THE TEACHER-TEXT INTERACTION IN MATHEMATICS INSTRUCTION: ELEMENTARY AND MIDDLE SCHOOL TEACHERS REDESIGN MATHEMATICS EXERCISES TO INCREASE COGNITIVE DEMAND

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ABSTRACT

Mathematics teachers modify problems for use in their classrooms but we know little about what exercises they choose to modify, in what ways they typically modify them, and what role teaching experience plays. Additionally, the modification of pre-written tasks can impact the mathematics students have the opportunity to learn, so it is important to learn more about this phenomenon. This qualitative, empirical study explored the ways that K-8 teachers interacted with mathematics curriculum materials with the goal of redesigning exercises into more cognitively demanding tasks. Seventy early career and experienced teachers participated in the study, which took place over the course of a school year. Data were collected in two stages, first from early career teachers and then with more experienced teachers. The collected data consisted of the original and redesigned tasks, teachers' written reflections on the work that they did, audio-recorded work sessions of third and sixth grade teacher groups, and the researcher's reflective journal. Analysis occurred within and between grade bands and teacher experience levels. Findings indicate that teachers tended to choose simple exercises to modify and typically redesigned them by making structural changes to the task, or by opening them up to become student explorations. Results of this study indicate that teachers may benefit from frameworks that detail a hierarchy of how children best develop mathematical understanding of concepts beyond basic number operations.

CHAPTER 1: INTRODUCTION

Problem Statement

The purpose of this study is to investigate how teachers re-design mathematics exercises into mathematics tasks in order to increase the cognitive load placed on their students. This is an important problem to investigate because despite the fact that teachers all adapt and modify curriculum on a regular basis (Ball & Feiman-Nemser, 1988; Elsaleh, 2010, Grossman & Thompson, 2004. & Nicol & Crespo, 2006), little is known about what mathematics exercises teachers choose to modify, what they consider as they go about making those changes, or what types of modifications they make. Research indicates that variations in adaptations have different effects on student learning (De Araujo, Jacobson, Singletary, Wilson, Lowe, & Marshall, 2013; Stodolsky & Grossman, 2000) which can impact the choices available to students in subsequent years. Therefore, it is important to learn about what teachers consider as they modify and adapt mathematics tasks for use in their classrooms so that teacher educators can effectively prepare teachers for this task.

Although teachers are often positioned as the central agents of change in mathematics education reform efforts, this positioning puts a large burden on them. Teachers are expected to understand mathematics content and how to make that content accessible to students. They also need to know what contributes to the rigor of a mathematics task and how to use specialized knowledge to design learning experiences for students that will increase their mathematical knowledge. In light of the scarcity of resources in schools, teachers may have little guidance as they make important decisions about how to modify and adapt existing, possibly deficient, curriculum materials (Heitin, 2015; National Council of Teachers of Mathematics, 2015) into rigorous mathematics tasks.

Teachers, however, are very creative, resilient, and smart people. They know that realization of the ambitious vision of mathematics achievement posed by National Council of Teachers of Mathematics and the Common Core State Standards for Mathematics will first occur with them before it filters through to students, their parents, and the public. They are also well aware that resources are limited. A part of teachers’ work, taking place long before the current reform effort, includes reading and analyzing standards, and making adaptations to curriculum materials and instruction techniques in order to best meet the needs of their students. Therefore, it is important to acknowledge and honor teachers’ professional knowledge of mathematics, pedagogy, and student development as we learn about how teachers modify and adapt mathematics exercises. To further understanding of this phenomenon, I propose the following research investigation.

Research Questions

The following research questions will guide my study as I strive to find out how teachers modify and adapt pre-written mathematics exercises to use in their classrooms:

1) In what ways do novice and experienced teachers re-design pre-written mathematics exercises to increase the level of student thinking?
2) What do teachers report as important considerations in the re-design of mathematics exercises?
3) What differences do I notice between the work of early-career and experienced teachers?

Why Study This Issue?

When I was a middle school mathematics teacher, I noticed that my grade level colleagues and I often enacted the same lesson in very different ways. We used the same textbook but our lessons were always a variation on what the text suggested and we often approached the topics differently. An example is that of teaching integer addition and subtraction. My grade-level colleague chose a direct route, providing students with the algorithmic rules, modeling problems, and guiding students as they practiced, where I chose a more round-about method of using number lines and chip boards to illustrate the concept to the students and to build understanding. We were both enacting lessons from the same textbook but interpreted them very differently and, as a result, our students received different instructional experiences.

Later, in my work supervising student teachers, I noticed variations in the way teacher candidates planned and modified lessons. For example, some would emphasize the procedural components of a lesson and completely bypass the problem-solving aspect while others would focus on the conceptual ideas but inadvertently provide scaffolds that would, in effect, convert the mathematics concepts into procedural problems. It caused me to wonder about how teachers make those instructional decisions. It made me want to learn more about what teachers think about and consider as they interact with mathematics curriculum materials and how the Common Core State Standards for Mathematics could be made more accessible to teachers as they plan learning experiences for their students. In preparation for this dissertation study, which was funded by a grant from the Arizona Board of Regents (project #ITQ0135-UA), I designed and conducted a small pilot study in an attempt to learn more. In the following section, I will outline a summary of the pilot study and describe how the results informed my dissertation.

Pilot Study

I worked with seven early career middle school mathematics teachers during the spring of 2013 with the purpose of understanding how they adapted mathematics exercises into problems to align with the Common Core State Standards for Mathematics. The group consisted of six females and one male who were all within their first three years of practice. Five were Caucasian, two Latino, and they ranged in age from mid-twenties to mid-forties. They all worked for the same public school district, located in a suburban area in the southwestern United States. The district used a traditional mathematics curriculum and a standards-referenced system of mathematics instruction. All of the teachers used the same district-adopted textbook in their instruction, along with various supplemental materials they found on their own.

Findings from Pilot Study

A theme that emerged from the pilot study was that the teachers inherently trusted the curriculum materials they used and were apprehensive about making any changes to them. Though they all incorporated supplemental materials into their instruction, those tended to also be used verbatim. They seemed to feel like they were not authorized or empowered to make changes to pre-written mathematics exercises and, though they expressed an understanding of the benefits of doing so, they struggled with how to go about it. They excitedly shared with one another the supplemental resources they found and enjoyed hearing how their colleagues enacted them, but tended not to question or critique the activities. Because this went against what I experienced in practice, I wondered if their struggle was due to their level of teaching experience. I found that this particular group of new teachers had limited opportunities to experience adapting mathematics exercises and those findings made me want to learn more about what they would do with such an opportunity.

Findings from the pilot study informed the design of my dissertation study in several ways. First, the teachers in the pilot study were all from the same school district and, as such, all used the same textbook series to teach their classes. The single set of curriculum materials did not provide a broad enough perspective from which they could work. For the current study, teachers from three area school districts, who all use different curriculum materials, were invited to participate. This provided a broader context from which to learn about the phenomenon in question.

Second, the sample size of seven middle school teachers was too small to generate the kind of collaboration and dialogue I had hoped for. Dividing into two groups felt superficial and contrived when the teachers were more united as a single group of seven. Therefore, for the current study, I invited both elementary and middle school teachers to participate, and recruited a much larger group of participants. By bringing many teachers, with diverse experience and perspectives, together to explore the act of modifying and adapting mathematics exercises I felt that a broader base of contextual knowledge could be obtained which would allow me to paint a more detailed picture of this phenomenon as it plays out in the field.

Finally, results from the pilot study made me wonder if teaching experience impacts the process of task modification. I was concerned, however, that if I included more experienced teachers in the conversation, they would take over the process and inadvertently silence the voices of novice teachers. As a solution, I created two different sessions; the first consisted of teachers with five or fewer years of experience, and the second consisted of teachers with between five and ten years of experience. This structure allowed me to understand potential differences in how the work of early career teachers differed from that of more experienced teachers.

Theoretical Frame

There are many lenses through which to view this problem and, as such, it is important to make explicit that I chose to view the literature outlined in this paper through the lens of teachers as designers (Brown, 2009). Brown suggested that teachers bring to life the ideas expressed in mathematics curriculum materials in ways unique to their own beliefs, background, and knowledge bases. Similar to how particular songs sound both similar and distinct when performed by different artists, the way teachers interact with curriculum materials, and take agency with their use, is a factor of how they position themselves in relation to those materials.

Although curriculum can be an important vehicle for reforming mathematics instruction, studies have indicated that curriculum adoption in itself asserts only minimal influence upon the practice of teachers (Ball & Cohen, 1996; Collopy, 2003; Fullan & Pomfret, 1977; Stein, Grover, & Henningsen, 1996). To achieve large-scale changes in mathematics learning, placing high-quality curriculum materials in the hands of teachers is but one component. We also need to understand what teachers do with those materials in order to bring about the desired change. Professional development, a common way to further the education of teachers, can be helpful in guiding the teacher text interaction but the variability in quality and frequency of professional development can be a barrier to consistent results. For this study, I chose to take the perspective that teachers are designers of instruction for which curriculum materials are tools.

This framework of teaching as design (Brown, 2009) provides an appropriate way to ground my observations and understand the phenomenon of the teacher text interaction.

Looking through the lens of teaching as a design activity presumes that the way teachers position themselves in relation to the tools of curriculum, (how they interact with curriculum materials), is inextricably connected to the impact the curriculum will make on student learning.

Prior Relevant Research

In the following sections I briefly summarize what previous scholars have learned about the teacher text interaction. In chapter two, I elaborate on extant research on curriculum, teachers, and the teacher-text interaction, along with specific examples of what we have learned from recent research.

We know that curriculum work presents many challenges for teachers (Drake & Sherin, 2009; Lloyd, 1999; & Remillard, 2000) and that they often use materials in ways not intended by the writers (Collopy, 2003; Remillard & Bryans, 2004). We know that, to some extent, teachers all adapt curriculum materials (Elsaleh, 2010), and variations in those adaptations can have different effects on student learning (De Araujo et al, 2013; Stodolsky & Grossman, 2000). The body of literature on teachers' use of curriculum materials is in the early stages and still growing (Lloyd, Remillard, Herbel-Eisenmann, 2009) and currently we have limited knowledge about *how* teachers make curriculum adaptation decisions to best serve the needs of their students (Taylor, 2013). Learning about how teachers go about adapting and modifying curriculum materials, and how we can support them in doing so, are important areas for research.

Overview of Methods

In this section, I will provide a brief overview of the methodology used in my study. Further details about the research design and data collection and analysis methods are provided in Chapter 3. This study is a qualitative investigation around a professional development intervention where teachers from several area districts came together to explore routine mathematics exercises and re-design them into more rigorous tasks that they individually enacted in their classrooms. Afterwards, the teachers came back together to reflect on both the re-designed tasks and the enactment of those tasks. Primary sources of data include the before-and-after task artifact work of the teachers, teacher written reflections on the process of task redesign, audio recordings that captured group conversations about the task re-design process and enactment details including discussion of the difficulties teachers' came across in the classroom, and refinements they said they would make to future enactments, and researcher notes.

Potential Benefits

The results of my study will contribute to the body of knowledge on the teacher-text interaction by answering the research questions listed above. By investigating what mathematics exercises teachers choose to re-design, what types of modifications they make, and what they consider as they re-designed tasks, my study will contribute to the understanding of the strengths teachers bring to curriculum adaption work, along with the challenges they face. I also suggest possible future research directions so that we can learn more about how future teacher education programs can best prepare potential teachers to take on mathematics teaching in a *standards-based* system, and what curriculum designers can do to create materials that complement the work of teachers.

Definition of Key Terms

The term 'curriculum' has been defined and used in various ways throughout history. From Levine's (1981) very narrow *body of courses* to Taylor's (1950) more general description of ones' *total active life of schooling*, it is a term that needs to be defined within the context it is used. For the purpose of this paper, I defined curriculum as the set of materials from which teachers create learning experiences for students, to instill within them the knowledge and skills they are expected to acquire as a result of their participation in a grade level or class. The use of curriculum to create these experiences represents an intentional design for learning situated within the needs of students (Toombs & Tierney, 1993, p.21).

Consistent with my theoretical frame, this definition of curriculum situates teaching, and the development of lessons, as a goal-oriented design activity, (Brown, Remillard, Herbel-Eisenman, & Lloyd, 2009; Geudet & Trouche, 2012). *Curriculum materials* will be defined as the *tools teachers use to design learning activities* (Brown & Edelson, 2003) and which guide the mathematics learning of students. The word *design* in this case will be used both as a noun, to denote the program of study, and a verb, to depict the actions taken by teachers to create the program of study.

Overview of Dissertation

Following this introduction, I will detail the research I did into the teacher-text interaction in an effort to answer the above mentioned research questions. After a review of the extant literature, I will discuss the research methodology I used in this study and go on to discuss my findings. Afterwards, I will conclude with discussion about the findings along with suggestions for further research.

CHAPTER TWO: LITERATURE REVIEW

Mathematics Teaching and Teachers

This review of the literature summarizes what researchers know about how teachers interact with curriculum materials in the design and facilitation of mathematics lessons for their students. First, I will outline what we know about curriculum and the subtle ways it serves to influence teachers in ways in which they may be unaware. I will discuss nuances such as how the teacher is positioned and addressed in the curriculum and how those nuances can serve to guide teachers into taking on roles in which the writers cast them. As an example, educative curriculum materials (Davis & Krajcik, 2005; 2014) position teachers as both facilitators of learning and as learners themselves, supporting both teachers and students in learning mathematics content. I describe how these curriculum features may contribute to what teachers perceive as important to curriculum re-design work, and may help teachers make better design decisions.

Second, mathematics tasks, when used strategically, are an important part of curriculum materials and also an essential component to developing students' high-level thinking and problem solving skills. Recent research that suggests there are patterns in how teachers interact with mathematics tasks and how they adapt tasks for use in their classrooms (Smith & Stein, 2006). Teachers draw on specialized knowledge in their re-design work, including an understanding of the content and how to make that content accessible to students. In some instances, teachers must understand how to choose and enact tasks in their classroom that drive particular pedagogical goals beyond the learning of mathematics. In the next section I define and summarize what we know about mathematics tasks and how teachers typically adapt them for use in their classrooms.

Third, and finally, I present results of empirical studies that have explored the teacher-text interaction specific to mathematics with both prospective and in-service teachers. I will highlight what previous scholars have identified as the key components teachers considered in mathematics task re-design work.

Curriculum

As stated in chapter one, *curriculum, texts,* and *curriculum materials* are the lessons, assignments, projects, readings, presentations, and media used by teachers to instruct students (Brown & Edelson, 2003; Remillard, 2005). I will use those terms synonymously in this paper to refer to these 'tools of the trade'. Whether it is the adopted curriculum of the district in which they work, or lessons obtained from a supplemental source, these are the resources that make up what teachers use to design learning experiences for their students.

However, curriculum is irrelevant in isolation. This irrelevancy is similar to the hammer that sits idle on a carpenter's tool bench. Objectively it is a tool and has a specific function but, until it is used, it remains insignificant to the job. Curriculum, like a tool, comes to life when the teacher and/or student begin to read, talk about, and build something with that object (Gueudet & Trouche, 2012) even if that something is abstract like knowledge. Consistent with Gueudet & Trouche's (2012) claim of curriculum as a 'lived' resource, other researchers describe the interplay between teachers and curriculum by how the teacher is positioned and addressed in the materials.

Teacher Positioning by Curriculum Materials

The mathematics curriculum chosen for adoption by a school or district should represent the ideological instructional goals held by that district for mathematics instruction. Therefore, nuances of the materials are important to consider in ensuring consistency with these goals. For instance, how do the curricula position teachers? Do they empower teachers to be the content expert or does that power remain with the writers? For instance, if a district's focus is on the development of student voice and choice, adopting a curriculum that allows for the development of those skills makes more sense than selecting a curriculum program in which the teachers and students are positioned as the passive receivers of mathematics content.

Mode of engagement.

The *mode of engagement* refers to the way textbook writers position their intended audience. It is based upon assumptions about whom the writers need the audience to be, in order to create the story the writers wish to tell (Ellsworth, 1997; Remillard, Gueudet, Pepin, & Trouche, 2012). Originating in the film industry, where keeping an audience engaged is directly related to profits, the idea was superimposed onto teaching by Ellsworth (1997) and furthered by Remillard and colleagues (2012) onto curriculum materials. The idea is not just to present content but also to present it in a way that maintains consistency with the mathematical story the writers wish to tell, which is often not made explicit.

The mode of engagement refers to the non-neutral privileging of certain instructional approaches over others and serves to elicit a particular kind of participation from the reader and subsequently rewards her for behaving in that manner (Ellsworth, 1997). In other words, through the mode of address, textbook writers create an idealized identity of who the teacher sees/wants to see herself to be, and then interacts with her as if she were that person. If the materials characterize the teacher as a competent professional, and treat her as such, she is more likely to react positively to the materials and continue to interact with them. In some ways, this treatment may serve to increase a teacher's progression towards her idealized teaching identity. However, if the materials do not effectively engage the teacher, she may lose interest and begin to look elsewhere for supplemental resources, possibly disrupting the cohesion of mathematics instruction.

Because teaching is a complex and dynamic endeavor, it is very difficult, perhaps impossible, for teachers to actually become the idealized version of the teacher depicted in an engaging curriculum. The mode of address, however, can help teachers work towards that identity by keeping them engaged with the content in the way the writer wants the mathematics to be positioned in the classroom.

A text may have multiple modes of engagement to serve several constructed teacher identities while keeping each of them actively engaged. For example, teachers who are more comfortable following the text as written are invited to sequentially follow along with the writers as they teach their class. Others, who may be skimming the material in order to find activities with which to challenge students, will appreciate the enrichment activities highlighted in the margins. Then there are those teachers who may be looking for remedial exercises to help students strengthen their fluency with a particular skill, which they will find neatly organized in the supplemental resource section. When the format is consistent and simple, teachers can develop a sense of how the curriculum is set up, making it easier for them to modify and adapt the material to build on what their particular students need.

Voice.

A text's *voice* also shapes the teacher-text interactions. Voice, as defined by Love and Pimm (1996) refers to how the authors represent themselves in the text and how they communicate with the audience. Herbel-Eisenmann (2007) and Remillard (2000) suggest that the voice of a textbook can be differentiated into one of two distinct categories of speaking: *through the teacher*, or *to the teacher*. The position taken by the authors and the voice they use to communicate with the teacher can cast her into the passive role of receiver, or empower her as the instructional authority in the classroom. Texts that provide teachers with what they need to do as they teach the lesson, without an explanation about why things are set up that way, speak *through* the teacher and are the most common representations of voice in curriculum materials. They position the teacher as enactor of the writer's ideas (Ziebarth, Hart, Marcus, Ritsema, Schoen, & Walker, 2009) and speak to her as such.

However, voice can also serve to empower the teacher by including her in a sort of collegial conversation. By talking *to* the teacher about the mathematics and the reasons the ideas are presented and sequenced the way they are, the authors serve as sort of a colleague. When the curriculum talks *to* the teacher, it affords the writers the opportunity to increase the teachers' content knowledge without diminishing her as a professional. Certainly, both types of voice are biased in their own ways, but the voice of a text can provide clues to the author's intentions, allowing schools and districts to make choices about curriculum that aligns to their idealized goals.

Educative curriculum.

Educative curriculum materials are defined as those that, through their design, promote teacher learning in conjunction with student learning (Davis & Krajcik, 2005). Though most curricula offer support for particular teaching strategies, educative materials differ in that they also guide teachers to develop their own knowledge about instructional decision-making and how they can apply different instructional strategies to support students.

Promising research into educative curriculum materials shows that when teachers and prospective teachers learn about curriculum as they work with it, they are more likely to develop increased content and pedagogical knowledge (Choppin, 2011; Drake, Land,

and Tyminski, 2014). These findings are supported by additional research in science education that found educative curriculum materials increased teachers' knowledge of the scientific process (Davis, Palincsar, Smith, Arias, and Kademian, 2017). So, the potential for educative curriculum to serve as a vehicle to increase student, as well as teacher, knowledge can have large ramifications for learning and task design. By working with educative curriculum materials, teachers may develop a better sense of how to modify tasks for their context-specific classroom needs. Another important part of any mathematics curriculum is the tasks with which the teacher engages students.

Mathematics Tasks

Mathematics tasks, as defined by Schoenfeld (1988) are those that require students to solve problems using mathematics. They differ from the more traditional *mathematics exercises,* which Schoenfeld (1988) defined as the routine application of mathematics procedures for the purpose of practicing that application. *Mathematics tasks* may have solutions that are not immediately obvious or require students to make connections between ideas, and draw conclusions from among a set of related problems. *Tasks* may offer students a richer mathematical experience than *exercises*. A problem that has many solutions is an example of a *task*. When a problem has many possible solutions, students must be granted the authority to place limits or parameters on a situation in order to create a solution that makes sense to them. In their research, Stein and Lane (1996) suggested that the enactment of high-level tasks is important to the development of students' capacity to think, reason, and problem-solve with mathematics.

Stein, Smith, Henningsen, & Silver (2009) moved beyond conceptualizing tasks as just the problems presented in curriculum materials to include the activities of the teacher as she sets up and enacts the task with students. As such, each written task can be thought of as two tasks. The first being the task as it appears in the curriculum materials and the second is that task being brought to life by the teacher during the planning and enactment of instruction. As such, no two teachers, no two classrooms will interpret and interact with the task in exactly the same way. Research indicates that many teachers enact high-level tasks in low-level ways, thereby decreasing the demands of the second task as they bring it to life during classroom enactment (Stein, Grover, & Henningsen, 1996; Hiebert, Gallimore, Garnier, Givvin, Hollingsworth & Jacobs, 2003; Hiebert, Stigler, Jacobs, Givvin, Garnier, & Smith, 2005). Implementing high-level tasks in the classroom can be difficult (Stein & Kim, 2009) so it is important that we understand what teachers naturally do with tasks.

Research suggests that if we want high levels of achievement in mathematics, it is imperative that teachers regularly engage students with cognitively complex tasks in high-level ways that challenge them to think deeply and make connections to the real world and to their prior learning in meaningful ways (Stein & Lane, 1996). Additional studies have shown that the levels at which students learn mathematics is influenced by the mathematics with which they engage (Hiebert & Wearne, 1997; Smith, Stein, Henningsen, 2000). This suggests that the types of tasks in a curriculum can have far-reaching implications about the type and amount of mathematics students ultimately learn.

However, every task is subject to interpretation by the teachers and that interpretation can significantly impact the way they are are posed to students and the learning that ensues. Sometimes teachers inadvertently take away the challenge of high-level tasks during enactment (Stein, et al, 1996; Hiebert et all, 2003; 2005). How can teachers increase their knowledge of the types of mathematics tasks that drive student achievement? Research-based tools can help teachers better interpret mathematics tasks for use in their classrooms. One such tool is the Mathematics Task Analysis Guide

Tool to guide understanding of tasks.

Smith and Stein (1998) developed the Mathematical Task Analysis Guide (TAG), as a tool to help teachers determine the complexity of mathematics tasks they enact in their classrooms. The TAG offers a comprehensive outline of what categorizes a task as high or low level and use of the framework may assist teachers with task selection/development. It outlines four categories of cognitive demand benchmarks; two levels of lower-demand categories, and two of higher-demand.

Memorization tasks are defined as those that require students to follow a memorized procedure or rule. *Procedures without connections tasks* require students to use procedurally based mathematics with little connection to the underlying conceptual component of the mathematics. *Procedures with connections tasks* still require the use of a procedure but remain connected to the underlying concepts. *Doing mathematics tasks* require extended thinking, reasoning, and problem solving in order to reach a solution (Smith & Stein, 1998). Smith and Stein's summary of the TAG, along with examples of each level, is shown in Table 2.1 below:

Table 2.1
Cognitive Demand/Task Analysis Guide, (Smith & Stein, 1998)

Lower-Level Demand	Higher-Level Demand
Memorization Tasks Complete the following multiplication facts in one minute or less. 2 x 3 = 5 x 4 = 10 x 6 = 4 x 7 = 8 x 10 = 8 x 4 = 9 x 5 = 3 x 4 = 5 x 5 = 6 x 8 = 7 x 9 = 2 x 6 = 3 x 9 = 8 x 7 = 9 x 2 =	Procedures with Connections Tasks About how big is 4/5 of this rectangle? Show your answer by shading in the rectangle. What other fractions are near 4/5 in size?
Procedures without Connections Tasks Solve each of the following. Show all your work. Check your answer with a calculator. 1. $8\overline{)96}$ 2. $7\overline{)452}$ 3. $6\overline{)3288}$ 4. $5\overline{)3412}$ 5. $10\overline{)4630}$ 6. $16\overline{)4952}$	Doing Mathematics Tasks The kindergarten class is coming to watch a play in our classroom. There are 20 students. In what different ways could we arrange the chairs for them so that all the rows are equal? The two third grade classes are going to watch our play in the cafeteria. There are 49 students all together. In what different ways could we arrange the chairs for them so that all the rows are equal? What do you notice about your solutions for the first two problems?

Teachers' Interaction with Curriculum

Despite the fact that teachers all adapt and modify curriculum on a regular basis (Ball & Cohen, 1996; Collopy, 2003; Davis & Krajcik, 2005; Elsaleh 2010), little is known about what mathematics exercises teachers choose to modify, how they go about making those changes, or what types of modifications they make. Research indicates that variations in adaptations have different effects on student learning (De Araujo et al, 2013; Stodolsky

& Grossman, 2000) so better understanding teachers' interactions with curricula is essential. When teachers work with and interpret curriculum materials, their work filters through their beliefs about mathematics, beliefs about teaching, their cultural backgrounds, and personal histories, among other things. This interpretation helps to explain why two teachers who use exactly the same curriculum materials may have substantially different variations of the enactment of that lesson. Though the writers can work to be as clear as possible with how they hope the material to be understood, ultimate understanding rests with the end users.

Research by Remillard (2005) outlines four ways that teachers typically interact with mathematics curriculum materials. First, teachers might adhere to the curriculum with *fidelity* by following the text exactly as written. They also might take a *participatory* view of the text, understanding curriculum use to be a collaboration, of sorts, with the writers. Finally, they might hold an *interpretive* view of the text, which assumes that that fidelity between classroom actions and the teachers guide is impossible so teachers *draw on the materials* to incorporate them into instruction, using them flexibly with their own design views. The perspective a teacher takes with curriculum materials may yield different results in student learning which can also be subtly driven by the mode of engagement and voice of a curriculum.

For my study, I positioned teachers to take an interpretive view of curriculum materials versus a fidelity or participatory view. I asked them to explore several different sets of materials and choose tasks for re-design. As such, they were not participating with the writers, nor were they following one curriculum with fidelity. Instead, I asked them to interpret the writer's intent with particular tasks and re-design them according to their own considerations and those of their group mates. It was a position that teachers may or may not typically take with comfort. However, through this view, I hoped to see evidence of teachers drawing upon the specialized knowledge research suggests teachers possess.

To summarize this section, curriculum materials are not neutral. They position teachers in certain ways and contain nuances such as the mode of engagement and voice, which serve to develop and perpetuate that position. Also, a text can speak through a teacher or to a teacher, characteristics that impact a teacher's mathematical identity and authority and guides how she interacts with the text. An educative curriculum is one that develops teacher knowledge in addition to student knowledge through the way it engages and addresses the teacher. The content of mathematics exercises and tasks can be characterized according to the level of thinking it requires of students and the Mathematics Task Analysis Guide can assist teachers in making those determinations. Finally, research on teacher-text interactions suggests that teachers might take one of several approaches to curriculum interaction including a fidelity view, a participatory view, or an interpretive view. Curriculum itself, and the way teachers interact with it have important ramifications on both student learning and the development of teacher pedagogical and content skills.

Teachers as Designers

The practice of teaching can be conceptualized as design work (Brown & Edelson, 2003), where teachers interact with both students and content to plan and facilitate effective and engaging mathematics lessons. Teachers' design work is situation specific as classrooms are very complex places, involving people and the emotional lives of those people over time in the presence of challenging mathematics content.

As they build and refine their practice over time, teachers amass a collection of ideas and strategies that they have found to be effective. This portfolio, or as referred to by Gueudet and Trouche (2012), this *document* represents a dynamic and living representation of a teacher's work. It contains the manifestation of the specialized knowledge teachers use in their work. Each teacher's document of work differs, stylistically and in content. It is unique and personal as each teacher's beliefs, background, culture and attitudes, (among other individual traits), which interact with and impact their design work.

Specialized Knowledge of Teachers

Shulman (1986) suggested that teachers develop pedagogical content knowledge (PCK) that allows them to understand how to teach content in ways that make it understandable to students, which could contribute to how they redesign mathematics tasks. Since that time, the field has grown to include the introduction of mathematics knowledge for teaching (Ball, 2003), pedagogical task knowledge (Liljedahl, 2007), and more recently, pedagogical design capacity (Brown, 2009). Such knowledge increases with experience and powerfully suggests that redesigning mathematics tasks and teaching is so much more than just telling. In the upcoming sections I describe several types of specialized knowledge and how that knowledge may contribute to task design work.

Mathematics knowledge for teaching.

Mathematical Knowledge for Teaching (MKT) is one form of specialized knowledge of teachers (Ball, Thames, & Phelps, 2008). Distinct from pedagogical content knowledge, MKT is the ability to make *mathematics* content accessible to students. MKT requires both an understanding of mathematics content and the knowledge and skills to make that content accessible to students. It requires the teacher to be able to anticipate the errors their students may make, and knowledge of how to guide them into correcting and understanding those errors (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008). MKT allows teachers to think creatively about how to design learning activities that connect with their particular students in their particular context.

Pedagogical task knowledge.

Another form of the specialized knowledge of teachers is Pedagogical Task Knowledge (PTK) (Liljedahl, Chernoff, & Zazkis, 2007). PTK is awareness of the mathematics in a task and how a task affords or constrains student learning in the pedagogical context the teacher wishes to leverage. It is the knowledge of how to use mathematics tasks for specific pedagogical reasons and requires teachers to have a deep understanding of their students' mathematical thinking. For example, if a teacher wishes to increase student-to-student academic conversation in class, she needs to understand what sort of task will drive that pedagogical goal.

Liljedahl et al (2007) identified the development of PTK as a recursive four-stage process consisting of: *predictive analysis, trial, reflective analysis*, and *adjustment* that increases with practice. During *predictive analysis*, teachers consider the mathematical goals of a task and how students might solve it. They also consider how the task will fulfill the pedagogical goals they have for the lesson. Whether for assessment, modeling group dynamics, or exploring the nature of mathematics for example, teachers consider how the task will play out in their classroom and the extent to which it will afford or constrain their pedagogical goals. *Trial* is the enactment of the task with students. Afterwards, teachers engage in *reflective analysis* where they make decisions about how well the task met the pedagogical and mathematical goals set by the teachers. Finally, *adjustment* occurs where teachers revise the task for the next iteration.

This process, positioned by Liljedahl et al (2007) as a way that PTK is developed, may also be understood as a process teachers go through as they interact with curriculum to re-design tasks. The idea is that perfection is never achieved; but we can work towards it through a recursive process that progressively yields new information. This process, detailed in chapter 3, also informed the design of my professional development intervention.

Pedagogical design capacity.

Pedagogical Design Capacity or PDC (Brown, 2002; Brown & Edelson, 2003; Brown, 2009) is another characteristic that teachers bring to their interaction with curriculum materials. Distinct from a the specialized knowledge of teachers, PDC is the capacity of a teacher to perceive and orchestrate curriculum resources within the context of the classroom to achieve the mathematical and pedagogical goals that lead to student achievement. It is the knowledge *that* versus knowledge *how* (Ryle, 1984). In other words, PDC is not whether or not teachers will design learning experiences; it is how those designs impact their students' mathematical learning. Increasing teachers' PDC may help them to make better design changes to mathematics tasks.

To summarize this section, we know that teachers draw on specialized knowledge as they interact with curriculum materials to design instructional experiences for their students. Their Pedagogical Design Capacity informs/impacts to what extent they may draw on Mathematical Knowledge for Teaching and Pedagogical Task Knowledge in the re-design of mathematics tasks. Through their work, teachers build a body of work over time that both influences and is influenced by experience.

Models of Mathematics Curriculum Interaction

There is a large body of research, both with prospective and in-service teachers into the teacher-text interaction specific to mathematics that can help us to understand how teachers re-design tasks and what they consider when doing so. I will begin with two studies enacted with in-service teachers, and close with two studies enacted with pre-service teachers. The work with in-service teachers provides information about the types of adaptations teachers make to mathematics tasks (Drake & Sherin, 2006) and the effects of a professional development workshop on curriculum enactment (Westwood-Taylor, 2015). The work with pre-service teachers highlights simultaneous practice with task design and actual student interaction through a pen-pal project (Crespo, 2003) designed to increase teacher content and pedagogical skills; and a task re-design assignment (Nicol & Crespo, 2006). The results from these studies summarize what we know about how teachers interact with curriculum materials and point to how we can use this knowledge to learn more about how teachers re-design tasks and what they consider as they do so.

Typical Adaptations Teachers Make to Tasks

As part of a larger study on elementary teachers implementation of, and learning from, a particular mathematics curriculum, Drake and Sherin (2006) extensively studied experienced teachers over the course of one school year to track the changes they made to the mathematics curriculum materials. Their overall findings pointed to three structural categories of adaptions categorized as omit, add, or substitute. Further analysis allowed them to break down the categories more specifically, into ten adaptation types, which are listed in Table 2.2 below:

Table 2.2
Ten Types of Adaptations Teachers Typically Make (Drake & Sherin, 2006)

Ten Types of Curriculum Adaptations
Changes in terminology
Changes in the order of activities
Changes in materials used
Changes in participant structures
Increasing student control over an activity
Increasing teacher control over an activity
Changes in the amount of time spent on an activity
Omitting problems
Subtracting from problems
Adding problems

The ten adaptation types were what the researchers found to be typical of the teachers in their study. Two experienced teachers were studied who taught at different grade levels. Linda was in her first year of using the adopted curriculum, having moved from teaching kindergarten the previous year, and Beth was in her second year of using the curriculum at the same grade level. As a way to learn more about what kinds of adaptations make up the categories in the table above, consider the following examples from these two teachers.

Being in her second year of using the curriculum, the two most common adaptations Beth made were *omissions* followed by *changes in terminology.* As an example of what was coded as a change in terminology, consider the action Beth took during a review of two-digit addition. In an attempt to link mathematics to science, Beth attempted to connect the word “digits” in the mathematical sense to the fingers and toes of dinosaurs, which students were studying in science. However, the change in terminology did not foster any increased student understanding of the mathematical concept. Although not successful, that adaptation contributed to Beth’s document of knowledge.

Linda, who was using the curriculum for the first time, most commonly made adaptations to the *amount of time spent on an activity.* Typically, she would extend the time spent on activities in dramatic ways, as, for example, during the introduction of teen numbers when she extended the activity from half a day to a full week. In follow up communications, Linda conveyed that the time extensions were usually done because students had not yet attained mastery of the topic.

This foundational research looked at experienced teacher interaction with a new curriculum before, during, and after lessons. This work informed my study, in which I looked at how teachers interacted with curriculum before instruction, in that the ten types of adaptations gave me a starting point with which to make sense of what I might see. Another important difference is that the researchers focused on a broad range of actions with curriculum while my focus was on what teachers do with mathematics exercises and tasks.

Secondary Teachers Assess and Adapt Curriculum Materials

In another study, Westwood-Taylor (2015) studied how four in-service secondary teachers used mathematics curriculum materials during planning and instruction both before and after a professional development intervention. The goal was to help teachers more effectively interact with curriculum materials, to "curriculum proof teachers" as her title suggests.

The intervention consisted of teachers collaborating to share and discuss materials they had assessed, adapted, and used with students around the Mathematics Curriculum Assessment and Adaptation (MCAA), a process designed by teachers that outlined three specific ways for teachers to deliberately interact with curriculum. The three ways were 1) prioritization of goals and identification of expected or possible ranges of student understanding, 2) assessment of how well curriculum will meet students' needs and maximize learning, and 3) adaptation of curriculum (if necessary), including supplementation or replacement of materials (pg. 299).

Results indicate that the teachers put more focus into considering students' mathematical needs and understandings when adapting tasks as a result of the intervention. The adaptations they made showed an increase in more flexible and deliberate curriculum adaptations that were also more student-centered. For example, one teacher created multiple versions of the assessments to include a range of scaffolding and allowed students to anonymously choose a version to complete. Rather than scoring items as correct or incorrect, the teacher looked at the level of understanding her students demonstrated based on the work they did. Higher levels of understanding were given more points.

This study informed my work in that the researcher was looking at a similar problem to the one I explored. Her work gave me insight into what I might see with in the work of the experienced teachers with whom I worked.

Practice Engaging Students with Tasks

Learning to teach mathematics by designing tasks for students could be a way to help teachers develop the skill of pedagogical task knowledge (Liljedahl et al, 2007). In order to learn more about how pre-service teachers pose tasks to students, researchers designed an empirical study that took place during a teacher education course where pre-service teachers were paired with fourth grade pen pals to exchange weekly letters investigating children's mathematical knowledge and disposition. The goal was to find out how pre-service teachers posed mathematical problems to students; how those practices changed over time and what factors contributed to the change.

Results indicate that, initially, pre-service teachers posed short, single answer problems in their pen pal letters, focused on making the problems familiar to students and easy to solve. Often, they selected problems without fully working them out first. They also worked to avoid student errors and remove ambiguities in the tasks they posed. However, later in the semester, pre-service teachers were more willing to pose more challenging tasks to students, a shift that was attributed to significant changes in their views and beliefs about worthwhile mathematical tasks.

Additional findings suggest a change in the teachers' view about student errors. Rather than seeing them as something to be avoided by trivializing tasks, the teachers seemed to learn that considering students' mathematical thinking and the errors they make could be a way to challenge and push student's mathematical thinking. Working directly with students as they interacted with curriculum, these prospective teachers were able to move beyond posing single answer, low-level problems to their students.

This study is important to my work because I explored the similarities and differences between the re-design work of early career and experienced teachers. It informed me about what mathematics teachers who are early in their career might do as they re-designed tasks.

Teachers Collect and Modify Mathematics Tasks

In another study of pre-service teachers' interaction with curriculum materials, researchers developed an assignment to understand what teachers considered as they re-designed mathematics tasks (Nicol and Crespo, 2006). For the first part of the assignment, students had to solve the ten problems and provide a written analysis of each related to content, grade-level appropriateness, and develop possible adaptations and extensions. For the second part of the assignment, they were asked to provide a general review of a textbook in use at a local district, exploring both the implicit and explicit learning theories presented in the material, how content was introduced, and various features of the book.

Results indicated that the pre-service teachers considered context that was familiar and meaningful to their students as one component of the tasks they chose to select. Some pre-service teachers chose tasks that were procedurally based, with one-step pathways to single solutions while others selected tasks that used a wider range of student skills and allowed for more imaginative interaction.

The adaptations they chose to make focused on using familiar names in the problem settings instead of the names in the original problems, or changing the context of the problem to make it more interesting and familiar to their particular students. Fewer adaptations made by the pre-service teachers focused on making the problems more complex and none of them were made more accessible to students of varying abilities.

This study informed my work in that it prepared me for what I might see in the work of the early career teachers as they re-designed tasks. ....

Summary

Curriculum and what teachers do with it is important (Lloyd et al, 2009) and directly impacts student learning (Elsalah, 2010). We know that mathematics teachers have several types of specialized knowledge that they use to effectively teach mathematics to students and a flexible capacity for using that knowledge (Ball et all, 2008; Brown et al, 2002, 2003, 2009; Liljedahl et al, 2007). As they draw on this knowledge, it has the potential to grow. Over time, teachers amass a collection of artifacts, documents, conversations, and experience, to build a personal comprehensive document of their work that constitutes the sum of their teaching practice (Gueudet & Trouche, 2012). Each teacher's document is a unique representation of their specific situation and is impacted by their individual characteristics and those of their students.

We know that an important aspect of this work is the interaction between the teacher and the mathematics curriculum materials. Curriculum materials that are educative (Davis & Krajcik, 2005) may also serve to increase teachers' specialized knowledge of mathematics instruction. The text can hold power over classroom instruction with the way it engages, and speaks to, the teacher (Herbel-Eisenmann, 2007; Remillard, 2000) which positions teachers to interact with the materials in ways pre-determined by the he authors.

However, research shows us that there are ways to develop within teachers the skill of interacting flexibility with curriculum materials (Crespo, 2003; Nicol & Crespo, 2006; Remillard, 2013; Westwood-Taylor, 2015) in ways that increase student learning. To add to the knowledge base of, and to better understand, the teacher-text interaction, we need to study, more extensively, this phenomenon.

CHAPTER THREE: METHODOLOGY

The purpose of this study is to gain a better understanding of the teacher text interaction specific to mathematics in order to inform teacher preparation and support teacher education programs. In this chapter, I describe the research design, the professional development setting upon which it was built, and the reason for my choice of methodology. I continue with details about how participants were recruited and selected, and clarify key terms and phrases I used to articulate the perspective I took for the study. Next I explain the data sources I collected, methods of data collection, and analysis. Data collection instruments are included in the appendices.

Research Design

I approached this qualitative investigation as a Thematic Analysis (Boyatzis, 1999; Roulston, 2001) in which I explored the mathematics tasks chosen and re-designed by teachers, the written reflections they wrote after every work session, and audio recorded data of select teacher workgroups. Qualitative investigation was appropriate as it helped me to see and understand the actions of the teachers as they interacted with mathematics curriculum materials, a context-specific phenomenon.

I analyzed the three sources of data separately, collectively, and across the sets. I used an inductive approach to the analysis, meaning that I moved from specific observations to more broad generalizations. I worked through each of the re-designed tasks, and listened to the audio-recorded conversations to be sure I understood the teachers' perspectives. This afforded me the opportunity to double check the transcribed data for accuracy, and to make sure I had not missed important details.

Positionality Statement

As both the researcher of this study and facilitator of the professional development intervention, I understood that I had to be mindful of how I positioned myself with the teacher participants. I made it clear that workshop participation was not contingent on participating in the study and took care to treat every teacher with respect and courtesy, whether or not they chose to participate in the study. I also made it clear to the teachers that their participation in the study would have no bearing on their employment or performance reviews. I took care to assign pseudonyms to every teacher and district in order to protect the privacy and identity of the participants and the districts.

Having been a middle school mathematics teacher myself for many years prior to this study, I frequently redesigned tasks to better suit the needs of my students. Further, with the implementation of the Common Core State Standards for Mathematics and the lack of materials aligned to the standards at the time, I thought redesigning tasks would be a skill that could empower teachers in the study to use their available resources in new ways to help students meet the new standards.

I acknowledge that I have strong feelings that certain types of tasks are better for students than others. For example, I feel that cognitively demanding tasks challenge all students at all levels and are instrumental to increasing student achievement. I also think that the Mathematics Task Analysis Guide (Smith & Stein, 1998) clearly articulated the differences between high and low demand tasks and I was confident that teachers would also feel that way.

I did not have any pre-conceived ideas about particular kind of adaptation being better than another and was not partial to any particular curriculum. However, I strongly believed that teachers would be able to redesign tasks in interesting and innovative ways and I was genuinely curious about what they would do

Thematic Analysis

Thematic Analysis is a qualitative data analysis tool that Boyzati (1999) characterized as one that is flexible enough to be used across different qualitative research methodologies. It involves looking across a data set to find repeated patterns of meaning (Braun & Clarke, 2006) and is used to reflect reality as well as delve under the surface of that reality. In Thematic Analysis, a researcher may choose to take an inductive or deductive approach to analysis and, though both are acceptable, the choice should be made explicit as the two approaches are unique and important in different ways for understanding data. A deductive approach to analysis begins with a specific theory that gets narrowed down into a testable hypothesis. It has been termed a "top down" approach. An inductive approach to analysis, on the other hand, has been termed a "bottom up" approach because it moves from specific observations to more broad themes. For this study, I took an inductive approach to data analysis.

Thematic analysis is a process that begins with coding data and organizing the themes or patterns within that data. According to Boyzati (1999), themes may be directly observable (the manifest level) or may be underlying the phenomenon at a more theoretical level (the latent level), so it is important for the researcher to denote what was done and how it was done in order to present a strong argument. I was careful to avoid taking a passive account of themes *emerging* in the data by acknowledging the active role I played in the development of codes and the way I identified patterns and themes. Both of these are important to the research questions under consideration and the way in which findings are presented (Braun & Clarke, 2006).

Although Braun & Clarke (2006) suggest that thematic analysis can be perceived as an inferior method of qualitative data analysis, they attribute this to it being widely used but poorly claimed or described, and suggest the aforementioned steps to strengthen the analysis. As the researcher in this study, I carefully and explicitly delineated the type and level of analysis I performed on each data source, and meticulously considered my own values and theoretical position on how the themes were identified and organized. Doing so highlighted the need to look further into the phenomenon so that I could develop a rich and robust understanding of how teachers go about re-designing mathematics tasks for use in their classrooms.

Setting and Participants

The population for this study consisted of new and experienced mathematics teachers employed by three different school districts in a suburban area in the southwestern United States who were part of the Common Core Collaborative (C3) Professional Development workshop. The workshop was funded by an *Arizona Improving Teacher Quality* (ITQ) supplemental award (Project #ITQ0135-UA) to support teachers with the implementation of the CCSSM. The school districts were all Title 1 districts, meaning that they served a large percentage of low-income families. The workshop was advertised through each district by internal district personal and interested teachers were instructed to email the researcher to reserve a spot. To gain a sense of the size of the school districts, the number of students enrolled in each participating district, along with the square miles the district served during the time of the study, is summarized in Table 3.1 below:

Table 3.1
Overview of School District Sizes by Number of Students and Area

District	Enrolled Students	Square Miles
Carhall	12,300	550
Ormont	17,000	100
Glasshurst	52,000	230

Response to the workshop was overwhelming; over 300 teachers emailed me to be included, which led to the addition of the second workshop. This afforded me the opportunity to create a two-stage study where I could gain a better understanding of how teaching experience plays into the re-design process. I chose teachers for each workshop through purposeful qualitative sampling based upon the pre-selected criteria of teacher experience. I organized teachers by experience level in the order in which they applied to be included in the workshop. I defined early-career teachers as those within their first five years of practice, and experienced teachers as those with between five and ten years experience. The first 40 early-career teachers on the list were invited to be part of the workshop for Stage One, and the first 40 experienced teachers for Stage Two. Twenty-three teachers from Stage One agreed to participate in the study and 40 teachers in Stage Two agreed. Table 3.2 below summarizes the number of teachers from each district by experience level and grade level taught at the time of the study.

Table 3.2
Teacher Participants by Grade Level and District in Stage One and Stage Two

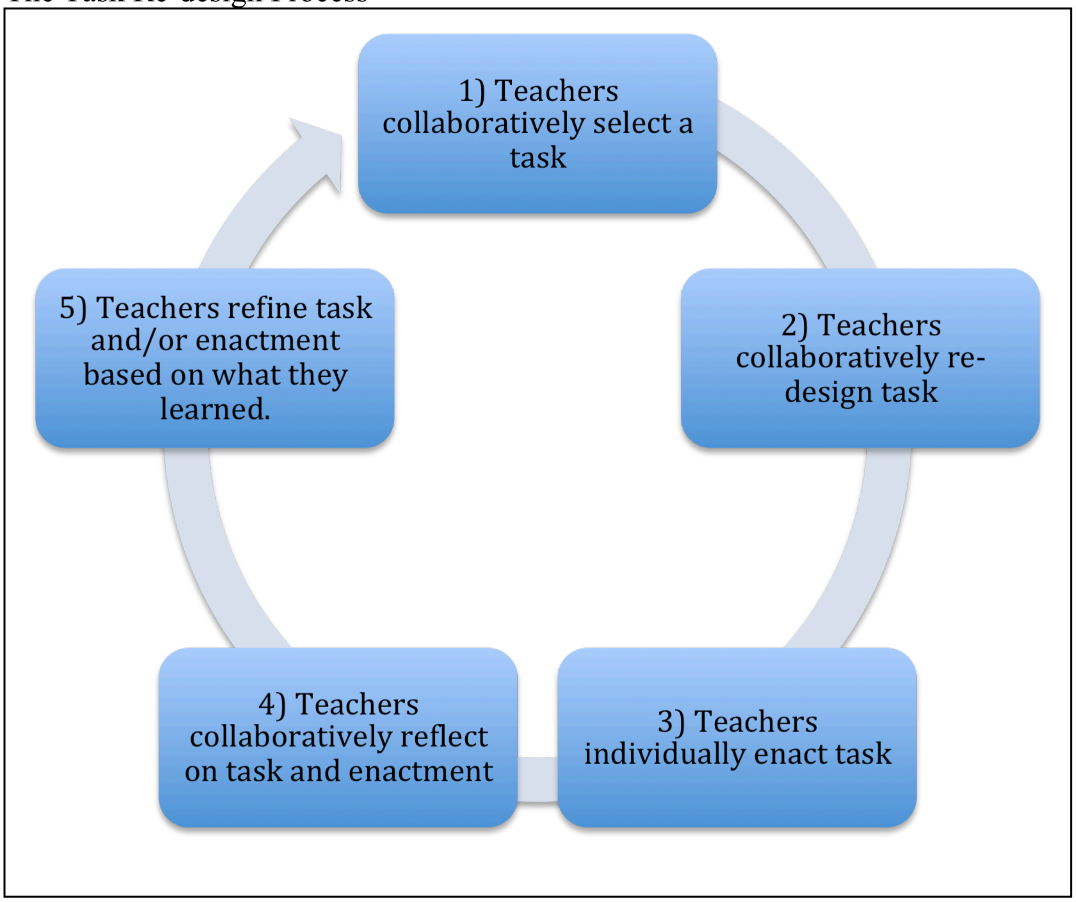
District	Early-Career Teachers			Experienced Teachers		
	K-2	3-5	6-8	K-2	3-5	6-8
Carhall District	4	4	1	6	6	3
Ormont District	4	2	1	1	3	2
Glasshurst District	1	3	3	7	10	2

Intervention

I organized and facilitated a professional development workshop over the course of a school year to guide teachers into a greater understanding of the Common Core Mathematics Standards, which were new at the time of this study, and rigor in mathematics. The workshop consisted of six meetings, every other week, over a period of twelve weeks and I facilitated two iterations of the workshop. Each session lasted approximately three hours and teachers were instructed in cognitively demanding tasks before working to redesign tasks in small grade level groups made up of teachers from three area school districts. I grouped the teachers this way to facilitate the sharing of curriculum materials, allow for a diverse make-up of perspectives from which teachers could approach the work, and foster camaraderie between teachers from different schools and districts.

I began each session by engaging the teachers with instruction on cognitive demand. We categorized a series of tasks according to the cognitive demand level with a card sort and I posed challenging tasks to teachers. A sample of the tasks with which I engaged teachers can be found in Appendix A. I began each session this way in order to provide guidance to teachers about cognitively demanding mathematics so that they would develop a frame of reference on which to base their redesign work. The overall goal of the workshop, however, was to task teachers with collaboratively deciding on mathematics exercises to re-design that would increase the level of cognitive demand placed upon their students. To do this, they had to interact interpretively with several curriculum materials to decide the intent of the writer and collectively select a task they felt they could re-design in a way that would increase the intellectual demand placed upon students. Then, between sessions, teachers agreed to enact the re-designed tasks in their classrooms. Part of each succeeding session was devoted to having the teachers share their enactment experience with the group, who would then collaborate about how to refine the task and/or its enactment based upon what they learned. The work informed their next cycle of task re-design. The Task Re-Design Process is illustrated in Figure 3.1 below:

Figure 3.1
The Task Re-design Process



My Task Re-Design Process graphic was informed by the work of Thanheiser, et al (2015) who implemented a similar model with pre-service teachers. My Task Re-Design Process differs from theirs in that I designed it for use with in-service teachers. As shown in the above graphic, a complete cycle of task re-writing consists of five phases: 1) Teachers peruse several different curriculum materials to choose a mathematics exercise to re-design; 2) Teachers collaboratively re-design the exercise into a mathematics task to use with their various student groups; 3) Teachers enact the task with their students; 4) Teachers come back together to collaboratively reflect on the re-designed task and its enactment; and 5) Teachers discuss how they would revise the task and/or enactment for future use.

Each group of teachers participated in at least two complete cycles of task re-design. In chapter two I defined the differences between mathematics tasks and mathematics exercises and in the next section I clarify what I mean by re-designing mathematics exercises into more cognitively demanding mathematics tasks.

Re-Designing Mathematics Tasks

For the purpose of this study, I defined *re-designing mathematics task*s as the process teachers go through as they interpret mathematics exercises that appear in curricular/instructional materials and adapt them in an attempt to make them more cognitively demanding for students (see Figure A). First, teachers utilized a variety of curriculum materials from their respective districts to collectively decide on a mathematics exercise to re-design. Then they collaboratively redesigned the task, where they had to think about the mathematics in the selected task and what it would initially require of students and how they could redesign it to increase the cognitive demand. Next, they enacted the task in their respective classrooms, paying attention to things they considered important. Finally, they came back together to discuss the enactment and any additional modifications they felt were necessary based on the enactment.

It is important to note that, in practice, teachers re-design mathematics exercises for a variety of reasons. Examples of these reasons include making the mathematics of the problem more accessible to students, or re-designing the language demands of the task to accommodate the needs of second language learners, among a few. For this study, to highlight the re-design process, I asked teachers to re-design tasks with the goal of increasing the cognitive demand placed upon their students. I chose this purpose as one that would challenge teachers at all levels of experience, potentially challenge students at all levels, as well as provide teachers with a specific reason to re-design tasks for the study, which I hoped, would cut across their different teaching styles and contexts to help me to understand the re-design phenomenon.

I realize that focusing the teachers' work in this way may have been a potential limitation of this study. As teachers worked to increase the cognitive demand of the tasks they redesigned, I may have limited my opportunity to learn about other kinds of revisions teachers make to tasks, such as those that would have made tasks more culturally relevant or revisions that focused on linguistic demand. However, given that I was specifically interested in how teachers could make the tasks more rigorous to meet the increased demands of the Common Core State Standards for Mathematics, focusing on cognitive demand was warranted.

Two Stage Study

As both the professional development facilitator and project researcher, I recruited study participants from the workshop attendees. I collected data across the two stages of the study, which gave me the opportunity to make slight modifications between the two workshops in order to extend my inquiry into the phenomenon. One of those modifications was the addition of audio recording the work sessions of two groups of experienced teachers during the second stage of the study. I decided to add this data source because the data collected in Stage One did not provide me with enough detail about how teachers negotiated the modifications they made. I found that as I circulated among the groups in

Stage One, their discussion would shift to questions or comments they had for me. Audiotaping the work sessions allowed me to listen in on their work without disrupting it. In the sections that follow, I provide an overview of both stages of the study, the data sources I collected, and how I analyzed them.

Overview of Stage One: Early Career Teachers

Stage One of the C3 workshop consisted of forty K-8 mathematics teachers within their first five years of teaching practice and took place in the first semester of the school year. Twenty-three teachers from the workshop, at least one from each grade level, agreed to participate in this study. Though most of the teachers' experience occurred in the grade level they were teaching at the time of the study, one was teaching both seventh and eighth grade classes during her first year of practice, and another, who was a second year teacher, had moved from second to fifth grade.

This group averaged 1.35 years of teaching experience and seven of the twenty-three teachers were just beginning their first year of practice. The group consisted of two males and twenty-one females. The three school districts had approximately equal representation in terms of the number of teachers from each. Most were general education teachers; one was an exceptional education teacher, and most worked with English Language Learners. For a summary of how each district was represented in the study relative to the number of teacher participants, see Table 3.3 below:

Table 3.3
District Representation Stage One: Early Career Teachers

District Name	Participants Phase One
Glasshurst	39% (n=9)
Ormont	30.5% (n=7)
Carhall	30.5% (n=7)

Overview of Stage Two: Experienced Teachers

Stage Two of the workshop consisted of forty K-8 mathematics teachers with between five and ten years of teaching experience, and took place during the second semester of the school year. All of teachers in the workshop agreed to participate in the research study. The group consisted of thirty-nine females and one male teacher who averaged 6 years of experience at the time of the study. Two of the teachers worked exclusively with students who had exceptional learning needs, one in a self-contained autism classroom, and the other with multiple grade levels as a resource teacher. Most of the teachers worked with English Language Learners. One teacher taught both seventh and eighth grade students in a traditional middle school setting, and another taught middle school mathematics at a district alternative school for students who needed behavioral support. For Stage Two of the study, fewer teachers from Ormont District applied so the number of teachers from that district is much lower than the others. For a summary of each district's representation in the study see Table 3.4 below:

Table 3.4
District Representation Stage Two: Experienced Teachers

District Name	Participants Phase Two
Glasshurst	47.5% (n=19)
Ormont	15% (n=6)
Carhall	37.5% (n=15)

Data Sources Stages One and Two

I collected four sources of data for this study: before-and-after mathematics tasks, teacher written reflections, audio-recorded work sessions, and a researcher journal where I kept reflective notes on each session.

Mathematics Tasks

First, I collected the before-and-after task re-design artifacts that the teachers worked on in each stage. I collected the mathematics exercises, as they originally appeared, along with the re-designed version of the problem as the teachers wrote them. The teachers used the Mathematics Task Rewrite Form located in Appendix B as a guide to organizing their re-design work. The form was printed on carbonless copy paper, which allowed teachers to each keep a copy for use during classroom enactment, and the originals to stay with me. All teachers, regardless of which session they were in, or which grade level they taught, used the same form. I collected between three to five tasks from each group. The stage one, early career teachers, chose to redesign 37 tasks, and the stage two, experienced teachers redesigned 38. The breakdown of the number of tasks chosen for re-design by teacher experience level, across both stages of the study is shown in Table 3.5:

Table 3.5
Number of Tasks Chosen For Redesign by Grade and Experience Level

Grade Level Band	Early Career Teachers	Experienced Teachers	Total Tasks chosen for re-design
K-2	19	14	33
3-5	13	19	32
6-8	5	5	10
Total	37	38	

Written Reflections

Second, I also collected the written reflections teachers wrote at the end of each session. In those, I asked teachers to write about the work they did during that session including what they considered as they re-designed tasks and what challenges they faced. The questions were open-ended and I wanted teachers to write freely for as long as they felt they needed to in order to answer the questions. The teachers usually answered the questions in a few phrases or sentences though there were times when they would write much more.

During the final session of the each workshop I asked teachers to reflect on what they felt was important to consider in the redesigning of mathematics tasks. During Stage One, I collected 23 teacher reflections and during Stage Two, I collected 40. A completed teacher reflection sample is located in Appendix C.

Audio-Recorded Work Sessions

Third, during Stage Two, I audio recorded two groups of teachers-a third grade teacher group and a middle school teacher group as they worked during each session. I selected the two groups for audio recording through opportunistic sampling. Since the research had already begun, this purposeful sampling allowed me to take advantage of unfolding events that helped me to better understand the phenomenon. Originally, I had planned on also recording a Kindergarten group, which would have provided data at the beginning, middle, and end level groups in the study but, due to equipment failure, the data was not useable.

All together, I recorded each teacher group three times for a total of six recordings that were each approximately one hour long. Content of the recordings consisted of 1) teacher conversations as they worked on the first task redesign session, 2) their conversations as they debriefed the first task enactment along with the second task redesign session, and 3) the debrief of the second task enactment. As such, I was able to capture one complete cycle of the Task Redesign Process for each group. I had the recordings transcribed by a professional transcription service and I listened to each tape as I read the transcripts to double-check them for accuracy before I began coding and analysis.

Researcher Reflective Journal

Fourth, I kept detailed notes in a researcher's journal that I compiled immediately after each session. These notes served to remind me about the events and occurrences of each work. I took note of the emotional mood and physical health of the group overall during each session and added details of any outside events taking place that may be impacting the mood. For example, one session was held on a night when the local university basketball team was well into the playoffs and I noted that the groups seemed more distracted. I also took note of things that teachers talked about and any questions they asked me after the session. I also used the journal as a way to sequentially record the events of each session as I recalled them. I included a sample journal entry in Appendix D.

Data Analysis

Mathematics Tasks - Coding

I coded the mathematics task artifacts in three ways. First I coded the mathematical structure or content of the task, both before and after the re-design. Next, I coded for changes in the level of cognitive demand before and after the re-design, and finally, I coded the types of modifications the teachers made.

Mathematical structure and content.

To begin I sorted the 75 tasks the teachers chose for redesign into two piles-those that were addition, subtraction, multiplication or division and those that were not. At this point I did not consider grade level band; instead I focused on the mathematics operations in the tasks. For those that were word problems, I coded the mathematical structure of the problem, using the CGI Framework for problem types (Carpenter & Fennema, 1992), located in Appendix E, as a starting point for the codes. If the structure of the problem did not fit any of the existing codes, I developed a new code.

Afterwards, I went through the same process a second time with the 81 tasks the teachers had redesigned. I used the same codes and added new codes for additional problem structures or mathematics content that came up in the modifications. Table 3.6 includes a list of coded used, and definitions.

Table 3.6
Definitions of Problem Structure and Content Codes

Code: Problem Type	Definition
Result Unknown (join or separate)	An addition or subtraction task where the sum or difference is unknown.
Change Unknown (join or separate)	An addition or subtraction task in which one of the addends is unknown.
Start Unknown (join or separate)	An addition or subtraction task in which the starting quantity is unknown.
Part-part-whole, whole unknown	Involves understanding quantities as being made up of two or more parts and where the whole is unknown.
Part-part-whole, one or more parts unknown	Involves understanding numbers as being made up of two or more parts and where one or more of the parts is unknown.
Multiplication Word Problems	Contextual word problems in which an operation is performed on two numbers to obtain a third. Also known as repeated addition.
Partitive Division	Equal sharing task in which the number of groups is known but the quantity in each group is not.
Quotative Division	Equal sharing task in which the number in each group is known but the number of groups is not.
Multi-step and non routine problems	Tasks that require many steps to reach a solution and whose steps may consist of several different mathematical operations.
Calculation Tasks- Add, Subtract, Multiply, Divide	Tasks that are numbers-only and do not include any context or words.
Counting Tasks	Tasks that require one-to-one correspondence of objects/ numbers and the understanding of how to count them.
Operations and Number Sense	Tasks involving use of the Order of Operations.
Factors and Multiples Word Problems	Least Common Multiple and Greatest Common Factor tasks.
Identification/Comparison of Fractions	Tasks in which fractions are identified or compared by size.
Fraction Operations	Add, subtract, multiply or divide fractions.
Patterns and Algebraic Thinking	Tasks that require the identification, generation or extension of numerical patterns.
Data/Statistics	Tasks that involve the collection, display, and/or interpretation of data.
Geometry	Tasks that involve measurement, spatial reasoning, and/or shapes.
Proportional Reasoning/Unit Rates	Tasks that involve multiplicative comparison and/or the determination of 'per one'.
Compare-how many more or fewer?	Tasks in which students are required to compare quantities in order to determine how many more or fewer of one over another.
Tasks with Many Possible Solutions	Tasks that have an unlimited number of solutions or lack parameters that would define a solution.

Once the coding was complete, I shared samples of the tasks with a mathematics education professor to calibrate and refine the coding structure and to ensure tasks were coded appropriately for their mathematical operation. Table 3.7 gives an example of how I coded a task by problem type.

Table 3.7
Original and Re-designed Firefly Task Coded by Problem Type.

Original Task**:** RESULT UNKNOWN *There were 6 fireflies inside the jar. Seven more flew inside. How many fireflies are inside the jar now?*
Re-written task: PART-PART-WHOLE, TWO PARTS UNKNOWN *There were 13 insects in the jar. Some were fireflies and some were mosquitos. How many of each could there be?*

I coded the original problem as a *join result unknown* problem because it is an addition problem with a clear action and students must solve for the total result. I coded the re-designed problem as *part-part-whole, two parts unknown* because students must determine the size of each part, given the total number of insects in the jar.

All together the teachers chose tasks that reflected 18 different problem structures or content areas and there were many situations where they began with one task but redesigned it into two. Table 3.8 below outlines how many tasks of each problem structure or content were included in the code. The difference in column totals reflects that teachers often redesigned one task into two.

Table 3.8
Mathematics Tasks by Problem Structure and Content:

Problem Structure	Chosen for Re-design	Number After Redesign
Join or Separate Result Unknown	13	2
Join or Separate Change Unknown	1	3
Join or Separate Start Unknown	0	2
Part-Part-Whole, Whole Unknown	8	1
Part-Part-Whole, One Part Unknown	3	2
Part-Part-Whole, Two or More Parts Unknown	1	13
Multiplication Word Problems	7	6
Division – Partitive or Quotative	5	9
Multi-Step and Non Routine Problems	3	3
Calculation Tasks – Add, Subtract, Multiply, Divide (no context provided)	3	1
Counting Tasks	1	1
Operations and Number Sense	4	5
Factors and Multiples Word Problems	1	0
Identification/Comparison of Fractions	4	4
Fraction Operations	5	3
Patterns and Algebraic Thinking	1	1
Data/Statistics	5	4
Geometry	6	6
Proportional Reasoning/Unit Rate	4	4
Compare-how many more or fewer	0	1
Tasks with Many Possible Solutions	0	10
Total	**75**	**81**

Once I coded all of the tasks by mathematical operation, I entered the data into ATLASti, qualitative data analysis software to organize and facilitate further coding and analysis of the mathematics tasks.

Cognitive demand.

Next, I used the Mathematics Task Analysis Guide (TAG), (Smith & Stein, 1998) to code the cognitive demand level of each original and re-designed mathematics task. The complete TAG can be found in Appendix F but Table 3.9 below, gives a general summary of the levels of cognitive demand:

Table 3.9
Summary of the Mathematics Task Analysis Guide (Smith & Stein, 1998)

Lower-Level Demands	Higher-Level Demands
Memorization Tasks: Tasks that can be solved from memory or those that are too simple for the use of an algorithm.	**Procedures With Connections Tasks**: Tasks that can solve procedurally but the use of the procedure purposefully leads to a deeper level of understanding of the mathematical ideas in the problem.
Procedures Without Connections Tasks: Tasks that can be solved algorithmically with no connection to the underlying concept.	**Doing Mathematics Tasks:** Tasks that require complex thinking and do not contain a predictable and well-rehearsed path to solution.

I went through and assigned each task a level of cognitive demand required for successful completion. Following Table 3.10 below, which gives an example of how I coded a task for level of cognitive demand, I explain why I coded it as such.

Table 3.10
Original and Re-designed Division Task Coded by Cognitive Demand

Original Task: Procedures without connections (low level of cognitive demand) *157 ÷ 18 = ___*
Re-written task: Procedures with connections (high level of cognitive demand) *Write a word problem and create a visual representation of the equation. Justify your ideas.*

Word Problem:	*Equation:* *157 ÷ 8 = ______*
Visual Representation:	*Justification of my ideas:*

I coded the original task to be of a low level of cognitive demand as a 'procedures without Connections' task according to the TAG. I chose this level because it is possible for students to follow a procedure and reach a solution without connecting the procedure to the underlying concept of division, according to the TAG.

The redesigned task requires more cognitive effort for students to successfully complete so I coded the redesigned task at a high level of cognitive demand; a ‘procedures with connections’ task according to the TAG. In the redesigned task, the student will have to justify their answer with words, a story problem, and a visual representation. According to the TAG, this takes the focus away from the use of an algorithm to find the correct solution to a deeper level of understanding of the concept of division by including multiple representations of the concept. Therefore, I categorized the re-write as a ‘Procedures With Connections’ task.

To increase validity, I shared subsets of approximately 25% of the data with mathematics and education professors on campus and in other states. I used a random number generator to decide which tasks to double-code and I did not include my own codes with the requests to the professors for review. In situations where we disagreed on the rating, we reviewed the tasks and discussed the reasons for our ratings until we could reach an agreement.

I compared the changes in cognitive demand between the original and re-designed tasks and justified my reasoning with the Mathematics Task Analysis Guide. I coded a change in cognitive demand only if the level moved from a low to a high level or vice versa. If the task moved within the low and high levels, I did not count it as a change in cognitive demand. In the above example, I coded the original problem as a *procedures without connections* task (level 2) and the re-designed task as a *procedures with connections task* (level 3). The re-written task is represented in multiple ways and the solutions require students to stay connected to the meaning of division. Therefore, I coded this re-designed task to have moved from a low to a high level of cognitive demand.

Types of adaptations.

To code the adaptations that teachers made as they re-designed tasks, I worked from the list of ten adaptations developed by Drake & Sherin (2006). I went through the data set looking for one type of adaptation at a time, which allowed me to code each type of adaptation as an individual occurrence. However, it was not possible to see some of the adaptations on their list without observing the lesson enactment, which I did not do. For example, I could not determine changes in the amount of time spent on an activity. Also, one code, 'adding problems' was too broad for what I needed so I created sub-categories to gain a better understanding of the adaptations teachers in this study made. So, in order to narrow my focus, I used only a subset of the Drake and Sherin (2006) codes, and added additional codes as needed. The complete list of the codes I used, along with the number of times I coded a task as such is presented in Table 3.11 below:

Table 3.11
Adaptation Codes (based on the work of Drake and Sherin (2006)

Types of Task Adaptation	Number
Changes in terminology	2
Changes in materials used	1
Increasing student control over an activity	28
Increasing teacher control over an activity	1
Omitting problems	10
Subtracting from problems	5
Adding problems to include context	8
Adding problems – students to develop questions	2
Adding problems –do more of the same	5
Adding problems – Student exploration	9
Adding problems – list all possibilities	7

Sometimes teachers made more than one type of adaptation to a task so I did not limit myself to one code per task. To illustrate, Table 3.12, below, is an example of how I coded tasks for adaptation types:

Table 3.12
Original and Re-designed Fraction Identification Task by Adaptation Type

Original Task: *Draw a line and label tic marks for benchmarks 0, 1/2, and 1. Estimate where you would put the fractions 5/8, 7/8, and 1/8.*
Re-written task: *Draw a number line from 0 to 1. Estimate and put a tic mark for ½. Estimate and put a tic mark to show a fraction that could come between 0 and ½. Estimate and show a fraction that could come between ½ and 1*

I coded this task to have three adaptations: 1) omitting a task, 2) increasing student control over the work, and 3) the addition of a task that made the problem more open to student ideas. The reasons I chose those adaptations as codes for this task are listed below:

1) Omitting Problems: In the re-written task, the placement of fractions 5/8, 7/8, and 1/8 was omitted.
2) Increase in student control: By having students estimate and show any fraction between 0 and ½ and ½ and 1 respectively, the students have more control over what fraction to work with.
3) Addition of a task-Student Exploration: The re-designed task asked students to place possible fractions between benchmarks but did not tell them which fractions to place.

This task was typical of several of the tasks in that the teachers frequently made more than one adaptation.

Mathematics Tasks - Analysis

For my thematic analysis of the mathematics tasks data, I began at the manifest level (Boyzati,1999) with patterns and themes that were directly observable in the data. I began by separating the tasks into the grade band levels of K-2, 3-5, and 6-8. Then I further separated the grade bands by teacher experience level. This organization helped me to identify patterns among the tasks. Next I organized the tasks and compiled the data according to changes in the mathematics operation of tasks before and after teachers redesigned them. This way I could determine patterns of adaptations within the grade level and teacher experience bands. Finally, I organized and compiled the mathematics task according to changes in the cognitive demand. This allowed me to see, within and across each grade level and teacher experience band, patterns in the way teachers changed the level of cognitive demand of the tasks they redesigned. My analysis of the mathematics tasks at this manifest level (Boyzati, 1999), was focused on items that were directly observable in the data set and was a matter of compiling the numbers and organizing them by grade and experience level to determine patterns in the changes.

Other analysis occurred at the latent level where I took a more intense study of the data to uncover patterns that were not immediately evident on the surface. My analysis was facilitated by ATLASti software program, which allowed me to reorganize the data in many different ways so that I could follow patterns I noticed in the data. Some of the patterns led to interesting themes. For example, I noticed that when teachers in the grades 3-5 band omitted something from an original task, they often gave students increased control over the work in the redesigned task. The increase in student control came from the teachers adding tasks that opened up the problem to include more student thinking and these combined modifications nearly always led to an increase in cognitive demand. So I organized the data in this manner to explore the trend further both within and across grade levels. So while my analysis was from an inductive approach and at the latent level, the way I organized the data led to a more manifest level of analysis in several ways to best understand the phenomenon under study. This analysis helped me to answer Research Question One: *In what ways do teachers modify pre-written mathematics exercises?*

Teacher Reflections - Coding

After each task re-design session, teachers were invited to reflect, in writing, on their work. Although teachers were encouraged to answer each question completely, in numerous instances teachers did not and some did not complete the reflections at all. A complete list of the questions I asked the teachers to reflect on can be found in Appendix

G. On the very last day of the workshop I asked both the early career and experienced teachers to respond, in writing, to the following two questions:

1) How do you make decisions about how to adapt mathematics tasks from prewritten curriculum materials? And
2) What sort of expertise or knowledge do you draw upon as you make these decisions?

I posed two additional questions to the experienced teacher groups on the final day of the workshop. Those were:

3) What skills with curriculum decisions have you develop with experience that would have been beneficial to have as a new teacher?
4) Do you think it's possible to teach these skills to new teachers? If so, how? If not, why not?

These four questions formed the basis of my analysis of the written reflections. The other questions I asked served to support my observations and provide the teacher perspective on the work as it was in progress.

Because most teachers did not identify their grade level on the final written reflections, I coded the 63 reflections all together, without separating by grade or experience level. To begin the coding process, I typed the teachers' responses into ATLASti so that I could organize them by question. Then I carefully read, and re-read, the responses to each question looking for words and phrases that came up frequently across the set of reflections. As an example, several teachers made reference to their students as important consideration in mathematics task redesign work. So I began by color-coding all references to students in the set. Table 3.13 below outlines a few examples of what I color-coded as a student reference:

Table 3.13
Sample Teacher Responses

Question: How do you make decisions about how to adapt mathematics tasks from prewritten materials?
"The most important skill is anticipating with students will struggle with."
"When I re-design tasks, I think about differentiating and how to have the students work more than me."
"Really thinking about the concepts I want the kids to learn, rather than the skills; this way I can plan on how to teach those concepts rather than practice and skills."
"I've learned, to some extent, how children will understand or misunderstand-which concepts are more difficult for them."

Once I had the data color-coded for student references, I saw that not all of the comments about students referenced them in the same way. To better understand what I saw, I parsed the responses out further by thinking more specifically about how the teachers referenced their students. I found that there were three broad ways the teachers mentioned their students as important considerations as listed in Table 3.14 below:

Table 3.14
Three Broad Ways Teachers Considered Students in Task Redesign Work

Theme	Definition of theme
Student thinking	References teachers made to how their students think mathematically.
Students' feelings and emotional needs	References teachers made to create emotional comfort and feelings of success within their students.
Teaching mathematics to students for understanding	References teachers made to teaching mathematics in ways that encouraged students to gain an understanding of the procedural and/or conceptual aspects of a task.

References to students came up repeatedly in the data but the references were not always to the same ideas. Parsing them out, I was able to see that references to students came in the three broad categories of *student thinking, students' feelings and emotional needs,* and *teaching mathematics to students for understanding.*

Once I coded the data, I re-read the teacher reflections to double check my coding scheme and to confirm that I accurately represented the data and exhausted every angle of looking at the data. For example, it was possible that teachers made modifications that potentially made the task easier to enact in class and by considering student thinking, they could have meant wanting to make a task easier for students. Therefore, I took care to look holistically at the reflection data within the context of the tasks to do the best I could to interpret the data the way the teachers' intended.

Teacher Reflections - Analysis

I analyzed the teacher reflection data within the context of the task re-design work sessions and, in stage two, the audio-recorded work sessions. What I mean by that is, whenever a teacher had put their name on the reflection, I matched it to the task the group worked on during that session to give me more insight into their words. As such, I was able to develop a deeper understanding of the reflection data within the context of the task the teachers worked on that day.

The three broad themes of student thinking, students' emotional needs, and teaching for understanding, underpinned my analysis of the teacher reflections as I kept in the forefront, the idea of teachers as designers. This analysis provided evidence of what these teachers, in this context, considered as they re-designed mathematics tasks and helped me to answer the following research question: *What do teachers consider as they re-design mathematics exercises?*

Audio Recordings – Coding

During Stage Two of the study, I audio-recorded an experienced third grade teacher group and an experienced sixth grade teacher group as they worked to re-design mathematics tasks and debriefed their enactment in the classroom.

Audiotaping allowed me to capture the conversation that occurred between teachers as they worked, without being a part of the conversation myself. They allowed me to explore the work sessions at a deeper, more latent level, than just an analysis of the tasks allowed. I recorded a total of six recordings; three each for third and sixth grade, each about one hour long.

I went through a number of steps to code the audio recording data. First, I de-identified the data and checked the transcripts for accuracy by listening to the recordings as I read the transcripts. Next, I looked for, and coded, ideas from the conversations that related to the themes established in the analysis of teachers' reflection: student thinking, students feelings and emotional needs, and teaching mathematics for understanding. Specifically, I listened to the teachers words as they worked for instances where they were indeed considering those ideas they said were important.

Next, I re-read the transcripts and looked for any other ideas that may have come up in the teachers' conversations but were not related to the themes mentioned in the teacher reflections. There were instances where teachers spent time comparing their respective districts to one another on things like curriculum and policy, and other times where they spent time talking about resources, both those that they brought with them to the workshop and others that they had found to be useful. There was discussion about the challenges teachers face with the number of tasks they are responsible for and the time pressures they face. However, there were no consistent patterns in those discussions as they were more about getting to know and relate to one another as they worked.

Audio Recordings –Analysis

I analyzed teacher's conversations in conjunction with the tasks that they redesigned. To begin, I gathered the tasks that went along with each portion of the transcript. Then I listened to the audiotape and worked through the redesign of each task as though I were a member of the group. I visualized myself as a silent member sitting at the table with the teachers. Afterwards, I referred to my reflective journal for any details I may have captured about the session which served to give me both the researcher and teacher perspective on the session.

Next, I went through the session again but this time through the transcribed conversations. When teachers brought up and talked about what they considered as they re-designed tasks, I highlighted those words to guide myself through the redesign process through the teachers' eyes. I then compared the highlighted conversations to the ideas the teachers talked about in their post-session reflections to develop, more fully, a complete picture of the work they did.

Finally, I followed their ideas through to the next task as a way to observe how their ideas progressed. In doing so, I was able to understand how teachers learned from one Task Re-Design Process and how they applied that learning to impact future task re-design sessions. I included the transcribed conversations with my findings to support my interpretation of their work. The analysis of the audio recorded conversations helped me to answer the following research question: *What do teachers consider as they re-design mathematics exercises?*

Researcher Reflective Journal – Coding and Analysis

I did not analyze my journal entries independent of other data. Instead, I referenced them throughout the coding and analysis of the tasks and teacher reflections to give me a better sense of the phenomenon as it occurred. That way, my journal provided details about the contextual situation of the work groups, reminded me of the details that occurred during the work sessions, and served as a form of triangulation.

For example, for one session scheduled on the last day of classes before an extended break, I wrote this in my journal:

> *I anticipate that many teachers will not attend tonight's session, as today was the last day of school for a week of spring break. I worry that the absences will impact the cohesiveness of each group's work. If that does happen, I am going to combine people into new groups and give them an alternative task this evening.*

However, I was wrong in that assessment as attendance that evening was 100% and, as I noted in my journal, the mood was that of "excitement, almost jubilance". As such, my journal helped to guide my understanding of the teacher's work within the context of the work itself.

Analysis Between Grade Levels

In order to develop an understanding of the ways in which teachers re-designed mathematics tasks, I organized the data by grade level bands. I separated the work of the early career and experienced teachers into the three groups of K-2, 3-5, and 6-8. This allowed me to identify similarities and differences within and between the work of the grade level teachers according to the problem structure and mathematics content of problems chosen for redesign and the types of modifications they made.

Cross Grade Task Analysis by Problem Structure

I compared the work of the K-2, 3-5 and 6-8 grade level band teachers according to the mathematics structure of tasks they chose for redesign. I looked at each grade level band individually and sorted the tasks teachers chose for redesign by the structure of the problem.

I organized the tasks into broad categories at first and looked for patterns in the problem structures that allowed me to further delineate them. For example, there were many *multiplication* tasks but some were contextual word problems while others were strictly number calculations. Also, there were not many *partitive division* and *quotative division* contextual tasks so combining those two into a general *division* category made more sense as long as I stayed consistent with keeping the numbers-only calculation tasks in their own category. Organizing the tasks like this helped me gain a better understanding of what types of tasks teachers at each grade level band chose for modification.

Additionally, organizing the tasks by grade level band made sense because, as expected, teachers at different grade levels chose very different mathematics operation tasks to redesign and this organization helped me to focus in on what types of tasks teachers chose to redesign.

Cross Grade Task Analysis by Types of Adaptations

Next, I did a comparison between the grade level groups on the types of adaptations the teachers made to the tasks. I used the version of Drake and Sherin's (2006) adaptations list to which I added and removed items to better fit the data I collected during my study.

My analysis included looking at problems for more than one modification. As I carefully studied each task to determine specifically what the teachers did to redesign each task, I found that it was quite common for teachers to perform multiple modifications to one task. For example, if teachers *omitted* a problem, it was very typical for them to make *additions* to that same problem. However, simply coding a modification as an addition created too broad a category for meaningful analysis. Therefore, I further delineated the additions teachers made to determine the specific types of additions teachers made and organized the data more specifically. This allowed me to better understand the phenomenon of how teachers modified problems in order to increase the level of cognitive demand required of their students.

Analysis Between Teacher Experience Levels

In addition to analyzing at the data by grade level band, I also analyzed it by teacher experience level. I analyzed, by teacher experience level, both the tasks according to the problem structures teachers tended to choose for modification, and the types of adaptations the teachers made to the tasks. To begin, I organized the mathematics operations of the tasks chosen for redesign by teacher experience level. Within that, I delineated the tasks first by problem structure so that I could better see and understand the similarities and differences and then by type of adaptation.

I looked at the problem structures of the tasks teachers chose for modification across the experience level sets to find patterns hat were consistently woven throughout the tasks. I noticed that the early career teachers frequently chose simple problems to redesign while the experienced teachers tended to choose for adaptation, tasks from a wider range of problem structures. It makes sense that teachers fairly new to the profession would choose problem structures that are easily modified into other problems types, while more experienced teachers may be more willing to take risks with a wider variety of problems. This evidences that teaching experience is important to the teacher text interaction specific to mathematics.

Next, I analyzed the types of adaptations both teacher groups made to the tasks as they worked to redesign them. This required only a slight change to the organization of the data as I already had the tasks separated by experience level. I reorganized the tasks so that I could better see and understand the types of adaptations the teachers typically made to tasks as they redesigned them. This helped me to understand that the early career teachers tended to make simple, structural changes to tasks. For example, many teachers chose to redesign *result unknown* problems and modified them into *part-part-whole, one part unknown* tasks. More experienced teachers, on the other hand, tended to make modifications that gave students more control over the intellectual work of the task. For example, it was typical of the experienced teachers to remove multiple-choice options and, instead, ask open-ended questions. They also were more likely to take routine problems and modify them into student explorations, which were often open-ended with multiple possible solutions. This again affirms that experience plays an important role in how teachers interact with mathematics curriculum materials. The more experienced teachers may felt more confident in making such changes.

This grade level and teacher experience level analysis helped me to answer the following research question: *What differences do I notice between the work of new and experienced teachers?*

Triangulation of Data Sources

Because one data source is not sufficient to provide a true understanding of the phenomenon under study, I triangulated the mathematics task data, the teachers' written reflections, the audiotaped teacher work sessions with my researcher reflective journal notes to develop a complete picture of the phenomenon of the teacher text interaction specific to mathematics. The process of triangulating across data sources helped me to gain assurance that I did not oversimplify the situation, overlook key meanings from the data (Stake, 2006), or miss important details. Triangulation of the data allowed me to synthesize the themes and patterns of each grade level band and experience level into an overall understanding of the phenomenon.

Summary

This point of this qualitative study was to explore the ways that early career and experienced teachers interact with mathematics curriculum materials to redesign tasks from prewritten materials. It was a two-stage study; Stage One consisted of teachers early in their careers, with less than five years of experience. Stage Two consisted of teachers with between five and ten years of experience. Stage One data was comprised of original and re-written mathematics tasks, teacher written reflections, and the researcher reflective journal. Stage Two data also consisted of that data but I included three audio-recorded work sessions of each a third grade and sixth grade teacher group. I analyzed the data both between and across the grade and experience level groups

For the study, I facilitated teachers through the Task Redesign Process, described on page 51. I asked teachers to take an interpretive view of several different curriculum materials and re-design tasks in ways that would increase the cognitive demand level required of their students. Once they collaboratively re-designed tasks, the teachers enacted them in their individual classrooms, and then came back together to share their experiences with their group members. At that point they discussed any revisions they would make prior to future enactments of the task. During this study, I collected and analyzed task data, teacher reflection data, and audiotaped work session data, which served to answer the following research questions:

1) In what ways do teachers modify pre-written mathematics exercises?
2) What did teachers report to consider as important as they re-design mathematics tasks?
3) What differences did I notice between the task re-design work of new and experienced teachers?

CHAPTER FOUR: FINDINGS

In this chapter, I outline results of my analysis of the mathematics tasks in order to answer the following research questions:

1) In what ways do novice and experienced teachers re-design pre-written mathematics exercises to increase the level of student thinking?
2) What do teachers report as important considerations in the re-design of mathematics exercises?
3) What differences do I notice between the work of early career and experienced teachers?

I begin by describing the general patterns I observed in the task redesign work of teachers at each grade level band: K-2, 3-5, and 6-8. In each section, I present the tasks teachers chose to re-design and the ways they re-designed them. I chose to present the patterns of modifications in the order of frequency for each grade level band in order to preserve the story of the teachers' work. For example, all grade band teachers made structural changes to tasks; for the K-2 groups, structural changes were the most common type of modification they made, but for the 6-8 groups, it was a far less common choice. Therefore, *structural change* as a modification is presented first in the K-2 section but much later in the 6-8 section. Second, I discuss the knowledge and resources teachers reported as important considerations in effectively making curriculum re-design decisions. Excerpts of audio-recorded interactions between teachers serve to support my understanding of the teacher-text interaction. Finally, I highlight the patterns and themes as they cut across experience and grade level.

Research Question 1: In What Ways Do Early-Career and Experienced Teachers Re-Design Pre-Written Mathematics Exercises?

In this section, I outline the number and type of tasks that teachers at each grade level band chose to re-design and the ways in which they re-designed them. I present the patterns I noticed both within and across the grade levels beginning (K-2, 3-5, and 6-8).

Types of Problems Chosen for Redesign

As expected, teachers in different grade level bands chose different types of problems to redesign. Table 4.1 provides an overview of the types of tasks teachers chose for modification at each of the three grade level bands.

Table 4.1
Problem Structure of Tasks Chosen for Redesign by Grade Level

Problem Structure	K-2	3-5	6-8
Join or Separate Result Unknown	13		
Join or Separate Change Unknown	1		
Part-Part-Whole, Whole Unknown	8		
Part-Part-Whole, One Part Unknown	2		1
Part-Part-Whole, Two or More Parts Unknown	1		
Multiplication Word Problems	1	6	
Division – Partitive or Quotative		5	
Multi-Step and Non Routine Problems		3	
Calculation Tasks – Add, Subtract, Multiply, Divide (no context provided)	1	1	1
Counting Tasks	1		
Operations and Number Sense		2	2
Factors and Multiples Word Problems			1
Identification/Comparison of Fractions		4	
Fraction Operations		4	1
Patterns and Algebraic Thinking		1	
Data/Statistics	4	1	
Geometry	1	3	2
Proportional Reasoning/Unit Rate		2	2
Total	**33**	**32**	**10**

The above table shows that the K-2 teachers mostly chose simple problems including *result unknown* and *part-part-whole, whole unknown* problem to redesign. It also shows that teachers in the grade 3-5 band chose a wider range of problems to redesign and that the grades 6-8 teachers chose problems to redesign from content areas important to their grade. In the following sections I will provide more detailed information about the problems teachers chose for redesign in each grade level band.

Modifications Teachers Made to Problems

Teachers in each grade level band redesigned problems in a variety of ways. I organized the adaptations they made in Table 4.2:

Table 4.2
Types of Task Adaptations by Grade Level Band

Types of Task Adaptation	K-2	3-5	6-8
Changes in terminology		2	
Changes in materials used		1	
Increasing student control over activity	13	12	3
Increasing teacher control over activity			1
Omitting problems	1	4	5
Subtracting from problems	1	5	
Adding problems to include context	2	5	1
Adding problems – students develop questions	2		
Adding problems – do more of the same	1	3	
Adding problems – student exploration		7	2
Adding problems – list all possibilities	3	4	
Add problems-multiple representations	3	4	5
Total	26	47	17

Teachers in the K-2 and 3-5 grade level bands commonly *increased student control over an activity* as a way to redesign problems. In the 6-8 grade level band, though they did so less frequently, teachers also redesigned problems by giving students more control over the intellectual work of the task.

I noticed that when teachers increased student control over the intellectual work of a task, they also made additional adaptations on those tasks that detailed how they would make that happen. So I explored the different combinations of adaptations that included increasing student control over the work to find out if there were any patterns in the pairing of certain adaptations. Table 4.3 below outlines the types of adaptation that occurred along with an increase in student control in each grade level band:

Table 4.3
Adaptations That Occurred with Increased Student Control:

Adaptations made with Increased Student Control	K-2	3-5	6-8
Add problems – students develop questions	2		
Add problems – multiple representations	1	2	1
Omit problems		4	1
Subtract from problems		5	
Add problems – list all possible answers	3	3	
Add problems – student explorations		6	2
Add problems - context		1	1
Changes in structure of problems	8		
Total	14	21	5

The K-2 grade level band teachers increased student control over the work by changing the structure of problems they redesigned. For instance, they frequently redesigned *result unknown* exercises into *part-part-whole, two or more parts unknown* tasks. This type of adaptation increased student control in that there was typically more than one possible solution.

The grades 3-5 teachers typically increased student control over the work by omitting questions in the original task and adding others. The types of questions they added were usually those that made the problem more contextual, asked for multiple representations, or asked student to list all possible answers. Similar to the work of the grades K-2 teachers, the types of questions added usually created more than one solution to the task.

The grades 6-8 teachers increased student control over the work by opening tasks up for student exploration. They redesigned the original problems to require students to create or build something and answer questions about it. Therefore, like the other two grade levels, the revised tasks had more than one solution depending on how students worked through it. In the following sections I will provide more detail on the tasks teachers chose for redesign and how they modified them.

Grades K-2 Teachers

As an overview, a total of 23 K-2 teachers participated in this study. Nine of them were in the early years of their career, and 14 were experienced teachers. In sum, they chose 33 tasks to re-design and created a total of 33 new tasks. They commonly re-designed mathematics tasks in two ways. First, they made structural changes to the tasks, taking, for example, a *result unknown* problem and re-designing it into a *part-part whole, two or more parts unknown* problem. Teachers made these structural changes 13 times, with the early career teachers responsible for nine. Second, the teachers re-designed problems that increased student control over the intellectual work of tasks. For example, some groups started with a problem with only one solution and re-designed it into a task with multiple possible solutions. They increased student control 12 times, with early career teachers being responsible for eight. Often, structural changes and increasing student control were overlapping modifications. Following Table 4.4 in which I present the before and after tasks teachers chose for redesign, I describe how the K-2 teachers often focused on increasing the cognitive demand of mathematics tasks through task re-design.

Table 4.4
Tasks chosen by K-2 Teachers for Re-design

Tasks chosen by the K-2 teacher groups, before and after re-design				
Problem Structure	Before		After	
	Number	Percent	Number	Percent
Result Unknown (Join or Separate)	13	39%	2	6%
Change Unknown (Join or Separate)	1	3%	3	9%
Start Unknown (Join or Separate)	0		2	6%
Part-Part-Whole, whole unknown	8	25%	1	3%
Part-Part-Whole, one part unknown	2	6%	1	3%
Part-Part-Whole, two or more parts unknown	1	3%	13	39%
Compare-how many more or fewer?	0		1	3%
Multiplication Word Problems	1	3%		
Division-Partitive/Quotative Word Problems	0		1	3%
Calculation-add, subtract, multiply, divide	1	3%	0	
Data/Statistics	4	12%	3	9%
Counting Tasks	1	3%	1	3%
Geometry	1	3%	0	
Tasks with Many Possible Solutions	0		5	16%
Totals	33	100%	33	100%

Structural Changes

Changing the structure of a problem was a common design choice for many K-2 teachers. These teachers frequently selected contextualized word problems of the simplest structures to re-design. *Result unknown* and *part-part-whole, whole unknown* problems were the most common choices for re-design. The K-2 teachers changed the structure of the tasks 13 times, re-designing them into more cognitively demanding problems that were not as easy for students to directly model or count. In the upcoming sections I provide examples of the teachers' work to highlight what tasks teachers chose for re-design and how they re-designed them.

Result unknown problems.

Result Unknown problems are the simplest type of problem for young children to solve because they include a clear action that can be directly modeled by students with drawings, counters, or their fingers (Carpenter & Fennema, 1992). Additionally the unknown quantity is the final quantity, which allows students to model the joining or separating actions in the story and the result is the solution. Result Unknown problems were the most commonly chosen mathematical operation chosen for redesign by the grades K-2 teachers. Of the 13 problems these teachers structurally redesigned, 11 were *result unknown* problems like the Library Book problem selected by a group of early-career kindergarten teachers in Figure 4.1 below:

Figure 4.1
Library Books – Original

The library had 2 picture books. Then Mr. Callahan gave the library 1 more. *How many picture books does the library have now?* *a) 4 b) 5 c) 3 d) 6*

This problem, categorized according to the CGI framework as *join result unknown*, is set in a context familiar to most students. Additionally, it includes multiple-choice options, and there is nothing to stop students from choosing an answer if they have not actually solved the problem. Also, the solution requires no explanation, and can be obtained by a guess. These features are what distinguish this problem as low-demand (Smith & Stein, 1998). Consider the re-designed the problem in which the teachers worked to increase the level of cognitive effort the problem would require of students shown in Figure 4.2 below:

Figure 4.2
Library Books – Redesigned

Mr. Callahan put one book on the shelf so there were 3 books on the shelf altogether. How many books were on the shelf before Mr. Callahan put his book down?

The teachers changed the structure of this problem to *join-start-unknown*. The total number of books on the shelf *after* Mr. Callahan added one is now given, and the problem asks for the number of books on the shelf before the action occurred. *Start-Unknown* problems are more difficult for children to directly model because the initial quantity is unknown.

In addition to the structural change, the teachers made two more design modifications. First, they opted to use the word 'one' in the problem rather than the numeral 1. The numeral 1 is a symbol that stands for the quantity. To move between words and numerals requires a basic understanding of how words and numerals are related. Also, the teachers omitted the multiple-choice option, requiring students to rely on their own reasoning to find a solution. According to the Mathematical Task Analysis Guide, the modifications these early-career teachers made to the task increased the cognitive demand of the task from a low to a high level. Altogether, the K-2 teachers increased the cognitive demand of 7 of the 13 *result unknown* problems they re-designed.

Another way teachers re-designed *result unknown* problems was to change them into *part-part-whole, two or more parts unknown* problems. One of the problems redesigned this way was selected by an early-career kindergarten teacher group and is presented in Figure 4.3 below:

Figure 4.3
Cookies - Original

There were 19 cookies on a plate. A girl ate one and then a boy ate another one. How many cookies are on the plate now?

This is a *separate-result-unknown* problem with clear action that students can directly model. The quantity of 19 is challenging for kindergarteners to work with but the context may be familiar enough to most and can act as a scaffold for the mathematics. In the following excerpt, one teacher reflected on the process her group went through as they re-designed the task. "We decided on this problem because we knew we could change the numbers and the question asked in the problem". The teachers intentionally chose this problem because they felt it would be easy for them to redesign. The re-designed task is shown in Figure 4.4 below:

Figure 4.4
Cookies - Redesigned

There were 6 cookies on a plate. A boy ate some and a girl ate some. *How many did the boy eat? How many did the girl eat?*

The problem is now a *part-part-whole problem, with both parts unknown,* and requires more intellectual engagement on the part of the student. *Part-part-whole* problems have no action verbs. Instead, they are about the relationship between parts and wholes. Despite the lack of action, students can model these, and research indicates that students find *part-part-whole, whole known* problems no more difficult than *result unknown* problems (Carpenter, Ansell, Franke, & Fennema, 1993). However, when the *parts* are unknown, the problem is more challenging.

Another important change is that the problem now has more than one correct answer, even though the question does not specifically ask students to find all possible solutions. This puts more of the intellectual load onto the student, giving them more control over how the work is done, and allows them to explore the problem to their own level of understanding. Students might find one or two possible answers, or they may come to realize that an organized list would help them see patterns in the numbers, pushing them to find all five of the possible solutions. At the very least, it could drive mathematical discussions between students who may see only their particular solution. Students also might reason about whether all six cookies were eaten, or whether some cookies remained on the plate. For these reasons, this problem is another in which the teachers increased the cognitive demand from a low to a high level.

Part-part-whole problems.

Two of the 13 tasks teachers structurally re-designed were *part-part-whole* problems like this example from an early career first grade teacher group shown in Figure 4.5 below:

Figure 4.5
Gifts - Original

Mrs. Cohen is buying gifts for her son and daughter. So far, she has bought a total of 15 gifts. 6 of the gifts are for her son. How many gifts are for her daughter?

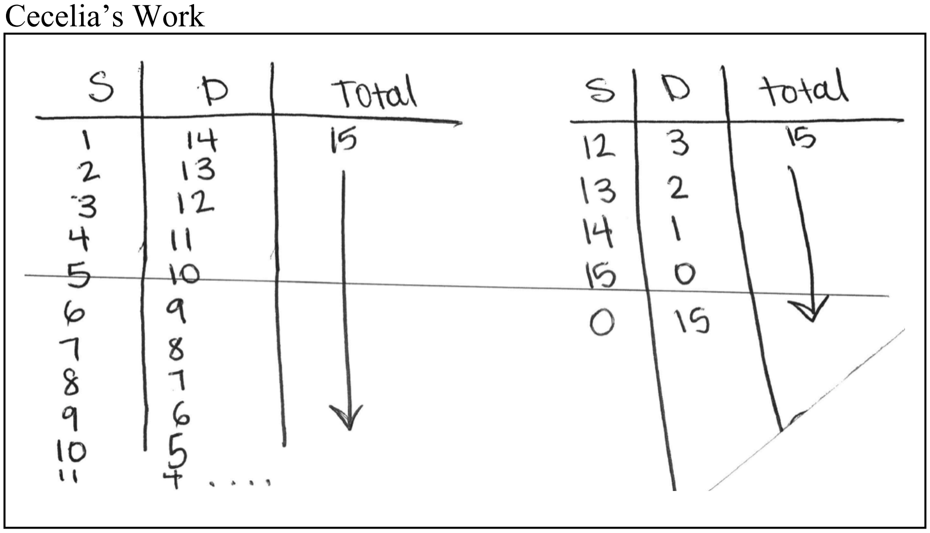
The lack of action in a *part-part whole, one part unknown* makes this a potentially challenging problem for first graders to solve. The goal is to find the missing part but it asks for no explanation. Consider the re-designed task, in which the focus shifts from finding one unknown part to finding both unknown parts in Figure 4.6 (*part-part-whole, two or more parts unknown* task):

Figure 4.6
Gifts - Redesigned

Mrs. Cohen bought 15 gifts total for her son and daughter (combined). How many gifts could be for her son and how many could be for her daughter? List all the possible combinations.

Now the students are asked to find all possible combinations of 15, which will require more intentional and systematic reasoning on the part of the student. In fact, Cecelia's work in Figure 4.7 shows that she thought very purposefully about how students might organize possible solutions as they work on this task:

Figure 4.7
Cecelia's Work



Certainly, Cecelia's work portrays only one possible way students might organize the task but by working through it, the teacher could assess the level of thinking this task would require of her students. The re-designed problem increases student control over the intellectual work of the task and shifts the focus from finding an unknown part to determining different ways to make the whole. This increased the cognitive demand of the task from a low to a high level.

Although changing the structure of a problem was a common design change these K-2 teachers made, they also made modifications that kept the structure of the original problem intact. For example, in the next section, I present changes focused on increased student control.

Increased Student Control

K-2 teachers made changes that increased student control over the intellectual work in 12 of the 33 tasks chosen for re-design. They did this by changing the structure of problems. For example, changing *result unknown* problems into *part-part-whole, two or more parts unknown* tasks.

Open-ended problems like these align with Schoenfeld's (1988) idea that cognitively demanding *mathematics tasks* are those whose solutions are not immediately obvious. They require students to make connections between ideas, draw conclusions and offer to students a richer mathematical experience than *mathematics exercises* in which students follow known procedures or rely on rote memorization for solutions. Table 4.5 outlines the number of times the K-2 teachers made this type of modification and the impact on the cognitive demand.

Table 4.5
K-2 Teachers Increase Student Control over the Intellectual work of Tasks

Grade Level	Number where student control was increased	Number of those where cognitive demand went from a low to high level
Kindergarten	5	4
First Grade	6	4
Second Grade	1	0

The K-2 teachers re-designed tasks to increase student control over the intellectual work 12 times and, in eight of those, they increased the cognitive demand of the task. Table 5 above shows that at least half of the tasks at each grade level, which were re-designed in this way, contributed to a more rigorous problem for students. In the next section I present examples from each grade level to highlight the ways in which teachers re-designed the tasks by increasing student control over the work.

Kindergarten examples.

Kindergarten teachers were the most successful at increasing student control over the intellectual work of a task and having that re-design contribute to an increase in cognitive demand. In one example, an early career group of kindergarten teachers focused on the naked number problem shown in Figure 4.8 below:

Figure 4.8
Calculation – Original

$6 + 4 =$ _____

This is a reasonable *calculation problem for* kindergarten since the focus of the grade is making ten. To increase student control over the work, the teachers re-designed the problem in Figure 4.9 as follows:

Figure 4.9
Calculation - Redesigned

I have two numbers whose sum is10. What could those two numbers be?

The task is still focused on the concept of making ten but the possibilities have been expanded. Students can explore the concept at a deeper level, with more embedded practice, and may find that organizing the information makes it easier to find solutions and highlight patterns between the numbers.

In another instance, an experienced kindergarten group over-scaffolded a data task and inadvertently decreased the cognitive demand. Consider the Favorite Foods problem in Figure 4.10 below:

Figure 4.10
Favorite Foods - Original

Draw your favorite food for lunch. Sort foods into categories. *Brainstorm which type of food do most students like? Least like?* *How many students like ______?*

This task has elements of a real-world investigation, as students need to collect and analyze data in order to answer the questions. In the re-designed task, presented in Figure 4.11 below, teachers added implementation notes in parenthesis to guide the classroom enactment.

Figure 4.11
Favorite Foods - Redesigned

Would you rather have chicken nuggets or pizza for lunch? (children physically move to a side of the room indicated for each choice). *Share with your partner why you chose _______.* *How many more students like _____ than ______?* *(Have students do a handshake with someone from the other*

Word problems involving comparisons (*how many more children like____ than* ____ can be challenging for kindergarteners as they reflect a high level of cognitive interaction (Carpenter, Fennema, & Franke, 1996). The one-to-one match up via handshake provides a way for students to organize and compare different quantities, but also has the potential to overly scaffold, or constrain student thinking. In a way, the re-designed task models for students how to solve the problem instead of asking them to generate possible solution strategies. Finally, while asking students to talk with their partner is an important way to develop their understanding of mathematics (Stein, 2007), having them talk about *why* they prefer one food over another shifts the focus away from the mathematics and decreases the required level of cognitive demand. Morgan illustrated this in her post-implementation teacher reflection:

> *I tried to have my students come up with one discussion about what their table liked the most. That turned into arguing of what their parents would let them eat or the taste of the food. By the time all was done, we did not complete the task. I lowered*

the thinking and made a graph for all the different types of food. Then they had to answer why they thought that food was the best.

Araceli, however, had a very different reaction:

This lesson was a great way to build a sense of community in my class as we learn more about each other by sharing perspectives and choices. Justifying why they (the students) chose something specific helps them to think critically.

Though the students were given increased control, to some extent, in the re-designed problem, it tended to draw them away from the mathematical ideas and decreased the level of cognitive demand required to solve.

First grade example.

The first grade teacher groups successfully increased the cognitive demand of four out of six tasks in which they increased student control over the work. *Plastic Shapes* was one of the exercises, a *part-part whole, whole unknown* problem chosen by an experienced group of first grade teachers presented in Figure 4.12 below:

Figure 4.12
Plastic Shapes – Original

You have two plastic shapes; one is a rectangle, the other a triangle. *How many sides are there all together?*

This task can let the teacher know if students understand the meaning of the word *side* in the context of geometrical shapes, and whether or not they know how to count, but does little else. The following re-designed task, in Figure 4.13, asks more of students:

Figure 4.13
Plastic Shapes - Redesigned

I have some plastic shapes in my cup with a total of ten sides.

What shapes could be in my cup?

In this *part-part whole, two or more parts unknown* task the meaning of sides becomes secondary to the exploration of figuring out what shapes could be in the cup. Students might approach the task by guessing and checking, or depending on their current level of knowledge, they may see that making an organized list will help them to reach a solution. By increasing students control over the work to the students, the task becomes more accessible to a wider range of students.

Second grade example.

In the following data/statistics problem, teachers were able to maintain a high level of cognitive rigor while increasing student control over the work. Consider this task selected by an early career second-grade teacher group in Figure 4.14 below:

Figure 4.14
Recycling – Original

Mike does his part to keep the Earth clean. He recycles items. He went through his recycling tub and sorted the items into groups. Create a graph of the data.

Recycled Items	
Cans	*7*
Plastic Bottles	*4*
Boxes	*2*
Paper	*3*

Students are to use the table to construct a graphical representation of the data. There is no scaffold or suggested frame for the graph. It is intellectually demanding, as students must decide how to construct the graph, which could mean students will come up with different solutions. To re-design the task and maintain the high level of cognitive demand, the group kept the original problem and added additional statements as shown in Figure 4.15 below:

Figure 4.15
Recycling - Redesigned

Mike does his part to keep the Earth clean. He recycles items. He went throughhis recycling tub and sorted the items into groups. Create a graph of the data.

Recycled Items	
Cans	*7*
Plastic Bottles	*4*
Boxes	*2*
Paper	*3*

Write 3 questions about the data in your graph.

Write 3 facts about the data in your graph.

These additions gave students a focused way to think about the data but kept the student in control over how to organize, interpret, and present the data in order to write questions and facts about it. Of course, students could come up with lower level identification questions such as "how many cans did Mike recycle?" but they might also come up with, "how many more cans and bottles did Mike recycle than boxes and paper?" After implementing the task in her classroom, Aileen wrote the following reflection:

Their (students') attitude was a lot different toward math compared to how it had been when just asked to answer questions about data. Students were using the vocabulary words and critically thinking about what the data meant by thinking of questions to ask about it.

By adding prompts that make students develop questions and facts about the data, students must think about the quantities and representations conceptually, considering what the data and graphs mean.

Tasks with Many Possible Solutions

As teachers worked with the intention of increasing the cognitive demand of problems by redesigning them, they sometimes created tasks that had many possible solutions. There were five instances where modifications the K-2 teachers made to tasks held this property. Four of the five were from the second grade teacher groups, one from an experienced group, and three from early career teacher groups. The pattern I noticed in these five tasks is that they all lacked parameters that made them difficult to deal with as written. But, the lack of parameters also gave the tasks the potential to engage young students in important thinking where they would need to make assumptions in order to find a solution. Take this example from an early career second grade group. The original problem is in Figure 4.16 as follows:

Figure 4.16
Pencils and Stickers - Original

A pencil costs 59¢ and a sticker costs 20¢. How much (do they cost) all together?

The original problem is a low-level *result unknown* problem where there is little ambiguity about what needs to be done, and the numbers do not require regrouping. The redesigned version of the task is as follows in Figure 4.17:

Figure 4.17
Pencils and Stickers - Redesigned

I bought a pencil and a sticker for 98¢ total. How much did they cost?

This task now asks only "how much did they cost?" Though the teachers' intent may have been to design a *part-part-whole, two or more parts unknown* task, they did not directly articulate that question. Second graders may see no other possible answer to this task besides "98¢" because that answers the question they were asked. Though the large number of possibilities might quickly discourage most students, it could also provide them the experience of wrestling with real world problems in which they have to determine and impose realistic parameters in order to reach a solution. Would students decide a base price of what each item could cost and work from there? Or would they begin with one cent and ninety-seven cents and consider every factor pair as a possibility? Because of the lack of parameters and the real world situation, the task, though possibly overwhelming for second graders, could be an interesting task with which to engage students. .

As another example from another early career second grade teacher group, these teachers began with a low level *result unknown* task as shown in Figure 4.18:

Figure 4.18
Apples – Original

Tom had 10 apples. Jack gave him 10 more. How many apples does Tom have now? *a) 60 b) 20 c) 80 d) 100*

The multiple-choice option and friendly numbers contributed to the low level categorization of this problem. Figure 4.19 shows how the teachers re-designed it:

Figure 4.19
Apples - Redesigned

Tom went to a farm and picked up 10 different kinds of apples, red, green, and yellow. As he reaches into the bag of apples, what is the probability of picking red, green, and yellow apples?

The teachers' goal seemed to have been to re-design this *result unknown* problem into a *probability* task in order to move from an easy to a more difficult concept but the task as written does not provide enough information about the number of each apple color in order for the student to find a solution. However, if students understand that they themselves are empowered to be the experts and place parameters onto the situation, the task holds the potential to be a way for young students to begin grappling with complex mathematical decisions. The task becomes rich when students decide how many of each color apple Jack has in his bag, which allows students to determine a solution.

Summary of Key Trends- K-2

Overall the teachers in the K-2 group chose the relatively simple *Result Unknown* and *Part-Part-Whole* problem types to re-design. They most commonly re-designed the structure of simple problems into more challenging mathematics operations that increased student control over the mathematics. They increased the cognitive demand of 18 of 33 tasks they modified, most often by making design changes that incorporated two adaptation-that of changing the structure of the mathematics operation of the task along with increasing student control over the intellectual work of the task. These teachers made design changes that maintained the cognitive demand level nine times and decreased it once. There were three tasks in which the redesign work of the teachers created potential tasks with many possible solutions.

Grades 3-5 Teachers

A total of 28 teachers of grades 3-5 from three different area school districts participated in this study; nine were in the early years of their practice and 19 were experienced teachers. All together they chose 32 tasks to re-design, from which they created 38 new tasks. Like the K-2 group, they most commonly re-designed tasks in two ways. Most often, they increased student control over the intellectual work of the task by omitting questions from the original task and adding others that opened up the number of possible responses students might have. All together, they modified tasks this way 12 times; experienced teachers being responsible for 11. Though less common, another way these grades 3-5 teachers re-designed task was to structurally change problems, often by changing multiplication problems into division problems. They did this five times and experienced teachers were responsible for four of the five.

Other types of changes the grades 3-5 teachers made include two terminology changes they made to tasks that had no impact on the mathematics or level of cognitive demand. They added questions to three tasks that asked students to do more of the same type of problem, which also had no impact on cognitive demand. Four times they added context that did not impact the mathematics; they asked for multiple representations twice, and created four tasks with many possible solutions.

Table 4.6 outlines the types of tasks teachers chose for re-design, and the structures of those tasks after re-design.

Table 4.6
Problem Structures Before and After Re-design

3-5 Tasks, before and after re-design-whole group comparison				
Problem Structure	Before		After	
	Number	Percent	Number	Percent
Calculation-add, subtract, multiply, divide	1	3%		
Operations and Number Sense	2	6%	2	4%
Multiplication Word Problems	6	19%	6	16%
Division-Partitive/Quotative Word Problems	5	16%	8	21%
Identification/Comparison of Fractions	4	13%	4	11%
Fraction Operations	4	13%	3	8%
Patterns and Algebraic Thinking	1	3%	1	3%
Data/Statistics	1	3%	1	3%
Geometry	3	9%	4	11%
Multi-Step and Non Routine Problems	3	9%	3	8%
Proportional Reasoning/Unit Rate	2	6%	2	4%
Tasks with Many Possible Solutions			4	11%
Totals	32	100%	38	100%

Increased Student Control

Like the K-2 teachers, the grades 3-5 teachers began with relatively simple problems and re-designed them in ways that increased the level of student control over the mathematics. This was most common with the early career teacher groups but was evident across the data. They did this 12 of 32 times, and in seven of those, they increased the cognitive demand from a low to a high level. Table 4.7 below outlines the number of tasks at each grade level where teachers' modifications increased student control and cognitive demand after which I provide examples of tasks from each of the grade level bands.

Table 4.7
Grades 3-5 Increased Student Control and Increased Cognitive Demand.

Grade Level	Number where student control was increased	Number of those where cognitive demand went from a low to high level
Third Grade	4	2
Fourth Grade	3	0
Fifth Grade	5	5

Third grade examples.

The third grade teacher groups increased student control over the work four times, they increased the cognitive demand from a low to a high level twice, and in the final two tasks, they maintained a high cognitive demand. The following task, in Figure 4.20 chosen for re-design by a group of early career third grade teachers, was categorized as requiring a high level of cognitive demand as shown below in Figure 4.20:

Figure 4.20
Amusement Park – Original

Amy saved her money for a trip to the amusement park. She had enough money to buy 100 tickets. Before she went to the park, she planned how many times she wanted to ride each ride.
Complete the table to show how many tickets Amy planned to use on her trip.
How many tickets does she have left?
How many arcade games can she play?

Rides	***Tickets Per Ride***	***Number of Times***	***Number of Tickets Total***
Roller Coaster	*8*	*2*	
Spinning Teacups	*7*	*2*	
Flying Swings	*6*	*4*	
Bumper Cars	*5*	*4*	
Ferris Wheel	*4*	*3*	
Bounce House	*3*	*2*	
Total Tickets Used			
Tickets Remaining			
Arcade Games	*2*		

This cognitively demanding, multi-step task is one that will encourage third graders to explore multiplication within a context exciting for many children. In Figure 4.21, below, the re-designed task shows how the teachers maintained the high level of cognitive demand:

Figure 4.21
Amusement Park - Redesigned

Amy saved her money for a trip to the amusement park. She had enough money to buy 100 tickets. Decide on how Amy will use her tickets.

Ride	*Tickets Per Ride*
Roller Coaster	*8*
Spinning Teacups	*7*
Flying Swings	*6*
Bumper Cars	*5*
Ferris Wheel	*4*
Bounce House	*3*
Arcade Games	*2*

To modify the task, the teachers made a few important changes. First, they asked students to explore the idea of planning Amy's trip to the amusement park, within the parameters of her having 100 tickets. They also omitted the original table; students now must devise their own strategy for keeping track of Amy's trip and they subtracted two questions from the problem, (*how many tickets does she have left?* and *how many arcade games can she play?*). This task is another example where the changes of omitting problems and adding problems correlated with an increase in student control over the task but did not change the cognitive demand.

In another example of how teachers increased student control over the work, an experienced third grade teacher group chose this cognitively demand task for re-design, shown below in Figure 4.22:

Figure 4.22
Party Planning – Original

Jack is setting up tables for a party. Each table has 6 chairs. How many chairs does he need for 10 tables?

The problem cannot be solved by mindlessly following a procedure-students will need to make sense of the situation and strategize in order to find a solution. When they re-designed this task in Figure 4.23 below, the teachers incorporated even more student choice:

Figure 4.23
Party Planning - Redesigned

Jack is setting up tables for a party. He is expecting 24 guests. He wants an equal number of chairs at each table. *What are some different ways he can arrange the room?* *Extension: If 12 more guests arrive, how could he add chairs to tables that are already there?*

The problem is still cognitively demanding but is now a division problem, where students need to explore how they might arrange furniture for a party, given that each table is to have an equal number of chairs. Students know that 24 guests are expected at the party, and that Jack wants the same number of chairs at each table, but no other information is provided. Students are asked to offer some different ways Jack can arrange the room. It requires a degree of intellectual interaction as students think about how many tables and chairs to set up for the party. Beyond providing an answer, this task assumes students are competent problem solvers and asks for their mathematical thoughts and ideas rather than asking for answers, which can be classified as correct or incorrect.

When reflecting on the enactment of this task, one of the teachers said that it highlighted to her how she had been underestimating her students. She said that, while they were still working towards basic fact fluency, this task gave her students lots of practice to help them build that fluency, and also provided them with the opportunity to practice working with their peers as a team. (Rishor Reflective Journal, Feb. 2015). This task evidences that experienced teachers tended to think about redesigning problems to increase cognitive demand in complex ways beyond changing the position of the unknown.

The final third-grade example is a task selected by an experienced third grade teacher group. The task began as a high-level *fraction identification* problem in which students are asked to label benchmark fractions on a number line, then to estimate the location of additional fractions as shown in Figure 4.24:

Figure 4.24
Number Line – Original

Draw a number line and label tic marks for benchmarks 0, 1/2, and 1. Estimate where you would put the fractions 5/8, 7/8, and 1/8.

In this task, students draw a number line and label three benchmark fractions of 0, ½, and 1. They reason about where to place additional fractions on that same number line. The teachers enhanced the redesigned task by asking students to create and place their own fractions on a number line, within specific parameters. In this way, the re-designed task in Figure 4.25 was also cognitively demanding:

Figure 4.25
Number Line - Redesigned

Draw a number line from 0 to 1. Estimate and put a tic mark for 1/2.
Estimate and put a tick mark to show a fraction that could come between 0 and 1/2.
Estimate and show a fraction that could come between 1/2 and 1.

In the new task, which is still a *fraction identification* problem, students are faced with three steps. First they must draw a number line and label where 0, ½, and 1 are. Then they need to estimate and label fractions that come between 0 and ½, and between ½ and 1. This is another task in which the teachers added an element of exploration, as students must choose their own fractions to place on the number line. Altogether, this task requires students to think intentionally about fractions and how they relate to benchmark fractions and to one another. Various solutions are possible. As a result, the re-designed task maintains the high level of cognitive effort required of students. In an upcoming section I will present excerpts of the teachers' conversations as they re-designed, and later debriefed the enactment of this task.

Fourth grade example.

Fourth grade teachers increased student control over the intellectual work in three tasks. Twice, an experienced teacher group began with high-level problems and opened them up to more student exploration, a change that contributed to maintaining the high level of cognitive demand. Figure 4.26 shows an example of one of those original tasks chosen by an experienced teacher group:

Figure 4.26 Penny Jar - Original

Here is a penny jar situation. Start with 1 penny. Add 3 pennies each round. How many pennies will be in the jar after 20 rounds?

This task is one that fourth grade students will probably need to manually work through using some sort of physical representation. There is no suggested procedure to follow but students will quickly come to see that they need a way to organize their work. They might approach this in a linear fashion, working through the 20 rounds in order to reach an answer. This task asks a lot of students, but the re-designed task, Figure 4.27, asks even more:

Figure 4.27
Penny Jar - Redesigned

Here's a penny jar situation. Start with 1 penny. Then add 3 pennies each day. How many days will it take you to have $2.68? Now start again, begin with 1 nickel, and add 3 nickels each day. How many days will it take you to have $2.68? Illustrate each solution in at least two different ways.

Though similar to the first problem, the teachers omitted the question of how many pennies will be in the jar after 20 rounds. The teachers added to the problem that students now must find how long it will take to reach a sum of $2.68 by following the pattern (it takes 90 days to reach $2.68 with pennies). When working through the same problem with nickels, students need to realize that it is not possible to get to the exact sum of $2.68, since that quantity is not divisible by five. They could build the physical pattern or estimate that it will take 1/5 the amount of time it took with pennies. Both option opens up student exploration and could generate some interesting class discussions as students play with the mathematics. Thinking about how to represent each solution two different ways is a modification that may serve to keep students intellectually engaged. This task is another example of teachers omitting problems and adding an exploration that also increased student control over the work.

Although the fourth grade teachers did not impact the cognitive demand of the tasks they re-designed in this manner, increasing student control over the mathematics maintained the high level of intellectual challenge for students.

Fifth grade example.

The experienced fifth grade teacher groups increased student control over the intellectual work in five tasks and, in all five, increased the cognitive demand of the tasks from a low to a high level. As an example of what they did, consider this computation problem in Figure 4.28 that an experienced teacher group chose for re-design:

Figure 4.28
Order of Operations - Original

Evaluate:
$7 \div (10 + 3 - 6)$

This is a low level procedural task in which students follow the order of operations to derive the one correct solution. The experienced fifth grade teacher group who decided to re-design this problem came up with a new version in which students have more control of the work through exploration shown in Table 4.29 below:

Figure 4.29
Order of Operations –Redesigned

Use the numbers 3, 6, 7, and 10, and any of the four operations of addition, subtraction, multiplication, and division to make the number 1. How many solutions can you create without using exponents?

The teachers omitted the original question to require a more thinking on the part of the student. There is no algorithmic way to find an answer to the new, exploratory question so instead of solving one problem, students will be practicing their fluency by solving many problems and, even if they cannot find more than one solution, they will have invested a good deal of intellectual effort into the mathematics of the task.

Structural Changes

As another re-design technique, the grades 3-5 teachers changed the structure of problems they chose for modification. They made five structural changes to task and all five were made to *multiplication* problems. Teachers changed four of the five multiplication problems to division problems and rendered one of them unsolvable. They increased the cognitive demand in three of the four problems. In the upcoming section I present the structural changes made by grades 3-5 teachers.

Multiplication problems.

Multiplication problems were the most common problem type that teachers chose for structural re-design and the third grade teacher groups redesigned all five of them; one by an early-career group and four by experienced teacher groups. To qualify as a structural change, an original task had to begin as one type of problem and end as a different type.

Twice the teachers chose to design a division problem from a multiplication problem as in this cognitively demanding task presented in Figure 4.30 chosen by an early career third grade teacher group:

Figure 4.30
Strawberries – Original

A restaurant ordered 3 crates of strawberries. There were 70 berries in each crate. How many strawberries in total did the restaurant order?

Students could solve this problem a number of ways, including the use of models, repeated addition, or by multiplying the two factors together for example. These teachers re-designed the problem by turning it into a partitive division problem as shown below in Figure 4.31:

Figure 4.31
Strawberries - Redesigned

A restaurant ordered 90 strawberries. They came in 3 crates. How many strawberries were in each crate?

In the re-designed task, students partition the strawberries into three groups to determine how many strawberries are in each crate. The teachers did not explicitly state that each crate should contain an equal number of strawberries which means students will need to make an assumption of that to be the case. The three continues to serve as the number of groups, but the number of strawberries has been changed from 70 to 90, a quantity more easily broken into three equal groups. No change in cognitive demand was evident and the high cognitive effort required in the original task is comparable to that of the revised task.

Other times teachers increased the cognitive demand of *multiplication* problems by re-designing them into *division* problems. As an example, consider this low level task in Figure 4.32 chosen for re-design by an experienced third grade teacher group:

Figure 4.32
Pencil Box – Original

Johnny has 3 boxes of pencils; each box has 4 pencils in it. How many pencils does Johnny have?

This is a fairly typical equal groups multiplication word problem that asks students to find the product of two numbers. While some students may be able to solve this problem by recalling the memorized multiplication fact of 3 x 4, it may require more work of other students. The teachers re-designed the task into a division problem as shown below in Figure 4.33:

Figure 4.33
Pencil Box - Redesigned

Joe has 12 pencils total. He needs to separate them equally into boxes. *How many ways could he separate them? What's the least number of boxes he can use? What's the most?*

In the re-designed task, students need to determine both the number *of* groups, and the number *in* each group - both quantities are unknown. Students also need to think about the problem in context because of the final two questions, "*What's the least number of boxes he can use*" and, "*What's the most?*"

As students begin to sort the pencils into groups, they must keep track of which quantity is the number of boxes, and which is the number of pencils in each box. Students need to organize their work to find multiple possible solutions, a skill that requires some degree of critical thinking and cognitive effort. As such, their re-designed task is more cognitively demanding than the original. In an upcoming section, I present teachers' conversations as they re-designed this problem.

Fraction operations problems.

Three of the thirty-two tasks teachers chose for re-design were *fraction operations* problems but they changed the structure of the problem in only one of them. In the following example, chosen for re-design by an experienced fifth grade teacher group; the teachers re-designed the *fraction operations* problem into a *multi-step non-routine* problem. The original problem is shown below in Figure 4.34:

Figure 4.34
Fraction Operation – Original

What is the difference between 3/5 and 2/10?

The teachers chose a problem that students might solve in a number of ways. Had they experience with the traditional algorithm used in the United States; students might create an equivalent fraction to the subtrahend (i.e., 3/5 = 6/10) and then subtract the numerators. Since the task is intended for fifth graders who, most likely, have been introduced to an algorithm, and only one fraction needs to be re-named, this problem requires a relatively low level of cognitive effort.

Consider the re-designed problem in Figure 4.35 that the teachers structurally changed to a multi-step, non-routine problem and to which they added context in two ways:

Figure 4.35
Fraction Operation – Redesigned

> *I had 3/5 of a dollar. I lost 20¢. What fraction of a dollar do I have left?*
>
> *How many different ways can you show how to solve this problem?*

The students now have to perform multiple steps in order to solve the problem, which goes beyond a typical fraction calculation task. The starting quantity is a fraction, and the amount of lost money is a decimal. The two different representations require students to attend to the problem deliberately by connecting the fractional and decimal representations. The teachers also re-designed this problem by adding a real-world context – money - and asked students to come up with multiple ways to solve the problem. Both of these re-design techniques contributed to making this a more cognitively demanding task for students.

Multi-step and non-routine problems.

The final type of problem the grades 3-5 teachers chose for re-design was *multi-step, non-routine*, selected three out of 32 times. All three started as low demand problems and two of them remained so after re-design. An interesting example redesigned by an experienced fourth grade teacher group is presented below in Figure 4.36:

Figure 4.36
Field Trip – Original

> *There are 19 students going on a field trip. They are taking a 7-seat bus.*
>
> *Each seat holds 3 students. Will there be enough seats for all 19 students?*

Though it is a contextual problem, it is posed as a yes-or-no question. Students can guess and have a 50% chance of getting the correct answer, which makes for a task that is not very intellectually demanding. The re-designed problem, in Figure 4.37, however, asks more of students:

Figure 4.37
Field Trip – Redesigned

19 children take a bus to the zoo. They are to sit 2 or 3 to a seat. There are 7 seats. How many seats will have 3 children seated on them? Use pictures, numbers, and words to justify your answer.

For the re-designed task, the teachers opted to replace the original problem with this well-known non-routine CGI problem (Carpenter, Ansell, Franke, &Weisbeck, 1993). This problem requires students to interact more with the mathematics in order to reach a solution. It is given that all children will have a seat on the bus, and the focus is on how many students will sit two to a seat, and how many will sit three to a seat. Students will need to explore the situation in order to find a solution. The answer must be justified with words, pictures and numbers. To facilitate student understanding of the multiple representations required, one teacher created a four-square graphic organizer for students and labeled the boxes for the word problem, the equation, the model, and the explanation (Rishor reflective journal, March 2015).

When reflecting on the enactment of this problem, one of the teachers noted: "Students immediately went to pictures (but) I would have preferred more styles of solving." She did not elaborate on what students did with the pictures or what other styles of solving she hoped to see. Another teacher remarked during the group debrief that her students drew seven boxes to represent seats and used counters to model the students. The multiple entry points into the task encourage student exploration and contribute to the intellectual rigor.

Tasks with Many Possible Solutions

Four of the problems chosen by early career teacher groups were re-designed into problems that had many possible solutions. Typically, the problems lacked parameters, which made them seem unsolvable, but could actually encourage students to assert their own parameters to determine a solution. Doing so could help students think logically about what makes sense, (beyond double-checking their final answers), and empower them to interact with mathematics as authorities in that they could impose their own limits onto problems. Following is an example of such a problem. The teachers began with a highly demanding task and recreated it into one with many possible solutions. Below in Figure 4.38 is the task chosen for re-design by a group of early career third grade teachers:

Figure 4.38
Bows – Original

Jackie has 30 feet of ribbon to make bows. How many bows can she make if each bow needs 6 feet of ribbon? Make a model. Find how many bows.

Consistent with other highly demanding tasks, the problem is contextually based and contains different entry points for students at various levels of understanding. For example, using a number line or bar model to represent the total length of ribbon, and then partitioning it into sections measuring six units each is one way students might approach this problem. Others may use knowledge of multiplication facts to find a solution. In Figure 4.41, the re-designed problem shows that the teachers made the problem more open-ended:

Figure 4.39
Bows - Redesigned

> *Jackie has 30 feet of ribbon to make bows. How many bows can she make if each bow is the same length?*

This is a more cognitively demanding task. It is open-ended and encourages students to explore various lengths of ribbon to make bows. The problem includes no information about how long a piece of ribbon must be in order to be able to make a bow, or what units can be used to partition the ribbon (e.g., feet? inches? fractions of an inch?). Also unstated is whether the entire length of ribbon must be used, or if it is okay to make, for example, seven bows of four feet teach and have two feet of ribbon left over. As a result of the lack of parameters, students need to think realistically about the context and impose their own ideas onto the task.

In a collaborative group setting, coming to a consensus about such questions could generate rich conversation among students who will have to justify their ideas to their peers as they argue for their point of view. As such, with their redesign efforts, the teachers created a task that could help students develop skills with abstract thinking. First, the students must interact with important mathematical ideas in a flexible way as they work out different solutions for the number of bows that could be made based on a certain length of ribbon. Second, the task could also drive students to refine their oral communication and justification skills as they work together to decide on a reasonable length of ribbon to make a bow. They could decide that a range of lengths would be acceptable and approach the problem from that perspective or they might impose parameters that are not inherent in the task itself. For example, students might decide that bows must have large loops and long tails which would impact the length of ribbon required. The point is that the redesigned task is a realistic, open-ended situation and could serve to advance students' skills with mathematics in ways that are authentic to approaching real life problems.

Summary of Key Trends 3-5

The grades 3-5 teachers increased the cognitive demand from a low to a high level in 13 of the 32 tasks they re-designed, maintained it in 15, and made changes that created potential modeling tasks four times. The early career teachers were responsible for one of the 13 modifications that increased cognitive demand and they also made all four of the modifications that had many possible solutions. Seven of the twelve re-designs that turned increased control over to the student were ones in which the teachers increased the cognitive demand. Three increases in cognitive demand were attributed to structural changes teachers made to the problem.

I found an interesting relationship within the 3-5 grade band but not the K-2 or the 6-8 bands between the adaptations these teachers typically made to task. Specifically, whenever the grades 3-5 teachers increased student control over the intellectual work of a task, it always went along with *omitting from* and *adding to* the task. However, it was only evident where the type of addition made turned problems into *student explorations*. I double-checked the data to make sure I understood it correctly. To do so, I looked at combinations of *omitting problems* paired with the other types of *additions* to tasks (see Table 3), to determine how those impacted cognitive demand. The other types of additions were *additions of context*, *asking for multiple representations*, or *doing more of the same type of problem*. I found that while some of the combinations resulted in increased cognitive demand some of the time, there was no consistent pattern like there was with the combination of *omitting problems* and *adding problems that turned the task into student exploration*s.

Grades 6-8 Teachers

A total of 12 middle grades teachers from three area school districts participated in this study. Seven of them were in the early years of their practice and five were more experienced teachers. Like the other groups, the middle grades teachers chose to re-design tasks from content areas central to the grade band, (i.e., *proportional reasoning, operations and number sense*, and *geometry* problems) and made changes that gave more control over the work to students. They chose a total of ten problems for redesign and ended up with ten new tasks after their re-design work.

In the next sections I present the tasks teachers chose to re-design and the ways in which they re-designed them. I begin with Table 4.8 below, which outlines the before-and-after structure of the middle grades tasks:

Table 4.8
Tasks Chosen for Re-design by the Grade 6-8 Teachers

6-8 Tasks, before and after re-design-whole group comparison				
Problem Structure	Before		After	
	Number	Percent	Number	Percent
Part-Part-Whole, one part unknown	1	10%	1	10%
Factors and Multiples Word Problems	1	10%		
Calculation-add, subtract, multiply, divide	1	10%	1	10%
Operations/number sense	2	20%	3	40%
Fraction Operations	1	10%		
Geometry	2	20%	1	10%
Proportional Reasoning/Unit Rate	2	20%	2	20%
Tasks with Many Possible Solutions			2	20%
Totals	10	100%	10	100%

Both early career and experienced grades 6-8 teachers took more time, during each session, to look through the materials everyone brought before deciding on a task to re-design. The most common design changes teachers made were to increase student control over the mathematics, which they did twice; and to ask for multiple representations of a solution, which they also did twice. Other changes they made include a structural change to one task and the inadvertent oversimplification of another. They redesigned two tasks to have many possible solutions. In the upcoming sections I present the tasks that the middle grade teachers re-designed and the ways in which they re-designed them.

Increased Student Control

In both of the tasks where teachers increased student control over the work, they were able to increase the cognitive demand from a low to a high level. Following Table 4.9 below outlines which groups made this type of modification, I present the tasks in which teachers increased student control over the work.

Table 4.9
Increased student control and increased cognitive demand

Grade Level	Number of tasks where student control was increased	Number of those where cognitive demand went from a low to high level
Sixth Grade	1	1
Seventh Grade	0	0
Eighth Grade	5	5

Sixth grade example.

Teachers in the middle school groups typically chose numbers-only calculation problems for redesign. For example, Figure 4.40 shows a problem chosen for re-design by a group of experienced sixth grade teachers. It begins as a low level, numbers-only division problem:

Figure 4.40
Division – Original

157 ÷ 18 =

There is no solution strategy suggested so students will need knowledge of how to solve division problems with double-digit divisors, and how to deal with remainders. However, there is no requirement for students to show, or explain, their solution strategy. The revised *calculation* task is below in Figure 4.41:

Figure 4.41
Division - Redesigned

Write a word problem, create a visual, and justify your solution:

Word problem	*Equation* *157 ÷ 18 =*
Visual	*Justification*

The re-designed problem now asks for a word problem, a visual depiction of the problem, and justification for the solution. Given that there is a remainder, students will need to think carefully about division in order to come up with three representations of the solution. As such, the problem has become more cognitively demanding.

Seventh grade example.

Two seventh grade teacher groups increased student control over two tasks by asking for multiple representations of the solution, and in both tasks increased the cognitive demand. They were chosen by two different groups, one an early career teacher group and the other by an experienced teacher group. Here, in Figure 4.42 is the *integer calculation* problem chosen by a group of early career teachers:

Figure 4.42
Integer Calculation – Original

7 – (-3) =

This low level, numbers-only problem asks students to find one correct solution. Typically, once students understand and know from memory a procedure for solving integer problems that is the only strategy they use unless specifically asked otherwise. Here is the re-designed *calculation* task in Figure 4.43:

Figure 4.43
Integer Calculation - Redesigned

Model the following: 7 – (-3)

With their re-design of this task, the teachers increased the level of cognitive demand from low to high. Students will need to think carefully about the meaning of this problem in order to accurately model the situation. Modeling will make the students' thinking visible and provide more formative assessment data to the teacher. Among other strategies, students could model the task with a vertical/horizontal number line, or by using two-color counters, for example.

Eighth grade example.

In another example of a design change that increased student control over the mathematics, an experienced eighth grade teacher group decided to re-design a *proportional reasoning* task embedded in *geometry* and shown in Figure 4.44:

Figure 4.44
Triangle – Original

Given a triangle and its image under dilation, explain how you can use a ruler to find the scale factor of the dilation.

As written, this is a very direct task. Students must find the solution and explain how they found it by using a ruler to determine the scale factor. It suggests, therefore, that students have already been introduced to this strategy and are being asked to apply and articulate it. No other entry point exists. As such, this is a low level task. Consider the re-designed task in Figure 4.45:

Figure 4.45
Triangle - Redesigned

Given 5 posters with original and dilated triangles, ruler, scissors, and graph paper, prove which are similar, congruent, or not a dilation.

The teachers re-designed the problem to require students to demonstrate a deeper understanding of similarity, congruence, and scaling to a) determine which of the five models fit which term, and b) justify those answers with proof. Teachers kept the problem structurally the same as a *proportional reasoning* task and gave students a numbers of tools with which to solve the problems. The tools provide for the students more than one entry point into the task, and puts the intellectual work of both finding solutions and constructing proof under the students' control. The re-designed task is more cognitively demanding than the original problem.

Over-Simplification of a Problem

The grades 6-8 teachers also redesigned a task and oversimplified the mathematics. The over-simplification decreased the cognitive demand of the problem. Figure 4.46 shows the task that an early career group of sixth-grade teachers chose for re-design:

Figure 4.46
Garden Task – Original

Courtney's uncle lives in the city and has rented a small rectangular parcel of land in order to have a vegetable garden. The dimensions of the parcel are 1.25 meters by 4.8 meters. *a) Find the area and perimeter of the garden.* *Her uncle has decided that he wants to dedicate 1/3 of the garden space to growing tomatoes; ¼ of the garden space for corn, and the rest of the space will be for carrots.* *b) What fraction of the total space will Courtney's uncle dedicate for carrots?*

This cognitively demanding task assumes that students both have, and can apply, a level of prior knowledge. There are two parts, a, and b. For part a, a *geometry and measurement* task, students are provided with two dimensions of a rectangular parcel of land, and asked to determine the area and perimeter of the parcel. According to the CCSSM, (www.corestandards.org) students received concrete instruction on area and perimeter in third grade and, since the end of fourth grade, have been expected to apply those formulas in real world and mathematical problems. Multiplying decimals is part of the fifth grade mathematics curriculum. So, even though this was posed as a sixth grade task, it is more of a fifth grade task.

Part b, asks students to demonstrate understanding that a unit fraction (1/b) is the quantity formed when a whole is partitioned into b equal parts (www.corestandards.org), also a third grade standard. Drawing an area model will require students to partition and label 1/3 of the rectangle as tomatoes. Then they will need to re-partition the rectangle into fourths and label 1/4 of the garden as corn. To figure out the fraction of the garden reserved for carrots, students will need to partition the rectangle into twelfths to determine that carrots will be planted in 5/12 of the garden. Consider the re-designed task in Figure 4.47 below:

Figure 4.47
Garden Task – Redesigned

Courtney's uncle has a parcel of land and has 6 meters of edging.

What would be the possible dimensions of the parcel?

The uncle wants to plant of the garden with 1/3 tomatoes, 1/4 of the garden with corn, and the rest for carrots. Sketch how this garden would look.

Part of the teachers' re-design strategy for part *a* of this task was to omit the area and perimeter questions about the parcel. They kept the problem structurally the same as a *geometry* task but they added the total length of the perimeter of the parcel, (six meters), and called it edging. They omitted the decimal side lengths and significantly decreased the size of the parcel. A total perimeter of six meters produces only one whole-number combination of dimension measurements when one assumes that all of the edging will be used in the garden (one meter by two meters).

To re-design part b of the original task, the teachers retained the original structure of the task, kept the fractional measurements of tomatoes and corn in the garden, but omitted the question about the fraction of the total space to be that planted with carrots. Instead, they asked students to sketch what the garden would look like. Students no longer have to figure out that carrots will be planted in 5/12 of the garden. Instead students can shade in the fractional part of the garden for tomatoes and corn and then label the rest of the rectangle as carrots, without including the fraction. The changes the teachers made have lowered the cognitive demand of the task.

In a post-session reflection, I asked this teacher group how they chose this task for revision, and one of the teachers replied in writing on behalf of the group, "we decided to choose a fractions task since so many students struggle with fractions". When asked how they went about the re-design work, they replied, "We discussed wording and ways to make the task more open-ended while maintaining focus on fractions. We changed our minds a few times and finally completed the re-design."

Tasks with Many Possible Solutions

The grade 6-8 teachers redesigned two tasks in ways that shifted them into tasks that had many possible solutions. One of which is this *factors/multiples* task in Figure 4.48 chosen for re-design by an early career teacher group and began as high-level task:

Figure 4.48
Factors and Multiples – Original

Irene and Simon are studying a set of new words for Spanish class. Irene decides to break the set into lists of eight words. Meanwhile, Simon creates lists of 12 words. What is the smallest number of words there could be on both of their lists?

This traditional, cognitively demanding LCM (least common multiple) problem can be solved in at least two ways. For example, by listing the multiples of each number and determining the first number common to both lists, or by finding the prime factorization of each number and then multiplying together the prime factors on both lists. Students might also reason their way through the task. There is no directive toward a particular solution strategy and the addition of context contributes to the high cognitive demand level.. Consider the re-designed task in Figure 4.49:

Figure 4.49
Factors and Multiples – Redesigned

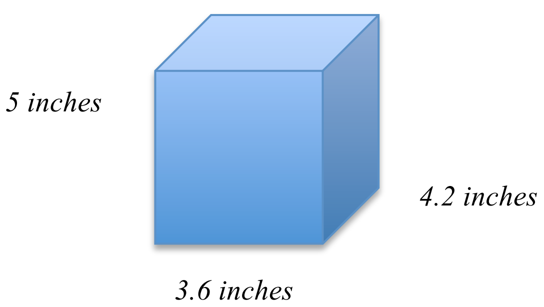
Irene and Simon are studying the same set of new words for Spanish class. Irene decides to break the set into lists of eight words. Meanwhile, Simon creates lists of 12 words. How many possible numbers are on the lists?

In the re-designed task, students are not constrained to the least common multiple and instead are faced with finding every multiple of eight and twelve. As written, the problem has an infinite number of solutions and would probably be overwhelming to many students. However, when we look at this problem as an open-ended task, lacking clear parameters, it becomes more interesting. If students see themselves as empowered to interact with the mathematics in a way that encourages them to think about and impose their own limits onto the lists of words, the problem becomes more reasonable. To do so would require students have a discussion of what a reasonable number of words would be to study at one time. This could lead to students needing to justify their positions and reach a consensus among different ideas-a sophisticated skill.

In the second example, an experienced sixth grade teacher group started with a low-level *geometry* task and increased the cognitive demand by turning control of the mathematics over to the students. Here, in Figure 4.50 is the original task:

Figure 4.50
Volume Task

What is the volume?



The objective of this problem seems to be student application of a procedure or previously memorized formula to determine the volume of a rectangular prism, and has one correct solution. The formula itself is not provided, so it is assumed that students have access, perhaps through prior memorization, or a copy of the formula for volume of a rectangular prism. Additionally, the task asks only that students find the volume, no explanation or justification of a solution is required. Consider the re-designed *geometry* task in Figure 4.51 in which much more is asked of students:

Figure 4.51
Volume Task – Redesigned

The Kraft Food Co. is going green! They have contacted you to re-design the packaging for their famous marshmallows. They now want to use recyclable boxes. The plant manager says that the new material costs $0.15 per square inch. You must design a box that doesn't squish the marshmallows and is cost efficient for Kraft to make. Justify your design.

Although the problem itself does not specify the size of the marshmallows, the teachers talked of providing a bag of marshmallows and explaining that students should design a package for the size and quantity of marshmallows in the bag (Rishor reflective journal March 2015). However, if teachers gave students the freedom to impose their own ideas on the size of the marshmallows in the box, it becomes a very cognitively demanding task. In order to determine the most cost-efficient type of package, students need skills in measurement, factorization, volume, surface area, and logical reasoning and by deciding on a marshmallow size, students can learn that multiple solutions are possible and that the size of the marshmallow is the variable. The problem is a more cognitively demanding task than the original.

When talking to me about the enactment of the task, one of the teachers, Heather said that she and her students realized during the task that the cost of $0.15 per square inch of material was too high and made any possible package too expensive to actually produce. However, she said she used that error could to guide students in a discussion sense making and what would be a more realistic price per square inch of material for the packaging, (Rishor reflective journal, Nov 2014).

Summary of Key Trends 6-8

The grades 6-8 selected ten problems for re-design. They chose from concept areas important to the grade band including *proportionality, geometry*, and *operations/number sense*. They increased the cognitive demand from low to high in two of the ten tasks by increasing student control over the intellectual work of the task. In both cases, the teachers turned the tasks into open-ended student exploration.

The teachers decreased the cognitive demand from high to low in one task, maintained the level, (either high or low), in two of the tasks with experienced teachers responsible for two of them. Two of the tasks were redesigned into tasks with many possible solutions.

Summary of Findings for Research Question One

I found three patterns in the ways that early-career and experienced teachers redesigned mathematics exercises to increase the cognitive demand. One such pattern was that they typically changed the structure of the problems. For example, from result unknown to start or change-unknown type problems, and from multiplication to division. Another common pattern was increasing student control over the intellectual work of the task. For example, opening problems up beyond multiple choice options or exercises having just one solution. There were differences in the frequency of the patterns between grade levels which will be addressed in an upcoming section.

Research Question 2: What Did Teachers Report as Important Considerations in the Re-design of Mathematics Exercises?

Teachers reflected in writing about what they considered to be important as they worked on re-designing mathematics tasks. Their words gave me insight into key components of this phenomenon. I asked but did not require, names or grade levels on the written reflections as a way to encourage teachers to be candid in their responses. For this reason, findings reflect key themes that arose across all teachers' reflections. The three most prominent themes were *knowledge of student thinking; knowledge of students' emotional needs;* and *teaching mathematics with understanding.*

As noted in chapter three, I audio-recorded the conversations of some teacher groups as they worked on task design. These conversations provided additional details about both the process of mathematics task re-design, and what factors teachers considered as they re-designed tasks and debriefed the enactments of those tasks. I found that each of the general themes noted above was also evident in teachers' conversations. In the sections that follow, I introduce each theme and describe how it was evidenced across teachers' written reflections, and then use excerpts from teachers' re-design conversations to further illustrate the theme. In the next section I present comments from the teacher's written reflections that helped me to determine the themes, and illustrate the manifestation of those themes through the audio-recorded teacher work groups.

Theme One: Knowledge of Student Thinking

A common idea teachers reflected upon as an important consideration to task redesign was *knowledge of student thinking*. Many teachers referenced knowledge of student thinking as a desire to create opportunities for students to critically think and reasoning within a task. Such tasks, they indicated, would encourage more communication between students during math class, surface misconceptions students may hold, and be a way to differentiate instruction, providing a wider range of student responses.

For example, some teachers talked about the importance of increasing student communication about mathematics. They wrote, "I want to make math more rigorous, with more student explanations", "I try to make things more cognitively demanding and I try to figure out materials that lend themselves to conversation starters”, and “I focus on higher order thinking and tasks that encourage talking about math”.

Other teachers framed knowledge of student thinking as understanding what students struggle with and what misconceptions they hold. For example, teachers wrote, “the most important skill is anticipating with students will struggle with”, and “to be able to identify rigor and know how to tweak problems; having the ability to turn student misunderstandings into *a-ha* moments for the class, and “knowledge of basic misconceptions, to scaffold to a basic enough level of knowledge”.

Another way teachers talked about knowledge of student thinking was the differentiation of tasks to meet the needs of students at their own levels of understanding. For example they wrote, “every day I look at each task and pre-plan how to re-design it for the different levels (of student understanding) in the class”, “when I re-design tasks, I think about differentiating and how to have the students work more than me”, and “I learned how to re-write tasks to let students do the work, not hold their hands all the time”. Their words indicate that consideration of student thinking was an important driver of this task re-design work.

In the following examples of the teachers’ work during task redesign and debrief sessions, I highlight the way the teachers’ considered knowledge of student thinking throughout the task redesign process.

Pencil Box Task Redesign

The experienced third grade teacher group chose the pencil box task for their first attempt at redesigning mathematics exercises into tasks and discussed opening up the task to include more student thinking and reasoning. The Pencil Box Task, originally presented on page 111 is also presented in Table 4.10 below:

Table 4.10
Pencil Box Task

Original: *Johnny has 3 boxes of pencils; each box has 4 pencils in it. How many pencils does Johnny have?*
Redesigned: *Johnny has 12 pencils total. He needs to separate them equally into boxes. How many ways could he separate them? What's the least number of boxes he can use? What's the most?*

The teachers decided that they wanted to start with a straightforward multiplication problem and re-design it with the goal of making it more open-ended to incorporate more student thinking and reasoning. In the following conversation, they talk about how to do that:

Natalie: Let's find a basic problem. Let's stick with the three times four is twelve. We can do…what would be a word problem for that? Johnny has—

Claudia: -Johnny has three boxes of pencils, and each box has four pencils in it. How many pencils did he have in all? In order to make it more difficult I think that it…

Jennifer: It would have to be something where—

Claudia: If it's three times four equals 12. We what do, I always use x but it's kind of hard. Then another one could be, or if you really want it to be open-ended we could say, Johnny has, for this one, Johnny has 12 pencils total. Okay, he has 12 pencils total. Then say, how many boxes? Wait, something about each. Okay, hold on…

Jennifer: Each box has four pencils. How many boxes?

Claudia: No, I don't want to give… I want to have two open ends total.

Jennifer: Oh, okay.

Claudia: Johnny has 12 pencils total. He needs to separate them equally.

Natalie: Separate them equally into boxes, how many different ways can he do that?

Claudia: In two boxes, yeah because it could be three pencils and four boxes or four pencils and three boxes.

Tammy: You can do two and two, or two, two, two.

Claudia: In two boxes, right. How many ways would he separate them?

Tammy: Mm-hmm.

Claudia: Because then it really equals 12 and they have to figure out the ways to equally divide them.

Natalie: It really gives them more opportunities to figure it out.

Tammy: Oh, I'm excited to use this problem in my class because that will be really fun. They'll have the counters and some kids will need them and some won't. Then when they get to five they'll be like oh it doesn't work.

In the above passage, there are several instances where teachers referred to a desire to create opportunities for students to engage in mathematical thinking reasoning. For example, in line 4, and again in line 13 Claudia indicated that she wanted to make the problem more complex. She said that she wanted to create a problem with two open ends, which would increase the level of student thinking required to reach a solution. Then, in line 25, Natalie commented on how the re-designed problem would provide more opportunities for students to use what they know to figure out the mathematics. Finally, in lines 26-28, Tammy anticipated how the differentiation they built in would enhance the variety of student thinking in her class. She expected that some students would want to use manipulatives while others would not, and looked forward to how much fun it would be. She predicted that her students would approach the problem systematically by starting with one pencil in each box, then put two pencils in each box, then three and so on. She noted that, as students tried to put five pencils in each box, a pivotal moment in the lesson could occur as students come to the realization that it is not possible to put five pencils in each box and have groups of equal sizes when you start with twelve pencils. These examples indicate that the teachers considered student thinking and differentiation as they redesigned this task

Pencil box task enactment debrief.

When the teachers debriefed the Pencil Box Task during the second session, they began by sharing their individual experiences and reflecting collectively on general trends they noticed during the task enactment. Though their goal was to use their knowledge of student thinking to open up the problem for students to do more thinking and reasoning, the teachers lamented that students struggled with the task more than they had anticipated and that actions they took during task enactment became a barrier to both the occurrence of more student thinking and to learning more about what misconceptions students had. As an example, consider this excerpt from their conversation:

1 *Claudia*: I think I probably gave too much support. It probably would have
2 been better had I not done all three of the problems. I really didn't think about
3 that. I don't think I would've had to give them too much guidance had I
4 separated it out.

Jennifer: I wanted to see what their misconceptions were, so I just let them grapple with it for a little while. I was shocked actually. I thought they would rock it and be like, "I can totally do this", but none of them wrote down exactly how many boxes they used.

Tammy: Mine started off a little bit slowly, then one mentioned using one and instead of letting him think about it and keep going, I said, what other factors can we use? instead of letting him come up with that language. I think I helped too much when I said, okay, we need more factors of 12, what else can we do? Instead, I should have just said, are there any other ways? Look at the numbers again.

Natalie: Mine would just figure out two ways they could separate it. Then they compared those two ways and labeled the most and the least. It's like, well, no. Give me as many ways as you can to separate them first.

During the redesign session, the teachers' conversation centered on the ways they inadvertently limited student reasoning even though their intent was to increase it. They described feeling like they had taken over the intellectual work of the task with the kind of help they provided to students. For example, in lines 1-3, Claudia described her regret at having posed all three questions to students at once and saw that as the cause of her students' confusion. In lines 8-13 Tammy described how she inadvertently over-scaffolded the task by asking leading questions, and in lines 14-16, Natalie talked about how she unintentionally funneled a student to the correct solution pathway rather than listening to his ideas about finding two possibilities and comparing those.

In lines 5-8 Jennifer described how she let the students grapple with the task in order to see what misconceptions they had. She mentioned that she thought the task would be easy for them and was surprised by what they did not understand. In lines 15-16 Natalie described how her students displayed their misunderstandings by not completing the problem even though they seemed to believe that they had. All of these examples indicate that during the enactment of the task, when they wanted to use the increased student thinking they created room for as a way to determine student misunderstandings, teachers inadvertently shifted the focus away from that initial goal with actions that they took.

The teachers' original goals in redesigning this task were to open up space for more student thinking so that they could see students' misunderstandings. They wanted the problem to be more challenging and open-ended so that students would have to work to figure things out. Then, during the debrief, they seemed to come to the conclusion that, while the task may have created more space for student reasoning, their actions of being directive and leading in their prompts actually prevented that from happening which subsequently prevented students from displaying any misunderstandings about the mathematics. Although their intent was to foster more student thinking with the re-designed task, the teachers saw their actions as a barrier to that thinking actually taking place and prohibited them from truly seeing what students did and did not understand.

Number Line Task Redesign

These teachers leveraged their understanding about student thinking as they redesigned additional tasks in subsequent professional development sessions. For instance, during the second task re-design session, the third grade teacher group seemed to draw upon their awareness of over-scaffolding the Pencil Box Task as they re-designed their next task to incorporate more student thinking in a way that would help them to see what students did not understand. The Number Line Task, originally presented on page 105 and listed in Table 4.11 below, was redesigned by teachers to open up a space for more student thinking and thus provide insight for the teachers into how their students reason mathematically, and into any misconceptions they may hold.

Table 4.11
Number Line Task

Original: *Draw a number line and label tic marks for benchmarks 0, 1/2, and 1. Estimate where you would put the fractions 5/8, 7/8, and 1/8.*
Redesigned: *Draw a number line from 0 to 1. Estimate and put a tic mark for 1/2. Estimate and put a tic mark to show a fraction that could come between 0 and 1/2. Estimate and show a fraction that could come between 1/2 and 1.*

In the interaction below, the teachers talk about their goal of opening up the task to increase student reasoning:

Tammy: Let's see… what if we had them just create their own fractions but only gave them the parameters of 'between zero and one half'?

Claudia: Then just tell them to put fractions on a number line and see what happens?

Natalie: Yeah. Then have them create a fraction and place it on the number line between half and one so they would have to do, like five-eighths, or they would have to create…if they do two-eighths, then it's not between half and one so… Oh, I don't know; you'd be having them somehow create their own something.

Natalie: Mmm-hmm. Like come up with three fractions or something they'd have to create.

Claudia: So our open-end would say, "draw a number line and label the benchmarks for zero, half, and one. Give me….place one fraction between zero and half, and one or two fractions between one-half and one.

Jennifer: and label them.

Claudia: and label them. They'd have to come up with what that looks like, put the fractions on the number line and justify why they belong there.

Natalie: I like that.

Tammy: That's open-ended. They have to think of it all by themselves. They would have to justify why it belongs there and if it belongs between zero and half or half and one.

Natalie: I like that.

The teachers wanted students to decide for themselves on which fractions to place on the number line in order to create something of their own rather than to follow a directive. This could be because they felt they gave too many directives during the last task. They also wanted to make the problem accessible to a wide range of student understanding. For example, in line 4, Claudia asks the group if they want to just pose a question to students and see what happens. They decided that students could choose any fraction within the range of zero and one-half, and then another between one-half and one. This would both create space for student thinking and allow a range of solutions to differentiate according to student understanding. In lines 23 Tammy summarizes how students will be in charge of the task and will create their own interpretation of the question; another form of differentiation.

This problem is more open-ended than the pencil box in that there are more possible answers and students will choose fractions that are within their comfort zone, which will reveal more about their level of understanding rather than having to find the set number of factor pairs that make twelve, like in the first task. As the teachers negotiated the re-design of their second task, they evidenced a renewed desire to open the task to student exploration and provide less teacher direction.

Number line task enactment debrief.

When the teachers reconvened after enacting the Number Line task with their classes, they were excited to share how it went in their classes. Their intent was to open up the task to more student thinking, differentiate for a range of student understanding, and identify students' misunderstandings. During enactment, though, their attention focused mainly on students' misunderstandings as evidenced in the following conversation:

1 *Natalie:* My students really struggled to remember their strategies.

2 But they did help each other out as well as share their frustrations.

3 I feel like I should have added a visual component to the problem.

4 They definitely could have benefited from visual prompting.

5 *Jennifer:* Math is pictures, words, and numbers. The pictures really

6 helped them and I was very happy with the way the students used

pictures to draw fraction models that helped them understand the placement on the number line.

Natalie: I wish I would've thought of that.

Claudia: I think it went okay. I think I should have the problem be a little more real-world relatable to help the understanding and to help the independently make models of the fractions without my help.

As the teachers discussed their experiences with enacting the redesigned task, they considered how students struggled in thinking about the mathematics and ways that they possibly could have supported student thinking. For example, in lines 1-4, Natalie lamented that students had trouble remembering strategies for placing fractions on a number line. She wished that she had included visual representations to help activate their prior knowledge. Jennifer, on the other hand, in lines 5-8, considered the way her students thought about math and included visuals in the enactment, which left her pleased with the results. In lines 10-13 Claudia shared that the task would have been better had she included some sort of real life context. Through the debrief of the task, teachers came to the conclusion that student misunderstandings could be mitigated with the use of visual representations and familiar, real world contexts. Sharing their experiences with enacting the task they collaboratively redesigned resulted in the teachers successfully learning more about student thinking.

Summary of Theme One: Knowledge of Student Thinking

Teachers in the study reflected in writing that when they redesigned tasks, they considered their students' mathematical thinking. As they redesigned tasks, they worked to create space for more student reasoning, discussed how to handle students' misconceptions, and how to support student thinking during task enactment. Though they did not always meet their goals, their focus did not deviate from doing so.

Theme Two: Knowledge of Students' Emotional Needs.

Teachers also expressed *knowledge of students' emotional needs* as an important consideration when re-designing mathematics tasks. This theme included references teachers made to students feelings and emotional comfort during mathematics activities including students' fear of being wrong and confidence in their mathematical competence. It was important to these teachers that students like mathematics and feel emotionally comfortable as they worked.

Responses in their written reflections included, "My students really enjoy hearing that they are doing high-level tasks- it increases their motivation", and "There are students that I cannot put together to work" and "each group has "kindness hearts"-if they fight too much, they give me their "hearts" and they don't like it" and "I try to create tasks that are relevant to students". For these teachers, the way their students felt about mathematics was an important consideration in the way they interacted with curriculum to redesign mathematics tasks. They wanted their students to feel happy and confident as problem solvers.

Pencil Box Task Enactment Debrief, Continued

As the teachers continued to debrief about the Pencil Box Task enactment, they noticed how hard it was for students to be wrong. They struggled with how to help students feel confident as they worked on tasks, and to understand that justification of their solutions was key. Some teachers noted that students were challenged by the re-designed (and more open-ended) mathematics tasks, and at times reluctant to generate their own strategies or take risks, which seemed to trouble the teachers greatly. For example, the following is from a continuation of the discussion about the enactment of the pencil box task.

Claudia: {Students are} terrified to be wrong. They never want to guess, they never want to try. They want to be told how to do it, like I am. I am just like that too. I need to be told how to do it because I don't want to be wrong, and they don't want to look dumb in front of people. It's scary to be wrong in front of your friends at this age. I think at any age. At first in kindergarten I don't think it's as installed in them but becomes more of a challenge as they get older. They don't want to play with the numbers. They don't want to try something and be wrong so they just wait for someone to tell them exactly how to do it or exactly what the answer is.

Natalie: They say I don't know and I'm like, yes you do.

Claudia: I think as a teacher you need to be able to create an environment for it to be okay to be wrong. You're wrong, here's a gold star, try again. Something like that, I don't know. Because they need to be okay, it's okay to be wrong and try a different way.

Tammy: It was just so interesting to finally give them that permission-to say, if it makes sense, do it. Yeah, they don't want to be wrong.

The teachers knew that students with a positive disposition towards mathematics are more open to learning but also reflected on how difficult it was to instill that disposition in students. In lines 1-9 Claudia's frustration comes through as she described students who were afraid to make mistakes and be wrong. Claudia connected this experience to her own feelings and she talked about how this being even more of an issue with older students, like her third graders. Then in lines 11-14 she described how the role of the teacher is paramount in creating an environment where it is okay, and even expected, to be wrong and make mistakes and for students to understand that they will still be emotionally safe.

Another example of wanting students to feel emotionally safe is evidenced in lines 15-16 where Tammy described feeling like she had to give students permission to be wrong, but how that permission seemed to help students engage in the work. Being fearful of taking risks in mathematics makes it very difficult for students to explore ideas. As such, students' emotional needs were an important consideration to these teachers as they re-designed mathematics tasks and attempted to find a balance between keeping students emotionally comfortable and challenging them with rigorous problems.

Number Line Task Enactment Debrief, Continued

Teachers also talked about students' feelings and emotional needs as they debriefed the enactment of the Number Line task. For example, in the conversation below, teachers discussed the uncomfortable frustration their students experienced as they worked on the Number Line Task:

> *Natalie:* They had to kind of re-learn estimating fractions from long ago and there was a lot of frustration.
>
> *Tammy:* Mine were frustrated, too, that a number line doesn't make complete concrete sense yet. I felt like it wasn't really rigorous but more of a trick question because there wasn't context added to the numbers. This helped me figure out how to get better at this, though.

The things teachers noticed and spoke about during the enactment of this task indicated that they paid attention to how students felt while doing mathematics. In line 1-

2 Natalie mentioned that there was a lot of frustration, which could have meant that her students were frustrated with trying to remember how to estimate fractions. In lines 3-5 Tammy echoed the thought, confirming that her students seemed to feel like they were being tricked rather than challenged. These instances indicated that teachers paid attention to the emotional reactions their students had to the re-designed task and considered how to develop within them a positive disposition towards about mathematics.

Summary of Theme Two: Knowledge of Students' Emotional Needs

Considering the emotional needs of their students by working to ensure students felt competent as mathematics problem solvers was important to teachers as they redesigned mathematics tasks. They paid careful attention to how students reacted to and engaged with tasks in the classroom and discussed how they could support students in finding productive and confident feelings as they worked through mathematics tasks.

Theme Three: Teaching Mathematics for Understanding

The third theme, *teaching mathematics for understanding*, also came up frequently in the teachers' written reflections as an important consideration when re-designing mathematics tasks. The term "understanding", however, is inherently ambiguous so I took that into account and coded references teachers made both to conceptual and procedural understanding with no requirement that they occur together as *teaching mathematics for understanding*.

Conceptual Understanding

One way teachers talked about teaching for understanding was to refer to their students' *understanding of the concept* of the lesson. For example, some of their written comments included "we try to move away from just the procedures to making sure students understand the concept and can apply their learning", and "really thinking about the concepts I want the kids to learn, rather than the skills; this way I can plan on how to teach those concepts rather than practice and skills”. Another teacher considers students’ “previous knowledge and what they should know; asking them why not how” and “Next time I will use something other than crackers to split up, like a liquid of some sort so that I can tie in measurement and volume” and “ I determine what I want them to learn in the end and work backwards”. Their words indicate that some teachers interpreted *teaching mathematics for understanding* to mean that students have learned both how to do the mathematics and the why behind the how. To illustrate this point, I refer back to the Division Task, originally presented on page 122 and in Table 4.12 below:

Table 4.12
Division Task

Original: 157 ÷ 18 =
Redesigned:

Equation:	Word Problem:
157 ÷ 18 =	
Visual:	**Justification:**

As teachers negotiated the redesign of the Division Task, their conversation evidenced how they interpreted *teaching mathematics for understanding*. For example, in this following conversation excerpt we see that conceptual understanding is what drove their redesign efforts:

Tori: What if we said—what is a problem? A hundred and fifty-seven divided by eighteen is eight, remainder thirteen. Come up with a word problem.

Heather: Okay. That would force them to think about the numbers, and what they mean, and what goes where, and what they're breaking up, and what remains.

Leila: I was thinking like 157 divided by 18. Even if we give them the answer.

Roberta: I wouldn't give them the answer.

Tori: I don't know if I would either because that forces them to solve it… and then use that information. That also gives you options when they solve it wrong. Some of them might solve it wrong and their visual, explanation, and word problem will be way different than everybody else's.

Heather: Okay, so we don't give the answer. We'll just put 157 divided by 18.

Leila: There's a visual.

Roberta: - and a justification.

Tori: - and justification.

Leila: Should we do that?

Heather: I like that.

Roberta: Yeah, let's do that. Just a number and they're gonna work out a visual, a word problem, and a justification.

Heather: That is definitely multilevel.

As they redesigned the problem, the teachers worked to build conceptual understanding into the task. In lines 1-4 Tori and Heather discussed how the redesign of a word problem from a division calculation problem would encourage students to think about the meaning of each number in the problem and would be a way for students to demonstrate their conceptual understanding of division. Then in lines 9-12, Tori argued that by not providing the answer to the students, they would have the opportunity to demonstrate a level of conceptual understanding that included procedural fluency with the correct solution, a visual representation of the situation, and a justification for their answer. These examples evidence that the teachers considered *teaching mathematics with understanding* to consist of students demonstrating conceptual understanding consisting of the combination of both how to solve the problem and why to solve it that way. This evidences that *teaching mathematics with understanding* is an important consideration in task redesign.

Procedural Understanding

However, teachers also made references to *teaching mathematics for understanding* that indicated a more procedurally based interpretation of the term *understanding.* For example, some teachers, when asked what students learned by engaging in the tasks, said the following: "Students seemed more excited to learn- they were getting the process", and "students paid more attention to the use of procedures and math vocabulary", and "I asked students to work independently-we discussed and decomposed the procedure before we got started but I did not provide additional support and they usually got the right answer". Their words indicate that some of the teachers interpreted *teaching mathematics for understanding* to be determining the correct solution through the application of a procedure.

So, although teachers reflected on a desire to teach for conceptual understanding, their conversations indicate that, for some of them, procedural fluency amounted to understanding the mathematics. Conversations captured during the redesign of The Exponent Task in Table 4.13 below, illustrate this occurrence.

Table 4.13
Exponent Task

Original: Circle the two expressions that are equivalent: a) $(6 + 4)^3$ b) $6^3 + 4^3$ c) $6 + 4^3$
Redesigned: Michael, Morgan, and Keegan are in the same class. The teacher asks the students to write and simplify the expression "6 plus 4 cubed". Here are the expressions each person wrote: Michael: $(6 + 4)^3$ Morgan: $6^3 + 4^3$ Keegan: $6 + 4^3$ Which students will get the correct answer? Which, if any, of the expressions are equivalent? Explain.

The teachers in this group decided, for their first task re-design, to work on a problem involving exponents because it was a concept area they felt all of their students needed to understand, and was a task that each one of them could enact in their classrooms despite working from pacing calendars that were sequenced differently. To illustrate how they considered *teaching mathematics for understanding* as procedural fluency, I draw on excerpts of conversations among the experienced middle school teachers as they redesigned the task.

Tori: What about using exponents?

Leila: We just finished that. That's easy. It's a good review. This is like what we were doing. Find the amount—let's see. Explore different ways to understand how to write an evaluate expressions with exponents.

Heather: Five squared, I think there's a lot of good discussion that can happen. Is that two fives? Is it five twos? How would you draw a picture? I don't know. I just feel like how would you visually represent five squared. I don't know.

Leila: We fought each other on it. It's not two times five, people.

Tori: Yeah. I still had somebody today do that.

Heather: I did a magic foldable, and I think I caught them with that one. That would be good, if we could figure out—how would we give them something to solve, give them the actual problem and then, instead ask them which one would be correct—

In lines 1-2 Tori and Leila suggested the idea of redesigning an exponent task and commented that it would be easy, (though it is not clear if she meant the teaching of exponents would be easy, or if it would be easy to redesign an exponent problem). In line 6, Heather talked about the rich discussion that could occur as students wrestled with the idea of exponents, but when she thought aloud about possible questions to pose to her students (is it two fives, or five twos), and wondered about how to make the problem visual, her ideas could have been about wanting her students to correctly apply a procedure to accurately reach the solution. This is followed up in lines 7-8, when Leila and Tori talked about how students continued to struggle with exponents even though they had already been introduced to the idea. Leila mentioned an argument between students about 5^2 being 2 x 5 versus 5 x 5, which indicates that *teaching mathematics for understanding* could mean students understand how to correctly solve exponential problems without necessarily understanding the idea of exponential growth. So even though teachers wrote about conceptual understanding as an important consideration in mathematics task redesign, ensuring that students learn and demonstrate procedural fluency may be how some of them interpreted *teaching mathematics for understanding*.

During the next session, the teachers debriefed the enactment of the re-designed Exponent Task. In excerpts of their conversation, the teachers discussed the conversations their students' engaged in to understand the mathematics:

Tori: Two students, Kim and Kenny couldn't figure out if it was it six plus four cubed or six plus four, cubed. They were going back and forth-between, if this is how it's written, which way do we write it. They were hung up on that. They were talking about how either one of these, depending on how this is being used, could then be, technically, the correct answer.

Heather: My literal ones were like, "Well, you read it from left to write, so six plus four, cubed, so that has to be this one." All the other ones are going, "No, because it says, 'Six plus four cubed,' so it has to be this one."

Tori: My kids were like, "But it doesn't say parentheses," and I'm like, "Does it always say parentheses? Have the ones that we've done in the past say

11 parentheses? No, they have a comma or random things.

12 *Heather*: Which is the correct answer?

13 *Tori*: We couldn't agree.

14 *Leila*: Well, yeah.

15 *Heather*: We honestly couldn't agree.

In lines 1-8 Tori and Heather outlined the rich conversation their students had as they worked through the task to negotiate its meaning. In lines 9-10 Tori mentioned that her students understood parenthesis to be a key component of the concept and how she redirected their understanding by asking them questions about previous exponential expressions they had engaged with. Then in lines 12-15 the teachers shifted their focus to the expectation that the problem contained one correct answer. To answer Tori's question in line 13, "which is the correct answer?", the teachers collectively replied that they could not agree on a solution. However, there was no further conversation about how (or if) the teachers resolved this issue to ensure that students understood, mathematically, how to translate the phrase to a numerical expression. This is additional evidence that *teaching mathematics for understanding*, which teachers considered important for redesigning mathematics tasks, may have been interpreted in more than one way.

Summary of Theme Three: Teaching Mathematics for Understanding

Teaching mathematics for understanding was something teachers considered as important to mathematics task redesign. However, teachers interpreted what understanding means in multiple ways. In some cases, they focused on the conceptual understanding of a topic but in others it seems they interpreted the term *understanding* to be equivalent to procedural fluency.

Summary of Findings for Research Question Two

The teachers in this study commonly reported that knowledge of their students was the most important consideration in re-designing mathematics tasks. Specifically, they reported *knowledge of student thinking*, *knowledge of students' emotional needs,* and *teaching mathematics for understanding* as important considerations in the re-design of tasks. Other responses from teachers about the knowledge they considered important to task re-design work were scattered between mathematics standards, available curriculum materials, and knowledge of cognitive demand, with no discernible pattern.

Research Question 3: What Differences Do I Notice Between the Work of Early Career and Experienced Teachers?

The two-stage design of this study allowed me to explore how early career teachers re-designed mathematics tasks as compared to their more experienced colleagues. First, I analyzed the types of tasks each experience level group chose for redesign and the typical adaptations they made to those tasks. Table 4.14 provides a summary of the types of tasks both the early career and experienced teachers chose for redesign.

Table 4.14
Operation of Tasks Chosen for Re-design by Teacher Experience Level

Problem Type	Early Career Teachers	Experienced Teachers
Join or Separate Result Unknown	12	1
Join or Separate Change Unknown		1
Part-Part-Whole, Whole Unknown	5	3
Part-Part-Whole, One Part Unknown	2	1
Part-Part-Whole, Two or More Parts Unknown	1	
Multiplication Word Problems	3	4
Division – Partitive or Quotative	3	2
Multi-Step and Non Routine Problems	1	2
Calculation Tasks – Add, Subtract, Multiply, Divide (no context provided)	1	2
Counting Tasks	1	
Operations and Number Sense	2	2
Factors and Multiples Word Problems	1	
Identification/Comparison of Fractions		4
Fraction Operations	1	4
Patterns and Algebraic Thinking		1
Data/Statistics	2	3
Geometry	2	4
Proportional Reasoning/Unit Rate	2	2
Total	**39**	**36**

The early career teachers tended to choose problems for redesign that were *result unknown*, or *part-part-whole* problems while the experienced teachers chose among a wider range of problems to redesign. There was no clustering around specific types of problems like with the early career teachers. This could be because experienced teachers may have felt more confidence with redesigning mathematics tasks and thus more willing to experiment with different problem types.

Next I organized the data to analyze the types of modifications teachers made to tasks as they redesigned them. Table 4.15 illustrates those modifications.

Table 4.15
Types of Task Adaptations by Teacher Experience Level:

Types of Task Adaptation	Early Career	Experienced
Changes in terminology	1	1
Changes in materials used		1
Increasing student control over activity	13	15
Increasing teacher control over activity		1
Omitting problems	5	5
Subtracting from problems	1	4
Adding problems to include context	1	7
Adding problems – students develop questions	2	
Adding problems – do more of the same	1	4
Adding problems – student exploration	1	7
Adding problems – list all possibilities	2	5
Add problems-multiple representations	5	7
Changes in the structure of problems	14	8
Total	46	65

Experienced teachers made a greater number of modifications to each task they chose to redesign. As such, even though the experienced teachers worked with fewer tasks overall, they made a greater number of modifications to those tasks. Both groups tended to make changes that increased student control over the intellectual work of the tasks in about equal amounts. Increasing student control over the work of the task was a typical adaptation made by all of the teachers in the study, regardless of grade level or experience level. The increase in student control was most often achieved by opening up the task to have more than one possible solution. In the next sections I provide more details about the differences I found between early career and experienced teachers' task redesign work.

Early Career Teachers Chose Simple Problems to Redesign

Early career teachers generally chose simple problems to redesign and typically increased the level of cognitive demand of tasks. Most commonly, their redesign strategy focused on making changes to the structure of problems, which contributed to increases in the cognitive demand of tasks. However, these early career teachers were also more likely than their experienced peers to render tasks unsolvable.

Structural changes to increase cognitive demand.

Early career teachers, those with five or fewer years of experience, made more structural changes to problems than did experienced teachers, and the majority of these structural changes contributed to an increase in cognitive demand. The trend was most prominent in the K-2 grade level band which may be because result unknown problems lend themselves more readily to structural changes than the broader range of problems older students experience. For example, the following *result unknown* task in Table 4.16 and originally presented on page 5 and chosen for redesign by a group of second grade teachers began as such:

Table 4.16
Cookies:

Cookies - original version: *There were 19 cookies on a plate. A girl ate one and then a boy ate another one. How many cookies are on the plate now?*

Result unknown problems are abundant in primary grades mathematics instruction and are structurally similar. Also, learning about the different variations of join and separate problems is often part of teacher preparation programs and are represented in the CCSSM. As such, redesigning such problems can be relatively straightforward by changing the position of the unknown.

However, the problems become more complex as children move into the intermediate grades and the abundance of result unknown type problems decreases. For example, the Garden Task, originally presented on pages 125 and again in in Table 4.17 below is one such task:

Table 4.17
Garden

Garden – Original version: *Courtney's uncle lives in the city and has rented a small rectangular parcel of land in order to have a vegetable garden. The dimensions of the parcel are 1.25 meters by 4.8 meters.* *a) Find the area and perimeter of the garden.* *Her uncle has decided that he wants to dedicate 1/3 of the garden space to growing tomatoes; ¼ of the garden space for corn, and the rest of the space will be for carrots.* b) *What fraction of the total space will Courtney's uncle dedicate for carrots?*
Garden - Redesigned: *Courtney's uncle has a parcel of land and has 6 meters of edging.* *What would be the possible dimensions of the parcel?* *The uncle wants to plant of the garden with 1/3 tomatoes, 1/4 of the garden with corn, and the rest for carrots. Sketch how this garden would look.*

The above problem does not contain as clear a path to redesign as a typical *result unknown* problem. As such, this type of problem may be potentially more difficult for an early career teacher to redesign than *result unknown* problems because changing the position of the unknown is not an option. Teachers could redesign it by using different numbers in the problem but that, unlike changing the position of the unknown in a result unknown task, would necessitate teachers to work out the new problem to make sure it makes sense and is operational. The predominance of *result unknown* problems in K-2 curriculum materials make it logical that so many K-2 teachers chose these types of task to redesign. Not only are they relatively easy to structurally change, they are also what the teachers have to work with in this grade level band.

More generally, early career teachers were just as likely as more experienced teachers to redesign tasks in ways that increased the cognitive demand. Both groups achieved the increased cognitive demand in various ways, including structural changes (changing the position of the unknown as noted above), elimination of guides, scaffolds, or multiple-choice answer options, and the addition of multiple representations (drawing models, justifying solutions). However, in many instances, the early career teachers achieved the increased cognitive demand through rather small changes, (i.e., change a result unknown problem, to a start or change unknown problem) whereas the more experienced teachers chose more complex problems to begin with.

Experienced Teachers Open Tasks to Student Exploration

The experienced teachers tended to choose more cognitively demanding problems to redesign and frequently made changes that increased student control over the intellectual work of the task. Increased student control usually gave students more choices to make in the task, which opened up space for students to engage in more critical thinking and reasoning. They also rendered fewer tasks unsolvable than their less experienced colleagues.

Increase student control to increase cognitive demand.

Experienced teachers commonly increased student control over the intellectual work of tasks with their redesign efforts. By that I mean that they opened up tasks to provide opportunities for students to take more agency over how to approach and engage with the task, which, made the problems more of an exploration. As an example of what I mean, consider the Party Planning problem, originally presented on page 104 and also in Table 4.18 below:

Table 4.18
Party Planning Task

Party Planning Original Version: *Jack is setting up tables for a party. Each table has 6 chairs. How many chairs does he need for 10 tables?*
Party Planning Redesigned: *Jack is setting up tables for a party. He is expecting 24 guests. He wants an equal number of chairs at each table.* *What are some different ways he can arrange the room?* *Extension: If 12 more guests arrive, how could he add chairs to tables that are already there?*

These examples evidence how experienced teachers redesigned problems to turn them into student explorations. Because of their experience, these teachers may redesign problems in their classrooms on a regular basis to incorporate more student activity and discussion. Therefore, these teachers may have felt competent and comfortable with redesigning tasks into student explorations that were more cognitively demanding than the original problems.

Tasks with Many Possible Solutions

The early career teachers and experienced teachers were just as likely than to create tasks with many possible solutions though it is possible they did not do so intentionally. For example, in some instance the redesigned problem lacked important information or parameters required for reaching a solution. Early career teachers may lack experience with revising mathematics tasks and, because of that, may have found it harder to think through what would be required for students to successfully solve a task. As such they may have revised tasks with the belief that they were making them more challenging, without realizing that they were not providing enough information for students to successfully solve the problem or creating problems with many, sometimes infinite, solutions.

Summary of Findings for Research Question Three

I noticed several important differences between the work of early career and experienced teachers. First, early career teachers chose simple problems to redesign and modified them in simple ways such as changing the position of the unknown. More experienced teachers, however, tended to choose problems for redesign that were already of a high level of cognitive demand. Both groups were equally likely to increase the cognitive demand of tasks. Experienced teachers increased student control over the intellectual work of tasks by making modifications that opened problems up into student exploration more often than did the early career teachers. Both groups created potential mathematical modeling problems.

An important factor in mathematics task redesign work is teacher experience. Experienced teachers have enacted tasks with students on numerous occasions and may have more of an understanding of what works with students and what does not. Also, they may be more confident than their early career peers to take risks because, perhaps, they know that if a lesson is not successful, there is always tomorrow.

CHAPTER FIVE: DISCUSSION AND CONCLUSIONS

Summary and Discussion of Findings, Conclusions, and Recommendations

Summary of Study

The purpose of this study is to learn about how beginning and experienced teachers interact with mathematics curriculum materials. This is important because teachers all adapt curriculum materials to some extent (Ball & Feiman-Nemser, 1988; Elsalah, 2010; Grossman & Thompson, 2004; Nicol & Crespo, 2006) and research indicates that these adaptations can impact student learning (De Araujo et al, 2013; Stodolsky & Grossman, 2000).

This study took place in the southwestern United States as a collaboration between teachers from three area school districts who were brought together to redesign mathematics exercises from existing curriculum materials into more cognitively demanding tasks. The study took place during the 2014-15 school year, which was the year the state began full implementation of the Common Core State Standards for Mathematics. However, curriculum materials aligned to the new standards were not yet widely available, thus tasking teachers to modify existing materials to meet new requirements. That meant that teachers and schools, overall, were on a learning curve, working to make sense of and understand what the new standards were asking of them while teaching with them. This transition proved to be an interesting and informative time to ask teachers to redesign mathematics exercises to the increased cognitive demand requirements of the Common Core standards. The data collected provided a snapshot of teachers' work while they were in the process of learning something new.

I framed this qualitative thematic analysis through the lens of teachers as designers (Brown, 2009) because that is the role I asked them to take. Data gathered include both the original and redesigned versions of the mathematics tasks artifacts, teachers' written reflections on the redesign work, audio-recorded teacher work sessions, and the researcher's reflective journal. Kindergarten through eighth grade teachers from three area school districts who were part of the Common Core Collaborative professional development workshop were invited to participate in the study. A total of 23 early career teachers and 40 experienced teachers agreed to do so.

This study addressed the following research question:

4) In what ways do novice and experienced teachers re-design pre-written mathematics exercises to increase the level of student thinking?
5) What do teachers report as important considerations in the re-design of mathematics exercises?
6) What differences do I notice between the work of early-career and experienced teachers?

Summary of Findings by Research Question

In this section, I summarize and interpret my findings for each research question, and then look across the findings to discuss implications for research, practice and for curriculum design. I then suggest future research possibilities.

Summary of Findings for Research Question One

The first research question of my study is as follows: *In what ways do novice and experienced teachers re-design pre-written mathematics exercises to increase the level of student thinking?* My analysis focused on how teachers modified and adapted pre-written mathematics exercises to increase the level of cognitive demand. They were to bring with them the mathematics curriculum they were using in their classes, and were asked to take an interpretive view of the curriculum materials in order to find an exercise to redesign. This allowed me to learn about what kind of exercises they tended to choose for redesign - an important understanding with which to support teachers who are beginning to develop the specialized knowledge they need for success.

I found that teachers typically chose structurally simple tasks (e.g., word problems with the result unknown) that were central to the work of their grade level band. One way the teachers in both the K-2 and 3-5 grade bands typically redesigned tasks was by changing the structure of the problem (i.e., the location of the unknown, or the number of unknown quantities). For example, teachers commonly changed result-unknown problems to part-part-whole, two or more parts unknown problems as a way to increase the cognitive demand required of students. Such modifications often resulted in more cognitively demanding tasks, as the redesign removes a clear action from the task, and adds an additional unknown quantity. Carpenter and Fennema (1992) outlined a framework of basic addition and subtraction word problems in which the task difficulty is impacted by the location of the unknown, the number of knowns, and the presence (or absence) of action in the problem. For examples, please refer to page 86.

Another common modification these teachers made was to change the mathematical operation of a task; for example, from multiplication to division, see page 110 for an example. Interestingly, these changes did not necessarily impact the cognitive demand of tasks. This is important to note, as the goal of this study was for teachers to increase the level of cognitive demand of mathematics tasks (to meet the requirements of the new standards). However, by routinely changing the mathematics operation of tasks, from multiplication to division, for example, teachers may affect students' opportunities to learn multiplication the way the curriculum writers suggest. For example, this finding strengthens previous research that suggests variations in teacher adaptations to tasks can create different learning experiences for students (Ball & Feiman-Nemser, 1988; Elsalah, 2010; Grossman & Thompson, 2004; Nicol & Crespo, 2006).

Another way teachers typically redesigned tasks was by opening them up to student exploration. What I mean by that is that teachers began with textbook-like tasks and created from them problems that would require students to investigate or explore in order to find a solution. This increased student control over the intellectual work of the task and allowed for any number of possible solution strategies. When students explore and make sense of mathematics on their own terms, they are *doing mathematics* as categorized by the Smith and Stein (1998) Task Analysis Framework, and are more likely to attain a deeper level of learning (Stein, Grover, & Henningsen, 1996; Schoenfeld, 1992; Smith & Stein, 2009) than they would had they spend their time on more procedural tasks. The tasks teachers redesigned in this way often contained more than one correct solution and had a variety of entry points from which students could approach. These new tasks reflected the views of mathematics learning as outlined in NCTM's Principles and Standards (2000) and the Common Core State Standards for Mathematics (CITE). The teachers in this study were able to redesign several tasks that provided many opportunities for "doing math", strengthening the research on teachers as designers (Brown, 2009).

However, rendering tasks unsolvable was also a common modification teachers often made. Unsolvable tasks made up 13% of all redesigned tasks. This typically occurred when teachers did not place sufficient parameters on a task to limit the number of possible solutions, or when they did not provide enough information for students to reach a solution.

Certainly, teachers did not intend to make tasks unsolvable. Instead, they may have been trying to create open-ended problems requiring a higher level of cognitive demand.

Designing problems that encourage students to make reasonable assumptions, wrestle with unknown quantities or undefined questions, or to determine that an unlimited number of solutions might be possible is certainly a way to increase the cognitive demand required of students. However, creating tasks that encompass those ideals is not easy. It is likely that teachers were taking steps to increase the level of student thinking without realizing the task they created would be very challenging (or even unsolvable) for young children.

These findings support research that argues that deep content knowledge along with knowledge of mathematics pedagogy are important components of the everyday work of teachers (Campbell, Nishio, Smith, Clark, Conant, Rust, Depiper, Frank, Griffin, & Choi, 2014). These results further suggest that educative curriculum materials (Davis & Kracik, 2005) could serve to increase teachers' skills with the planning of effective lessons (Beyer & Davis, 2009). For example, teachers in this study redesigned the division task on page 122 by asking students to find a quotient to a given equation, come up with a picture to represent the problem, a story context, and a justification for their solution. While this design change certainly increased the level of thinking required of students and encouraged them to reason about division in a deeper way, it is not the only way teachers could have modified the task. They might have redesigned it by asking students to explain the mathematical situation in two different contexts using both words and diagrams. This kind of modification would require a different sort of thinking from students; they would need to think about the task from both the *measurement* and *partitive* perspectives. Not to say that teachers would not have come up with this type of modification on their own, they very well might have. However, an educative curriculum could support teachers with these kinds of instructional decisions by teaching them how students build an understanding of division through the typical progressions students take that lead to deep understanding. When teachers have a deep understanding of how mathematics concepts build, they may be better positioned to adapt tasks to meet the needs of their students.

These findings also indicate that the design work teachers do with curriculum adaptation is not trivial (Brown, 2009; Drake & Sherin, 2009). We know that teachers modify pre-written curriculum to some extent as they design instructional experiences for their students. We also know that sometimes teachers modify curriculum materials in less productive ways for many reasons (Drake & Sherin, 2006). The fact that teachers in this study both increased the cognitive demand of some tasks and rendered other tasks unsolvable fortifies the argument that teachers need support with learning how to effectively interact with curriculum materials (Beyer & Davis, 2009; Garrison Wilhelm, 2014).

Summary of Findings for Research Question Two

The second research question of my study is as follows: *What do teachers report as important considerations in the re-design of mathematics exercises?* This question helped me to gain perspective on the things that these teachers thought about and considered as they modified mathematics exercises. This is important because knowledge of what teachers believe to be important as they interact with curriculum materials can guide our understanding of how best to support them in that work.

The teachers in this study talked and wrote about how much they valued their students' ways of thinking as they redesigned mathematics tasks. They wanted their students to reason mathematically and talk to one another about their reasoning and ideas.

They expressed distinctly their belief that when children talk to each other and reason their way through challenging problems, it can serve to surface their misconceptions and drive differentiation in instruction (Gresham & Shannon, 2017; National Council of Teachers of Mathematics, 2000; Stein & Lane, 1996; Stigler & Hiebert, 2004).

Teachers also reported knowledge of students' emotional needs as an important consideration to the adaptation of mathematics exercises. Teachers wanted students to feel emotionally comfortable engaging with mathematics and were aware that students often have a fear of mathematics (Ramirez, Gunderson, Levine, & Beilock, 2012). One group was audio-recorded talking extensively about their students' insecurities about being wrong (page142), and worked to modify tasks in ways that would encourage their students to develop feelings of competence and confidence with mathematics. This is important because research into math anxiety with elementary school children suggests that, similar to older students, younger students can experience math anxiety to the level that it impedes achievement (Beilock, 2008). Additional research suggests that the discourse and collaboration that occur as students wrestle with high-level mathematics problems can serve to build student confidence and motivation to further engage with mathematics (Miller, 2013). As teachers worked to redesign problems into higher-level tasks, their actions were consistent with what the research indicates to be effective for promoting feelings of confidence in their students (Beilock, 2008; Miller, 2013). Additional research suggests that students experience increases in academic and social skills when they can discuss concepts and refine ideas as they give and receive encouragement from one another (Walshaw & Anthony, 2008).

However, it is possible that teachers respond to students who quickly become frustrated with challenging mathematics in ways that may inadvertently perpetuate that anxiety (Beilock, 2008). Teachers may interpret student anxiety as emotional discomfort and decide to modify a task to accommodate the emotional needs of that student and end up designing away the mathematical challenge. Teachers might present an over-simplified version of the task thus removing opportunity from the students to fully interact with, and learn challenging mathematics (Charalambous, 2010). In other words, these findings suggest that teachers, and the decisions they make about learning designs, are very important.

I also found that teachers considered teaching mathematics for understanding to be a significant component of redesigning mathematics tasks. Teachers frequently referred to conceptual understanding as the focus of their teaching mathematics, a focus which is consistently mentioned in the research as fundamental to student learning (Ball et al, 2008; Carpenter et al, 1993; Hill et al, 2008; & Shulman, 1992). However, teachers work from the perspective of their own understandings, which may impact how they interpret the main ideas in a mathematics task (Brown, 2009; Lowenburg-Ball & Cohen, 1996; & Remillard, 2005) and how they present them to their students. The current reform effort asks teachers to teach mathematics in ways different from how they may have learned which, for many teachers, may have centered around surface-level memorization of algorithms. It is a big shift that asks a lot of teachers and can be a significant transition for even seasoned teachers.

Overall, teachers placed high importance on the way their students think and feel about mathematics. They wanted students to feel comfortable and confident while learning rigorous mathematics and they wanted students to be successful. They wanted to teach mathematics in ways that helped their students attain conceptual understanding of mathematics. However, knowing one wants to redesign tasks in this manner does not always equate to actually knowing how do so, especially for teachers new to the profession. These findings indicate a need for a structured way to support teachers to meet the needs of their students while maintaining the rigor of learning. Curriculum materials could serve to guide teachers to modify tasks in ways that are conducive to students' needs for emotional security while maintaining a rigorous and productive learning trajectory, a point I will return to in an upcoming section.

Summary of Findings for Research Question Three

The third research question of my study is as follows: *What differences do I notice between the work of early-career and experienced teachers?* This question helped me to understand the role experience plays in mathematics task redesign work. I found that early career teachers were more likely than experienced teachers to choose structurally simple problems for redesign. They were also more likely to change the structure of the mathematics operation as a redesign strategy, but were also less likely than their experienced peers to open tasks up into explorations.

These findings support research that indicates novice teachers' pedagogical reasoning skills are less developed than their experienced peers (Livingston & Borko, 1989). Early career teachers tend to be more focused on implementing structured lessons as written (Westerman,1991; Drake & Sherin, 2009), rather than thinking flexibly about how they can adapt them to better fit the needs of their particular students. We know that novice teachers struggle to move fluidly between different interpretations and representations of concepts (Cleary & Grover, 1994; Davis & Renert, 2013). It makes sense that these teachers could benefit from guidance about how to effectively redesign tasks perhaps including the use of frameworks that guide teachers as they learn these valuable skills.

Experienced teachers in this study were more likely than their novice peers to open mathematics exercises up into student-led explorations to increase student control over the intellectual work of the task. This supports research that suggests experienced teachers tend to be more relaxed and confident about straying from the textbook to experiment with new ideas and activities (Davis & Renert, 2013; Meier & Lubinski, 2006). It is also consistent with literature that suggests more experienced teachers tend to exhibit higher levels of confidence with probing students thinking and handling incorrect solutions (Meier & Lubinski, 2006). Less experienced teachers tend to think about learning from a management perspective and are more focused on making sure that they themselves understand the mathematics (Sherin & Drake, 2009; Livingston & Borko, 1989) before thinking about how students will understand it.

Research indicates that new teachers benefit from induction programs that help them develop skills with student questioning practices in mathematics (Ingersoll & Strong, 2011). Such support could also benefit them during the planning process as they design the tasks and questions they will pose to students during enactment. This is important because teaching mathematics with tasks is very different from teaching with a traditional direct instruction model. It requires teachers to hone a different skill set that includes the development of pedagogical task knowledge (PTK). I will return to this point in an upcoming section.

Limitations

For this study I asked teachers to take an interpretive view of curriculum materials in order to increase the cognitive demand of tasks, which may not have reflected design changes they would make on their own. Perhaps teachers found changing the structure of tasks by changing the location of and/or number of unknowns, or changing the mathematics operation to be an efficient way to increase the cognitive demand, even though they may not make those kinds of modifications during the implementation of a particular curriculum program. I realize that focusing the teachers' work in this way may be a limitation of this study. As a result, I missed an opportunity to learn about other kinds of revisions that teachers make to tasks, such as those that make tasks more culturally relevant or focus on linguistic demand. However, given the research questions I worked to answer, I felt this was a necessary limitation.

An additional limitation of this study is that the phenomenon was studied over only two semesters. I do not know how sustainable the learning was for teachers and whether or not they continued the practice of adapting tasks in this manner. Research suggests that professional development needs to take place on a continuing basis over the long term in order to have true impact (Desimone, 2011). Thus it is possible that, once the workshops ended, so did teachers' interest in redesigning tasks. Additionally, better quality curriculum materials have since appeared on the market, reducing the necessity for teachers to redesign pre-written tasks to meet the demands of the new standards.

Finally, as I did not observe how the redesigned tasks played out in the classroom, I do not know whether or not the high cognitive demand level of the tasks was maintained throughout enactment. Teachers reported on how the enactment went but a limitation of this study is that it did not include mathematics task enactment.

Discussion

I began this study with three questions. 1) in what ways do novice and experienced teachers redesign prewritten mathematics exercises to increase the level of student thinking? 2) what do teachers report as important to consider in the redesign of mathematics exercises? And 3) what differences do I notice between the work of early career and experienced teachers? Research suggests that all teachers, to some extent, redesign mathematics exercises from curriculum materials for use in their classrooms (Ball & Feiman-Nemser, 1988; Elsaleh, 2010; Grossman & Thompson, 2004; Nicol & Crespo, 2006), and the types of adaptations they make have different effects on student learning (De Araujo, Jacobson, Singletary, Wilson, Lowe; Marshall, 2013; & Stodolsky & Grossman, 2000). Findings from my study suggest that we have a lot to learn from teachers themselves about how curriculum interaction influences the instructional design process. In the next section, I discuss what I learned as a result of this study and suggest future directions.

Educative Curriculum as a Tool

Frameworks as a tool for organization.

First, I found that teachers in this study chose simple exercises to redesign and often modified them by changing the structure of the problem. Though only a few teachers expressed having received prior instruction with the use of frameworks for task structures (such as the framework from CGI, (Carpenter & Fennema, 1992), or problem types (www.corestandards.org)), teachers seemed to be using the principles of these frameworks as they redesigned tasks. Carpenter and Fennema (1996) found that these sorts of framework (e.g., charts of problem structures) supported teachers in understanding how elementary students' mathematics learning developed. Frameworks could also support teachers *during* instruction to guide instructional decision making. In my study it was quite common for K-2 teachers to modify problem structures to increase the cognitive demand by creating *part-part-whole tasks* from *result unknown* exercises. Also, the grade 3-5 groups almost always redesigned *multiplication* exercises into *division* tasks, again by varying the unknown quantity. These findings raise questions about if and how frameworks for problem structures may support teachers in this redesign work.

What was even more interesting to me, however, was how much the grades 6-8 teachers seemed to struggle with redesigning tasks. It may be more difficult to redesign a proportional reasoning problem than it is to redesign a result unknown problem, but it also may be that access (or lack of access) to frameworks for these types of problems made a difference. What I mean by that is, for example, that the CGI Framework for addition and subtraction problems demonstrates that by changing the position of the unknown, a mathematics task can elicit a different sort of thinking from students. But similar frameworks do not exist for all content areas. This does not mean that teachers lack the knowledge of how student learning develops in ratio and proportion; instead, it could mean that they do indeed have an informal knowledge base about it but do not yet know how to formalize that knowledge into a more organized, accessible resource.

The grades 6-8 teachers spent more time than the other groups going through the pooled resources and chose to redesign a wider variety of tasks than their peers. They were just as likely, though, to choose operations/number sense exercises for redesign, as they were to choose geometry or proportional reasoning exercises. They tended to redesign problems in ways that increased student control over the intellectual work of the task rather than by changing the structure of the problem as the elementary teachers did. Perhaps giving students more control over a task is one way mathematics knowledge develops with adolescents and this redesign strategy sprung from the teachers' informal knowledge about how students learn.

The current literature base has not yet reached consensus about how students develop understanding of some middle grade mathematics concepts. For instance, one recent study suggests that students begin to develop proportional reasoning *by using pictures* (Ruchti & Bennett, 2013), while another one suggests that students first need to understand that *ratios can be thought of as composed units*, for example, for every one of these, there are four of those (Rathouz, Cengiz, Krebs & Rubenstein, 2014). Additionally, the CCSSM Progression Documents (www.commoncoretools.files.wordpress.com) suggest students begin their study of ratio and proportion by focusing on *the use of ratio language*. Of course, this is an example from just one domain and does not indicate that every concept in middle grades mathematics contains such discrepancies. The grades 6-8 teachers in this study typically redesigned mathematics exercises by opening them up to include multiple entry points and gave students more choice in the problems. This gave students more control over how to make sense of complex ideas, but may not have been aligned with research-based recommendations for developing deep understanding.

One way to think about these findings is that middle grades teachers possess an informal knowledge base about how students' mathematical ideas develop during adolescence, perhaps part of MKT (Ball, Thames, & Phelps, 2008) but no formal way yet, to organize this knowledge. Additionally, ratio and proportion is known for being difficult to teach (Thompson & Saldanha, 2003), and the wide variety of approaches to the topic as mentioned above may indicate that we, as mathematics education scholars, may not yet fully understand how best to develop concepts like ratio and proportion in middle grades students. This lack of formal consensus among researchers could contribute to more confusion among teachers who may turn to research for answers to these questions. However, this lack of consensus could also have led the teachers to make the tasks more open, giving students choices about how they approach the problems.

Curriculum as a tool for teacher learning.

Another way to interpret these findings is that the middle school mathematics teachers in this study struggled with the knowledge of *how* to redesign tasks. Teaching is a profession where many (perhaps most) of the required skills are learned in practice and interacting with curriculum materials is a required skill. In their study on teachers' interaction with curriculum, Empson and Junk (2014) found that teachers attributed their own solid knowledge of multiplication strategies to the *Investigations* mathematics curriculum. The findings suggest that the educative curriculum of *Investigations* guided these elementary teachers into gaining a deeper understanding of mathematics than they may have achieved on their own. The *Investigations* curriculum explicitly instructed teachers about different ways students might understand multi digit multiplication, thereby increasing the content knowledge of teachers and, by extension, students. The writers outlined each representation in a manner that teachers could connect to and learn from.

It is possible that the grades 6-8 teachers in this study, after exploring several different curriculum programs, each of which most likely had a different voice (Love & Pimm, 1996; Herbel-Eisenmann, 2007; Remillard, 2000) and mode of engagement (Ellsworth, 1997; Remillard et al, 2012), were less clear about how to develop students' ideas. Perhaps they would have benefited from an educative curriculum that included teacher instruction on connections between different concepts and ideas, and a framework that presented the overall learning trajectory of the ideas central to the grade level.

For example, in their study of how elementary teachers use educative curriculum materials, Beyer and Davis (2009) designed two different support structures for groups of pre-service teachers to use as they analyzed and planned science lessons. The first was lesson-specific support and guided the teacher through the planning of a particular lesson. The second consisted of more general supports consisting of principles and pedagogical ideas and strategies. They found that the teachers in the general support group focused in on the principles and applied them to other lessons, while the group who received lesson-specific support, on the other hand may have not transferred the support beyond the lesson in which it was located. So while there is still much to be learned about the most beneficial components of an educative curriculum and how to effectively leverage them, evidence suggests that providing teachers the reasoning behind particular curriculum components can support teacher learning of content and pedagogy (Beyer & Davis, 2009; Davis & Kracjik, 2005).

Curriculum as a tool to support students emotional needs

I also found that teachers consider students' thinking and emotional needs as key factors in deciding how to redesign mathematics exercises. They want students to engage in tasks that require them to reason mathematically and communicate about their ideas with their peers. They expressed using knowledge of student thinking to differentiate instruction for their students but did not elaborate on how, exactly, they would do that. In a comprehensive study of education in the United States, the most commonly reported professional development need expressed by teachers was that of differentiation of instruction, and the implementation of problem solving and argumentation in the classroom (Hamilton, Kaufman, Steecher, Naftel, Robbins, Thompson, Garber, Faxon-Mills and Opfer, 2016). The teachers in my study seemed to have the same desire.

Teachers understand the importance of such skills but may need support with developing them. This raises questions about whether or not an educative curriculum could help teachers with differentiation of instruction for both students' current understandings and their emotional affect towards mathematics. What would such support look like? We know that math anxiety is common among students (Beilock, 2008) and can impede their learning. We also know that engaging with, and talking about, meaningful mathematics problems may be a way to build students' confidence with mathematics (Miller, 2013). Is it possible then for a curriculum to be the vehicle serving to educate teachers about how to implement such ideas?

Teachers as Designers

For this study I chose to look at mathematics teachers and the act of teaching through the lens of teachers as designers (Brown, 2009). Doing so allowed me to understand how teachers interact with curriculum materials in the particular ways I have outlined in this paper. However, to promote lasting change in mathematics instruction, it does not really matter how researchers like myself choose to view teachers and the act of teaching. What really matters is how teachers see themselves and the work that they do.

Some teachers see themselves as delivery agents, responsible for carrying out lessons as they appear in curriculum materials. Others see themselves as crowd controllers, believing that the job of a teacher is to keep students silent and compliant in their seats for the duration of the class. Additionally, school systems hold various beliefs of what the role of the mathematics teacher consists of. For teachers to perceive themselves as instructional designers, they must first be empowered to adopt the mindset that they are instructional leaders and that *to teach mathematics* means to become an instructional designer. When teachers fully embrace this ideology, learning about how they interact with curriculum materials to design rigorous mathematics tasks for their students will become even more interesting than it was for me during this study.

Current research into the idea of teachers as designers is gaining momentum. The Design Thinking Framework from the Institute of Design at Stanford offers is a way for teachers to frame tasks in order to elicit thought-provoking learning (http://dschool.stanford.edu/). Early research into the process shows promising results. In their study, Bush, Karp, Cox, Cook, Albanese and Karp (2018) used the framework to task a class of intermediate grade students with designing a prosthetic hand and arm for a kindergarten student in a neighboring school district. Their results indicate that, though the framework takes time to teach to students, it is time well spent as they found students to readily engage with the task and the design process.

Implications for Mathematics Education Research

As mathematics education researchers, we hold the great responsibility of supporting teachers in the daunting tasks of learning to teach and effectively educating the youth of our society. We hold a different perspective from that of the teacher, but as front line professionals, theirs is a perspective we need to consider. It allows us insight into their complex world. Our work must always intertwine with the work of teachers as we listen to them to determine what we can do to support and enhance both the practice of teachers and the profession in general. There are three areas I believe may benefit from further research and will help us achieve a better understanding of the teacher text interaction. The first is research into what middle school teachers know about how students' develop ratio and proportional understanding. The second is research into the specialized knowledge of pedagogical task knowledge (Liljedahl et al, 2007) teachers use in their work. The third is research into how teachers' beliefs and attitudes impact their practice. Such research could help mathematics education researchers better understand what support teachers need with curriculum interaction and task design. It could also help teachers develop a better understanding of their craft and empower them to more fully embrace the importance of their role. In the next section I discuss each in more detail.

The Common Core State Standards for Mathematics has placed an increased focus on the topic of ratio and proportion in the middle grades. Though, historically, it has been part of the curriculum (McCallum, 2015) it was typically taught with the use of memorized algorithms that students struggled to retain due to the lack of connection to conceptual understanding (Lobato & Ellis, 2010). The new standards require students to interact with the concepts of ratio and proportion through tables, coordinate plane graphing, equations, diagrams, and verbal descriptions of proportional relationships (Common Core State Standards for Mathematics, 2010). These changes seem more likely to drive student understanding. However, it would be interesting to learn from teachers directly how the new standards impacted their practice and beliefs, how they see conceptual understanding developing in their students, and what struggles they face (or are facing) in these areas. Certainly there are many mathematics areas that would also be interesting to explore but the significant changes to the Ratio and Proportion concepts at the middle school level would be, in my opinion, extremely interesting.

Additionally, further research into how the specialized knowledge of teachers takes root could further what we know about how teachers develop. Specifically, a better understanding of the development of pedagogical task knowledge (Liljedahl et al, 2007) would be interesting. Framing a mathematics task as three tasks, (the task as written, the task as interpreted and modified by the teacher, and the task as enacted by the teacher), illustrates the complexity of the work of teachers. When mathematics education researchers more fully understand the way teachers decide how to use particular mathematics tasks to drive pedagogical goals, we can better support their work with mathematics task interaction.

Finally, learning more about how the beliefs and attitudes of teachers are reflected in their practice and how those beliefs are established or changed may point to ways in which we can support teachers with seeing themselves as instructional designers.

Implications for Practice

There exists a tension in the teacher-text interaction in that teachers draw upon their own personal resources to make design decision (Brown, 2009; Remillard, 2005) resulting in a variety of modifications that may or may not be productive for student learning. As an example from this study, teachers redesigned the exponent task (page 148), with the desire to increase students' conceptual understanding of exponents. They wanted students to talk about the meaning of exponents and though they were successful at engaging students in mathematical argument, the task itself was still procedural in nature.

As such, when the time comes to choose a curriculum program, school leadership should look carefully at their goals and consider the adoption of an educative curriculum program that has the potential to enhance teacher as well as student learning in the pursuit of their goals. When we conceptualize teachers as designers, and curriculum materials as tools with which teachers work to create learning experiences for students, it frees us to look at curriculum in a different way. However, merely putting an educative curriculum program in the hands of teachers is not enough to drive the large-scale change we seek (Ball & Cohen, 1996; Collopy, 2003; Davis & Krajcik, 2005). Teachers need support with the interaction of the materials in order to effectively and efficiently utilize the educative components of the curriculum (Beyer & Davis, 2009).

We know that mathematics teachers' access specialized forms of knowledge in their work such as pedagogical task knowledge (PTK), (Liljedahl et al, 2007) and mathematics knowledge for teaching (MKT) (Ball, Thames, & Phelps, 2008), but we have not yet reached consensus on exactly how that knowledge is developed. Perhaps the first step in supporting the development of this knowledge is to increase teachers pedagogical design capacity (PDC) (Brown, 2009), discussed on chapter 37. Recall that PDC is a teachers' ability to utilize and mobilize instructional resources in the pursuit of student learning and maybe interventions described in this study can serve to build capacity. Once the capacity is built, perhaps PTK and MKT can more easily flourish.

This study suggests that teachers have the capacity for pedagogical design work (PDC) but that it needs to be developed. The teachers in this study at all grade levels elegantly redesigned many tasks in ways that made learning mathematics rigorous and engaging for students. They did this by intentionally interacting with curriculum materials around the specific purpose of creating cognitively demanding tasks.

However, having teachers redesign mathematics problems from pre-written curriculum materials is not something that we want teachers to engage in as a rule because the design work of teachers is different from that of curriculum developers. Teachers interact with the tools of curriculum to plan challenging and fun mathematics lessons but should not be expected to build the tools with which they work. Instead, the process of redesigning tasks may serve to guide teachers to a deeper understanding of what makes a mathematics task cognitively demanding. With this deeper understanding, teachers may feel more empowered to choose tasks for their students that require a high level of thinking.

It may also serve to help them see themselves as instructional designers who use curriculum materials as tools to serve the needs of their particular students. Additionally, intentional curriculum interaction practice might support teachers with the development of specialized teaching knowledge. The Task Redesign Cycle could serve as a useful learning tool in helping teachers to develop the skills they need to be effective instructional designers. However, I do not recommend teachers redesign every task they come across. Interacting with and redesigning tasks may serve to educate teachers but is not a form of practice as students need a mathematics education that is cohesive and, as this paper illustrates, modifications teachers make to tasks can change the mathematical goal of a task. At the very least, more research into this area is needed.

Implications for Curriculum Design

Research suggests that more experienced teachers tend to move fluidly between representations of concepts as they facilitate such discussion (Davis & Renert, 2013) which indicates that they probably think through the possible solutions students may come up with during the planning process. Davis and Renert (2013) go on to suggest that these experienced teachers often do not even realize the depth of their skills with this fluidity. This suggests that, once teachers learn how move fluidly between ideas and representations, it becomes an automatic skill for them. Although we do not know exactly how those skills develop, educative curriculum that supports teachers in thinking through all possible student solutions may serve to increase teachers' development of this important skill.

Final Thoughts - Future Possibilities

Enacting any prewritten curriculum program with complete fidelity is akin to microwaving a frozen lunch and serving it directly to students (Shawer, et al, 2008). Mathematics curriculum writers expect teachers to interact with materials in a flexible way, working to modify and adapt tasks to meet the needs of their particular students. Research suggests that, for better or worse, it is teachers who moderate the effects of any curriculum program (Westwood-Taylor, 2016). However, such adaptation work is not trivial or straightforward. Teachers need support to interact effectively with curriculum materials in order to create meaningful lessons for students.

Students deserve competent, confident, and creative teacher-designers in charge of creating meaningful instructional activities for them. Teachers deserve the support of curriculum that respects their professionalism and includes them in the reasoning behind decisions made in the materials. Teachers positioned and supported as Instructional Designers can learn to interact with curriculum in ways that increase student curiosity and interest in mathematics, leading to more meaningful learning. Every teacher deserves to be listened to and supported as they increase their pedagogical design capacity and become true agents of change.

APPENDICES

Appendix A: Tasks teachers engaged with during the professional development workshops.

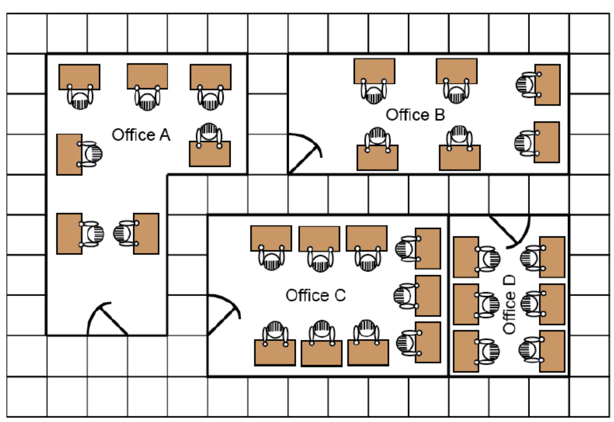
Leo the Rabbit, (youcubed.org, 2014)

Leo the Rabbit

Leo the Rabbit is climbing up a flight of 10 steps. Leo can only hop up 1 or 2 steps each time he hops. He never hops down, only up. How many different ways can Leo hop up the flight of 10 steps? Provide evidence to justify your thinking.

Office Space Problem, Bowland Charitable Trust, 2008 (bowland math, 2008)

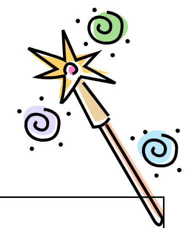
Modelling and explaining ***Sharing office space***



This drawing shows four offices in a factory. The workers complain that some offices are more crowded than others. How could the workers be rearranged with the minimum of fuss, so that the offices are equally crowded?

Appendix B: Task Re-Write Form

Common Core Collaborative Fall 2014 CCSSM Problem Re-Write



Names:____

Standard Code:
Original Problem and Source:
Problem re-write:

Appendix C: Sample Teacher Reflection

April 9, 2015 C3 Spring 2015 Day Five

Please reflect on, and answer the following questions.

1) How do you make decisions about how to adapt mathematics tasks from prewritten curriculum materials? I try to adapt materials in a way that promotes conversation and justification of ideas & solutions.

2) What sort of expertise, or knowledge, do you draw on as you make those decisions? Content knowledge. Knowledge of my students, (where they are ... where they need to be...) I also consider what materials I have available.

3) What skills with curriculum decisions have you developed with experience that would have been beneficial to have as a new teacher? The ability to predict possible misconceptions, to evaluate the cognitive demands required ahead of time.

4) Do you think it's possible to teach those skills to new teachers? If so, how? If not, why not? I do think it's possible to teach through doing the teaching following the decisions as well as reflecting; however, I feel refinement of these skills comes with experience & reflection.

Appendix D: Sample Researcher Reflective Journal Entry (Feb 2015)

Though I have not yet listened to the recording, I had my undergraduate assistant, turn on the recorders at a third grade table and the middle school table, when they began doing task redesign work. I will listen to those today and hopefully they are okay and give me more information than the question form I used for the first workshop. I heard all of the groups engaged in discussion so I really hope I was able to capture what they did to redesign the task.

Prior to that, when we first got to the curriculum part, I was amazed at how many different materials came out. Teachers from Ormont said that they have no set adopted curriculum that it is all site based. Another teacher from Carhall said she was directed to use a particular curriculum but that she hated it and has not complied.

Another teacher said that they were told NOT to use a certain geometry text, and one teacher said that her district had made a "do not use" list of curriculum books. So she spent all summer constructing a learning progression with logical sequencing only to find out that, when she came back, they were mandated to use a particular book because it was aligned to their internal testing program.

A teacher at the middle school table said her principal paid for a few teacher editions and some student editions of a particular book so that the teachers can make copies of the student consumable book. The student book looked pretty cool but I didn't get to examine the problems in it; the thing that looked cool was that students could write in it, it was pretty thick so it seemed you'd be able to see the progression of the conceptual foundation building. She had no idea what the rest of the district was using for math. These groups spent most of their time exploring the different materials at their table.

Another MS teacher brought a traditional curriculum book but said she was not happy with it. She said that they stay piled in her closet and that she uses the things she finds online and things that she makes up to teach her class. The frustration level of these teachers is incredible and I am staggered by the amount of work they must be doing.

Appendix E: Word Problem Chart Based on Cognitively Guided Instruction Problem Types (Carpenter & Fennema, 1992).

Joining Problems		
Join (Result Unknown) 6 + 3 = ___	**Join (Change Unknown) 4 + = 7**	**Join (Start Unknown) __ + 4 = 6**
Mr. Smith had 6 cookies. Suzy gave him 3 more cookies. How many cookies does Mr. Smith have now?	Mr. Smith had 4 cookies. Suzy gave him some more. Then, Mr. Smith had 7 cookies. How many cookies did Suzy give Mr. Smith?	Mr. Smith had some cookies. Suzy gave him 4 more cookies. Then, he had 6 cookies. How many cookies did Mr. Smith start with?

Separating Problems		
Separate (Result Unknown) 7 - 4 = ___	**Separate (Change Unknown) 5 - = 1**	**Separate (Start Unknown) __ - 4 = 4**
Mr. Smith had 7 cookies. He gave 4 of them to Suzy. How many cookies did Mr. Smith have left?	Mr. Smith had 5 cookies. He gave some to Suzy. Then, he had 1 cookie left. How many cookies did Mr. Smith give to Suzy?	Mr. Smith had some cookies. He gave 4 to Suzy. Then, he had 4 cookies left. How many cookies did Mr. Smith have to start with?

Part-Part-Whole Problems	
Part - Part - Whole (Whole Unknown) **6 + 3 = ___**	**Part - Part - Whole (Part Unknown)** **7 - 4 =__ or 4 + _ =7**
Mr. Smith had 6 white cookies and 3 pink cookies. How many cookies did Mr. Smith have altogether?	Mr. Smith had 7 cookies. 4 were pink and the rest were white. How many white cookies did Mr. Smith have?

Comparing Problems		
Compare (Difference Unknown) 5 - 3 = **or 3 + = 5**	**Compare (Quantity Unknown)** **3 + 2 = _**	**Compare (Referent Unknown)** **8 - 5 = __**
Mr. Smith had 5 cookies. Suzy had 3 cookies. How many more cookies did Mr. Smith have than Suzy?	Mr. Smith had 3 cookies. Suzy had 2 more cookies than Mr. Smith. How many cookies did Suzy have?	Mr. Smith had 8 cookies. He had 5 more than Suzy. How many cookies did Suzy have?

Multiplying and Dividing Problems		
Multiplication 3 x 3 =	**Measurement Division** **9 ÷ 3 =**	**Partitive Division** **12 ÷ 3 =**
Mr. Smith had 3 piles of cookies. There were 3 cookies in each pile. How many cookies did Mr. Smith have?	Mr. Smith had 9 cookies. He put 3 cookies in each box. How many boxes did he need?	Mr. Smith had 12 cookies. He wanted to give them to 3 friends. How many cookies did each friend get?

Appendix F: Task Analysis Guide (Smith & Stein, 1998)

Lower-Level Demands	**Higher-Level Demands**
Memorization Tasks • Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	Procedures With Connections Tasks • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad, general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	Doing Mathematics Tasks • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one's own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Appendix G: Questions Teachers Were Asked to Reflect in Writing

What skills with curriculum decisions have you developed with experience that would have been beneficial to have as a new teacher
What challenges did your group face as your re-designed mathematics tasks?
How do you make decisions about how to adapt mathematics tasks from pre-written curriculum materials?
What sort of expertise or knowledge do you draw on as you make those decisions?
What type of assistance did your students request as they worked through the task?
What sort of assistance did you provide students as they worked through the task?
What sort of support do you think teachers need in order to redesign mathematics tasks?
How productive were your students as they worked through the task?
What challenges did your students face as they worked through the task?

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