attitude tracking. The main challenge is to ensure the system stability and enable the hybrid control framework makes full use of the advantages of the three algorithms to improve the robustness and accuracy. The rest of this paper is organized as follows. First, we introduce the Mars entry dynamical model with uncertainty/disturbance and the attitude control architecture utilized in this paper. Second, the terminal sliding mode attitude control law is presented for slow-loop. Third, the super-twisting sliding mode disturbance observer based terminal sliding mode attitude control with anti-windup is proposed for fast-loop. Fourth, simulation results and analysis are presented in detail to demonstrate the effectiveness of the proposed approach. Finally, the conclusion of our work is summarized.

Problem formulation

Mars entry attitude dynamical model

The guidance command of Mars entry vehicle is achieved through modulating attitude with RCS. The impact of the Martian rotation is ignored here. The following nonlinear rigid body dynamics equations are used to describe the Mars entry vehicle attitude dynamics. In order to be compatible with the new configuration of future Mars entry vehicle and facilitate to improve the control strategy proposed in this paper, a group of dynamics equations with general form is adopted here (Jiang et al. 2015; Mooij 2013).

$$\dot{\alpha} = q - \tan\beta(p\cos\alpha + r\sin\alpha) + \frac{1}{MV\cos\beta}\left(-L + Mg\cos\gamma\cos\mu - T_{rx}\sin\alpha + T_{rz}\cos\alpha\right) \tag{1}$$

$$\dot{\beta} = -r\cos\alpha + p\sin\alpha + \frac{1}{MV}(Mg\cos\gamma\sin\mu - T_{rx}\sin\beta\cos\alpha + T_{ry}\cos\beta - T_{rz}\sin\beta\sin\alpha) \tag{2}$$

$$\dot{\mu} = \sec\beta(p\cos\alpha + r\sin\alpha) + \frac{1}{MV}\big[L\tan\gamma\sin\mu + L\tan\beta - Mg\cos\gamma\cos\mu\tan\beta +$$

$$(T_{rx}\sin\alpha - T_{rz}\cos\alpha)(\tan\gamma\sin\mu + \tan\beta) - (T_{rx}\cos\alpha + T_{rz}\sin\alpha)\tan\gamma\cos\mu\sin\beta + T_{ry}\tan\gamma\cos\mu\big] \tag{3}$$

$$\dot{p} = \frac{I_{yy} - I_{zz}}{I_{xx}}qr + \frac{1}{I_{xx}}(-\dot{I}_{xx}p + l_{Tr}) \tag{4}$$

$$\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}}pr + \frac{1}{I_{yy}}(-\dot{I}_{yy}q + m_{Tr}) \tag{5}$$

$$\dot{r} = \frac{I_{yy} - I_{xx}}{I_{zz}}pq + \frac{1}{I_{zz}}(-\dot{I}_{zz}r + n_{Tr}) \tag{6}$$

where V is the vehicle's velocity; $\gamma, \alpha, \beta, \mu$ are the flight path angle, angle of attack, angle of sideslip, and roll angle, respectively; p, q, r are the roll rate, pitch rate, and yaw rate, respectively; M is the mass of the entry vehicle; L, D are respectively the aerodynamic lift force and drag force which defined as $L = \frac{1}{2}\rho V^2 C_L S_{ref}, D = \frac{1}{2}\rho V^2 C_D S_{ref}$, and ρ, C_L, C_D, S_{ref} are Martian atmospheric density, aerodynamic lift and drag coefficients and reference platform area of the vehicle; T_{rx}, T_{ry}, T_{rz} are the tri-axial thrusts produced by RCS, respectively; l_{Tr}, m_{Tr}, n_{Tr} are the rolling torque, pitching torque, and yawing torque produced by RCS, respectively; I_{xx}, I_{yy}, I_{zz} are the rotational inertia for x, y, z axis, respectively; $\dot{I}_{xx}, \dot{I}_{yy}, \dot{I}_{zz}$ are the variation rate of rotational inertia for x, y, z axis, respectively.

Note that the RCS thrusters' actions will lead to the change of the mass of the Mars entry vehicle, and then the center of mass and the rotational inertia for each axis of the entry vehicle will change accordingly. At the same time, the absolute value of the attitude angles α, β, μ of Mars entry vehicle are always less than $\pi/2$ in practice, and therefore it would not cause singularity and unlimited control torque (Atkins 2014; Paul et al. 2008).

Attitude control architecture

Generally, for Mars entry vehicle, the response speed of attitude angular rate is about one-order faster than the response speed of attitude angle (Atkins 2014; Jiang et al. 2014; Jiang et al. 2015; Paul et al. 2008). Therefore, according to the time scale separation principle, the attitude control system can be divided into fast-loop (i.e. attitude angular rate control loop) and slow-loop (i.e. attitude angle control loop). As shown in Fig.1, two TSMC are respectively designed for fast loop and slow loop. For the Mars entry vehicle, the control authority is usually restricted during the atmospheric entry process. It is prone to result in saturation control problem, which means too large control value to be realized and the system performance will be inevitably degraded. Fast loop nominal controller using terminal sliding mode control, only the sliding mode surface needs to be designed. So the external anti-windup (AW) system is employed to design the actuator input saturation compensator. For the attitude control system, external interference is usually the disturbance torque. The disturbance torque has an obvious impact on attitude angular rate, and a slight influence on attitude angle. Therefore, a super-twisting SMDO is designed for disturbance identification in fast-loop, and the corresponding compensation control is also designed. The main control goal is: under the consideration of restricted control authority and uncertain disturbance, enabling attitude angle $\boldsymbol{\Omega}$ rapidly accurately tracks the reference guidance command $\boldsymbol{\Omega}_c$ via proper control torque command $\boldsymbol{M}_c$. Attitude control is to complete the mapping of the guidance command to the control torque. The RCS is to realize the mapping of the control torque to the attitude thrusters' ignition logic.

Fig. 1. Control architecture for Mars entry vehicle.

The system state of slow-loop is $\boldsymbol{\Omega}=[\alpha,\beta,\mu]^{\mathrm{T}}$. According to the equations (1) ~ (3), the slow-loop attitude dynamical model can be rewritten as an affine nonlinear system:

$$\dot{\boldsymbol{\Omega}}=\boldsymbol{f}_s+\Delta\boldsymbol{f}_s+\boldsymbol{g}_s\boldsymbol{\omega} \tag{7}$$

where state vector function $\boldsymbol{f}_s=[f_\alpha,f_\beta,f_\mu]^{\mathrm{T}}$, $f_\alpha=(-L+Mg\cos\gamma\cos\mu)/(MV\cos\beta)$, $f_\beta=Mg\cos\gamma\sin\mu/(MV)$, $f_\mu=(L\tan\beta+L\tan\gamma\sin\mu-Mg\cos\gamma\cos\mu\tan\beta)/(MV)$, $\boldsymbol{\omega}=[p,q,r]^{\mathrm{T}}$, $\boldsymbol{g}_s=\begin{bmatrix}-\cos\alpha\tan\beta & 1 & -\sin\alpha\tan\beta\\ \sin\alpha & 0 & -\cos\alpha\\ -\cos\alpha\sec\beta & 0 & -\sin\alpha\sec\beta\end{bmatrix}$. For further practical situation, model uncertainty is added here， and $\Delta\boldsymbol{f}_s$ is the uncertainty of $\boldsymbol{f}_s$. Because the expression of $\boldsymbol{g}_s$ is only related to Euler angles, the uncertainty of $\boldsymbol{g}_s$ can be ignored here.

The system state of fast-loop is $\boldsymbol{\omega}=[p,q,r]^{\mathrm{T}}$. Based on the equations (4) ~ (6), the fast-loop attitude dynamical model can be rewritten as an affine nonlinear system:

$$\dot{\boldsymbol{\omega}}=\boldsymbol{f}_f+\Delta\boldsymbol{f}_f+\boldsymbol{g}_f\boldsymbol{M}_c+\boldsymbol{d} \tag{8}$$

where state vector function $\boldsymbol{f}_f=[f_p,f_q,f_r]^{\mathrm{T}}$, $f_p=\frac{I_{yy}-I_{zz}}{I_{xx}}qr+\frac{1}{I_{xx}}\left(-\dot{I}_{xx}p+l_{Tr}\right)$, $f_q=\frac{I_{zz}-I_{xx}}{I_{yy}}pr+\frac{1}{I_{yy}}\left(-\dot{I}_{yy}q+m_{Tr}\right)$, $f_r=\frac{I_{yy}-I_{xx}}{I_{zz}}pq+\frac{1}{I_{zz}}\left(-\dot{I}_{zz}r+n_{Tr}\right)$, $\boldsymbol{g}_f=\begin{bmatrix}I_{xx} & & \\ & I_{yy} & \\ & & I_{zz}\end{bmatrix}^{-1}$, $\boldsymbol{M}_c$ denotes the control torque, $\Delta\boldsymbol{f}_f$ is the uncertainty of the $\boldsymbol{f}_f$, $\boldsymbol{d}$ is external disturbance torque.

Slow-loop terminal sliding mode controller design

In slow-loop subsystem, attitude angular rate is treated as the control variable. Slow-loop starts to response when fast-loop has already reached its stable state. The influence of parameter uncertainty in slow-loop is eliminated via online adaptive identify the boundary of disturbance. For the convenience of controller design, the following assumption is taken into account.

Let us assume that the conditions for the TSMC existence hold. Because the magnitude of the uncertainty items in actual Mars missions is always not infinitely great, according to the existing studies (Dai et al. 2017; Furfaro et al. 2012; Jiang et al. 2014; Lei et al. 2016; Li et al. 2015), the magnitude of uncertainty terms is usually assumed to satisfy $\|\Delta\boldsymbol{f}_s\|\le\eta_1\|\boldsymbol{f}_s\|$, η_1 is a positive real constant and $0<\eta_1<1$, and $\Delta\boldsymbol{f}_s$ and $\boldsymbol{f}_s$ are usually assumed to be mutually independent. According to Eq.(7) and limited actuator capability of the Mars entry vehicle, there exist positive real constants η_2 and η, such that $\|\Delta\boldsymbol{f}_s\|\le\eta_1\|\boldsymbol{f}_s\|\le\eta_2\|\dot{\boldsymbol{\Omega}}\|\le\eta\|\boldsymbol{\Omega}\|$. Therefore, an assumption can be given as follows.

Assumption 1. The model uncertainty term $\Delta \boldsymbol{f}_s$ is independent from the state vector function $\boldsymbol{f}_s$, and the magnitude of $\Delta \boldsymbol{f}_s$ is bounded, thus there exist a positive real constant η, such that $\|\Delta \boldsymbol{f}_s\| \le \eta \|\boldsymbol{\Omega}\|$.

The slow-loop controller is designed to enable $\boldsymbol{\Omega}$ well track the reference attitude command $\boldsymbol{\Omega}_c$, and output a reference attitude angular rate command $\boldsymbol{\omega}_c$ to the fast-loop subsystem. At the same time, the robustness of closed-loop system also should be ensured.

To obtain the finite-time convergence of the attitude angle tracking-error on the sliding surface, the terminal sliding mode surface is defined (Song et al. 2014; Yang et al; 2013)

$$\boldsymbol{s} = \boldsymbol{\Omega}_e + \int_0^t (a_1 \boldsymbol{\Omega}_e + b_1 \boldsymbol{\Omega}_e^{q_1/p_1}) d\tau \tag{9}$$

where $\boldsymbol{\Omega}_e = \boldsymbol{\Omega} - \boldsymbol{\Omega}_c$ denotes attitude tracking error; $a_1 > 0$, $b_1 > 0$, $p_1 > q_1$; p_1 and q_1 are positive odd integers.

The time derivative of (9) is

$$\dot{\boldsymbol{s}} = \dot{\boldsymbol{\Omega}}_e + a_1 \boldsymbol{\Omega}_e + b_1 \boldsymbol{\Omega}_e^{q_1/p_1} = \dot{\boldsymbol{\Omega}} - \dot{\boldsymbol{\Omega}}_c + a_1 \boldsymbol{\Omega}_e + b_1 \boldsymbol{\Omega}_e^{q_1/p_1} \tag{10}$$

Then, substituting (7) into (10) yields

$$\dot{\boldsymbol{s}} = \boldsymbol{f}_s + \Delta \boldsymbol{f}_s + \boldsymbol{g}_s \boldsymbol{\omega} - \dot{\boldsymbol{\Omega}}_c + a_1 \boldsymbol{\Omega}_e + b_1 \boldsymbol{\Omega}_e^{q_1/p_1} \tag{11}$$

A reaching law is employed to improve the reaching dynamic quality. Let

$$\dot{\boldsymbol{s}} = \Delta \boldsymbol{f}_s - \hat{\eta} \|\boldsymbol{\Omega}\| \operatorname{sgn}(\boldsymbol{s}) - \frac{1}{2} \boldsymbol{s} - \frac{1}{2} \boldsymbol{s}^k \tag{12}$$

where $\boldsymbol{s}^k = \left[s_1^k, s_2^k, s_3^k \right]^{\mathrm{T}}$, $s_i^k = |s_i|^k \operatorname{sgn}(s_i)$, $i = 1,2,3$, and $k > 0$.

Note that, $\operatorname{sgn}(\cdot)$ is a continuous function adopted to replace the typical "sign" function to weaken the chattering from the controllers designed in this paper. All the $\operatorname{sgn}(\cdot)$ function mentioned in this paper is defined like the following general form:

$$\operatorname{sgn}(s_i) = \frac{s_i}{\|s_i\| + \Delta_s} \tag{13}$$

where Δ_s is a small positive numeric value ($0 < \Delta_s << 1$).

Thus, a terminal sliding mode controller for the slow-loop subsystem is presented by Theorem 1.

Theorem 1. Consider the slow-loop system (7) satisfying Assumption 1, with the proposed terminal sliding mode surface (9) and reaching law (12), if the control law is designed as

$$\boldsymbol{\omega}_c = \boldsymbol{g}_s^{-1} \left(-\boldsymbol{f}_s + \dot{\boldsymbol{\Omega}}_c - a_1 \boldsymbol{\Omega}_e - b_1 \boldsymbol{\Omega}_e^{q_1/p_1} - \hat{\eta} \|\boldsymbol{\Omega}\| \operatorname{sgn}(\boldsymbol{s}) - \frac{1}{2} \boldsymbol{s} - \frac{1}{2} \boldsymbol{s}^k \right) \tag{14}$$

where the adaptive regulation law of parameter η yields

$$\dot{\hat{\eta}} = m^{-1}\|\boldsymbol{s}\|\|\boldsymbol{\Omega}\| \tag{15}$$

where $\hat{\eta}$ is the estimation of η, $m > 0$; then the slow-loop subsystem is asymptotically stable.

Proof. Choose a bounded Lyapunov function as

$$V = \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s} + \frac{1}{2}m\tilde{\eta}^2 \tag{16}$$

where $\tilde{\eta} = \hat{\eta} - \eta$. From Assumption 1, Eq.(12) and Eq.(15), the time derivative of (16) is

$$
\begin{aligned}
\dot{V} &= \boldsymbol{s}^{\mathrm{T}}\dot{\boldsymbol{s}} + m\tilde{\eta}\dot{\tilde{\eta}} \\
&= \boldsymbol{s}^{\mathrm{T}}(\Delta\boldsymbol{f}_s - \hat{\eta}\|\boldsymbol{\Omega}\|\operatorname{sgn}(\boldsymbol{s}) - \frac{1}{2}\boldsymbol{s} - \frac{1}{2}\boldsymbol{s}^k) + \tilde{\eta}\|\boldsymbol{s}\|\|\boldsymbol{\Omega}\| \\
&\le \|\boldsymbol{s}\|\|\Delta\boldsymbol{f}_s\| - \hat{\eta}\|\boldsymbol{\Omega}\|\|\boldsymbol{s}\| - \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s} - \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}^k + \tilde{\eta}\|\boldsymbol{s}\|\|\boldsymbol{\Omega}\| \\
&\le \eta\|\boldsymbol{\Omega}\|\|\boldsymbol{s}\| - \hat{\eta}\|\boldsymbol{\Omega}\|\|\boldsymbol{s}\| - \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s} - \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}^k + \tilde{\eta}\|\boldsymbol{s}\|\|\boldsymbol{\Omega}\| \\
&= -\frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s} - \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}^k
\end{aligned} \tag{17}
$$

According to Eq.(9), $\dot{V} = 0$ if and only if $\boldsymbol{\Omega}_e = \boldsymbol{0}$, otherwise $\dot{V} < 0$. Hence, the slow-loop subsystem is asymptotically stable. This completes the proof. □

It also indicates that tracking error can converge to zero gradually. The convergence time is determined by the design parameters a_1, b_1, p_1, q_1 (see Appendix B).

Remark 1. Although the fact that $\boldsymbol{\omega}$ is not able to reach $\boldsymbol{\omega}_c$ instantly, the attitude tracking system is still stable. In fact, $\boldsymbol{\omega} \ne \boldsymbol{\omega}_c$ means $\dot{\boldsymbol{\Omega}}_e \ne \boldsymbol{0}$. According to Eq.(10), when $\dot{\boldsymbol{\Omega}}_e \ne \boldsymbol{0}$, $\boldsymbol{\Omega}_e$ could be either $\boldsymbol{\Omega}_e = \boldsymbol{0}$ or $\boldsymbol{\Omega}_e \ne \boldsymbol{0}$. From Eq.(9) and Eq.(17), the Lyapunov function would be negative semi-definite, therefore the attitude tracking system (7) is stable.

Fast-loop SMDO-TSMC-AW design

Nominal terminal sliding mode controller of fast-loop

For the convenience of subsequent fast-loop controller design with anti-windup and super-twisting SMDO, a nominal fast-loop terminal sliding mode controller is firstly designed here. Based on (8), the nominal fast-loop subsystem without uncertainties/disturbances can be described as an affine nonlinear system:

$$\dot{\boldsymbol{\omega}} = \boldsymbol{f}_f + \boldsymbol{g}_f\boldsymbol{M}_{c0} \tag{18}$$

Slow-loop terminal sliding mode controller outputs attitude angular rate $\boldsymbol{\omega}$ which is the reference command $\boldsymbol{\omega_c}$ of fast-loop subsystem. Nominal fast-loop terminal sliding mode controller is required to output the control torque $\boldsymbol{M}_{c0}$.

The terminal sliding mode surface of fast-loop subsystem is selected as (Song et al. 2014; Yang et al; 2013)

$$\boldsymbol{s}_{20} = \boldsymbol{\omega}_{e0} + \int_0^t (a_{20}\boldsymbol{\omega}_{e0} + b_{20}\boldsymbol{\omega}_{e0}{}^{q_{20}/p_{20}})d\tau \tag{19}$$

where $\boldsymbol{\omega}_{e0} = \boldsymbol{\omega} - \boldsymbol{\omega}_c$ denotes nominal attitude angular rate tracking error, $a_{20} > 0$, $b_{20} > 0$, $p_{20} > q_{20}$; p_{20} and q_{20} are positive odd integers.

Taking time derivative of (19) gives

$$\dot{\boldsymbol{s}}_{20} = \dot{\boldsymbol{\omega}}_{e0} + a_{20}\boldsymbol{\omega}_{e0} + b_{20}\boldsymbol{\omega}_{e0}^{q_{20}/p_{20}} \tag{20}$$

Reaching law is chosen as

$$\dot{\boldsymbol{s}}_{20} = -\varepsilon \operatorname{sgn}(\boldsymbol{s}_{20}) - k\boldsymbol{s}_{20} \tag{21}$$

where ε and k are positive real parameters.

Thus, a terminal sliding mode controller for nominal fast-loop subsystem is presented by Theorem 2.

Theorem 2. Consider nominal fast-loop subsystem (18) without uncertainty and disturbance, with the proposed terminal sliding mode surface (19) and reaching law (21), if the control law is designed as

$$\boldsymbol{M}_{c0} = -\boldsymbol{g}_f^{-1}\left(\boldsymbol{f}_f - \dot{\boldsymbol{\omega}}_{c0} + a_{20}\boldsymbol{\omega}_{e0} + b_{20}\boldsymbol{\omega}_{e0}^{q_{20}/p_{20}} + \varepsilon \operatorname{sgn}(\boldsymbol{s}_{20}) + k\boldsymbol{s}_{20}\right) \tag{22}$$

then the fast-loop nominal control is asymptotically stable.

Proof. Choose a bounded Lyapunov function as

$$V = \frac{1}{2}\boldsymbol{s}_{20}^{\mathrm{T}}\boldsymbol{s}_{20} \tag{23}$$

The time derivative of (23) is

$$\dot{V} = \boldsymbol{s}_{20}^{\mathrm{T}}\dot{\boldsymbol{s}}_{20} \tag{24}$$

Substituting (21) into (24), we obtain

$$\dot{V} = \boldsymbol{s}_{20}^{\mathrm{T}}\left(-\varepsilon \operatorname{sgn}(\boldsymbol{s}_{20}) - k\boldsymbol{s}_{20}\right) < 0 \tag{25}$$

Hence, the fast-loop control is asymptotically stable. This completes the proof. □

Predictably, the control law for the subsequent compound fast-loop subsystem may have the similar form with (22).

Anti-windup system design

The aim of the anti-windup system is to implement the actuator input saturation compensation control according to the command $\boldsymbol{M}_c$. Based on state nonlinear equations (4) ~ (6), the fast-loop attitude dynamical model can be rewritten as an affine nonlinear system as follows.

$$\dot{\boldsymbol{\omega}} = \boldsymbol{f}_f + \Delta\boldsymbol{f}_f + \boldsymbol{g}_f \boldsymbol{M}_c + \boldsymbol{d} \tag{26}$$

where $\boldsymbol{M}_c$ denotes control torque (i.e. the torque needed to implement the attitude angle $\boldsymbol{\Omega}$ tracking command $\boldsymbol{\Omega}_c$). $\Delta\boldsymbol{f}_f$ is uncertainty of $\boldsymbol{f}_f$, $\boldsymbol{d}$ denotes external disturbance torque. To cope with model uncertainty and external disturbance, $\boldsymbol{D}_f = \Delta\boldsymbol{f}_f + \boldsymbol{d}$ denotes composite disturbance. For the convenience of controller design, the following assumption is taken into account.

Let us assume that the conditions for the anti-windup system existence hold. The uncertainty item $\Delta\boldsymbol{f}_f$ in the vehicle's dynamics and the external disturbance item $\boldsymbol{d}$ are considered to be independent from system states herein (i.e. $\Delta\boldsymbol{f}_f$ and $\boldsymbol{d}$ are not functions of system states), because both of them are usually unpredicted or unmodeled in advance (Lei et al. 2016; Li et al. 2015). In actual Mars missions, the magnitude of the uncertainty and disturbance is always not infinitely great, the magnitude of uncertainty and disturbance is usually assumed to be bounded in guidance and control system design for Mars entry according to the existing studies (Dai et al. 2017; Furfaro et al. 2012; Jiang et al. 2014; Lei et al. 2016; Li et al. 2015). Therefore, an assumption can be given as follows.

Assumption 2. The composite disturbance $\boldsymbol{D}_f$ is bounded by some positive real constants $\boldsymbol{D}$, i.e. $\boldsymbol{D}_f = \left[D_{f1}, D_{f2}, D_{f3}\right]^{\mathrm{T}}$ is bounded by $\left|D_{fi}\right| \le D_i > 0$, $\boldsymbol{D} = \left[D_1, D_2, D_3\right]^{\mathrm{T}}$, $i = 1, 2, 3$.

In order to eliminate the limited control capability degrades the system performance. The anti-windup compensator is designed as

$$\dot{\boldsymbol{\omega}}_{aw} = \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) + \boldsymbol{g}_f \cdot \left(\boldsymbol{h}(\boldsymbol{\omega}_{aw}) + \Delta\boldsymbol{u}\right) \tag{27}$$

where $\boldsymbol{\omega}_{aw}$ is the anti-windup compensation angular rate, the function $\boldsymbol{h}(\boldsymbol{\omega}_{aw})$ entering the control input plays the same role as the state-feedback term (Herrmann et al. 2010). Hence, $\boldsymbol{h}(\boldsymbol{\omega}_{aw})$ is chosen such that when saturation dose not occur.

Choose a Lyapunov function as

$$V(\boldsymbol{\omega}_{aw}) = \boldsymbol{\omega}_{aw}^{\mathrm{T}} \boldsymbol{P}_{aw} \boldsymbol{\omega}_{aw} \tag{28}$$

where $\boldsymbol{P}_{aw}$ is a positive definite matrix. Then, $\boldsymbol{h}(\boldsymbol{\omega}_{aw})$ can be designed as

$$\boldsymbol{h}(\boldsymbol{\omega}_{aw}) = -\boldsymbol{g}_f^{\mathrm{T}} \cdot \frac{\partial V(\boldsymbol{\omega}_{aw})}{\partial \boldsymbol{\omega}_{aw}} = -2 \cdot \boldsymbol{g}_f^{\mathrm{T}} \boldsymbol{P}_{aw} \boldsymbol{\omega}_{aw} \tag{29}$$

As shown in Fig.1, the outputs of the anti-windup compensation system are acquired as follows

$$\boldsymbol{v_1} = \boldsymbol{\omega_{aw}} \tag{30}$$

$$\boldsymbol{v_2} = \boldsymbol{h}(\boldsymbol{\omega_{aw}}) + \boldsymbol{g}_f^{-1} \cdot \boldsymbol{f}_f(\boldsymbol{\omega_{aw}}) \tag{31}$$

Fast-loop terminal sliding mode control with anti-windup system

As mentioned above, the terminal sliding mode surface for fast-loop subsystem is chosen as

$$\boldsymbol{s}_2 = \boldsymbol{\omega_e} + \int_0^t (a_2 \boldsymbol{\omega_e} + b_2 \boldsymbol{\omega_e}^{q_2/p_2}) d\tau \tag{32}$$

where $\boldsymbol{\omega_e} = \boldsymbol{\omega}_{e0} - \boldsymbol{\omega_{aw}} = \boldsymbol{\omega} - \boldsymbol{\omega}_c - \boldsymbol{\omega_{aw}}$ denotes tracking error; $a_2 > 0$, $b_2 > 0$, $p_2 > q_2$; p_2 and q_2 are positive odd integers. One of the advantages of TSMC is the reachability of the sliding surface (Guo et al. 2016).

From (26) and (37), the time derivative of (32) is

$$\begin{aligned}
\dot{\boldsymbol{s}}_2 &= \dot{\boldsymbol{\omega}}_e + a_2 \boldsymbol{\omega_e} + b_2 \boldsymbol{\omega_e}^{q_2/p_2} \\
&= \dot{\boldsymbol{\omega}} - \dot{\boldsymbol{\omega}}_c - \dot{\boldsymbol{\omega}}_{aw} + a_2 \boldsymbol{\omega_e} + b_2 \boldsymbol{\omega_e}^{q_2/p_2} \\
&= \boldsymbol{f}_f + \boldsymbol{g}_f \boldsymbol{M_c} - \dot{\boldsymbol{\omega}}_c - \boldsymbol{f}_f(\boldsymbol{\omega_{aw}}) - \boldsymbol{g}_f \cdot \boldsymbol{h}(\boldsymbol{\omega_{aw}}) - \boldsymbol{g}_f \Delta \boldsymbol{u} + a_2 \boldsymbol{\omega_e} + b_2 \boldsymbol{\omega_e}^{q_2/p_2} \\
&= \boldsymbol{f}_f + \boldsymbol{g}_f \boldsymbol{M}_c - \dot{\boldsymbol{\omega}}_c - \boldsymbol{f}_f(\boldsymbol{\omega_{aw}}) - \boldsymbol{g}_f \cdot \boldsymbol{h}(\boldsymbol{\omega_{aw}}) + a_2 \boldsymbol{\omega_e} + b_2 \boldsymbol{\omega_e}^{q_2/p_2}
\end{aligned} \tag{33}$$

Reaching law is designed as:

$$\dot{\boldsymbol{s}}_2 = -\boldsymbol{K}\boldsymbol{s}_2 \tag{34}$$

where $\boldsymbol{K}$ denotes the gain of the reaching law. In fact, the response speed of the fast-loop is deferent from that of the slow-loop, i.e. the speed for the state errors in the fast-loop converge to zero are deferent from that of the slow-loop. Therefore, under the premise of ensuring the suppression of buffeting, the reaching law employed by the fast-loop is deferent from that of the slow-loop.

Thus, based on the nominal control law (22), a terminal sliding mode controller with anti-windup for fast-loop subsystem is presented by Theorem 3.

Theorem 3. Consider fast-loop subsystem (26) satisfying Assumption 2, with the proposed terminal sliding mode surface (32) and reaching law (34), if the control law is designed as

$$\boldsymbol{M}_c = -\boldsymbol{g}_f^{-1} \left(\boldsymbol{f}_f - \dot{\boldsymbol{\omega}}_c - \boldsymbol{f}_f(\boldsymbol{\omega_{aw}}) - \boldsymbol{g}_f \cdot \boldsymbol{h}(\boldsymbol{\omega_{aw}}) + a_2 \boldsymbol{\omega_e} + b_2 \boldsymbol{\omega_e}^{q_2/p_2} + \boldsymbol{K}\boldsymbol{s}_2 \right) \tag{35}$$

then the fast-loop terminal sliding mode control with anti-windup system is asymptotically stable and the tracking error $\boldsymbol{\omega_e}$ can converge to zero asymptotically.

Proof. Given Lyapunov function $V(\boldsymbol{\omega_{aw}}) > 0$, there exist $\alpha_i > 0$, $i = 1,2,3,4$, they yield to

$$0 < \alpha_1 \left\| \boldsymbol{\omega}_{aw} \right\| \le V(\boldsymbol{\omega}_{aw}) \le \alpha_2 \left\| \boldsymbol{\omega}_{aw} \right\| \tag{36}$$

$$\frac{\partial V(\boldsymbol{\omega}_{aw})}{\partial \boldsymbol{\omega}_{aw}} \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) \le -\alpha_3 \left\| \boldsymbol{\omega}_{aw} \right\| \tag{37}$$

$$\left\| \frac{\partial V(\boldsymbol{\omega}_{aw})}{\partial \boldsymbol{\omega}_{aw}} \right\| \le \alpha_4 \left\| \boldsymbol{\omega}_{aw} \right\| \tag{38}$$

As state variable $\boldsymbol{\omega}_{aw}$ finally reaches its stable state, according to (29), $\boldsymbol{h}(\boldsymbol{\omega}_{aw})$ will also reach its stable state. In order to facilitate the proof, we consider the stability of system (26) when $\boldsymbol{h}(\boldsymbol{\omega}_{aw}) = \boldsymbol{0}$, then we have

$$\dot{V}(\boldsymbol{\omega}_{aw}) = \frac{\partial V(\boldsymbol{\omega}_{aw})}{\partial \boldsymbol{\omega}_{aw}} \dot{\boldsymbol{\omega}}_{aw} = \frac{\partial V(\boldsymbol{\omega}_{aw})}{\partial \boldsymbol{\omega}_{aw}} \left(\boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) + \boldsymbol{g}_f \Delta \boldsymbol{u} \right) \tag{39}$$

In fact, $\Delta \boldsymbol{u}$ can be treated as a function of $\boldsymbol{M}_c$, denote $\Delta \boldsymbol{u} = \boldsymbol{\Psi}(\boldsymbol{M}_c)$. Combing (33), (34), (37) and (38), we arrive at $\alpha_3 \left\| \boldsymbol{\omega}_{aw} \right\| \ge \alpha_4 \left\| \boldsymbol{\omega}_{aw} \right\| \left\| \boldsymbol{g}_f \right\| \left\| \boldsymbol{g}_f^{-1} \right\| \left\| \dot{\boldsymbol{\omega}}_c + \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) - \boldsymbol{f}_f - a_2 \boldsymbol{\omega}_e - b_2 \boldsymbol{\omega}_e^{q_2/p_2} - \boldsymbol{K}\boldsymbol{s}_2 \right\|$, there is

$$\dot{V}(\boldsymbol{\omega}_{aw}) = \frac{\partial V(\boldsymbol{\omega}_{aw})}{\partial \boldsymbol{\omega}_{aw}} \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) + \frac{\partial V(\boldsymbol{\omega}_{aw})}{\partial \boldsymbol{\omega}_{aw}} \boldsymbol{g}_f \Delta \boldsymbol{u} \le -\alpha_3 \left\| \boldsymbol{\omega}_{aw} \right\| + \alpha_4 \left\| \boldsymbol{\omega}_{aw} \right\| \left\| \boldsymbol{g}_f \right\| \left\| \boldsymbol{\Psi}(\boldsymbol{M}_c) \right\|$$

$$\le -\alpha_3 \left\| \boldsymbol{\omega}_{aw} \right\| + \alpha_4 \left\| \boldsymbol{\omega}_{aw} \right\| \left\| \boldsymbol{g}_f \right\| \left\| \boldsymbol{g}_f^{-1} \right\| \left\| \dot{\boldsymbol{\omega}}_c + \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) - \boldsymbol{f}_f - a_2 \boldsymbol{\omega}_e - b_2 \boldsymbol{\omega}_e^{q_2/p_2} - \boldsymbol{K}\boldsymbol{s}_2 \right\| < 0 \tag{40}$$

Therefore, as $\boldsymbol{\omega}_e$ converges to zero, $\dot{V}(\boldsymbol{\omega}_{aw}) < 0$. This completes the proof. □

Super-twisting sliding mode disturbance observer design

The external disturbance is only considered to be in the fast-loop subsystem, during Mars hypersonic entry process, because the disturbance torque has more influence on the angular rate while few influence on the attitude angle. In fact, it is impossible to predict the composite interference, much less the variation rate of the external disturbance. Because the advantages of the super-twisting SMDO illustrated as follows, the super-twisting SMDO is introduced to estimate the disturbance in the fast-loop and corresponding compensation control law is derived in this section. a) As one of the second order sliding mode control algorithms, it can avoid the chattering problem of a first order sliding mode control, and also inherits the advantages of the latter (Huang et al. 2015). b) Since the super-twisting algorithm contains a discontinuous function under the integral, chattering can be attenuated (Pico et al. 2013). It is more helpful for combine with the terminal sliding mode controller than using other observers. c) Super-twisting algorithm does not require time derivative data of any sliding mode variables, so it has fewer adaptive learning parameters than neural network (Jiang et al. 2015; Li et al. 2015; Zhang et al. 2015).

The super-twisting controller *u* is calculated as follows (Huang et al. 2015; Pico et al. 2013; Jiang et al. 2015):

$$\begin{cases} u = -\lambda_1 |\sigma| \operatorname{sgn}(\sigma) + u_1 \\ \dot{u}_1 = -\lambda_2 \operatorname{sgn}(\sigma) \end{cases} \tag{41}$$

where σ is the sliding mode surface, while $\dot{\sigma}$ is not required.

Theorem 4. Suppose that Assumption 2 hold, for the composite disturbance in fast-loop nonlinear system (26), if the super-twisting SMDO is designed as

$$\begin{cases} \boldsymbol{\sigma} = \boldsymbol{\omega} + \boldsymbol{z} \\ \dot{\boldsymbol{z}} = -\boldsymbol{f_f} - \boldsymbol{g_f}\boldsymbol{u} - \boldsymbol{v} \\ \hat{\boldsymbol{D}}_{\boldsymbol{f}} = \boldsymbol{v} \end{cases} \tag{42}$$

where

$$\boldsymbol{v} = \boldsymbol{l_1} \cdot |\boldsymbol{\sigma}|^{\frac{1}{2}} \cdot \operatorname{sgn}(\boldsymbol{\sigma}) + \boldsymbol{l_2} \cdot \int \operatorname{sgn}(\boldsymbol{\sigma}) d\boldsymbol{\tau} \tag{43}$$

Adaptive law of parameter $\boldsymbol{l_1}$ is

$$\dot{l}_{1i} = \rho_i \|\omega_i\| \|\sigma_i\| \tag{44}$$

Relationship between $\boldsymbol{l_2}$ and $\boldsymbol{l_1}$ is

$$l_{2i} = \frac{\varepsilon}{2} l_{1i} + \frac{1}{2}\varepsilon^2 + \frac{1}{2}\lambda \tag{45}$$

where $\boldsymbol{\sigma}$ is assist sliding mode surface, $\boldsymbol{\omega}$ is the system state of the fast-loop, $\boldsymbol{z}$ is the state of the observer, $\boldsymbol{v}$ is the input of assist control, λ and ε are positive real number, *i* denotes the ***i***-th component in a vector, $\rho_i > 0$. And there exist a conservative design satisfies $\delta_i |\sigma_i|^{\frac{1}{2}} > D_i$, $\delta_i > 0$. Then, the assist sliding mode surface $\boldsymbol{\sigma}$ can converged to zero asymptotically, and $\boldsymbol{v}$ can be a precise estimation of composite disturbance item $\boldsymbol{D_f}$ (i.e. $\hat{\boldsymbol{D}}_{\boldsymbol{f}} = \boldsymbol{v}$). In order to facilitate the subsequent proof, Lemma 1 and Remark 1 are introduced as follows.

Lemma 1 (Moreno et al. 2012)**.** Consider the nonlinear system as follows:

$$\begin{cases} \dot{x}_1 = -\iota |x_1|^{\frac{1}{2}} \operatorname{sgn}(x_1) + x_2 + \xi(t) \\ \dot{x}_2 = -\varpi \operatorname{sgn}(x_1) \end{cases} \tag{46}$$

This equation can be rewritten as $\dot{x}_1 + \iota |x_1|^{\frac{1}{2}} \operatorname{sgn}(x_1) + \varpi \int \operatorname{sgn}(x_1) d\tau = \xi(t)$, $\xi(t)$ denotes the disturbance item. $\iota > 0$, $\varpi > 0$, the values of parameters ι and ϖ can be selected according to the rules presented by Moreno et al. (2012).

Remark 2. Let $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\sigma}}$ respectively match x_1 and $\dot{x}_1$ in Lemma 1, if parameters $\boldsymbol{l_1}$ and $\boldsymbol{l_2}$ are designed to enable the sliding mode surface $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\sigma}}$ converge to zero asymptotically, then $\boldsymbol{l_1}$ reaches its stable value $\boldsymbol{l_{1_0}}$. Thus, the observed composite disturbance $\hat{\boldsymbol{D}}_{\boldsymbol{f}}$ could uniform converges to its real value (Jiang et al. 2015).

Proof. For Theorem 4, from (8) and (42), time derivation of $\boldsymbol{\sigma}$ is

$$\dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{s}} + \dot{\boldsymbol{z}} = \boldsymbol{f_f} + \boldsymbol{g_f}\boldsymbol{u} + \boldsymbol{D_f} - \boldsymbol{f_f} - \boldsymbol{g_f}\boldsymbol{u} - \boldsymbol{v} = \boldsymbol{D_f} - \boldsymbol{v} \tag{47}$$

After transposition, that is

$$\dot{\boldsymbol{\sigma}} + \boldsymbol{v} = \boldsymbol{D_f} \tag{48}$$

Substituting (43) into (48), and rewriting (48) as the form of (46), that is

$$\begin{cases} \dot{\boldsymbol{\sigma}} = -\boldsymbol{l_1}.|\boldsymbol{\sigma}|^{\frac{1}{2}}.\mathrm{sgn}(\boldsymbol{\sigma}) + \boldsymbol{\upsilon} + \boldsymbol{D_f} \\ \dot{\boldsymbol{\upsilon}} = -\boldsymbol{l_2}.\mathrm{sgn}(\boldsymbol{\sigma}) \end{cases} \tag{49}$$

Let $\boldsymbol{\varsigma_i} = \begin{pmatrix} \varsigma_{i1} \\ \varsigma_{i2} \end{pmatrix} = \begin{pmatrix} |\sigma_i|^{\frac{1}{2}}\mathrm{sgn}(\sigma_i) \\ \upsilon_i \end{pmatrix}$, we have $\boldsymbol{\varsigma_i}^2 = \varsigma_{i1}{}^2 + \varsigma_{i2}{}^2 = |\sigma_i| + \upsilon_i{}^2$, $\mathrm{sgn}(\sigma_i) = \mathrm{sgn}(\varsigma_{i1})$, $|\varsigma_{i1}| = |\sigma_i|^{\frac{1}{2}}$. Then, according to (49), we have:

$$\begin{cases} \dot{\varsigma}_{i1} = \dfrac{1}{2|\sigma_i|^{\frac{1}{2}}}\left(-l_{1i}\varsigma_{i1} + \varsigma_{i2} + D_{fi} \right) \\ \dot{\varsigma}_{i2} = -l_{2i}\mathrm{sgn}(\sigma_i) = -l_{2i}\mathrm{sgn}(\varsigma_{i1}) = -\dfrac{1}{|\sigma_i|^{\frac{1}{2}}} l_{2i}\varsigma_{i1} \end{cases} \tag{50}$$

That is

$$\dot{\boldsymbol{\varsigma}}_{\boldsymbol{i}} = \frac{1}{|\sigma_i|^{\frac{1}{2}}} \begin{bmatrix} -\dfrac{l_{1i}}{2} & \dfrac{1}{2} \\ -l_{2i} & 0 \end{bmatrix} \boldsymbol{\varsigma_i} + \frac{1}{|\sigma_i|^{\frac{1}{2}}} \begin{bmatrix} \dfrac{1}{2} \\ 0 \end{bmatrix} D_{fi} \tag{51}$$

It can be rewritten as

$$\dot{\boldsymbol{\varsigma}}_{\boldsymbol{i}} = \boldsymbol{A_i}\boldsymbol{\varsigma_i} + \boldsymbol{B_i} D_{fi} \tag{52}$$

where $\boldsymbol{A_i} = \dfrac{1}{|\sigma_i|^{\frac{1}{2}}} \begin{bmatrix} -\dfrac{l_{1i}}{2} & \dfrac{1}{2} \\ -l_{2i} & 0 \end{bmatrix}$, $\boldsymbol{B_i} = \dfrac{1}{|\sigma_i|^{\frac{1}{2}}} \begin{bmatrix} \dfrac{1}{2} \\ 0 \end{bmatrix}$.

In fact, ς_{i1} and ς_{i2} can converge to zero asymptotically.

We select a Lyapunov function as

$$V = \boldsymbol{\varsigma_i}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{\varsigma_i} + \frac{1}{2\kappa}(l_{1i} - \hat{l}_{1i})^2 \tag{53}$$

where κ denotes any positive real number, l_{1i} is the ***i***-th component of $\boldsymbol{l}_1$, $\hat{l}_{1i}$ is the estimation of l_{1i} according to (44).

Let $V_0 = \boldsymbol{\varsigma_i}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{\varsigma_i}$, then (53) can be rewritten as:

$$V = V_0 + \frac{1}{2\kappa}(l_{1i} - \hat{l}_{1i})^2 \tag{54}$$

where $\boldsymbol{P}=\begin{bmatrix}\lambda+\varepsilon^2 & -\varepsilon\\ -\varepsilon & 1\end{bmatrix}$, λ and ε denote any positive real number. Therefore, $\boldsymbol{P}$ is a positive definite matrix.

Based on (52) ~ (54), time derivation of V_0 is

$$\dot{V}_0=\dot{\boldsymbol{\varsigma}}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma}_i+\boldsymbol{\varsigma}_i^{\mathrm{T}}\boldsymbol{P}\dot{\boldsymbol{\varsigma}}_i=\boldsymbol{\varsigma}_i^{\mathrm{T}}(\boldsymbol{P}\boldsymbol{A}_i+\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P})\boldsymbol{\varsigma}_i+2D_{fi}\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma}_i$$

$$=-\frac{1}{|\sigma_i|^{\frac{1}{2}}}\boldsymbol{\varsigma}_i^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{\varsigma}_i+2D_{fi}\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma}_i \tag{55}$$

According to (51) and (52), we can obtain

$$\boldsymbol{Q}=\begin{bmatrix}l_{1i}(\lambda+\varepsilon^2)-2l_{2i}\varepsilon & l_{2i}-\frac{l_{1i}}{2}\varepsilon-\frac{1}{2}(\lambda+\varepsilon^2)\\ * & \varepsilon\end{bmatrix} \tag{56}$$

$$2\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P}=\frac{1}{|\sigma_i|^{\frac{1}{2}}}\begin{bmatrix}\lambda+\varepsilon^2 & -\varepsilon\end{bmatrix} \tag{57}$$

Then, let $\boldsymbol{G}=-\begin{bmatrix}\lambda+\varepsilon^2 & -\varepsilon\end{bmatrix}$, we have

$$\dot{V}_0=-\frac{1}{|\sigma_i|^{\frac{1}{2}}}\boldsymbol{\varsigma}_i^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{\varsigma}_i-\frac{D_{fi}}{|\sigma_i|^{\frac{1}{2}}}\boldsymbol{G}\boldsymbol{\varsigma}_i \tag{58}$$

Let $\dot{V}_0\le-\frac{1}{|\sigma_i|^{\frac{1}{2}}}\varsigma_i^{\mathrm{T}}\tilde{\boldsymbol{Q}}\varsigma_i=-\frac{1}{|\sigma_i|^{\frac{1}{2}}}\varsigma_i^{\mathrm{T}}(\boldsymbol{Q}+\hat{\boldsymbol{Q}})\varsigma_i$, because if $|D_{fi}|\le\delta_i|\sigma_i|^{\frac{1}{2}}$, $\delta_i>0$, then

$$\hat{\boldsymbol{Q}}=-\frac{1}{|\sigma_i|^{\frac{1}{2}}}\begin{bmatrix}-\delta_i(\lambda+\varepsilon^2) & \frac{1}{2}\delta_i\varepsilon\\ * & 0\end{bmatrix} \tag{59}$$

Thus,

$$\tilde{\boldsymbol{Q}}=\boldsymbol{Q}+\hat{\boldsymbol{Q}}=\begin{bmatrix}(l_{1i}-\delta_i)(\lambda+\varepsilon^2)-2l_{2i}\varepsilon & l_{2i}-\frac{1}{2}\varepsilon l_{1i}-\frac{1}{2}(\lambda+\varepsilon^2)+\frac{1}{2}\delta_i\varepsilon\\ * & \varepsilon\end{bmatrix} \tag{60}$$

Let

$$l_{2i}=\frac{1}{2}\varepsilon l_{1i}+\frac{1}{2}\varepsilon^2+\frac{1}{2}\lambda \tag{61}$$

Substituting (61) into (60), then we get

$$\tilde{\boldsymbol{Q}}=\begin{bmatrix}l_{1i}\lambda-(\varepsilon+\delta_i)(\lambda+\varepsilon^2) & \frac{1}{2}\delta_i\varepsilon\\ * & \varepsilon\end{bmatrix} \tag{62}$$

Let $\chi_{\min}(\cdot)$ and $\chi_{\max}(\cdot)$ respectively denote the minimum and maximum Eigen values of a matrix. Thus, when $l_{1i} \geq \dfrac{(\varepsilon+\delta_i)(\lambda+\varepsilon^2)+\dfrac{\varepsilon}{2}(\delta_i^{\,2}+1)}{\lambda}$, $\chi_{\min}(\tilde{\boldsymbol{Q}}) \geq \dfrac{\varepsilon}{2}$, we have

$$\dot{V}_0 = -\frac{\chi_{\min}(\tilde{\boldsymbol{Q}})\chi_{\min}^{\frac{1}{2}}(\boldsymbol{P})}{\chi_{\max}(\boldsymbol{P})}V_0^{\frac{1}{2}} \tag{63}$$

That is $\dot{V}_0 \leq 0$. Let $\vartheta = \dfrac{\chi_{\min}(\tilde{\boldsymbol{Q}})\chi_{\min}^{\frac{1}{2}}(\boldsymbol{P})}{\chi_{\max}(\boldsymbol{P})}$, that is $\vartheta > 0$, then we get

$$\dot{V} = \dot{V}_0 + \frac{1}{\kappa}(l_{1i} - l_{1_0 i})\dot{l}_{1i} = -\vartheta V_0^{\frac{1}{2}} + \frac{1}{\kappa}(l_{1i} - \hat{l}_{1i})\dot{l}_{1i} \leq 0 \tag{64}$$

Obviously, $\dot{V} \leq 0$ means the assist sliding mode surface $\boldsymbol{\sigma}$ can converge to zero asymptotically when the observed composite disturbance $\hat{\boldsymbol{D}}_f$ uniform converges to its real value. This completes the proof. □

SMDO-AW-TSMC design

According to the output of the SMDO, we easily get the compensation control law as follows for the composite disturbance (i.e. model uncertainties and external disturbance).

$$\boldsymbol{M}_1 = -\boldsymbol{g}_f^{-1}\hat{\boldsymbol{D}}_f \tag{65}$$

Hence, combining with the AW-TSMC control law (35), the final SMDO-AW-TSMC control law of the fast-loop subsystem can be obtained as

$$\boldsymbol{M}_c = -\boldsymbol{g}_f^{-1}\left(\boldsymbol{f}_f - \dot{\boldsymbol{\omega}}_c - \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) - \boldsymbol{g}_f \cdot \boldsymbol{h}(\boldsymbol{\omega}_{aw}) + a_2\boldsymbol{\omega}_e + b_2\boldsymbol{\omega}_e^{q_2/p_2} + \boldsymbol{K}\boldsymbol{s}_2 + \hat{\boldsymbol{D}}_f\right) \tag{66}$$

where $\boldsymbol{M}_c = [l_{ctrl}, m_{ctrl}, n_{ctrl}]^{\mathrm{T}}$ denotes the control torques on rolling, pitching, and yawing.

Theorem 5. Consider the fast-loop subsystem (8) satisfying Assumption 1 and Assumption 2, the hybrid SMDO-AW-TSMC control law (66) guarantees that the fast-loop subsystem is asymptotically stable.

Proof. Substituting the control law (66) into the fast-loop state equation (26), we get

$$\dot{\boldsymbol{\omega}} - \dot{\boldsymbol{\omega}}_c = -a_2\boldsymbol{\omega}_e - b_2\boldsymbol{\omega}_e^{q_2/p_2} + \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) + \boldsymbol{g}_f\boldsymbol{h}(\boldsymbol{\omega}_{aw}) - \boldsymbol{K}\cdot\boldsymbol{s}_2 + \boldsymbol{D}_f - \hat{\boldsymbol{D}}_f \tag{67}$$

It can be rewritten as

$$\dot{\boldsymbol{\omega}}_e = -\boldsymbol{A}_f\boldsymbol{\omega}_e + \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) + \boldsymbol{g}_f\boldsymbol{h}(\boldsymbol{\omega}_{aw}) - \boldsymbol{K}\cdot\boldsymbol{s}_2 + \boldsymbol{D}_f - \hat{\boldsymbol{D}}_f \tag{68}$$

where $-\boldsymbol{A}_f$ is a diagonal Hurwitz matrix, it satisfies

$$\boldsymbol{P}_f\boldsymbol{A}_f + \boldsymbol{A}_f^{\mathrm{T}}\boldsymbol{P}_f = -\boldsymbol{Q}_f \tag{69}$$

where $\boldsymbol{Q_f} = \boldsymbol{Q_f}^{\mathrm{T}} > 0$, $\boldsymbol{P_f}$ is a real symmetric matrix which can be factorized as $\boldsymbol{P}_f = \boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}$.

We expand the closed-loop error vector as

$$\boldsymbol{\Psi_f} = [\boldsymbol{\omega}_e^{\mathrm{T}} \quad (\boldsymbol{L}\boldsymbol{\varsigma_i})^{\mathrm{T}}]^{\mathrm{T}} \tag{70}$$

where $\boldsymbol{\varsigma_i} = \begin{pmatrix} \varsigma_{i1} \\ \varsigma_{i2} \end{pmatrix} = \begin{pmatrix} |\sigma_i|^{\frac{1}{2}} \operatorname{sgn}(\sigma_i) \\ \upsilon_i \end{pmatrix}$ is defined in the front of Eq.(50).

The Lyapunov function of the entire closed-loop system is chosen as

$$V_{\boldsymbol{\Psi}_f} = \boldsymbol{\Psi}_f^{\mathrm{T}} \overline{\boldsymbol{P}}_f \boldsymbol{\Psi}_f \tag{71}$$

where $\overline{\boldsymbol{P}}_f = \frac{1}{2}\begin{bmatrix} \boldsymbol{P_f} & 0 \\ 0 & \boldsymbol{I}_{2\times 2} \end{bmatrix}$. Then, time derivation of $V_{\boldsymbol{\Psi}_f}$ is

$$\begin{aligned} \dot{V}_{\boldsymbol{\Psi}_f} &= \frac{1}{2}(\dot{\boldsymbol{\omega}}_e^{\mathrm{T}} \boldsymbol{P_f} \boldsymbol{\omega}_e + \boldsymbol{\omega}_e^{\mathrm{T}} \boldsymbol{P_f} \dot{\boldsymbol{\omega}}_e) + \frac{1}{2}(\dot{\boldsymbol{\varsigma}}_i^{\mathrm{T}} \boldsymbol{P} \boldsymbol{\varsigma_i} + \boldsymbol{\varsigma}_i^{\mathrm{T}} \boldsymbol{P} \dot{\boldsymbol{\varsigma}}_i) \\ &\le -\frac{1}{2}\chi_{\min}(\boldsymbol{Q_f})\|\boldsymbol{\omega}_e\|^2 + \frac{1}{2}(\dot{\boldsymbol{\varsigma}}_i^{\mathrm{T}} \boldsymbol{P} \boldsymbol{\varsigma_i} + \boldsymbol{\varsigma}_i^{\mathrm{T}} \boldsymbol{P} \dot{\boldsymbol{\varsigma}}_i) \\ &= -\frac{1}{2}\chi_{\min}(\boldsymbol{Q_f})\|\boldsymbol{\omega}_e\|^2 + \frac{1}{2}\dot{V}_0 \end{aligned} \tag{72}$$

where the minimum eigen value of $\boldsymbol{Q_f}$ is $\chi_{\min}(\boldsymbol{Q_f}) > 0$, then $-\frac{1}{2}\chi_{\min}(\boldsymbol{Q_f})\|\boldsymbol{\omega}_e\|^2 \le 0$. According to Eq.(63), there is $\dot{V}_0 < 0$. Thus, $\dot{V}_{\boldsymbol{\Psi}_f} < 0$.

Hence, the fast-loop subsystem is asymptotically stable by the control law (66). This completes the proof. □

Remark 3. Although the fact that $\boldsymbol{\omega}$ is not able to reach $\boldsymbol{\omega}_c$ instantly, the attitude tracking system is still stable. In fact, $\boldsymbol{\omega} \ne \boldsymbol{\omega}_c$ means $\boldsymbol{\omega}_e \ne \boldsymbol{0}$. According to Eq.(72), when $\boldsymbol{\omega}_e \ne \boldsymbol{0}$, the Lyapunov function would be negative definite, therefore the attitude tracking system (8) is still asymptotically stable.

Remark 4. In practice, from the actuator point of view, the control torques will be finally realized through proper RCS thrusters' ignition allocation. This implementation process can be expressed as follows.

$$\boldsymbol{M_c} = \boldsymbol{k}_c \boldsymbol{T}_c \boldsymbol{L}_c + \boldsymbol{\zeta}_c \tag{73}$$

where $\boldsymbol{k_c}$ is a 0-1 flag vector which denotes the thrusters' ignition allocation with different thrusts, $\boldsymbol{T}_c$ is a thrust magnitude vector which maps different thrusters, $\boldsymbol{L}_c$ is an effective force arm magnitude vector which matches each thrusters' location and thrust directions, $\boldsymbol{\zeta}_c$ denotes the implementation error vector. Obviously, it is determined by a special RCS configuration.

Remark 5. The control law derived in this paper neither focuses on a specific RCS configuration, nor pay close attention to RCS dynamical realization process. For more general, a restricted tri-axial control torques module is adopted to replace the RCS implementation process and reflect the limited control authority. Here, the restricted actuator module has the following form:

$$\boldsymbol{M}_{cc}(i)=\begin{cases}\boldsymbol{M}_{cc}(i)_{\max} & \text{if} \quad \boldsymbol{M}_{c}(i)>\boldsymbol{M}_{cc}(i)_{\max} \\ \boldsymbol{M}_{c}(i) & \text{if} \quad -\boldsymbol{M}_{cc}(i)_{\max}\le \boldsymbol{M}_{c}(i)\le \boldsymbol{M}_{cc}(i)_{\max} \\ -\boldsymbol{M}_{cc}(i)_{\max} & \text{if} \quad \boldsymbol{M}_{c}(i)<-\boldsymbol{M}_{cc}(i)_{\max}\end{cases} \tag{74}$$

where $\boldsymbol{M}_{cc}$ is the actual control torques output from the RCS, $\boldsymbol{M}_{c}(i)$ and $\boldsymbol{M}_{cc}(i)$ respectively denote the ***i***-axial element of $\boldsymbol{M}_{c}$ and $\boldsymbol{M}_{cc}$, $i=x,y,z$. $\boldsymbol{M}_{cc}(i)_{\max}$ is the available maximum value of $\boldsymbol{M}_{cc}(i)$, it can be further formulated as $\boldsymbol{M}_{cc}(i)_{\max}=\boldsymbol{T}_{r}(i)_{\max}\cdot \boldsymbol{L}_{ref}(i)$, where $\boldsymbol{T}_{r}(i)_{\max}$ and $\boldsymbol{L}_{ref}(i)$ respectively denotes corresponding maximum thrust and reference force arm.

Numerical simulations

In order to validate the effectiveness of the super-twisting sliding mode disturbance observer based terminal sliding mode control with anti-windup (SMDO-TSMC-AW) algorithm for Mars entry attitude tracking addressed in this paper, four simulation cases and analysis have been carried out in the MATLAB/Simulink environment (2010b 64bit version) on a laptop PC of Intel Core (TM) i7-3610QM CPU @ 2.3 GHz.

The reference velocity and attitude profiles adopted here are similar with the reference (Jiang et al. 2015). Mars atmospheric density model, gravity field model, and aerodynamic parameters of entry vehicle are the same as the reference (Li et al. 2015). According to the reference (Lei et al. 2016), the mass of entry vehicle is set to be 900kg, the rotational inertias are set as $I_{xx}=2983\,\text{kg}\cdot\text{m}^2$, $I_{yy}=4909\,\text{kg}\cdot\text{m}^2$, $I_{zz}=5683\,\text{kg}\cdot\text{m}^2$; the initial conditions are set as $\alpha=1^{\circ}$, $\beta=2.5^{\circ}$, $\mu=4^{\circ}$, $p=q=r=0$. Commanded attitude is set as $(\alpha_c,\beta_c,\mu_c)=(3^{\circ},0^{\circ},0^{\circ})$. Disturbance torques are set as: $d_1=10\cdot(\sin(4t)+0.2)$, $d_2=200\cdot(\sin(11t)-0.6)$, $d_3=200\cdot(\sin(5t)+0.2)$. Slow-loop control parameters are set as: $a_1=2$, $b_1=1$, $q_1=7$, $p_1=9$, $k=0.78$. Fast-loop control parameters are set as: $a_2=1$, $b_2=3$, $q_2=7$, $p_2=9$, $\boldsymbol{K}=\text{diag}(15,15,15)$, initial value of l_1 is set as 10. Adaptive law parameter $m=4000$. The available upper bounds for RCS control forces on each axis are set to be 150N and reference force arm is set to be 2.15m. To simulate the model uncertainties, 5% random errors are intentionally introduced to the nominal aerodynamic forces. Because the attitude tracking response generally costs a very short time during Mars entry, the planned simulation time span is supposed to be 7 seconds here. The rest simulation parameters are set as our preceding work (Jiang et al. 2015). The simulation results are shown as follows.

Case 1: purely terminal sliding mode controller

First, two classical TSMC are used for the slow-loop and fast-loop subsystem respectively, without disturbance observer and anti-windup compensator. The attitude tracking control process under both model uncertainties and external disturbances is shown in Fig.2. The expected attitude can be roughly tracked within bad accuracy. The tracking errors are not well convergent. Entry vehicle' attitude trembles frequently. What's worse, the RCS thrust commands are almost saturated all the time. This attitude tracking process is too perilous, which indicates the classical TSMC using the parameters set here for Mars entry attitude tracking has bad robustness and adaptability under larger model uncertainties and external disturbances. As is well known, the TSMC has a good robustness and asymptotic stability in the presence of certain external disturbance. Thus, the possible reasons include: a) the actuators' input saturation leads to attitude tremble; b) the robustness of TSMC is still insufficient to cope with larger model uncertainties and external disturbances during Mars entry when only limited control torques can be used. Based on the above inference, other assistant algorithms should be introduced to enhance the performance of attitude tracking during Mars entry.

Fig. 2. Mars entry attitude tracking based on classical terminal sliding mode controller.

Case 2: hybrid terminal sliding mode controller with anti-windup compensator

In the second case, the TSMC and AW are employed in the attitude tracking architecture, just like the Fig.1 without the disturbance observer. The sliding mode control (SMC) approach mentioned in reference (Zhang et al. 2013) is employed in this simulation case for performance comparison. The attitude tracking control process in the presence of both model uncertainties and external disturbances is shown in Fig.3. The expected attitude can be well tracked within higher accuracy. The attitude tracking process is smooth, and the tracking errors are well convergent. Entry vehicle' attitude is asymptotic stable. As expected, the RCS thrust commands almost have no saturation. The simulation results show that the hybrid TSMC with anti-windup compensator can effectively improve the robustness and accuracy of attitude tracking during Mars entry under larger model uncertainties and external disturbances when the available control torques are restricted. On the other hand, the convergence time (i.e. stable time) is relatively long, and the overshoot is relatively large. The possible reasons are: a) the anti-windup system can effectively compensate the attitude tremble caused by the actuators' input saturation; b) the variations of both uncertainties and disturbances are usually unknown for both the TSMC and the anti-windup compensator, which inevitably causes overshoot. So we guess that the performance of attitude tracking control will be further enhanced if the uncertainties and disturbances can be identified via sliding mode disturbance observer in this framework.

Fig. 3. Mars entry attitude tracking based on terminal sliding mode controller with anti-windup compensator.

Case 3: hybrid terminal sliding mode controller with super-twisting sliding mode disturbance observer

In the third case, the TSMC and super-twisting SMDO are employed in the attitude tracking architecture, just like the Fig.1 without the anti-windup system. The super-twisting SMDO approach mentioned in reference (Jiang et al. 2015) is employed in this simulation case for performance comparison. Here, complete attitude profile commands (instead of a single expected attitude) are intentionally provided to highlight the effect of the super-twisting SMDO. Fig.4 shows the nominal response (i.e. without model uncertainties and external disturbances) and the actual response (i.e. with model uncertainties and external disturbances) of the fast-loop subsystem. The attitude tracking control process with SMDO is smooth and stable, while the attitude tracking control process without SMDO is oscillating. It also can be found that, in the fast-loop subsystem, the compensation control based on the SMDO is superimposed on the nominal response. The reference attitude profile can be well tracked within higher accuracy, and the overshoots are smaller than that of the TSMC with anti-windup. Thus, the hybrid TSMC with SMDO can effectively improve the robustness and accuracy of attitude tracking during Mars entry under larger model uncertainties and external disturbances. Reversely, the compensation control oscillates large relatively. The possible reason is that the control scope has not been strictly restricted. So we guess that the performance of attitude tracking control will be further enhanced if the anti-windup compensator is introduced in this architecture.

Fig. 4. Mars entry attitude tracking based on terminal sliding mode controller with sliding mode disturbance observer.

Case 4: super-twisting SMDO based TSMC with AW

In the fourth case, the hybrid SMDO-TSMC-AW algorithm is adopted to track the reference attitude profile. The SMC and super-twisting SMDO approaches mentioned in references (Zhang et al. 2013, Jiang et al. 2015) are employed in this simulation case for performance comparison. As expected, the attitude tracking control process for Mars entry vehicle in the presence of both model uncertainties and external disturbances has the best performance than the three cases above, just as shown in Fig.5. The TSMC can provide a basic robust tracking control based on the conservative model uncertainties. The SMDO can timely observe the external unknown disturbances and feedback them to the control system, its corresponding compensation control is able to counteract the adverse effects caused by disturbances.

The AW can effectively compensate the attitude tremble caused by the actuators' input saturation. They ensure the reference attitude profile to be tracked rapidly, accurately and smoothly.

Fig. 5. Mars entry attitude tracking based on terminal sliding mode controller with sliding mode disturbance observer and anti-windup compensator.

Conclusions

The robust attitude tracking control problem for Mars entry vehicle with limited control capability under uncertainty and external disturbance is investigated in this paper. Based on super-twisting sliding mode disturbance observer, terminal sliding mode control, and anti-windup techniques, a hybrid SMDO-TSMC-AW algorithm is developed. SMDO is adopted to identify and compensate external disturbance, and then TSMC is used to overcome the adverse effect caused by uncertainty. At the same time, AW is employed to suppress actuators' input saturation. The superior performance of this control approach is verified by numerical simulations compared with the previous works (Jiang et al. 2015; Zhang et al. 2013). In order to obtain a more practical attitude control performance, the RCS control allocation logic and the thrusters' configuration of Mars entry vehicle should be more carefully considered. This issue is left for the future study.

Appendix A. Stability proof of the whole system including fast-loop and slow-loop subsystems.

To clarify the stability of the whole attitude tracking control system, Theorem 6 is provided herein and its proof is given accordingly as follows.

Theorem 6. Consider the closed-loop system equations (7) and (8) satisfying Assumptions 1 and 2, where the fast-loop control law $\boldsymbol{M}_c$ is given by (66) with parameter adaptive laws of (21), (30), (31), (34) and (42), and the slow-loop control law $\boldsymbol{\omega}_c$ is given by (14) with the parameter adaptive laws of (12) and (15). Then, the whole system is asymptotically stable.

Proof. Choose a Lyapunov function as

$$V=\frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}+\frac{1}{2}m\tilde{\eta}^{2}+\boldsymbol{\Psi}_f^{\mathrm{T}}\overline{\boldsymbol{P}}_f\boldsymbol{\Psi}_f \tag{75}$$

where $\boldsymbol{s}=\boldsymbol{\Omega}_e+\int_0^t(a_1\boldsymbol{\Omega}_e+b_1\boldsymbol{\Omega}_e{}^{q_1/p_1})d\tau$, $m>0$, $\tilde{\eta}=\hat{\eta}-\eta$, $\overline{\boldsymbol{P}}_f=\frac{1}{2}\begin{bmatrix}\boldsymbol{P}_f & 0\\ 0 & \boldsymbol{I}_{2\times2}\end{bmatrix}$, $\boldsymbol{\Psi}_f=[\boldsymbol{\omega}_e^{\mathrm{T}}\quad(\boldsymbol{L}\varsigma_i)^{\mathrm{T}}]^{\mathrm{T}}$ (see the text for their detailed definitions).

Differentiating (75) with respect to time yields

$$\dot{V} = \boldsymbol{s}^{\mathrm{T}}\dot{\boldsymbol{s}} + m\tilde{\eta}\dot{\tilde{\eta}} + \frac{1}{2}(\dot{\boldsymbol{\omega}}_e^{\mathrm{T}}\boldsymbol{P_f}\boldsymbol{\omega}_e + \boldsymbol{\omega}_e^{\mathrm{T}}\boldsymbol{P_f}\dot{\boldsymbol{\omega}}_e) + \frac{1}{2}(\dot{\boldsymbol{\varsigma}}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma_i} + \boldsymbol{\varsigma_i}^{\mathrm{T}}\boldsymbol{P}\dot{\boldsymbol{\varsigma}}_i) \tag{76}$$

Substituting (12), (15) and (55) into (76), then

$$\dot{V} = \boldsymbol{s}^{\mathrm{T}}(\Delta\boldsymbol{f_s} - \hat{\eta}\|\boldsymbol{\Omega}\|\mathrm{sgn}(\boldsymbol{s}) - \frac{1}{2}\boldsymbol{s} - \frac{1}{2}\boldsymbol{s}^k) + \tilde{\eta}\|\boldsymbol{s}\|\|\boldsymbol{\Omega}\| + \frac{1}{2}(\dot{\boldsymbol{\omega}}_e^{\mathrm{T}}\boldsymbol{P_f}\boldsymbol{\omega}_e + \boldsymbol{\omega}_e^{\mathrm{T}}\boldsymbol{P_f}\dot{\boldsymbol{\omega}}_e) + \frac{1}{2}\left(-\frac{1}{|\sigma_i|^{\frac{1}{2}}}\boldsymbol{\varsigma_i}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{\varsigma_i} + 2D_{fi}\boldsymbol{B_i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma_i}\right) \tag{77}$$

Substituting (63) and (69) into (77), then

$$\dot{V} \le \boldsymbol{s}^{\mathrm{T}}(\Delta\boldsymbol{f_s} - \hat{\eta}\|\boldsymbol{\Omega}\|\mathrm{sgn}(\boldsymbol{s}) - \frac{1}{2}\boldsymbol{s} - \frac{1}{2}\boldsymbol{s}^k) + \tilde{\eta}\|\boldsymbol{s}\|\|\boldsymbol{\Omega}\| - \frac{1}{2}\chi_{\min}(\boldsymbol{Q_f})\|\boldsymbol{\omega}_e\|^2 - \frac{1}{2}\cdot\frac{\chi_{\min}(\tilde{\boldsymbol{Q}})\chi_{\min}^{\frac{1}{2}}(\boldsymbol{P})}{\chi_{\max}(\boldsymbol{P})}\left(\boldsymbol{\varsigma_i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma_i}\right)^{\frac{1}{2}} \tag{78}$$

According to Assumptions 1 and 2, the following result is obtained

$$\begin{aligned}\dot{V} &\le \|\boldsymbol{s}\|\|\Delta\boldsymbol{f_s}\| - \hat{\eta}\|\boldsymbol{\Omega}\|\|\boldsymbol{s}\| - \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s} - \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}^k + \tilde{\eta}\|\boldsymbol{s}\|\|\boldsymbol{\Omega}\| - \frac{1}{2}\chi_{\min}(\boldsymbol{Q_f})\|\boldsymbol{\omega}_e\|^2 - \frac{1}{2}\cdot\frac{\chi_{\min}(\tilde{\boldsymbol{Q}})\chi_{\min}^{\frac{1}{2}}(\boldsymbol{P})}{\chi_{\max}(\boldsymbol{P})}\left(\boldsymbol{\varsigma_i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma_i}\right)^{\frac{1}{2}} \\ &\le \eta\|\boldsymbol{\Omega}\|\|\boldsymbol{s}\| - \hat{\eta}\|\boldsymbol{\Omega}\|\|\boldsymbol{s}\| - \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s} - \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}^k + \tilde{\eta}\|\boldsymbol{s}\|\|\boldsymbol{\Omega}\| - \frac{1}{2}\chi_{\min}(\boldsymbol{Q_f})\|\boldsymbol{\omega}_e\|^2 - \frac{1}{2}\cdot\frac{\chi_{\min}(\tilde{\boldsymbol{Q}})\chi_{\min}^{\frac{1}{2}}(\boldsymbol{P})}{\chi_{\max}(\boldsymbol{P})}\left(\boldsymbol{\varsigma_i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma_i}\right)^{\frac{1}{2}} \\ &= -\frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s} - \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}^k - \frac{1}{2}\chi_{\min}(\boldsymbol{Q_f})\|\boldsymbol{\omega}_e\|^2 - \frac{1}{2}\cdot\frac{\chi_{\min}(\tilde{\boldsymbol{Q}})\chi_{\min}^{\frac{1}{2}}(\boldsymbol{P})}{\chi_{\max}(\boldsymbol{P})}\left(\boldsymbol{\varsigma_i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma_i}\right)^{\frac{1}{2}}\end{aligned} \tag{79}$$

In fact, $\boldsymbol{P}, \boldsymbol{Q}_f, \tilde{\boldsymbol{Q}}$ are positive definite matrices (mentioned in the text), so $-\frac{1}{2}\chi_{\min}(\boldsymbol{Q_f})\|\boldsymbol{\omega}_e\|^2 \le 0$, $-\frac{1}{2}\frac{\chi_{\min}(\tilde{\boldsymbol{Q}})\chi_{\min}^{\frac{1}{2}}(\boldsymbol{P})}{\chi_{\max}(\boldsymbol{P})}\left(\boldsymbol{\varsigma_i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma_i}\right)^{\frac{1}{2}} \le 0$. Therefore, we can conclude that $\dot{V} \le 0$.

Hence, the whole closed system including slow-loop and fast-loop is asymptotically stable. This completes the proof.

□

Appendix B. Selection of TSMC design parameters and their effects.

In order to ensure rapid and accurate convergence, the design parameters of the TSMC need to be properly selected. Such as a_1, b_1, p_1, q_1, $a_{20}, b_{20}, p_{20}, q_{20}$, and a_2, b_2, p_2, q_2. In this paper, these parameters are selected following some rules in general form illustrated as follows. Note that parameters are denoted by a general form (a, b, p, q) herein.

Considering the sliding mode form of TSMC presented in this study, and let its integral item to be zero, that is

$$\dot{x} + ax + bx^{q/p} = 0 \tag{80}$$

where x denotes the system state variable, a, b are positive constants, and p, q are positive odd constants and $q < p$.

Let's multiply both sides of Eq.(80) by $x^{-q/p}$, then we have

$$x^{-q/p}\dot{x} + ax^{1-q/p} + b = 0 \tag{81}$$

Let $y = x^{1-q/p}$, we have

$$\dot{y} = \frac{p-q}{p} x^{1-q/p}\dot{x} \tag{82}$$

According to Eqs.(81) and (82), we can obtain

$$\dot{y} + a\frac{p-q}{p} y = -b\frac{p-q}{p} \tag{83}$$

Solving Eq.(83), that is

$$y = e^{-a\frac{p-q}{p}t}\left(x(0)^{1-q/p} + b/a\right) - b/a \tag{84}$$

Then, substituting $y = x^{1-q/p}$, that is

$$a\frac{p-q}{p}t = \ln\frac{x(0)^{1-q/p} + b/a}{x^{1-q/p} + b/a} \tag{85}$$

Thus, the convergence time of system state is

$$t_s = \frac{p}{a(p-q)}\ln\frac{ax(0)^{1-q/p} + b}{b} \tag{86}$$

It can be found from Eq.(86) that the convergence time is subject to a, b, p, q and initial state $x(0)$. Therefore, the parameters a, b, p, q can be selected for a proper convergence time, and the convergence time can be calculated via Eq.(86).

To show the effect of these design parameters (a, b, p, q) on the convergence time, some numerical results are listed in Table 1 under the conditions of $x(0) = 2$, $q = 5$, and $p = 7$.

Table 1. Example results of a, b and t_s.

In order to further discuss the effects of the design parameters, Eq.(80) is rewritten as

$$\dot{x} = -ax - bx^{q/p} \tag{87}$$

It can be found from Eq.(87) that (a) when system state is far from the balance point, convergence speed is mainly determined by $-bx^{q/p}$; (b) when system state is near the balance point, convergence speed is mostly subject to $-ax$.

Therefore, the design parameters a,b would affect the convergence speed in different situations. (c) By choosing a sufficient small value for q/p, it can urge the system state to reach a sufficient small neighborhood near the sliding mode surface, and then converge to the balance state along the sliding mode surface. Hence, parameters p,q would affect the accuracy of the convergence.

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