GENERALIZED SPATIAL MODULATION SYMBOL DESIGN FOR CORRELATED RICIAN FADING MIMO CHANNELS

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ABSTRACT

In generalized spatial modulation (GSM), information is conveyed both by the indices of multiple activated antennas and the modulation symbols they transmit. GSM includes generalized space shift keying (GSSK) and spatial modulation (SM) as special cases. In a multiple-input multiple-output (MIMO) system with correlated antennas, a large number of possible GSM symbol sets exists, and the use of a particular set affects the error performance. This problem has been addressed recently for visible light communication systems using an iterative combinatorial symbol search algorithm. This paper investigates the adaptation of the this iterative algorithm for GSM symbol design in MIMO radio frequency systems. Several approaches to calculating the inter-symbol distances are introduced. The performance of the designed GSM, GSSK, and SM symbol sets is compared. The effects of the Rician fading channel parameters and the spectral efficiency are investigated.

INTRODUCTION

Multiple input multiple output (MIMO) systems offer several benefits that include multiplexing and diversity. The use of MIMO techniques, however, also gives rise to new challenges. MIMO systems suffer from antenna correlations, more complex transmitter and receiver processing, more transmit radio frequency (RF) chains, and increased hardware complexity. One modulation class that can reduce the number of required transmit chains is the spatial modulation technique, where data is simultaneously conveyed by active antenna positions and their transmitted signals. Examples of this class include the generalized spatial modulation (GSM) [1], generalized space-shift keying (GSSK) [2], and spatial modulation (SM) [3].

In GSM, each channel use for data transmission activates only N_a antennas out of a total of N_t available transmit antennas. Each active antenna transmits a symbol from an M-ary quadrature amplitude modulation (QAM) alphabet. Note that SM is a special case of GSM where $N_a = 1$, while GSSK is a special case where $M = 1$. In GSM systems, a total of $C = M^{N_a}\binom{N_t}{N_a}$ symbols are available. In practice, however, the number of symbols used for communication has to be a power of two so that the spectral efficiency r is a positive real integer. Defining $C' = 2^r$, there are $\binom{C}{C'}$ symbol sets to choose from. As an example, with $N_t = 16, N_a = 3, M = 16$ and $r = 3$ bits per channel use (bpcu), there are $C = 3.6 \times 10^4$ symbols, and 6.7×10^{31} symbol sets to choose from. Brute force symbol set selection of this scale is unfeasible. Hence, it is necessary to use low complexity symbol set selection methods.

In [4], a symbol search tree (SST) is developed to efficiently solve the symbol set selection problem for a visible light communication (VLC) GSSK system. The SST finds the globally optimal symbol set, but has high implementation complexity for large N_t and N_a. In [5], the SST is extended to RF GSSK systems with correlated antennas. These two papers demonstrate that error performance gains of many dBs is achievable over random symbol set selection for both RF and VLC GSSK systems. In [6], a symbol set generation tree (SSGT) is developed for GSM symbol set selection for VLC systems. Despite significant complexity reduction, the SSGT remains unfeasible for even moderate values of N_t, N_a, M, and r. To reduce complexity, an iterative combinatorial generalized spatial modulation (ICGSM) algorithm is developed in [7] to efficiently solve the GSM symbol set selection problem for both camera and photodiode receiver based VLC GSM systems. While the selected set in ICGSM is not guaranteed to be absolutely optimal, the ICGSM algorithm is shown to provide large gains over existing VLC GSM symbol design algorithms in error performance, and matches the error performance of the globally optimal symbol set for simple scenarios [7].

The contributions of this paper are as follows. First, we explore and adapt the ICGSM algorithm presented in [7] for VLC systems to RF in correlated Rician fading MIMO channels. Second, several approaches to calculating inter-symbol distances are developed. These include the Rician channel-based design (RCD) method which considers both the line-of-sight (LOS) and scattered channel components in symbol design and is shown to provide gains of up to 7 dB over random symbol set selections. Third, we study the effect of various Rician K-factor and spectral efficiencies on the error performance. Finally, our examples compare the error performance of GSM, SM, and GSSK in the Rician fading channel.

Notations: We use bold lower case letters for column vectors, and bold upper case letters for matrices. For a set of vectors, $\mathbf{b}_i$ denotes the i-th vector, and $b_{i,m}$ is the m-th element of vector $\mathbf{b}_i$. For a matrix $\mathbf{A}$, $A_{m,n}$ denotes the (m, n)-th element and $\mathbf{A}^{1/2}$ denotes its square root. For a set Ω, $\Omega(i)$ denotes its i-th element and $|\Omega|$ denotes its cardinality. The notations $[\cdot]^T$, $[\cdot]^H$, $||\cdot||_l$, $\lfloor\cdot\rfloor$ and $E[\cdot]$ denote transpose, the Hermitian operator, the l-norm, the floor function, and expectation respectively. We denote $\binom{n}{k}$ as $n!/(n-k)!k!$ and $\mathcal{CN}(m, \sigma^2)$ as the complex Gaussian distribution with mean m and variance σ^2.

SYSTEM MODEL

We consider a MIMO system with a uniform linear array (ULA) of N_t transmit antennas and a receive ULA of N_r antennas. The distances between adjacent antennas in the transmitter and receiver arrays are d_t and d_r respectively. Let θ_t and θ_r denote the angles of the transmitter and the receiver arrays with respect to the LOS path as shown in Fig. 1. For simplicity, we assume that the transmitter and receiver ULAs are oriented parallel to each other such that $\theta_t = \theta_r = \theta$.

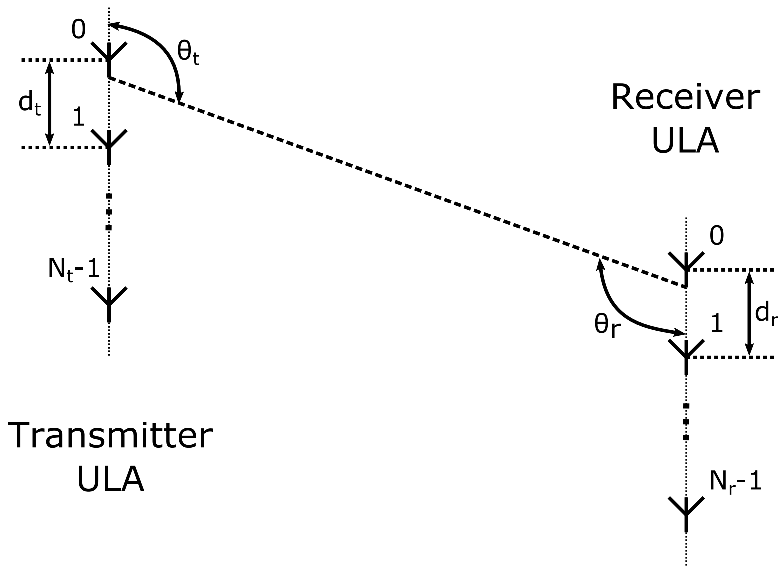

Figure 1: A MIMO system with N_t transmit antennas and N_r receive antennas.

In GSM, only N_a out of the N_t transmit antennas are activated in each channel use. These N_a antennas are referred to as the active antennas. Each active antenna transmits a symbol from an M-ary QAM alphabet $\mathcal{B}$. We denote $\mathcal{U}$ as the set of all GSM symbols, and $\mathbf{u}_i = [u_{i,1}, u_{i,2}, \ldots, u_{i,N_t}]^T \in \mathcal{U}$ as the i-th GSM symbol, where $u_{i,j} \in \mathcal{B}$ for active antennas and $u_{i,j} = 0$ otherwise. The received signal vector $\mathbf{y}$ of size N_r can then be modeled as

$$\mathbf{y} = \gamma \mathbf{H}\mathbf{u} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ is the channel matrix, $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2)$ is the i.i.d. complex Gaussian noise vector, and $\gamma = 1/\sqrt{E[\mathbf{u}_i^H \mathbf{u}_i]}$ is a transmit power normalization constant where $E[\cdot]$ denotes averaging over the selected symbol set Ω. For Rician channels, the channel matrix can be expressed as [8], [9],

$$\mathbf{H} = \sqrt{\frac{K}{1+K}}\mathbf{G} + \sqrt{\frac{1}{1+K}}\mathbf{R}^{1/2}\mathbf{W}\mathbf{T}^{1/2}, \tag{2}$$

where K is the Rician K-factor, $\mathbf{G}$ is the LOS channel component, $\mathbf{W}$ is the scattered channel component with elements $W_{i,j} \sim \mathcal{CN}(0, 1)$, and $\mathbf{R}$ and $\mathbf{T}$ are the receive and transmit antenna correlation matrices respectively. It is known that the LOS channel component can be calculated as

$$\mathbf{G} = \mathbf{s}_R \mathbf{s}_T^H, \tag{3}$$

where $\mathbf{s}_R = [1, \ldots, \exp(-(j2\pi d_r/\lambda)(N_r - 1)\cos\theta_r)]^T$ and $\mathbf{s}_T = [1, \ldots, \exp(-(j2\pi d_t/\lambda)(N_t - 1)\cos\theta_t)]^T$ are the receiver and transmitter steering vectors respectively. The elements of $\mathbf{R}$ and $\mathbf{T}$ express the correlation between receive and transmit antenna pairs, calculated for antennas i and j as

$$R_{i,j} = \alpha_r^{|i-j|} \quad and \quad T_{i,j} = \alpha_t^{|i-j|}, \tag{4}$$

where α_r and α_t represent the correlation between neighboring antennas in the receive and transmit ULAs respectively. The signal-to-noise ratio (SNR) is defined as $\text{SNR} = \gamma^2 N_a E_s/\sigma^2$ where E_s is

the average symbol energy of the QAM alphabet.
We assume perfect knowledge of the channel matrix $\mathbf{H}$ in receiver detection, and the receiver employs the maximum likelihood (ML) detector as

$$\hat{\mathbf{u}} = \underset{\mathbf{u}}{\mathrm{argmax}} \parallel \mathbf{y} - \mathbf{H}\mathbf{u} \parallel_2^2, \tag{5}$$

where $\hat{\mathbf{u}}$ is the detected GSM symbol.

SYMBOL SET DESIGN

The cardinality of the set of GSM symbols is $|\mathcal{U}| = M^{N_a}\binom{N_t}{N_a}$, making it highly complex to choose a set of 2^r symbols when $2^r << C$. The ICGSM algorithm developed in [7] addresses this problem for VLC systems. The ICGSM algorithm improves the error performance of a set of symbols by maximizing the minimum Euclidean distance between symbols in the set. We introduce the inter-symbol distance matrices $\mathbf{D}$ and $\mathbf{D}'$, whose elements $D_{i,j} = \parallel \mathbf{H}(\mathbf{u}_i - \mathbf{u}_j) \parallel_2$, $\mathbf{u}_i, \mathbf{u}_j \in \Omega$, and $D'_{i,j} = \parallel \mathbf{H}(\mathbf{u}_i - \mathbf{u}_j) \parallel_2$, $\mathbf{u}_i, \mathbf{u}_j \in \Omega'$, are respectively the inter-symbol distances. The algorithm can be summarized into four main steps as depicted in Fig. 2.

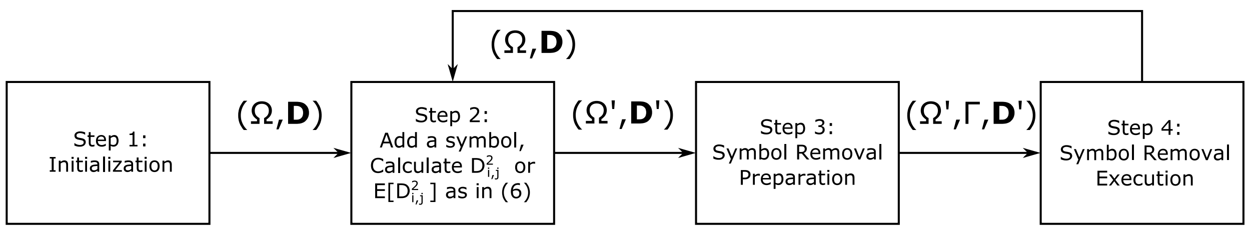

Figure 2: A depiction of the ICGSM algorithm.

1. Step 1 - Initialization:
   A random set Ω of $C' = 2^r$ symbols is chosen and its complement set Λ containing all GSM symbols that are not in Ω is created. The inter-symbol distance is calculated for each pair of symbols $\mathbf{u}_i, \mathbf{u}_j \in \Omega$ as $D_{i,j} = \parallel \mathbf{H}(\mathbf{u}_i - \mathbf{u}_j) \parallel_2$.

2. Step 2 - Symbol Inclusion:
   The first symbol in Λ is added to the set Ω to create Ω' and is removed from Λ. The inter-symbol distances $D_{i,j}$ are updated to reflect the inclusion of the new symbol.

3. Step 3 - Symbol Removal Preparation:
   Let D_{min} and D'_{min} be the minimum inter-symbol distances in symbol sets Ω and Ω' respectively. Define a new set Γ, and add all symbol pairs in Ω' that have the inter-symbol distance of D'_{min} to Γ.

4. Step 4 - Symbol Removal Execution:
   For $k = 1, 2, \ldots, |\Gamma|$, create the set $\Omega^{(k)}$ that consists of all symbols in Ω' except $\Gamma(k)$ and determine its minimum inter-symbol distance $D_{min}^{(k)}$. Update Ω as the set that offers the maximum $D_{min}^{(k)}$, and add the removed symbol to the end of Λ. If a tie occurs, then handle the tie as described in [7].

Steps 2-4 are repeated until $C - 2^r$ iterations are completed without D_{min} changing. Note that any symbol that is included that would reduce D_{min} can also be removed in the same iteration. As a result, D_{min} cannot decrease with iteration in the ICGSM algorithm. In order to solve the GSM symbol design problem for RF GSM systems, we now discuss the steps necessary to adapt the ICGSM algorithm for use in Rician fading RF systems.

In VLC systems, the channel matrix $\mathbf{H}$ is deterministic and can be exactly known during symbol design. In contrast, practical RF channels have both LOS and scattered channel components. Designing symbols based on instantaneous channel values is computationally too intense. Therefore, a straightforward symbol design method would be to perform the GSM symbol design considering only the LOS component. However, this design method can be problematic for low Rician K-factors where the scattered channel component is dominant. Thus, we propose to optimize the average minimum inter-symbol distance so that the LOS and scattered channel components are considered proportionally in the ICGSM algorithm. The average squared inter-symbol distance is calculated as

$$\begin{aligned} E[D_{i,j}^2] &= E[(\mathbf{u}_i - \mathbf{u}_j)^H \mathbf{H}^H \mathbf{H} (\mathbf{u}_i - \mathbf{u}_j)] \\ &= \frac{K}{1+K} \parallel \mathbf{G}(\mathbf{u}_i - \mathbf{u}_j) \parallel_2^2 + \frac{1}{1+K} (\mathbf{u}_i - \mathbf{u}_j)^H (\mathbf{T}^{1/2})^H E[\mathbf{W}^H \mathbf{R} \mathbf{W}] \mathbf{T}^{1/2} (\mathbf{u}_i - \mathbf{u}_j) \\ &= \frac{K}{1+K} \parallel \mathbf{G}(\mathbf{u}_i - \mathbf{u}_j) \parallel_2^2 + \frac{N_r}{1+K} (\mathbf{u}_i - \mathbf{u}_j)^H \mathbf{T} (\mathbf{u}_i - \mathbf{u}_j). \end{aligned} \tag{6}$$

Thus, $D_{i,j}^2$ can be replaced with $E[D_{i,j}^2]$ in the ICGSM algorithm. Note that an efficient calculation of $E[D_{i,j}^2]$ can be achieved by precalculating certain terms. We expand $E[D_{i,j}^2]$ as

$$\begin{aligned} E[D_{i,j}^2] = &\frac{K}{1+K} \left(\mathbf{u}_i^H \mathbf{G}^H \mathbf{G} \mathbf{u}_i + \mathbf{u}_j^H \mathbf{G}^H \mathbf{G} \mathbf{u}_j - 2\mathrm{Re}\{\mathbf{u}_i^H \mathbf{G}^H \mathbf{G} \mathbf{u}_j\} \right) \\ &+ \frac{N_r}{1+K} \left(\mathbf{u}_i^H \mathbf{T} \mathbf{u}_i + \mathbf{u}_j^H \mathbf{T} \mathbf{u}_j - 2\mathrm{Re}\{\mathbf{u}_i^H \mathbf{T} \mathbf{u}_j\} \right), \end{aligned} \tag{7}$$

where Re$\{\cdot\}$ denotes the real part of a complex number. The terms $\mathbf{u}_i^H \mathbf{G}^H \mathbf{G} \mathbf{u}_i$, $\mathbf{u}_i^H \mathbf{G}^H \mathbf{G}$, $\mathbf{u}_i^H \mathbf{T} \mathbf{u}_i$, and $\mathbf{u}_i^H \mathbf{T}$ can be calculated and stored prior to the beginning of the ICGSM algorithm and then subsequently used for efficient calculation of $E[D_{i,j}^2]$. In addition to handling the scattered channel component, θ may be unknown at the transmitter during symbol design. As such, we introduce the averaged LOS matrix $\bar{\mathbf{G}}$ by numerically averaging $\mathbf{G}$ over $\theta \in (\theta' - \delta, \theta' + \delta)$, where θ' is the center of the $\bar{\mathbf{G}}$ averaging window and δ controls the averaging window size.

We now propose the following four methods of the ICGSM algorithm implementation for RF GSM symbol design.

1. Method 1: Line-of-sight channel-based design (LCD)
   In this approach, the inter-symbol distances are calculated as $D_{i,j} = \parallel \mathbf{G}(\mathbf{u}_i - \mathbf{u}_j) \parallel_2$. This approach to symbol design requires only the perfect knowledge of $\theta = \theta_t = \theta_r$.

2. Method 2: Rician channel-based design (RCD)
   Inter-symbol distances $D_{i,j}$ are replaced with $E[D_{i,j}^2]$ in the ICGSM algorithm as calculated in (6). Knowledge of the parameters $\theta = \theta_t = \theta_r$, transmit correlation factor (α_t), and K are required for the symbol design.

3. Method 3: Averaged line-of-sight channel-based design (ALCD)
   Inter-symbol distances are calculated as $D_{i,j} = \| \bar{\mathbf{G}}(\mathbf{u}_i - \mathbf{u}_j) \|_2$. This symbol design method requires only the specification of a range of possible θ values. Thus, ALCD requires the least amount of prior knowledge about the channel.

4. Method 4: Averaged Rician channel-based design (ARCD)
   This method uses $E[D_{i,j}^2]$ in place of $D_{i,j}^2$, and the LOS matrix $\mathbf{G}$ is replaced with $\bar{\mathbf{G}}$ in (6) to calculate $E[D_{i,j}^2]$. ARCD symbol design requires specification of a range of possible θ values, transmit correlation factor (α_t), and K.

RESULTS AND DISCUSSION

In this section, we present symbol error rate (SER) simulation results for various scenarios. For simplicity, we fix $d_r = d_t = \lambda/2$, $\alpha_r = \alpha_t = \alpha$, and $\alpha = 0.5$ in all scenarios.

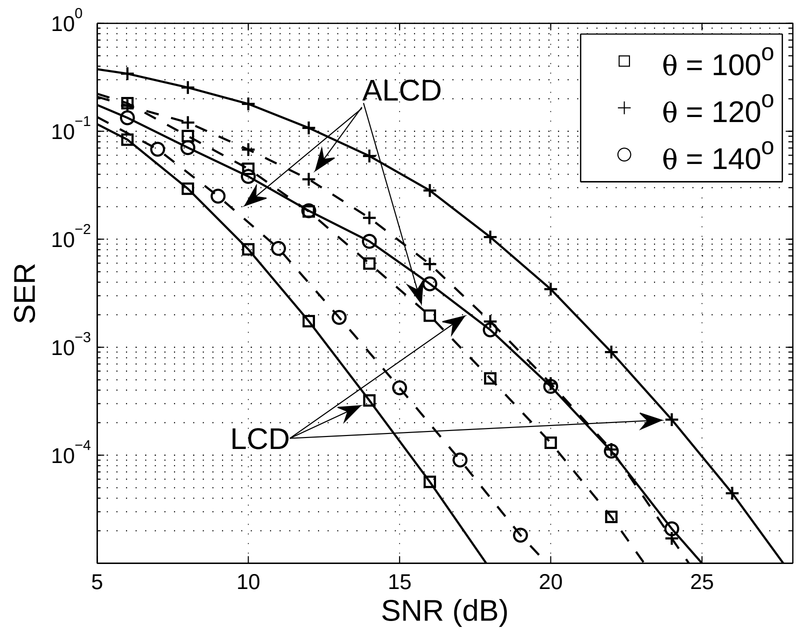

Figure 3: SER performance results for $\theta = 100^o, 120^o$, and 140^o.

Figure 3 demonstrates the SER performance comparison between the LCD and ALCD symbol designs. We make a distinction between the true θ and the value of θ (denoted as $\hat{\theta}$) used in the symbol design. The LCD symbol design uses $\hat{\theta} = 100^o$, but the SER results are shown for $\theta = 100^o, 120^o$, and 140^o. The ALCD symbol set uses $\theta' = 100^o$ and $\delta = 45^o$, and its SER performance is evaluated for $\theta = 100^o, 120^o$, and 140^o. We use $N_t = 4$, $N_r = 4$, $N_a = 2$, $M = 4$, $r = 4$ bpcu, and $K = 10$. The LCD symbol set offers the best performance at $\theta = \hat{\theta} = 100^o$ with a gain of 5 dB over the ALCD symbol set. This is because the LCD optimizes the selected symbol set for the specific LOS channel matrix with $\theta = 100^o$. Conversely, the ALCD symbol set outperforms the LCD symbol set for $\theta = 120^o$ and $\theta = 140^o$ by 3 and 5 dB respectively. This is to be expected as the ALCD symbol set is optimized for the average LOS matrix over the range $(55^o, 145^o)$.

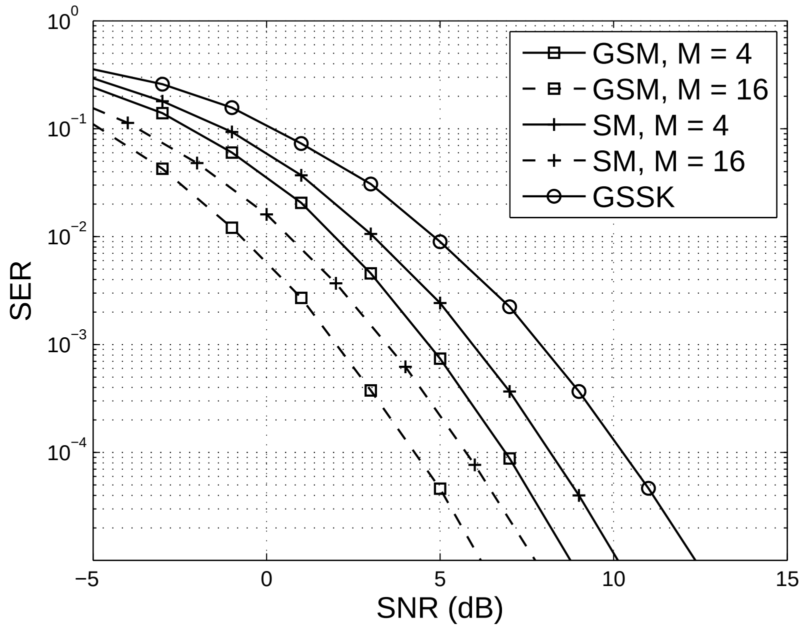

Figure 4: SER results for GSM, SM, and GSSK when $r = 3$ bpcu. All designs use the RCD symbol design method.

Fig. 4 compares the SER performance of GSM, SM, and GSSK. The RCD method is used in symbol design. The figure uses $N_t = 8$, $N_r = 8$, $N_a = 2$, $r = 3$ bpcu, $K = 1$, $\theta = 100^o$, except for SM which uses $N_a = 1$ while keeping the other parameters same as in GSM and GSSK. For GSM and SM, results are shown for both $M = 4$ and $M = 16$. For a given M, GSM offers the best performance followed by SM with GSSK offering the worst performance. Increasing M is shown to improve performance for a given set of parameters. GSM with $M = 16$ outperforms GSM with $M = 4$ by 3 dB, and SM with $M = 16$ outperforms SM with $M = 4$ by 2 dB.

Figure 5 demonstrates the SER performance of the four design methods. We use $N_t = 4$, $N_r = 4$, $N_a = 2$, $M = 4$, $r = 4$ bpcu, $K = 10$, and $\theta = 100^o$. The channel averaging uses $\theta' = 100^o$ and $\delta = 45^o$. The RCD method offers the best performance followed by the LCD method. The RCD and LCD methods offer a gain of 5 dB and 3 dB over the ARCD and ALCD methods respectively. This is because the LCD and RCD methods are designed specifically for the LOS channel with $\theta = 100^o$ whereas ALCD and ARCD use the averaged LOS channel matrix in symbol design. Because the LCD method ignores the scattered portion of the signal in symbol design, the RCD method performs better, offering a gain of 1 dB.

The SER performance of the RCD method is compared to the average SER performance of random symbol sets in Fig. 6 for $K = 1$ and 10. We use $N_t = 8$, $N_r = 8$, $N_a = 2$, $M = 4$, $r = 4$ bpcu, and $\theta = 100^o$. The average SER performance results of a random symbol set are obtained by averaging the SER performance of a randomly generated symbol set over 20 random sets. The gain of the RCD method over the average random symbol set performance is 4 dB and 7 dB for $K = 1$ and 10 respectively. This demonstrates that symbol design becomes increasingly important as K increases.

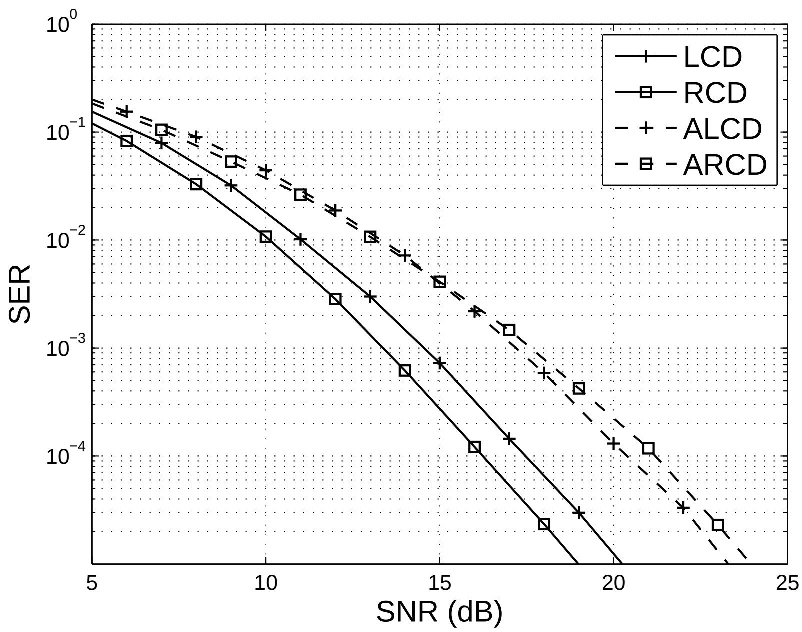

Figure 5: SER performance of the LCD, RCD, ALCD, and ARCD methods.

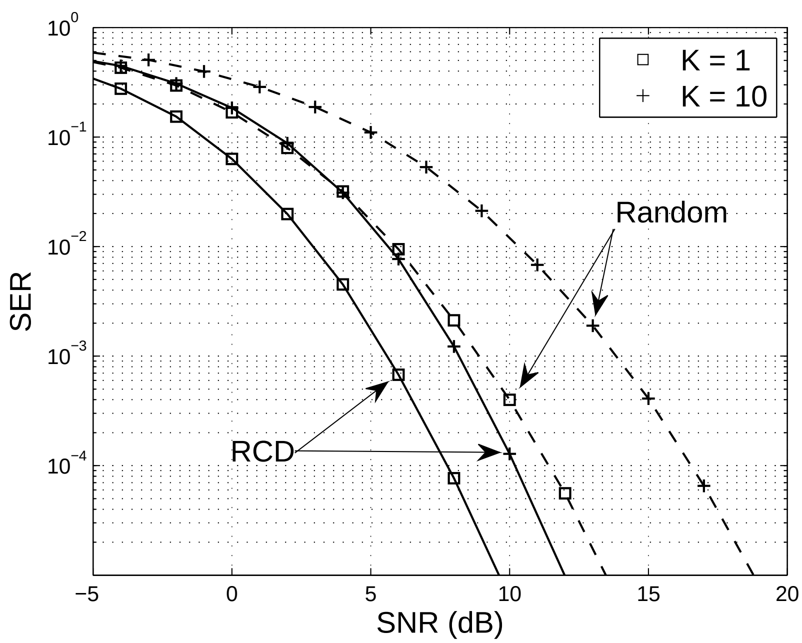

Figure 6: SER performance of the RCD method for $K = 1$ and 10.

In Fig. 7, the SER performance of the RCD method is compared against the performance of the average random symbol sets for $r = 3$ and 5 bpcu. We use $N_t = 4$, $N_r = 4$, $N_a = 2$, $M = 4$, $K = 1$, and $\theta = 100^o$. The average random symbol set SER performance is obtained using the same method as in Fig. 6. Both the RCD and the average random symbol set SER performance

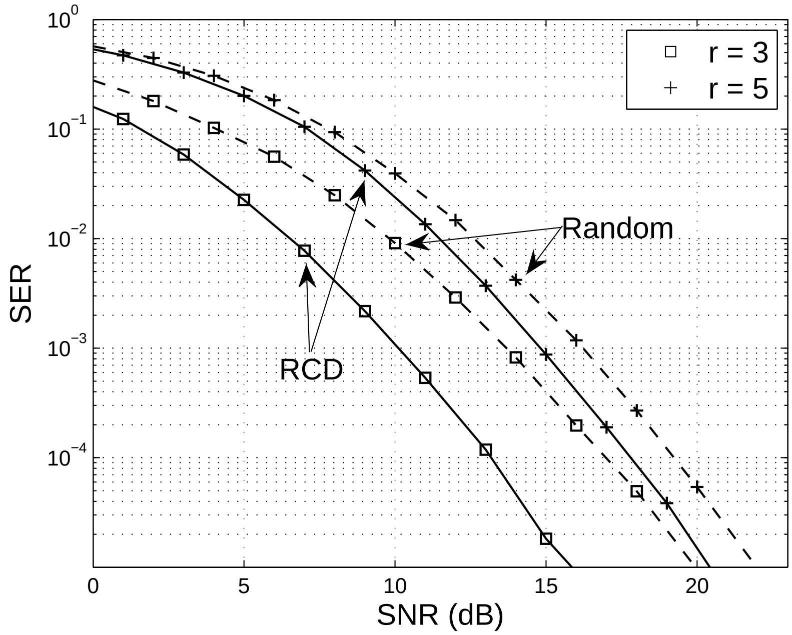

Figure 7: SER performance of the RCD method for $r = 3$ and 5 bpcu.

are found to degrade with increasing r. As r increases, the gain of the RCD method over the average random symbol set SER performance decreases significantly. The gains obtained by the RCD method are 4 dB and 2 dB for $r = 3$ and 5 bpcu respectively. Typically, very little gain is obtained from symbol set design when $r = r_{max}$ where $r_{max} = \lfloor \log_2 C \rfloor$ bpcu is the maximum obtainable spectral efficiency for a particular physical scenario. Note that the maximum spectral efficiency in this scenario is $r_{max} = 6$ bpcu. Though not shown here, performing a symbol set design for $r = r_{max}$ in this scenario offers a gain of around 0.5 dB over the average random symbol set SER performance.

CONCLUSION

The ICGSM algorithm is adapted to RF GSM systems to design GSM symbols in correlated MIMO Rician channels. Several design methods are proposed and their SER performance is compared. The RCD method is shown to provide the best error performance by considering both the LOS and the scattered channel components in symbol design. We study the effect of the spectral efficiency r on SER performance and find that the RCD method can offer large gains for low values of r. The effect of the Rician K-factor on SER performance is also investigated, and it is found that the gain obtained by designing a symbol set increases significantly with increasing Rician K-factor. Numerical results are presented demonstrating that our symbol design algorithm can provide gains of up to 7 dBs over the average random symbol set SER performance. Finally, the SER performance of GSM, SM, and GSSK is compared for the same spectral efficiency. It is found that GSM offers the best SER performance, and increasing M significantly improves SER performance at the cost of increased symbol design complexity.

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