ON THE PERFORMANCE OF FILTER BASED EQUALIZERS FOR 16APSK IN AERONAUTICAL TELEMETRY ENVIRONMENT

Farah Arabian
Brigham Young University
Department of Electrical and Computer Engineering
Provo, UT, 84602
farah.arabian@gmail.com

Faculty Advisor:
Dr. Michael Rice

ABSTRACT

16APSK is a candidate modulation for aeronautical telemetry because it has better spectral efficiency than SOQPSK-TG, but requires a linear RF power amplifier. This paper investigates the BER performance of filter-based equalizers for 16APSK operating over multipath channels measured at Edwards AFB. The results show that decision feedback equalizers outperform the other equalizers and are capable of providing excellent multipath mitigation.

INTRODUCTION

In aeronautical telemetry, inter-symbol interference (ISI is defined as unwanted distortion by other symbols on the desired symbol) prevents reliable information transmission at the highest possible data rate. ISI may be caused by a frequency selective communication channel or by filtering and pulse shaping at the transmitter. ISI can be alleviated by equalizers. Equalizers are discrete time systems.

The purpose of this paper is the performance evaluation of filter-based equalizers for 16APSK modulation. 16APSK is a candidate modulation for improving spectral efficiency in aeronautical telemetry environment [1]. The BER performance is evaluated over four multipath channels measured at Edwards AFB [2]. The equalization techniques that are explored in this paper are filter-based equalizers, which comprise the ZF (zero forcing) equalizer, the MMSE (minimum mean-squared error) equalizer and the DFE (decision feedback equalizer).

This paper is organized as follows. In the mathematical formulation section, we review the observation models used to compute equalizer coefficients. In the equalization techniques section we derive mathematical expressions for the filter-based equalizers coefficients. In the simulation section, we show that the DFE has the best performance from bit error rate (BER) point of view and the ZF equalizer has the worst performance for the channels examined in the simulations.

Notation: The symbol (.)’ denotes conjugate transpose (or Hermitian). A bold uppercase letter denotes a matrix. A bold lower case letter denotes a vector. For a square matrix the matrix inverse is indicated as $(.)^{-1}$.

MATHEMATICAL FORMULATION

The received signal $r(t)$ may be represented as

$$r(t) = \sum_k I_k h(t - kT_s) + w(t), \tag{1}$$

where I_k is a symbol drawn from the 16APSK constellation; T_s is the inverse of the symbol rate R_s (symbols/second); $h(t) = g(t) * h_c(t)$ where $g(t)$ is pulse shape and $h_c(t)$ is channel impulse response; $w(t)$ is a circularly symmetric complex-valued wide-sense stationary normal random process with zero mean and power spectral density $2N_0$ W/Hz. Maximum likelihood detection applies a matched filter with impulse response $h^*(-t)$ in the receiver. The matched filter output is

$$y(t) = r(t) * h^*(-t) = \sum_k I_k x(t - kT_s) + v(t), \tag{2}$$

where $x(t) = h(t) * h^*(-t)$ and $v(t) = w(t) * h^*(-t)$ is correlated noise. The symbol-spaced samples of the matched filter output are

$$y_n = y(nT_s) = \sum_k I_k x_{n-k} + v_n. \tag{3}$$

Equation (3) represents the Ungerboeck observation model [3]. In this model noise samples v_n are correlated because the channel matched filter does not satisfy the Nyquist no-ISI condition. The power spectral density (PSD) of sequence v_n is

$$S_v(z) = 2N_0 X(z), \tag{4}$$

where $X(z)$ is the z transform of x_n. Because x_n is an autocorrelation function, the equivalent discrete time channel in the Ungerboeck observation model is two-sided (non-causal), with length $2L + 1$ and it also possesses conjugate symmetry $x_{-n} = x_n^*$, which means the roots of $X(z)$ occur in conjugate reciprocal pairs. Consequently, each root of $X(z)$ inside the unit circle in the z plane has a companion outside the unit circle. The roots of $X(z)$ are shown in Figure 1 for all simulated channels examined in this paper.

The Forney observation model [4] shown in Figure 2 was the first classic development. The Forney observation model applies a discrete time noise whitening filter to equation (3). The noise whitening filter is derived as follows. $X(z)$ in equation (4) may be factored as [5], [6],

$$X(z) = 2N_0 F(z) F^*(1/z^*) \tag{5}$$

where the roots of $F(z)$ are all the roots of $X(z)$ that are inside the unit circle and the roots of $F^*(1/z^*)$ are all the roots of $X(z)$ that are outside the unit circle. Equation (5) is called a “spectral

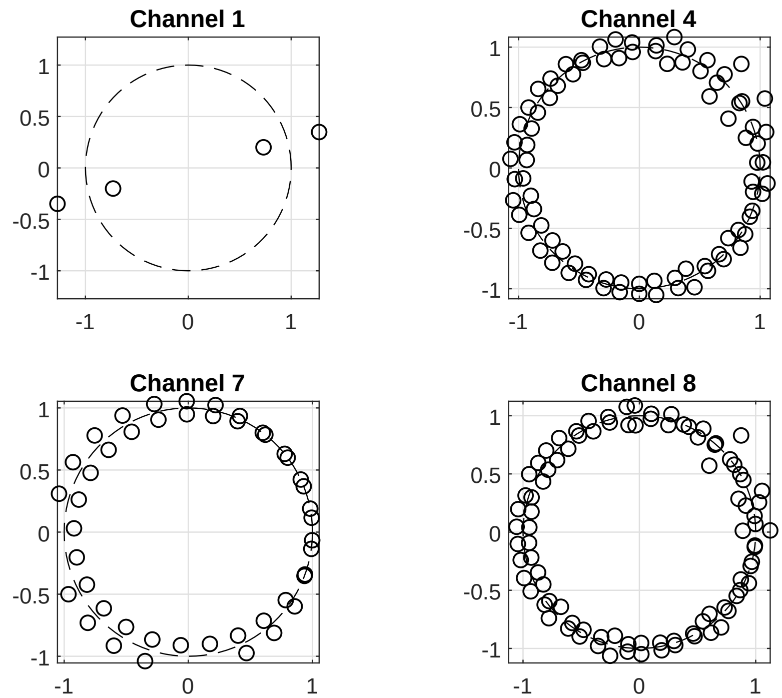

Figure 1: The roots representation of the equivalent discrete time channel in the Ungerboeck observation model for all simulated channels in frequency domain

factorization". The noise whitening filter is $1/F^*(1/z^*)$. Applying the noise whitening filter to equation (3) produces

$$u_k = \sum_{n=0}^{L} f_n I_{k-n} + \eta_k, \tag{6}$$

where $L + 1$ is the length of the channel in the FOM, η_n is circularly-symmetric complex-valued normal random variables with zero mean and common variance $2N_0$ W/Hz. The system in Figure 2 can be re-expressed as the system in Figure 3 by "using equivalent discrete time channel" definition.

The output of the Forney observation model is the input to the equalizer. Equalizers generally are categorized as linear and nonlinear. Two commonly used linear equalizers are ZF equalizer

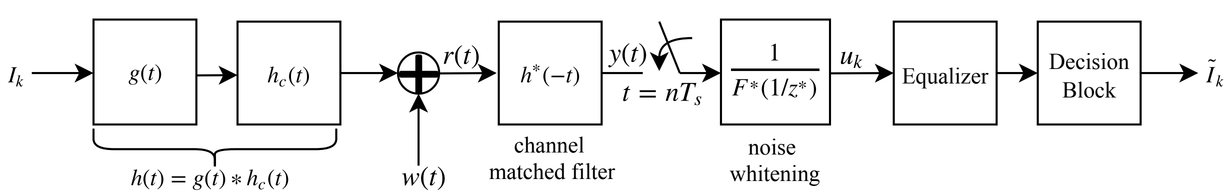

Figure 2: The Forney observation model.

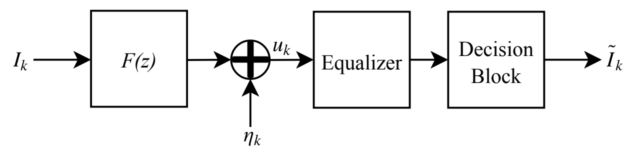

Figure 3: The equivalent discrete time system of the Forney observation model.

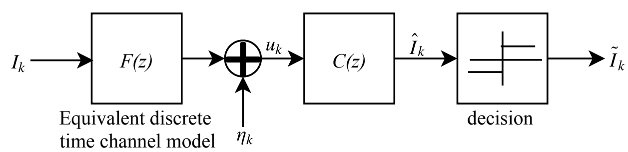

Figure 4: Linear equalizers (ZF and MMSE) block diagram.

and MMSE equalizer. Well-known nonlinear equalizers are DFE and MLSE (maximum likelihood sequence estimation). These optimization criteria are used to compute the equalizer coefficients.

In the following section, filter-based equalizers are reviewed. After the review, the performance of these equlizers with 16APSK is explored.

EQUALIZATION TECHNIQUES

A. Zero Forcing (ZF) Equalizer

The coefficients of the ZF equalizer are computed to minimize the ISI at the equalizer output. This technique is also called peak distortion criterion [5]. Figure 4 outlines a linear equalizer with the equivalent discrete time channel model. Equalizers defined by a filter coefficients vector $\begin{bmatrix} -c_{-K} & \dots & 0 & \dots & c_K \end{bmatrix}'$. The zero forcing criterion tries to minimize inter-symbol interference by using inverse of the channel impulse response:

$$F(z)C(z) = 1, \tag{7}$$

where $F(z)$ is z transform of the equivalent discrete time channel impulse response, $C(z)$ is z transform of equalizer coefficients.

If at some frequencies gain of the channel magnitude is low then, the ZF equalizer tries to boost its gain to compensate. This action also boosts the noise which accompanies the received signal. Consequently, the signal to noise ratio (SNR) is degraded. It will be worse when the channel has one or more nulls in its discrete-time Fourier transform (DTFT) $|H(e^{j\omega})|$."Null" here means less than 0.1 times the maximum value of $|H(e^{j\omega})|$.

The criterion (7) has a significant drawback: if the channel frequency impulse response is FIR then the inverse of that will be infinite impulse response (IIR), which is not desirable. Thus, to find coefficients of a ZF equalizer we use an FIR approximation to an IIR inverse filter. The coefficients for a ZF equalizer should be chosen to satisfy:

$$\sum_{j=-K}^{K} c_j f_{k-j} \approx \delta_k, \qquad -K \leq k \leq L+K, \tag{8}$$

where δ_k is the Kronecker delta function. The optimum ZF equalizer coefficients are given by

$$\mathbf{c} = \underset{\mathbf{c}}{\operatorname{argmin}} \sum_{k=-K}^{K+L} \left| \sum_{j=-K}^{K} c_j f_{k-j} - \delta_k \right|^2 . \tag{9}$$

Equation (8) can be written in a matrix-vector form:

$$\begin{bmatrix} f_0 & 0 & 0 & \cdots & 0 \\ f_1 & f_0 & 0 & \cdots & 0 \\ f_2 & f_1 & f_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ f_L & f_{L-1} & f_{L-2} & \cdots & f_0 \\ 0 & f_L & f_{L-1} & \cdots & f_1 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & \cdots & f_L \end{bmatrix} \begin{bmatrix} c_{-K} \\ \vdots \\ c_0 \\ \vdots \\ c_K \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{10}$$

which is equivalent to

$$\mathbf{Fc} = \mathbf{q}. \tag{11}$$

Equation (11) is an overdetermined system, whose least-square solution is given by the left pseudo-inverse [6].

$$\mathbf{c} = (\mathbf{F}'\mathbf{F})^{-1}\mathbf{F}'\mathbf{q}. \tag{12}$$

B. Minimum Mean-Squared Error (MMSE) Equalizer

Figure 4 is applicable to the MMSE equalizer because the MMSE equalizer also is a linear equalizer. The MMSE criterion chooses the filter coefficients to minimize the mean square of the error

$$\mathcal{E} = \{|e(k)|^2\} \tag{13}$$

where the error e_k is

$$e(k) = I_k - \hat{I}_k = I_k - \sum_{j=-K}^{K} c_j u_{k-j}. \tag{14}$$

The MMSE criterion considers both ISI and noise in equalizers coefficients calculation. Therefore, noise enhancement in MMSE equalizer is less than with the ZF equalizer. The coefficients that minimize (13) are given by

$$\mathbf{c} = \left[\mathbf{G_f} + \frac{\sigma^2}{E_b}\mathbf{I} \right]^{-1} \boldsymbol{\xi}, \tag{15}$$

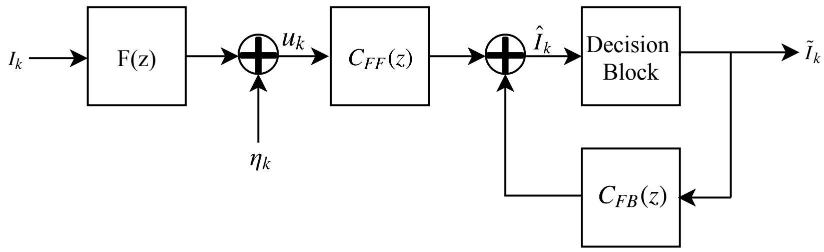

Figure 5: A block diagram of the decision feedback equalizer

where $\mathbf{I}$ is the identity matrix,

$$\mathbf{G_f} = \begin{bmatrix} G_f(0) & G_f(-1) & \cdots & \\ G_f(1) & G_f(0) & & \\ & & \ddots & \\ & & & G_f(0) \end{bmatrix}, \tag{16}$$

where

$$G_f(k) = \sum_{j=0}^{L} f_j f_{j-k}^*, \tag{17}$$

and

$$\boldsymbol{\xi} = \begin{bmatrix} f_L & f_{L-1} & \cdots & f_0 \end{bmatrix}'. \tag{18}$$

C. Decision Feedback Equalizer (DFE)

The DFE outperforms the linear equalizers because the DFE does not use the inverse of the channel. A block diagram of the DFE is shown in Figure 5. The received signal u_k passes through the feed-forward (FF) filter, which is an FIR filter with transfer function $C_{FF}(z)$. The output of the FF filter is combined with the output of feedback (FB) filter with transfer function $C_{FB}(z)$. The input to the decision block is

$$\hat{I}_k = \sum_{j=-K_1}^{0} c_j u_{k-j} + \sum_{j\prime=1}^{K_2} c_{j\prime} \tilde{I}_{k-j\prime}, \tag{19}$$

where $K_1 + 1$ is the FF filter length and K_2 is the FB filter length. The first term of the right-hand-side of (19) represents the action of the FF filter that reduces, but not eliminates, ISI from the received signal. The second term, which is related to the FB filter, reproduces residual ISI in the output of the FF filter and subtracts it out of the FF filter output in the hope of producing an ISI-free input $\hat{\mathbf{I}}_k$ to the decision block. Because the FB filter operates on symbols decisions, it does not enhance the noise. If the decisions are correct then this filter operates on noise free data which are actually the true symbols.

The feed-forward filter is designed using the MMSE criterion. The FF filter coefficients are given by

$$\mathbf{c}_{\text{FF}} = \left[\mathbf{R} + \frac{2\sigma^2}{E_b}\mathbf{I}\right]^{-1} \boldsymbol{\xi}_1, \tag{20}$$

where

$$\mathbf{c}_{\text{FF}} = \begin{bmatrix} c_{-K_1} \\ c_{-K_1+1} \\ \vdots \\ c_0 \end{bmatrix}, \tag{21}$$

$\mathbf{I}$ is the identity matrix,

$$\mathbf{R} = \begin{bmatrix} R(-K_1,-K_1) & R(-K_1,-K_1+1) & \cdots & R(-K_1,0) \\ R(-K_1+1,-K_1) & R(-K_1+1,-K_1+1) & \cdots & R(-K_1+1,0) \\ \vdots & \vdots & \ddots & \vdots \\ R(0,-K_1) & R(0,-K_1+1) & \cdots & R(0,0) \end{bmatrix}, \tag{22}$$

where

$$R(l,j) = \sum_{m=0}^{-l} f_m^* f_{m+l-j}, \quad l = -K_1, \cdots, 0 \quad ; \quad j = -K_1, \ldots, 0, \tag{23}$$

and

$$\boldsymbol{\xi}_1 = \begin{bmatrix} f_{K_1} & f_{K_1-1} & \cdots & f_0 \end{bmatrix}'. \tag{24}$$

The coefficients of FB filter are

$$c_m = -\sum_{j=-K_1}^{0} c_j f_{m-j}, \qquad 1 \le m \le K_2. \tag{25}$$

The relationship (25) can be expressed in the matrix-vector form:

$$\mathbf{c}_{\text{FB}} = \mathbf{W}\mathbf{c}_{\text{FF}} \tag{26}$$

where

$$\mathbf{W} = \begin{bmatrix} f_{1+K_1} & f_{K_1} & f_{K_1-1} & \cdots & f_1 \\ f_{2+K_1} & f_{1+K_1} & f_{K_1} & \cdots & f_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{K_2+K_1} & f_{K_2+K_1-1} & f_{K_2+K_1-2} & \cdots & f_{K_2} \end{bmatrix}. \tag{27}$$

SIMULATION RESULTS

We simulate BER performance of equalized 16APSK over mulitpath channels measured at Edwards AFB. The 16-APSK constellation is shown in Figure 7. The constellation is parameterized

Table 1: Time domain properties of explored channels in the simulation with FOM

Channel	Channel Name	L
Channel 1	Taxiway E	3
Channel 4	Takeoff on 22L	44
Channel 7	Cords Road	26
Channel 8	Black Mountain	47

by the ratio of radii $\gamma = r_2/r_1$ and the phase angle ϕ. The parameters used in the simulations are those that minimize peak of E_b/N_0 [7]: $\gamma = 2.46$ and $\phi = \pi/12$. The pulse shape $g(t)$ is the square-root raised cosine (SRRC) pulse shape with 50% excess bandwidth [8]. The mulitpath channels are described in Figure 6 and Table 1. Observations:

- The plots in Figure 6 show that 16APSK is more spectrally efficient than SOQPSK. However, 16APSK requires back-off with non-linear power amplifiers because the peak to average power ratio of 16APSK is greater than one.
- The DFE has absolutely the best performance if the feedback filter input is the true symbols. The reason is that DFE, unlike the ZF and MMSE equalizers does not use channel inversion. Moreover when the feedback filter input is the correct symbols, the ISI is zero.
- For all the channels explored in this paper, DFE using the symbol decisions for the feedback filter input outperforms the linear equalizers.
- Over the four channels, the MMSE equalizer has better BER performance than the ZF equalizer. Using the MMSE equalizer instead of the DFE results in a performance loss at least 1 dB. Moreover, the DFE improves BER much better than linear equalizers. However, the DFE brings more complexity to the system. So, there is a trade-off between BER improvement and complexity.
- Because channel 7 has the deepest nulls among all four channels, it has the worst BER performance. This is expected because filter based equalizers are strongly dependent on the channel characteristics.

CONCLUSIONS

Over the four simulated channels, the DFE has the greatest performance from the BER point of view and the ZF equalizer has the worst performance due to using the channel inversion. Some channels need long equalizers to achieve good performance, such as Channels 4, 7 and 8. To reduce the computational load for these kinds of channels, frequency domain equalizers may be used. In other words, the computational complexity of implementing long equalizers filters can be reduced by performing equalization in the DTFT domain. The BER performance of filter-based equalizers strongly depends on the channel characteristics. For example, a channel with deeper nulls has worse BER performance.

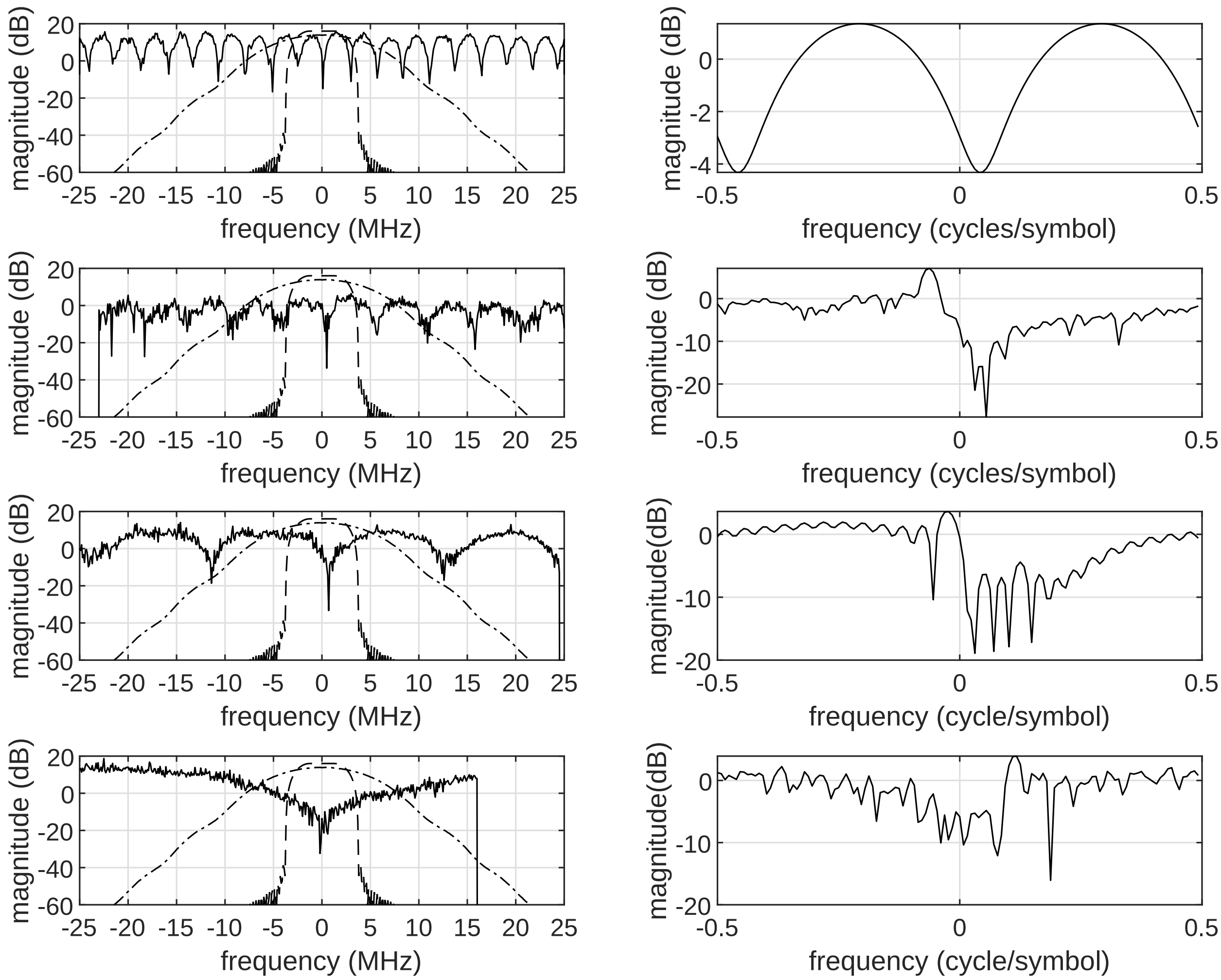

Figure 6: Frequency domain characteristics of the channels used in the simulations. Each row is specified for one channel that is shown in the table 1. The first column shows transfer functions (solid line), the power spectral density of 16APSK (dashed line), and the power spectral density of SOQPSK-TG (dash-dot line) for channels 1, 4, 7, and 8, respectively. The second column shows the transfer function of the corresponding Forney observation model.

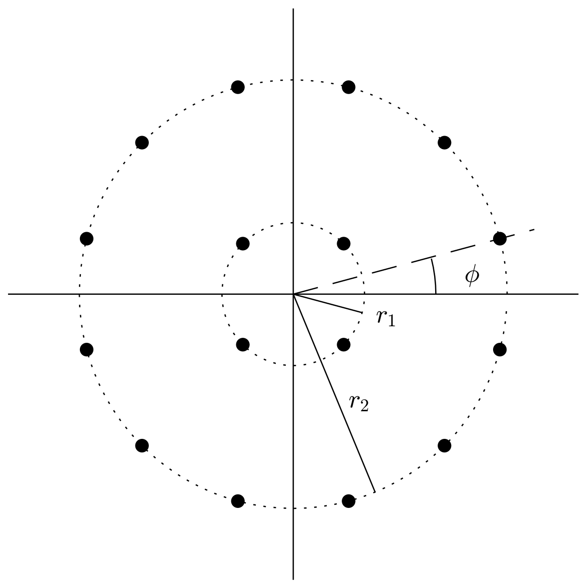

Figure 7: The 16-APSK constellation from the DVB-S2 standard [9]

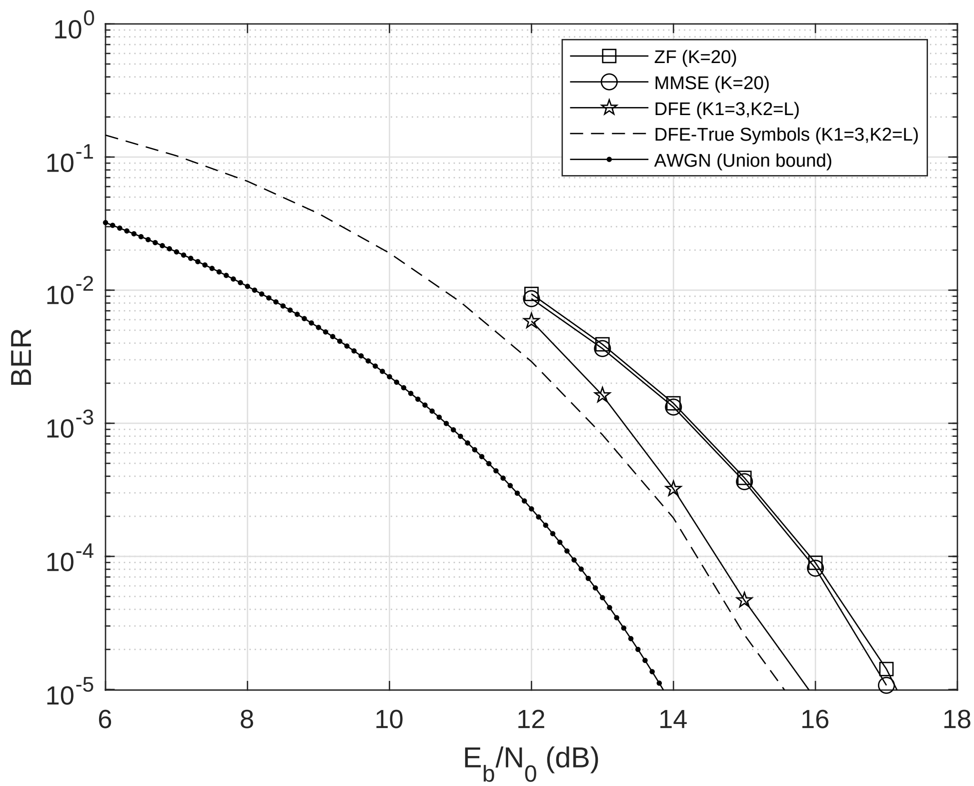

Figure 8: Simulation result for the filter-based equlaizers with FOM: Channel 1

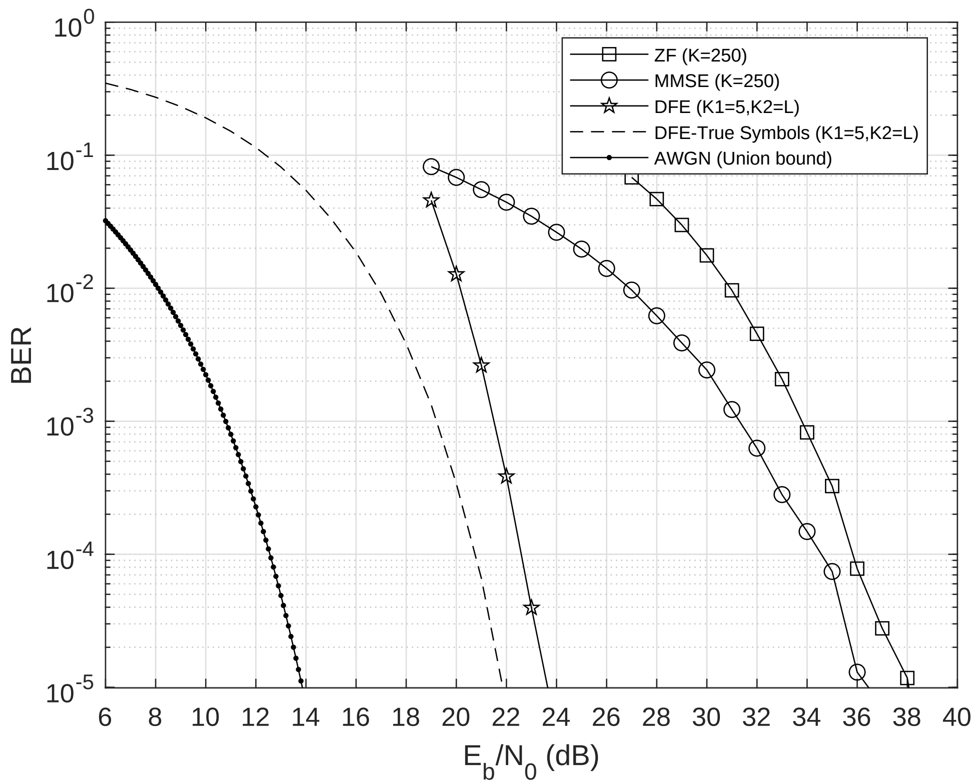

Figure 9: Simulation result for the filter-based equlaizers with FOM: Channel 4

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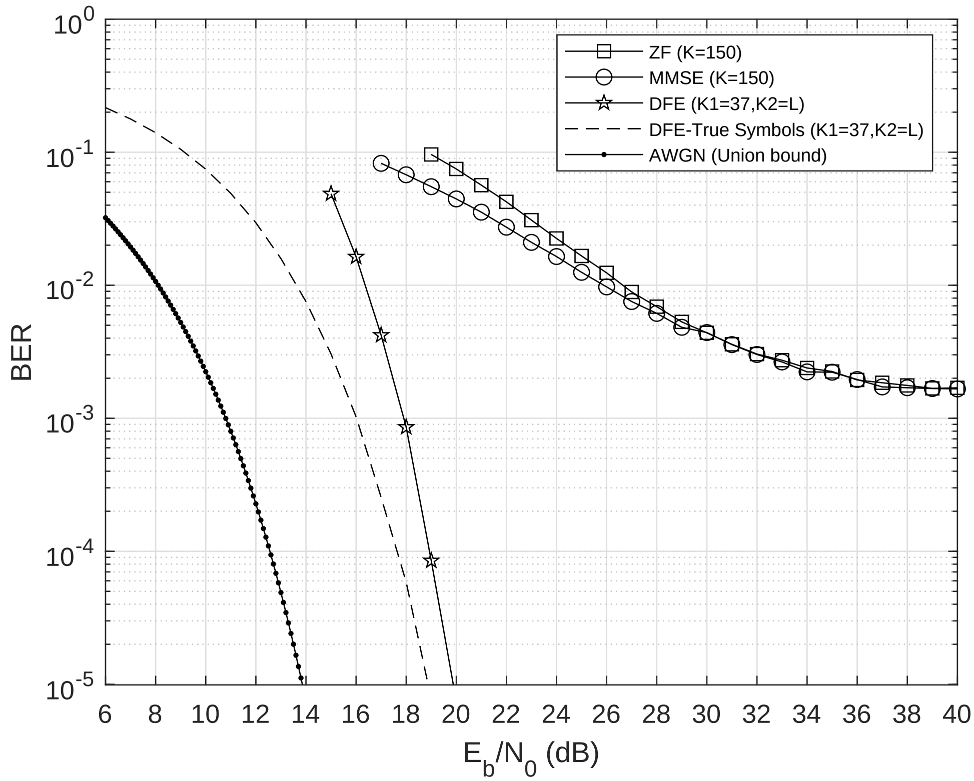

Figure 10: Simulation result for the filter-based equlaizers with FOM: Channel 7

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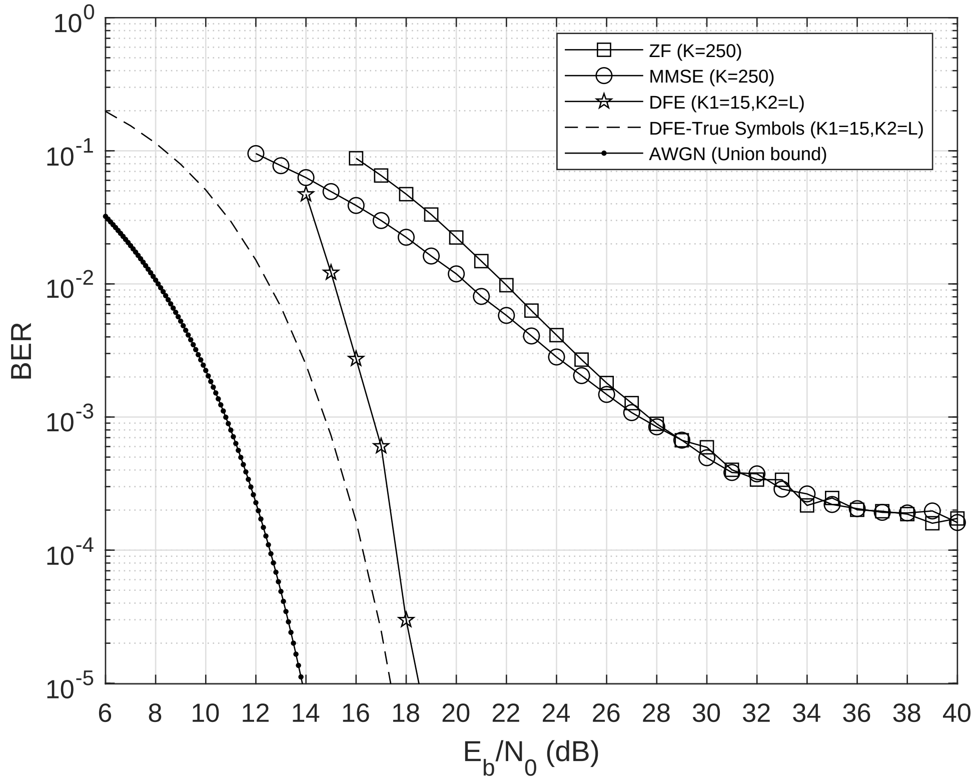

Figure 11: Simulation result for the filter-based equlaizers with FOM: Channel 8

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