

1 **Importance of variability in initial soil moisture and rainfalls on slope stability**

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26 27 **ABSTRACT**

28
29 A first-order moment analysis is developed to investigate the temporal and spatial
30 propagation of uncertainty of slope stability during rainfall, considering spatial variabilities in
31 initial soil water pressure and soil hydraulic properties, and temporal variability of rainfall.
32 Results of the analysis indicate that the uncertainties resulting from variabilities in initial soil
33 pore water pressure distributions and rainfalls are comparable with that from the variability in
34 soil hydraulic properties. Further, the evolution of slope stability uncertainty is driven by the
35 mean flow field, and a localized large-uncertainty zone along the slope profile could form,
36 leading to a localized low-reliability zone, which may lead to the failure of the slope. In
37 particular, when the slope is close to saturation, the reliability of the stability analysis of any
38 elevation of the slope is low even at early rainfall times. On the other hand, when the slope is
39 unsaturated and heavy rainfalls occur, the low-reliability zone exists at shallow parts of the slope
40 at early times. The results also show that greater unreliability exists at shallow depths at early
41 times when the rainfall has a descending trend in comparison with uniform and increasing trend.
42 Lastly, the low-reliability zone is always near the impermeable bedrock if rainfall persists.

43 44 **Keywords:**

45 Slope stability uncertainty; Variability; Initial soil pore water pressure; Rainfall characteristics;
46 Large-uncertainty zone; Low-reliability zone

47

48 1 INTRODUCTION

49 Rainfall-induced landslides are one of the most severe natural disasters(Ng and Shi, 1998;
50 Ng et al., 2001). Significant landslide disasters triggered by rainfalls have been reported annually
51 in many parts of the world such as Brazil, Italy, South Africa, Japan and China(Chowdhury and
52 Flentje, 2002). Infiltration of precipitation increases soil moisture content, decreases matric
53 suction of unsaturated soils or creates positive pore water pressure, and in turn, decreases the
54 shear strength of soils, and consequently, leads to landslides. Preventions for geohazards such as
55 slope failures and landslides, therefore, require reliable evaluations of slope stability under
56 rainfall infiltrations.

57 Rainfall-induced landslides are attributed to geologic characteristics, topography, initial
58 soil pore water pressure, and precipitation of the slope area. Due to the heterogeneous nature of
59 the geology and our inability to characterize them, an increasing number of researches has
60 focused on the variabilities of soil properties and their effects on the slope stability (e.g., Ali et
61 al., 2014; Cai et al., 2017c, 2017d, 2017a; Cho, 2014; Griffiths et al., 2011; Gui et al., 2000). On
62 the other hand, precipitation and initial soil pore water pressure generally exhibit a high degree
63 of temporal and spatial variabilities. For instance, it has been widely recognized that rainfall
64 intensity is significantly higher on the escarpment relative to the coastal plain(Chowdhury and
65 Flentje, 2002). In addition, rainfall processes are known to vary widely in time. That is,
66 spatiotemporal variability is an intrinsic characteristic of climate(Paolini et al., 2005).
67 Furthermore, the distribution of initial soil pore water pressures in the slope also varies spatially
68 due to variabilities in prior rainfalls and heterogeneities of geology.

69 Previous studies have focused on the variabilities in rainfalls or initial soil pore water
70 pressure distributions. For example, D’Odorico et al.(2005) showed that the temporal variations

71 of rainfall intensity affect stability, and the rainfall pattern with a peak at the end of the rainfall
72 process has a stronger destabilizing effect than a constant rainfall. Minder et al.(2009) studied the
73 impact of spatial rainfall heterogeneity on landslide susceptibility based on a numerical model
74 and pointed out that the heterogeneous rainfall decreased slope stability. Ruetter et al.(2014)
75 investigated the effects of rainfall variabilities and initial soil hydraulic conditions on the
76 statistics of locations, the numbers, and the released volumes of landslides at a catchment. They
77 suggested that fewer landslides were triggered under dry initial soil conditions than the wet
78 conditions, and rainfall heterogeneity may be an important missing link required for landslide
79 prediction.

80 Since it is practically impossible to characterize the variabilities in initial soil pore water
81 pressure and rainfalls in detail, uncertainties in the evaluation of slope stability exist. For this
82 reason, these variabilities deserve particular attention. The understanding of these variabilities
83 and their effects on slope stability is still relatively limited at present. For instance, in most
84 studies, rainfall data is obtained via spatially and temporally averaging approaches or
85 interpolation using sparse rain gauge measurements or coarse rainfall radar information over
86 large areas(von Ruetter et al., 2014). The initial soil pore water pressure is frequently assumed to
87 be spatially invariant (e.g., Cho, 2014) or to be those obtained by steady-state simulations under
88 antecedent rainfalls(e.g., Cai et al., 2017b). In these approaches, the initial soil pore water
89 pressure and rainfall are considered as deterministic, and their uncertainties are ignored. These
90 simplifications lead to inaccurate predictions of landslides, which hamper mitigation procedures.

91 During this rainfall-infiltration process in a slope, the effect of spatial variability in initial
92 soil pore water pressure and soil properties, and temporal varying rainfall on the slope stability
93 evolves with time and space. As a result, it becomes important to know where and when the

94 potential slip surfaces have relatively large uncertainties of their stabilities and whether these
95 uncertainties are significant enough to impact the slope stability. These issues are seldom
96 investigated. Therefore, we are compelled to conduct a probabilistic analysis of the interaction
97 between temporal and spatial distributions of initial soil pore water pressures, rainfalls, soil
98 properties, and slope stability.

99 The primary objective of this study is to propose a stochastic approach for a better
100 understanding and prediction of the temporal and spatial evolution of slope stability uncertainty
101 caused by variabilities in initial soil pore water pressure distributions, rainfalls, and soil
102 hydraulic properties and its importance in slope stability evaluations during rainfall.

103 This paper is organized as follows. The basic equations for describing the one-
104 dimensional vertical seepage and evaluation of infinite slope stability and deterministic analysis
105 of slope stability are presented in section 2. Section 3 first presents the first-order moment
106 approach and the approach to quantitatively represent the spatial variability in initial hydraulic
107 conditions and soil hydraulic properties and the temporal variability of rainfalls during the
108 rainfall process. Then, it describes a probabilistic analysis of slope stability. Simulation results
109 for different scenarios are discussed in section 4 with respect to the effects of the initial soil pore
110 water pressure, rainfall intensity, rainfall duration and rainfall pattern on uncertainty
111 propagations of slope stability and the importance of these uncertainties on slope stability. The
112 article then draws concludes in section 5.

113

114 **2 DETERMINISTIC SLOPE STABILITY ANALYSIS UNDER RAINFALL**

115 *2.1 Governing equations for seepage analysis*

116 The rainfall infiltration process in the infinite slope ([Fig. 1](#)) is assumed to be described by
117 a one-dimensional governing vertical flow equation([Yeh et al., 2015](#)):

$$118 \quad \frac{\partial}{\partial z} \left(K(h) \left(\frac{\partial h}{\partial z} + 1 \right) \right) = [\eta S_s + C(h)] \frac{\partial h}{\partial t} \quad (1)$$

119 where z denotes the coordinate along the vertical z -axis (positive upward); h is the pressure
 120 head; $K(h)$ is the hydraulic conductivity; S_s is the specific storage; $C(h)$ denotes the
 121 moisture capacity term; t denotes time; η is the saturation index. h is a positive value if the
 122 medium is fully saturated and is negative if the medium is unsaturated. $K(h)$ varies with
 123 pressure head under unsaturated conditions. S_s represents the percentage of water released
 124 from a unit volume of fully saturated porous media under a unit decline in hydraulic head. On the
 125 other hand, $C(h) = \left. \frac{\partial \theta}{\partial h} \right|_h$ is the change in moisture content in a unit volume of the porous
 126 medium under a unit change of negative pressure head, when the medium is unsaturated. While
 127 S_s and $C(h)$ are similar in definition, the physical mechanisms they represented are entirely
 128 different. S_s is related to the compressibility of porous media and water while the medium
 129 remains fully saturated, whereas $C(h)$ represents desaturation or saturation of the pores in the
 130 medium. On the right-hand side of Eq. (1), η is set to 1 if the medium is saturated and 0 if the
 131 medium is unsaturated.

132 Eq. (1) is subjected to an initial condition:

$$133 \quad h(z, 0) = h_0(z) \quad (2a)$$

134 where h_0 is the prescribed pressure head at the location z at the initial time.

135 The boundary conditions for Eq. (1) are: at the land surface, a prescribed pressure head

136 h_b at the time t is assigned to the top boundary to represent rainfall for the infinite slope:

$$137 \quad h(H, t) = h_b(t) \quad (2b)$$

138 while at slope base, no flux boundary is utilized:

$$139 \quad K(h) \left(\frac{\partial h}{\partial z} + 1 \right) \Big|_{(0,t)} = 0 \quad (2c)$$

140 To simulate flow in a hillslope using the governing equation and its initial and boundary

141 conditions, the hydraulic conductivity-pressure constitutive relationship by Mualem(1976) and

142 the moisture-pressure head constitutive relationship by van Genuchten(1980), known as the

143 MVG model, are adopted. They are listed below:

$$144 \quad K(h) = \begin{cases} K_s \left(1 - (\alpha_1 |h|)^{\alpha_2 - 1} \left[1 + (\alpha_1 |h|)^{\alpha_2} \right]^{-\alpha_3} \right)^2 \left[1 + (\alpha_1 |h|)^{\alpha_2} \right]^{-\alpha_3/2} & h < 0 \\ K_s & h \geq 0 \end{cases} \quad (3)$$

$$145 \quad \theta(h) = \begin{cases} (\theta_s - \theta_r) \left[1 + (\alpha_1 |h|)^{\alpha_2} \right]^{-\alpha_3} + \theta_r & h < 0 \\ \theta_s & h \geq 0 \end{cases} \quad (4)$$

146 where α_1 , α_2 and α_3 are soil parameters and $\alpha_3 = 1 - 1/\alpha_2$; K_s is the saturated hydraulic

147 conductivity; θ_s and θ_r denote the saturated and residual volumetric moisture contents,

148 respectively.

149 **2.2 Governing equations for slope stability analysis**

150 The factor of safety along i th potential slip surface (i.e., FS_i) of an infinite slope has
 151 been widely evaluated using the limit equilibrium model (LEM) with the unified effective stress
 152 under both saturated and unsaturated conditions(Lu and Godt, 2008). If we let the pore air
 153 pressure u_a be atmospheric pressure(i.e., $u_a = 0$), FS_i can be expressed as (e.g., Ali et al.,
 154 2014; Cho, 2014; Griffiths et al., 2011; Li et al., 2014):

$$\begin{aligned}
 FS_i &= \frac{((H - z_i)\gamma_i \cos^2 \beta - \sigma_i^s) \tan \phi'_i + c'_i}{(H - z_i)\gamma_i \sin \beta \cos \beta} \\
 &= \left(\frac{1}{\tan \beta} + \frac{-\sigma_i^s}{(H - z_i)\gamma_i \sin \beta \cos \beta} \right) \tan \phi'_i + \frac{c'_i}{(H - z_i)\gamma_i \sin \beta \cos \beta} \\
 &(0 \leq z_i < H, i = 1, \dots, n) \tag{5}
 \end{aligned}$$

157 where n is the total number of potential slip surfaces; β is the slope inclination; γ_i is the
 158 averaged total unit weight above i th potential slip surface; H denotes the vertical distance of
 159 soils from the slope base to the land surface; c'_i and ϕ'_i are the effective cohesion and the
 160 effective soil friction angle at i th potential slip surface, and z_i is the elevation (positive upward)
 161 of i th potential slip surface (see Fig. 1); σ_i^s represents the effective negative pore water pressure
 162 under unsaturated conditions or effective positive pore water pressure when under saturated
 163 conditions at i th potential slip surface(Lu and Godt, 2008).

164 According to Lu and Godt(2008), σ_i^s can be expressed as:

$$\sigma_i^s = -\frac{\theta_i - \theta_r}{\theta_s - \theta_r} (u_a - u_{wi}) = -S_{ei} (u_a - u_{wi}) \tag{6}$$

166 where u_{wi} , S_{ei} and θ_i are the pore water pressure, the effective water saturation and the
 167 volumetric moisture content at i th potential slip surface, respectively. The relationship between
 168 u_w and h is $u_w = h\gamma_w$. $\sigma_i^s = S_{ei}u_{wi} < 0$ is for unsaturated conditions ($u_{wi} < 0$), and $\sigma_i^s = u_{wi} \geq 0$ is
 169 for saturated conditions ($u_{wi} \geq 0$). Via this unified effective stress theory, Eq. (5) can account for
 170 both the reduction in matric suction and the development of positive pore water pressure in a
 171 continuous form(Cho, 2014; Lu and Godt, 2008).

172 In this study, the variation in unit weight resulting from changes in moisture content
 173 during infiltration is evaluated by integration of the moisture content profile above the potential
 174 slip surface. That is, the total unit weight γ_i can be expressed as follows:

$$175 \quad \gamma_i = \frac{1}{H - z_i} \int_{z_i}^H (\gamma_d + \theta(z)\gamma_w) dz \quad (0 \leq z_i < H) \quad (7)$$

176 where γ_d is the dry unit weight of the soil; γ_w is the unit weight of water.

177 **2.3 Setup of model**

178 Catastrophic shallow landslides frequently occur under heavy rainfall in weathered
 179 granite residual soil slopes in Korea and the southeast coast of China. The method developed
 180 here, therefore, is applied to the weathered granite residual soil slopes as an example. Generally,
 181 the hillslope in these areas can be treated as an unsaturated shallow layered slope above an
 182 impermeable bedrock, and the one-dimensional infinite slope model is appropriate for the
 183 simulation (e.g., Ali et al., 2014; Cai et al., 2017d, 2017a, 2017c; Cho, 2014; Griffiths et al.,
 184 2011; Li et al., 2014; Zhang et al., 2014). The geometrical parameters and all the hydraulic and
 185 mechanical parameters for this investigation are listed in Table 1. They are adopted from the data

186 listed in Cho (2014), which are representative of typical weathered granite soils in Seochang,
 187 Korea.

188 The entire infinite slope is discretized into 40 elements in the vertical direction,
 189 representing 40 potential slip surfaces (i.e., $n=40$) with an interval (Δz) of 0.05 m. These
 190 potential slip surfaces are numbered 1 to 40 from the slope base to the land surface (see Fig. 1).
 191 In addition, the initial pressure head distribution in the slope is -2 m through the depth with the
 192 impermeable boundary located at the slope base. As rainfall occurs, a specific pressure head, h_b ,
 193 which is a function of t , is assigned to the top boundary to represent a variable rainfall.

194 A finite element analysis code (available at <http://tian.hwr.arizona.edu/downloads>, Yeh et
 195 al., 1993) is employed to simulate the one-dimensional vertical seepage described by Eq. (1),
 196 subjected to the initial and boundary conditions described by Eq. (2). Subsequently, computed
 197 vertical profiles of pore water pressure, effective water saturation and moisture content from
 198 transient finite element seepage analyses are used as inputs for calculating FS_i ($i=1, \dots, n$) at
 199 each time step by Eq. (5).

200 **2.4 Deterministic analysis**

201 First, a deterministic transient seepage analysis is conducted, by using the mean values of
 202 parameters (i.e., μ_{h_0} , μ_{h_b} and μ_{K_s}) listed in Table 1, to study rainfall infiltration into the
 203 infinite slope with the impermeable boundary located at the slope base. The vertical profiles of
 204 pore water pressure, h , and the corresponding factor of safety FS_i at six different times (0.01,
 205 0.1, 0.25, 0.5, 0.75, 1 day) due to a given h_b (-0.1 m) are displayed in Fig. 2a and 2b, respectively.
 206 As shown in Fig. 2a, the wetting front propagates as rainfall infiltration continues until it reaches

207 the slope base. The matric suction of the soil behind the wetting front gradually reduces over
208 time, and the soil becomes fully saturated and soil water pressure becomes positive and increases
209 afterward. The effect of infiltration on slope stability is illustrated in Fig. 2b, which shows that
210 the advance of the wetting front decreases the factor of safety at each potential slip surface. At 1
211 day, the factor of safety of this slope decreases below 1 (i.e., unstable slope).

212 The above analysis based on the mean values of parameters provides a reference to
213 illustrate the effects of variability of h_0 , h_b and K_s in a heterogeneous slope. First, realizations
214 of random fields of h_0 , h_b and K_s (Figs. 2c, 2d) are generated using the Karhunen-Loève (K-L)
215 expansion method (e.g., Ghanem and Spanos, 1991; Jiang et al., 2015; Lu and Zhang, 2007) with
216 the given spatial statistics listed in Table 1. h_0 and h_b are assumed to be normally distributed
217 while K_s is assumed to be log-normally distributed (e.g., Brejda et al., 2000; Fenton and
218 Griffiths, 2008; Griffiths et al., 2011; Jiang et al., 2015; Li et al., 2014; Parkin et al., 1988;
219 Parkin and Robinson, 1992; Phoon and Kulhawy, 1999). Note that various types of models, e.g.,
220 the multiplicative random cascade models (Gupta and Waymire, 1993; Menabde and Sivapalan,
221 2000), have been proposed in the literature for modeling rainfall time series. However, in this
222 study, we treat h_b time series as a stochastic temporal process to illustrate the effect of
223 variability in h_b . The generated realization (see Fig. 2d) is sufficient to capture observed
224 features of measured rainfall data reported in Stern and Coe (1984) and Menabde and
225 Sivapalan (2000) such as intermittency and correlation. In addition, considering the fact that the
226 distributions of h_0 , K_s and h_b are generally unknown, in the study, h_0 is randomly generated
227 and assumed to be uncorrelated with K_s and h_b at previous time periods (i.e., rainfall history).

228 If these factors are known, correlation between these parameters could reduce the uncertainty of
229 h_0 . For this reason, this assumption is deemed realistic.

230 Subsequently, the flow fields are simulated, stability analyses are conducted, and the
231 results are displayed in Figs. 2e and 2f. According to the figures, the simulated h and FS_i
232 profiles dramatically differ from those based on mean values of parameters (Figs. 2a and 2b),
233 which represent the most likely h and FS_i profiles at different times. For example, the shape of
234 h and FS_i profiles in Figs. 2e and 2f are erratic at 0.01 day due to the variability of h_0 . As the
235 infiltration continues, their deviations from the most likely profiles at the corresponding times
236 become larger. Such large differences thus may pose a threat to the stability of the slope (e.g., in
237 Fig. 2f, the FS_i 's around Elevation 1.3m are smaller than 1). For this reason, a slope stability
238 analysis should consider uncertainties in the variability of antecedent moisture content, in the
239 temporal variability of rainfalls and in the spatial variability of soil properties (e.g., K_s).

240

241 3 PROBABILISTIC SLOPE STABILITY ANALYSIS

242 In order to address the uncertainty issue as presented above, a first-order moment
243 approach (Cai et al., 2017d, 2016) is developed and described below.

244 3.1 First-order estimation of slope stability uncertainty

245 The initial pressure head, h_0 , which is directly related to soil moisture content, varies
246 spatially and its distribution is unknown. We can treat it as a spatial random field. This means the
247 initial pressure head $h_0(z_i)$ at each location in the slope is a random variable, which has a mean

248 value and a variance. The collection of these random variables (i.e., $h_0(z_i)$, $i=1, \dots, n$) forms the
 249 random field and is characterized by a joint probability distribution (see [Yeh et al., 2015](#)).
 250 Therefore, $h_0(z_i)$ is expressed in terms of the mean and perturbations:

$$251 \quad h_0(z_i) = \mu_{h_0} + p_{h_0}(z_i) \quad (i = 1, \dots, n) \quad (8a)$$

252 where μ_{h_0} is the mean of h_0 and $p_{h_0}(z_i)$ denotes the perturbation of h_0 at the location z_i .

253 The pressure head h_b at the land surface, which is related to rainfall, changes with time
 254 t and its distribution is also unknown. Thus, h_b at each time t_l is considered as a random
 255 variable, denoted as $h_b(t_l)$, which has a mean value and a variance, representing the uncertainty
 256 due to the temporal variability as well as lack of measurements. Consequently, h_b can be
 257 expressed as:

$$258 \quad h_b(t_l) = \mu_{h_b}(t_l) + p_{h_b}(t_l) \quad (l = 1, \dots, m) \quad (8b)$$

259 where $\mu_{h_b}(t_l)$ is the time-varying mean of h_b , $p_{h_b}(t_l)$ denotes the perturbation of h_b at the
 260 time t_l and m denotes the total number of discrete time.

261 In addition, the medium property K_s of the slope is also considered as a spatial random
 262 field, which can be expressed in the form of the mean and perturbations:

$$263 \quad K_s(z_i) = \mu_{K_s} + p_{K_s}(z_i) \quad (i = 1, \dots, n) \quad (8c)$$

264 where μ_{K_s} is the mean of K_s and $p_{K_s}(z_i)$ denotes the perturbation of K_s at location, z_i .

265 Likewise, the uncertainty in the FS_i is represented by $FS_i = \mu_{FS_i} + p_{FS_i}$, where μ_{FS_i} is
 266 the mean and p_{FS_i} is the perturbation. Expanding the FS_i in Eq. (5) in a Taylor series about the
 267 mean values of parameters, neglecting second-order and higher order terms, and subtracting the
 268 mean part from both sides, the FS_i at i th potential slip surface at a given time t can be
 269 approximately expressed in a matrix form as:

$$270 \quad \mathbf{p}_{FS}(t) = \mathbf{J}_{FS h_0}(t) \mathbf{p}_{h_0} + \mathbf{J}_{FS h_b}(t) \mathbf{p}_{h_b} + \mathbf{J}_{FS K_s}(t) \mathbf{p}_{K_s} \quad (9)$$

271 where \mathbf{p}_{FS} , \mathbf{p}_{h_0} and \mathbf{p}_{K_s} are $n \times 1$ vectors; \mathbf{p}_{h_b} is a $m \times 1$ vector. $\mathbf{J}_{FS h_0}$ and $\mathbf{J}_{FS K_s}$ is a $n \times n$
 272 Jacobian matrix, representing the change in FS_i ($i = 1, \dots, n$) at i th potential slip surface at the
 273 time t due to a unit change in h_0 and K_s , respectively, at the location z_j ($j = 1, \dots, n$) in the
 274 domain; $\mathbf{J}_{FS h_b}$ is a $n \times m$ Jacobian matrix, each term in the matrix denotes the sensitivity of
 275 $FS(z_i, t)$ ($i = 1, \dots, n$) to the change of h_b at the time t_l ($l = 1, \dots, m$). Due to the
 276 nonlinearity, these sensitivities of the FS_i with respect to h_0 , h_b or K_s need to be evaluated
 277 numerically. In this study, the perturbation approach is employed. We refer to Cai et al.(2017d)
 278 for derivations of this sensitivity analysis approach.

279 In this study, we assume that the spatially varying initial pressure head, h_0 , the time-
 280 varying boundary condition, h_b and the medium property K_s are mutually independent of each
 281 other. Therefore, multiplying Eq. (9) by itself on both sides and taking the expected value of the
 282 product lead to the corresponding FS_i auto-covariance matrix \mathbf{R}_{FSFS} :

$$283 \quad \mathbf{R}_{FSFS}(t) = \mathbf{J}_{FS h_0}(t) \mathbf{R}_{h_0 h_0} \mathbf{J}_{FS h_0}^T(t) + \mathbf{J}_{FS h_b}(t) \mathbf{R}_{h_b h_b} \mathbf{J}_{FS h_b}^T(t) + \mathbf{J}_{FS K_s}(t) \mathbf{R}_{K_s K_s} \mathbf{J}_{FS K_s}^T(t) \quad (10)$$

284 where the superscript T denotes the transpose. $\mathbf{R}_{h_0h_0}$ and $\mathbf{R}_{K_sK_s}$ are $n \times n$ auto-covariance
 285 matrices for h_0 and K_s , respectively; $\mathbf{R}_{h_bh_b}$ is an $m \times m$ auto-covariance matrix for h_b . $\mathbf{R}_{h_0h_0}$
 286 and $\mathbf{R}_{K_sK_s}$ are modeled using an exponential function (e.g., [Cai et al., 2016](#)) with the spatial
 287 correlation scale $\lambda_s = \lambda_{h_0} = \lambda_{K_s}$ in the z direction. Spatial correlation scale represents the distance
 288 within which the parameters (i.e., $h_0(z_i)$ or $K_s(z_i)$) are correlated in space. $\mathbf{R}_{h_bh_b}$ is modeled
 289 using the exponential function with a temporal correlation scale $\lambda_t = \lambda_{h_b}$. This temporal
 290 correlation scale represents the time interval within which the h_b at different times are
 291 correlated in time. The diagonal components of \mathbf{R}_{FSFS} are the FS_i variances ($i=1, \dots, n$) at the
 292 time t , which are denoted as $\sigma_{FS}^2(z_i, t)$ ($i=1, \dots, n$). The $\sigma_{FS}^2(z_i, t)$ represents the uncertainty in
 293 FS_i at i th potential slip surface at the time t , due to variabilities in h_0 at z_j ($j=1, \dots, n$), h_b at
 294 t_l ($l=1, \dots, m$) and K_s at z_j ($j=1, \dots, n$).

295 **3.2 Reliability of slope stability evaluation**

296 In the probabilistic slope stability analysis, we use the reliability index (e.g., [Christian et](#)
 297 [al., 1994; Li et al., 2014](#)) to represent the reliability of slope stability evaluation. The reliability
 298 index is defined as:

$$299 \quad \beta_i(t) = (\mu_{FS_i}(t) - 1) / \sigma_{FS_i}(t) \quad (11)$$

300 where μ_{FS_i} and σ_{FS_i} are the mean and standard deviation of FS_i at the i^{th} potential slip surface at
 301 the time t , respectively. The reliability index describes the reliability of stability evaluations at
 302 certain parts at the time t in the slope. It is defined as the ratio of the deviation of the mean FS

303 from the limit equilibrium state value of 1.0 at a location i to the FS standard deviation at the
 304 location. According to Eq. (11), if the β_i at the i^{th} surface is greater than 1, the standard deviation
 305 of the FS_i is small compared with the deviation between the mean FS_i and the critical
 306 equilibrium value 1. Therefore, the reliability of the slope is large. In other words, the
 307 uncertainty in slope stability due to variabilities in h_0 , h_b and K_s is negligible and the μ_{FS_i}
 308 value, thus, is adequate to represent the slope stability. On the contrary, if the β_i is smaller than
 309 1 at the time t , the distance between μ_{FS_i} and 1 is smaller than σ_{FS_i} . Hence, the uncertainty in
 310 slope stability at the time t is significant. This uncertainty is very likely to threaten the reliability
 311 of slope stability evaluations and deserves special attention.

312 The μ_{FS_i} ($i=1, \dots, n$) at each time step is evaluated using mean values of parameters and
 313 the σ_{FS_i} ($i=1, \dots, n$) at each time step is estimated using the first order moment approach
 314 abovementioned. Afterward, β_i ($i=1, \dots, n$) at each time step during rainfall is calculated by Eq.
 315 (11).

316 **3.3 Probabilistic analysis for the base set**

317 Here, a case with $\mu_{h_0} = -2$ m, $\mu_{h_b} = -0.1$ m, $\mu_{K_s} = 0.2592$ m/d, $\text{COV}_{h_0} = \text{COV}_{h_b} =$
 318 $\text{COV}_{K_s} = 1.0$, $\lambda_{h_0} = 0.3$ m, $\lambda_{h_b} = 0.3$ day and $\lambda_{K_s} = 0.3$ m is considered as the base case. Other
 319 parameters are listed in **Table 1**, and the rainfall duration is 1 day.

320 The spatial distributions of σ_{FS_i} and β_i at every potential slip surface of the slope under
 321 the variability of all parameters (h_0 , h_b and K_s) at four different times (0.01, 0.25, 0.5, 1 day)

322 are depicted in Fig. 3a and Fig. 3b. The contribution to σ_{FS_i} from the variability of each
 323 parameter is presented in a normalized form in Figs. 3c, 3d, and 3e. As illustrated in Figs. 3a, 3c,
 324 3d, and 3e, it is evident that the propagations of slope stability uncertainty due to impacts of
 325 variabilities in h_0 , h_b and K_s are driven by the mean flow field (see Fig. 2a,b). Since the mean
 326 value of rainfall intensity is larger than the value of initial pore water pressure and gravity,
 327 rainfall infiltrates into the slope and the uncertainties propagate downwards. At an early stage of
 328 infiltration (e.g., $\tau = 0.25$ day), the values of σ_{FS_i} 's at shallow depths are larger than those at
 329 deep parts. Further, the variations of h_0 and K_s contribute to σ_{FS_i} more than the variation of
 330 h_b does. Moreover, the variations of h_0 is the major uncertainty sources to σ_{FS_i} 's at deep parts
 331 of the slope. The variations of h_b and K_s only contribute to σ_{FS_i} at shallow depths.

332 At the late stage of infiltration (e.g., $\tau = 1$ day), the values of σ_{FS_i} at shallow depths
 333 decline and become smaller than those at deep parts. Meanwhile, the contribution of the variation
 334 of h_0 to σ_{FS_i} 's diminishes, while the contributions from the variations of h_b and K_s increase
 335 and become larger than that of h_0 . These results manifest the influence of the variability in
 336 initial hydraulic condition at early times but its influence gradually attenuates as time progresses.
 337 On the other hand, the impact of the variability of rainfall continues to grow as the rainfall
 338 continues. Notice that although the contribution to σ_{FS_i} from variation of K_s is generally
 339 largest among these three uncertainty sources, the impacts on σ_{FS_i} from variations of h_0 and
 340 h_b are comparable with those from the variation of K_s and cannot be ignored. This finding

341 illuminates the importance of characterization of the variability in initial hydraulic conditions
 342 and rainfalls in slope stability evaluation.

343 Notice that a localized large-uncertainty zone (i.e., the one along the solid purple line
 344 with gradient symbols at elevation 1.2m in Fig. 3a) moves from shallow parts to deep parts of the
 345 slope and accumulates above the impermeable bedrock. This large-uncertainty zone greatly
 346 impacts the reliability of slope stability evaluations and deserves special attention. Fig. 3b
 347 indicates the rainfall infiltration decreases the β_i of each potential slip surface. For example, the
 348 minimum value of β_i 's is smaller than 1 at 0.25 day around elevation 1.2m at shallow parts of
 349 the slope, and it decreases as infiltration continues. Overall, localized low-reliability zones form
 350 and move downwards during the rainfall infiltration process.

351

352 **4 INFLUENCES OF MEAN VALUES OF h_0 AND h_b**

353 *4.1 Effects of initial pore water pressure*

354 To investigate the effects of mean initial pore water pressure on slope reliability, three
 355 cases with μ_{h_0} equals -1, -2 and -3 m, respectively, are considered. The COV_{h_0} 's of the three
 356 cases are set to the same, 1.0, and the COV_{h_b} , COV_{K_s} , λ_{h_0} , λ_{h_b} and λ_{K_s} of these cases are
 357 1.0, 1.0, 0.3 m 0.3 day and 0.3 m, respectively. μ_{h_b} is set as -0.1 m, μ_{K_s} is 0.2592 m/d and the
 358 rainfall duration is 1 day.

359 As shown in Fig. 4a, where the μ_{FS_i} profiles at $t=0.25$ day for the three cases are
 360 plotted, when the negative value of mean initial pore water pressure μ_{h_0} approaches zero, the

361 mean values of FS_i 's of every potential slip surface decrease significantly and become closer to
362 the limit equilibrium state (i.e., 1).

363 The σ_{FS_i} profiles at $t=0.25$ day for the three cases due to variations of all parameters
364 (h_0 , h_b and K_s) are displayed in Fig. 4b. The relative contribution from each parameter to
365 σ_{FS_i} is shown in Fig. A.1. Figs. 4b and A.1 indicate that μ_{h_0} significantly influences the
366 distributions of slope stability uncertainty induced by variabilities in initial hydraulic conditions,
367 rainfalls, and soil hydraulic properties. That is, as μ_{h_0} increases (less negative), the localized
368 large-uncertainty zone propagates rapidly to the deep parts of the slope under the same rainfall
369 intensity owing to the high hydraulic conductivity resulting from high μ_{h_0} . In addition, the
370 maximum value of the uncertainty in the zone becomes smaller as μ_{h_0} increases. This result is
371 consistent with the finding that the variance in pressure head generally decreases as soils become
372 close to saturation, as explained by the moisture-dependent variability theory reported by Yeh et
373 al.(1985a, 1985b, 1985c).

374 The reliability index β_i profiles at $t=0.25$ day for the three cases are shown in Fig. 4c.
375 Apparently, under the fixed rainfall intensity and time, the less saturation the slope is, the greater
376 uncertainties of stabilities are near the shallow parts of slopes, and the minimum value of the
377 localized reliability zone becomes smaller. All these factors lead to the high probability of failure
378 at shallow parts of the less saturated slopes.

379 On the other hand, albeit the influence of variations in initial hydraulic conditions,
380 rainfalls and soil hydraulic properties becomes smaller as the slope is close to saturation, the
381 mean values of FS_i 's become small and the propagation of the impact of variations is speeded

382 up. For these reasons, the likelihood of failure of the potential slip surfaces at any elevation
 383 increases. One typical realization of this situation is illustrated in Fig. 4d. This realization is
 384 generated with abovementioned statistics except μ_{h_0} is set as -1m.

385 **4.2 Effects of rainfall intensity**

386 Here, the effects of the rainfall intensity on slope reliability are studied through three
 387 cases with μ_{h_b} equals 0.1, -0.1 and -1 m, respectively. COV_{h_0} , COV_{h_b} , COV_{K_s} , λ_{h_0} , λ_{h_b}
 388 and λ_{K_s} of these cases are 1.0, 1.0, 1.0, 0.3 m, 0.3 days and 0.3 m, respectively. The mean initial
 389 pore water pressure μ_{h_0} is set to be -2 m, μ_{K_s} is 0.2592 m/d and the rainfall duration is 1 day.

390 The μ_{FS_i} profiles at $\tau = 0.25$ day for the three cases are illustrated in Fig. 5a. As expected,
 391 the increase of the mean rainfall intensity μ_{h_b} leads to decreases in μ_{FS_i} 's toward 1. Fig. 5b
 392 shows the σ_{FS_i} profiles at $\tau = 0.25$ day for the three cases due to variations of all parameters (h_0 ,
 393 h_b and K_s) while the relative contribution from each parameter to σ_{FS_i} is shown in Fig. A.2.
 394 As illustrated in these figures, μ_{h_b} has significant impacts on the distributions of slope stability
 395 uncertainty. The increase in μ_{h_b} leads to increasing trends in the propagation depth of σ_{FS_i} 's
 396 within the same time. In addition, given the same mean initial pore water pressure μ_{h_0} , larger
 397 μ_{h_b} leads to a larger maximum value of the localized large-uncertainty zone. Moreover, when the
 398 mean rainfall intensity μ_{h_b} becomes smaller and closer to μ_{h_0} , the localized large-uncertainty
 399 zone gradually vanishes. These findings indicate that a small rainfall intensity given limited
 400 times may fail to bring uncertainties into the slope. Under such circumstance, the σ_{FS_i} 's in the

401 slope are mainly due to the variation of h_0 . Fig. 5c depicts the reliability index β_i profiles at t
 402 $=0.25$ day for the three cases. As illustrated in Fig. 5c, given the same μ_{h_0} , the localized low-
 403 reliability zone gradually forms and the minimum value of the localized low-reliability zone
 404 becomes smaller as μ_{h_b} increases. This indicates that small rainfall intensities with short
 405 duration pose no threat to slope stability. On the other hand, under heavy rainfall conditions, the
 406 localized large-uncertainty zone first takes place at shallow parts of slopes: it impacts the
 407 reliability of slope stability evaluation and should be carefully treated. One typical realization of
 408 this situation is depicted in Fig. 5d. This realization is generated with abovementioned statistics
 409 except μ_{h_b} is set as 0.1 m.

410 **4.3 Effects of rainfall duration**

411 In this section, we investigate the effects of rainfall duration on slope reliability. Four
 412 cases are examined, which include two types of rainfall (namely, short-duration, intense rainfall
 413 (SDIR) and long-duration, mild rainfall (LDMR)) on the slope under two initial hydraulic
 414 conditions. The SDIR is defined as the rainfall with a rainfall duration $t_{rainfall}=0.2$ day and μ_{h_b}
 415 $=-0.1$ m, while the LDMR is rainfall with $t_{rainfall}=2$ days and $\mu_{h_b}=-1$ m. Two mean initial pore
 416 water pressure μ_{h_0} values (i.e., -2 m and -3 m, respectively) are considered. COV_{h_0} , COV_{h_b} ,
 417 COV_{K_s} , λ_{h_0} , λ_{h_b} and λ_{K_s} of these cases are 1.0, 1.0, 1.0, 0.3 m, 0.3 days and 0.3 m,
 418 respectively. μ_{K_s} is 0.2592 m/d and the rainfall duration is 1 day.

419 The resultant reliability index β_i profiles for the four cases at $t=0.2$ day, $t=1$ day and
 420 $t=2$ days, are displayed in Figs. 6a, 6b, and 6c, respectively. The time periods, $t=0.2$ day and
 421 $t=2$ days, represent the time when the rain stops for SDIR and LDMR, respectively.

422 At 0.2 day, the rain stops in the case of SDIR while the rain continues in the case of
 423 LDMR. The resultant reliability profiles are plotted in Fig. 6a. The solid green lines with delta
 424 triangles and solid black lines with circles are for the LDMR cases with initial mean pressure
 425 heads of -2m and -3m, respectively. The reliabilities of the slope under LDMR are large. In spite
 426 of the initial mean pressure heads, SDIR leads to localized low-reliability zone at shallow depths
 427 of the slope as illustrated by the solid red lines with rectangles and solid blue lines with gradient
 428 triangles in the figure. Formation of this zone is expected since an intense rainfall leads to a
 429 decrease of the mean values of FS_i 's and large variations in slope stability (σ_{FS_i}) within short
 430 times (see Fig. 5).

431 At $t=1$ day in Fig. 6b and $t=2$ days in Fig. 6c, the reliabilities still maintain relatively
 432 high at shallow depths under LDMR, and the reliabilities at shallow parts of the slope under
 433 SDIR rises due to the cease of rainfall and it becomes larger than those under LDMR. In addition,
 434 both SDIR and LDMR lead to a localized low-reliability zone at deep parts of the slope due to
 435 greater saturation. As a result, attention should be given to the uncertainties accumulated above
 436 the impermeable bedrock, where become critical to the slope stability evaluation, even if it is
 437 long after the heavy rainfall stops or under a mild rainfall. We show one typical realization of
 438 this situation in Fig. 6d, which is generated with abovementioned statistics under LDMR.

439 **4.4 Effects of rainfall pattern**

440 As reported by previous studies(e.g., Ng et al., 2001; Zhang et al., 2014), the rainfall
 441 pattern can significantly influence the slope stability even when the rainfall amount is the same.
 442 For this reason, we examine the effects of rainfall pattern on slope reliability. Specifically, the
 443 stability, uncertainty, and reliability along the slope profile are calculated for three rainfall
 444 patterns (namely, ascending rainfall with μ_{h_b} increased from -0.2 to 0 m; uniform rainfall with
 445 μ_{h_b} equals -0.1 m and descending rainfall with μ_{h_b} decreased from 0 to -0.2 m). The COV_{h_b} 's
 446 of these three cases are set to the same, 1.0. Note that in the case of ascending rainfall and the
 447 case of descending rainfall, μ_{h_b} varies with times. Hence the COV_{h_b} 's of these two cases are
 448 defined as the ratio of σ_{h_b} to the value of μ_{h_b} in the mid-time of the rainfall duration(i.e., 1 day).
 449 In addition, COV_{h_0} , COV_{K_s} , λ_{h_0} , λ_{h_b} and λ_{K_s} of these cases are 1.0, 1.0, 0.3 m, 0.3 days
 450 and 0.3 m, respectively. μ_{h_0} is -2 m and μ_{K_s} is 0.2592 m/d.

451 The μ_{FS_i} profiles at $t=0.25$ day under the three rainfall patterns are plotted in Fig. 7a.
 452 They indicate that the μ_{FS_i} profile corresponding to the descending rainfall is the closest to 1 at
 453 the upper part of the slope. The plot the σ_{FS_i} profiles at $t=0.25$ day under the three rainfall
 454 patterns are presented in Fig. 7b. The relative contribution from each parameter to σ_{FS_i} is shown
 455 in Fig. A.3. As indicated by these figures, the rainfall pattern can significantly affect the
 456 distribution of the slope stability uncertainty. Both the propagation depth of σ_{FS_i} 's and the
 457 maximum value of the localized large-uncertainty zone corresponding to the descending rainfall
 458 are the largest among the three rainfall patterns at a given time. The reliability index β_i profiles
 459 in Fig. 7c demonstrate that the descending rainfall pattern destabilizes slopes the most with the

460 minimum value of the localized low-reliability zone. One typical realization generated with
461 abovementioned statistics under the descending rainfall pattern is illustrated in Fig. 7d. These
462 results are likely owing to the fact that the descending rainfall with large rainfall intensities at
463 early time enhances infiltration and leads great uncertainties.

464

465 5 CONCLUSIONS

466 This study demonstrates that the uncertainties due to variabilities in initial soil pore water
467 pressure distributions and rainfalls are as significant as that due to the variabilities in soil
468 hydraulic properties, which have been emphasized by many previous studies. As a consequence,
469 our study stresses the fact that to better evaluations of slope stability, the temporal and spatial
470 propagation of slope stability uncertainty due to variabilities in initial soil pore water pressure
471 distributions and rainfalls should also be considered.

472 In addition, this study demonstrates that the propagation of slope stability uncertainty is
473 driven by the propagation of the mean flow field during the rainfall infiltration process. A
474 localized large-uncertainty zone along the slope stability profile could form and lead to the
475 existence of a localized low-reliability zone. Further, it demonstrates that the build-up of the
476 localized large-uncertainty zone or the localized low-reliability zone is greatly influenced by the
477 prior knowledge of the mean of initial pore water pressure, the rainfall intensity, the rainfall
478 duration, and the rainfall pattern.

479 The situations in which the uncertainties of slope stability are likely to impact the
480 reliability of slope stability evaluations, along with the critical time and location are presented in
481 Table 2. This indicates the location where the low-reliability zone forms and the time when this
482 zone starts to impact the slope under different situations. Under such situations, the uncertainty

483 plays a critical role in slope stability evaluation and requires special attention. That is, further
484 investigation or monitoring measures should be implemented to the location of the low-reliability
485 zone to reduce the uncertainty (e.g., [Cai et al., 2017c](#)). As a result, any unfavorable condition can
486 be detected immediately before it is too late for prevention.

487 Note that the above analysis is based on one-dimensional infinite slope model such that
488 the flow is restricted to vertical flow. For this reason, infiltrated water cannot detour from low
489 K_s zones, and the pressure behind the zones must build up significantly to push water flow
490 through. In other words, this one-dimensional flow may have amplified effects of hydraulic
491 heterogeneity. Multi-dimensional flow models may reduce this magnification.

492 At last, the climate of a region dictates typical ranges of the soil moistures and rainfall
493 intensity and rainfall duration during a specific season. Compiling historical records of spatial
494 distribution of soil moistures and temporal variation of rainfalls at the region can yield the
495 general statistical description of initial hydraulic conditions and rainfalls. This information could
496 facilitate better evaluation of slope stability during any possible rainfall events.

497

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615 **List of Figure Captions:**616 **Fig. 1** An infinite slope model.617 **Fig. 2** Results of deterministic analyses.618 **Fig. 3** σ_{FS_i} 's and β_i 's at different potential slip surfaces at the selected times for the base set.619 **Fig. 4** μ_{FS_i} 's, σ_{FS_i} 's and β_i 's at different potential slip surfaces at the selected time($t=0.25$ day)
620 with different mean initial pore water pressure distribution.621 **Fig. 5** μ_{FS_i} 's, σ_{FS_i} 's and β_i 's at different potential slip surfaces at the selected time($t=0.25$ day)
622 under different rainfall intensities.623 **Fig. 6** β_i 's at different potential slip surfaces at different times with different rainfall durations.624 **Fig. 7** μ_{FS_i} 's, σ_{FS_i} 's and β_i 's at different potential slip surfaces at the selected time($t=0.25$ day)
625 under different rainfall patterns.

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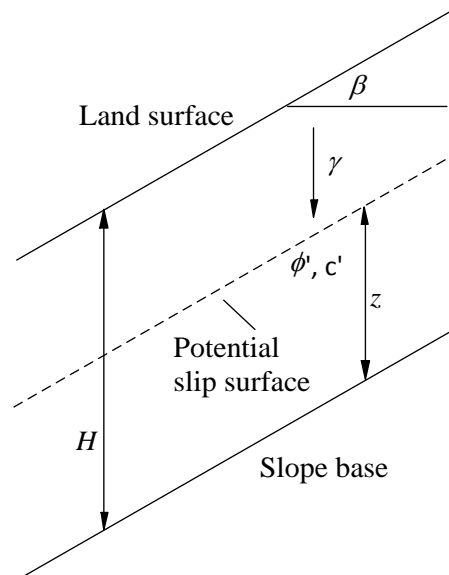
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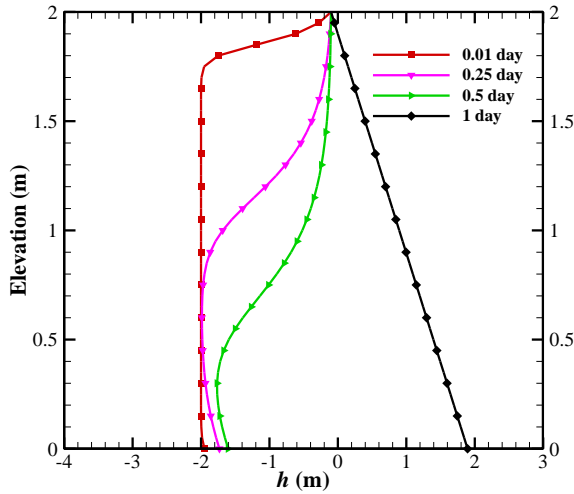
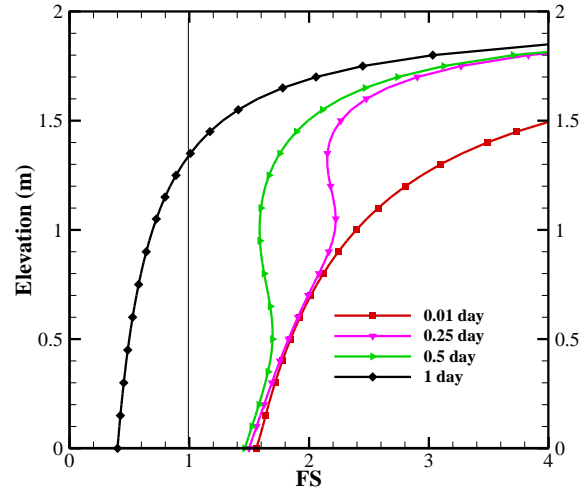
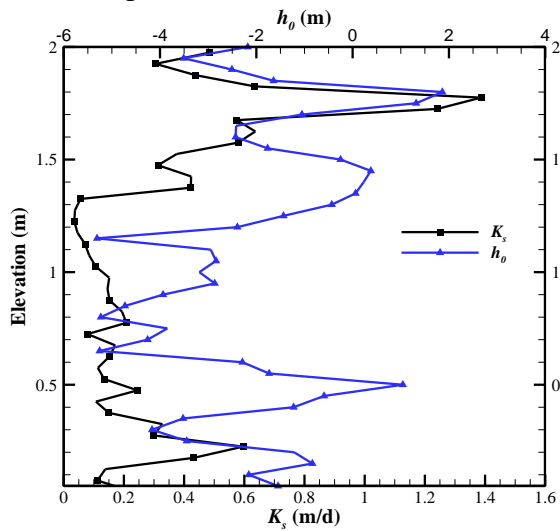
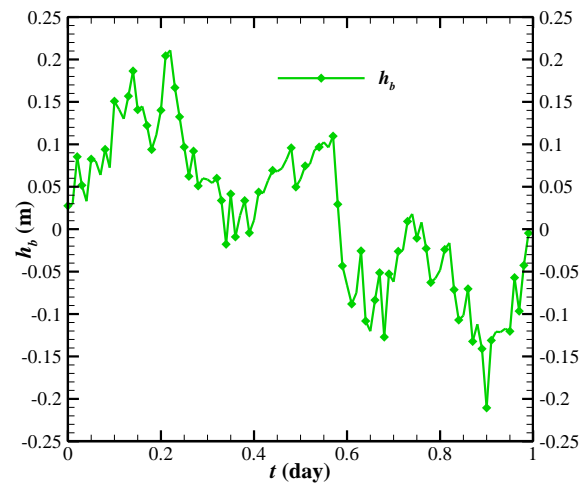
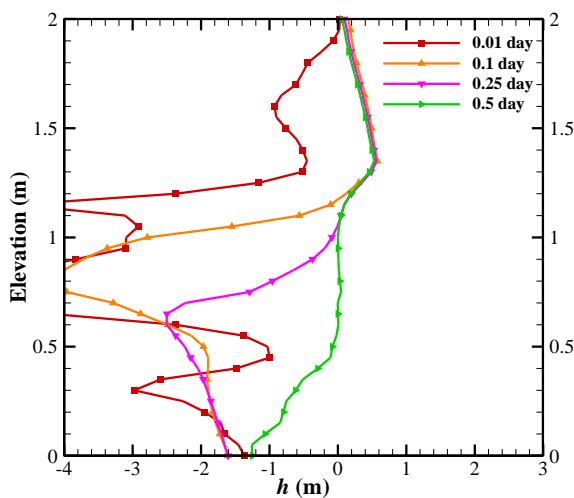
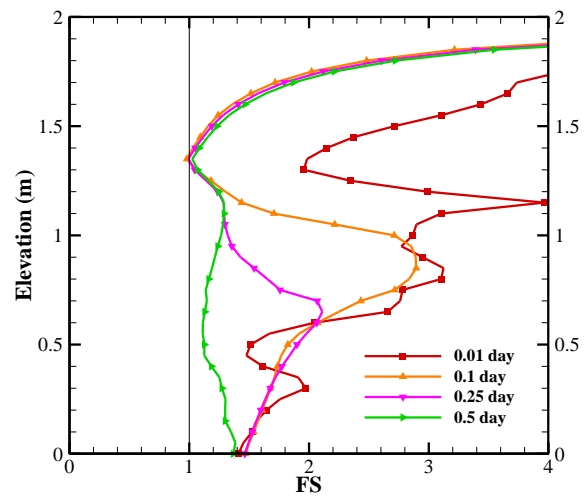


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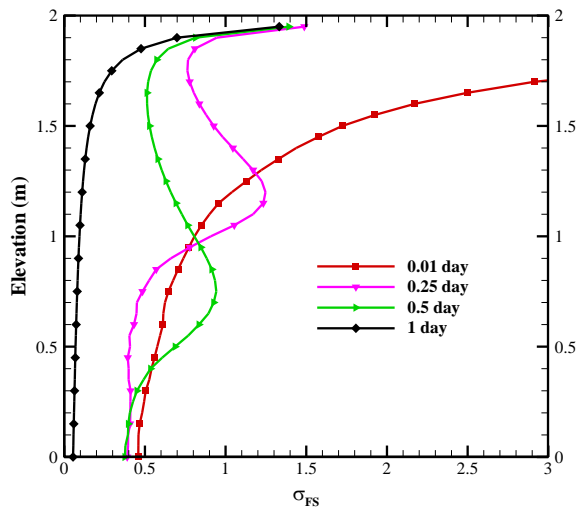
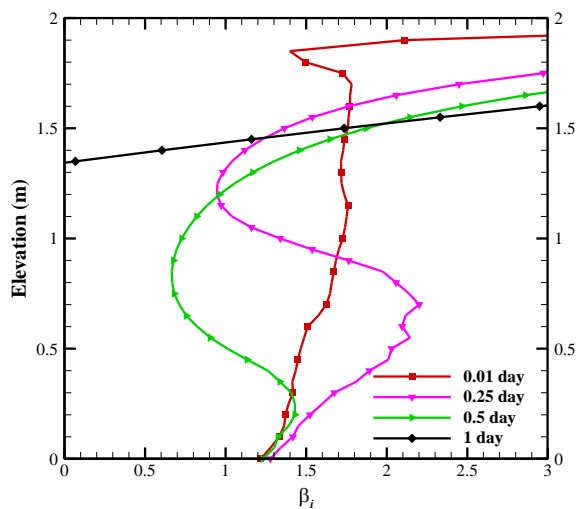
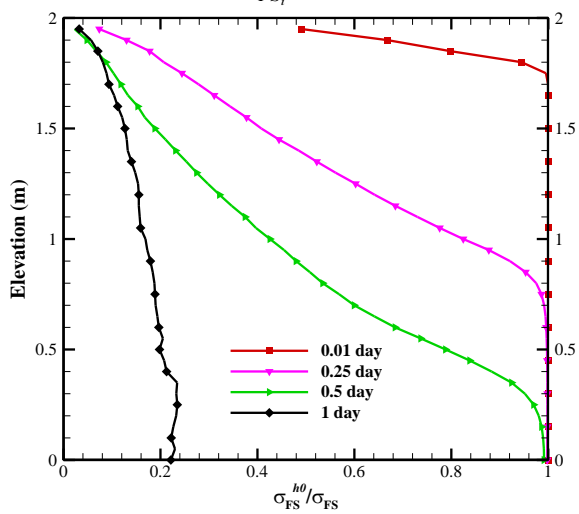
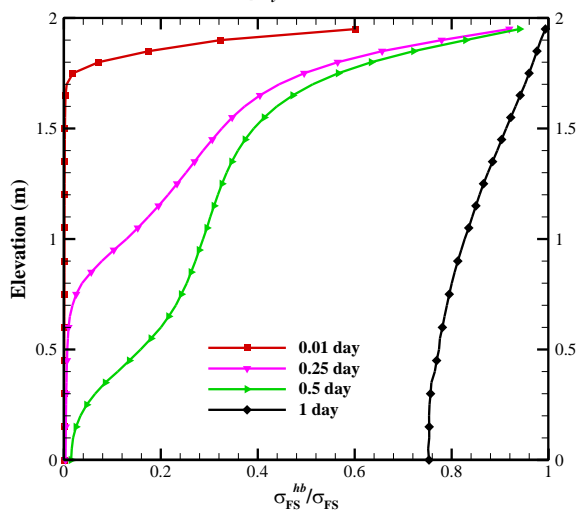
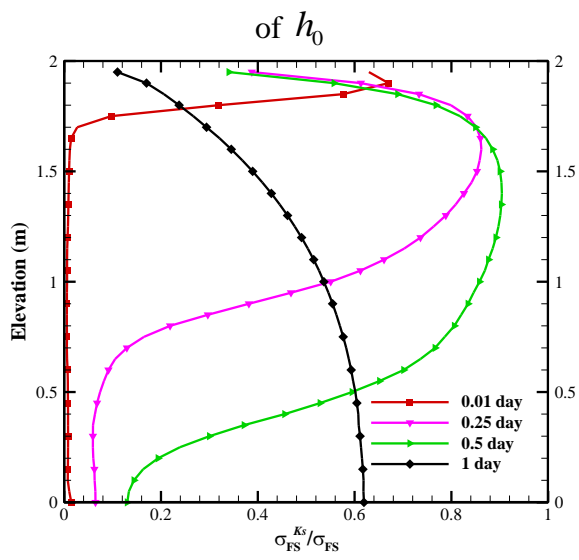
Fig. 1 An infinite slope model.

(a) h profile evaluated at mean values(b) FS_i profile evaluated at mean values(c) K_s and h_0 profiles(d) h_b profile(e) h profile generated using typical realizations of K_s , h_0 and h_b (f) FS_i profile generated using typical realizations of K_s , h_0 and h_b

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Fig. 2 Results of deterministic analyses.

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(a) σ_{FS_i} profiles(b) β_i profiles(c) Normalized σ_{FS_i} profiles due to variation of h_0 (d) Normalized σ_{FS_i} profiles due to variation of h_b (e) Normalized σ_{FS_i} profiles due to variation of K_s

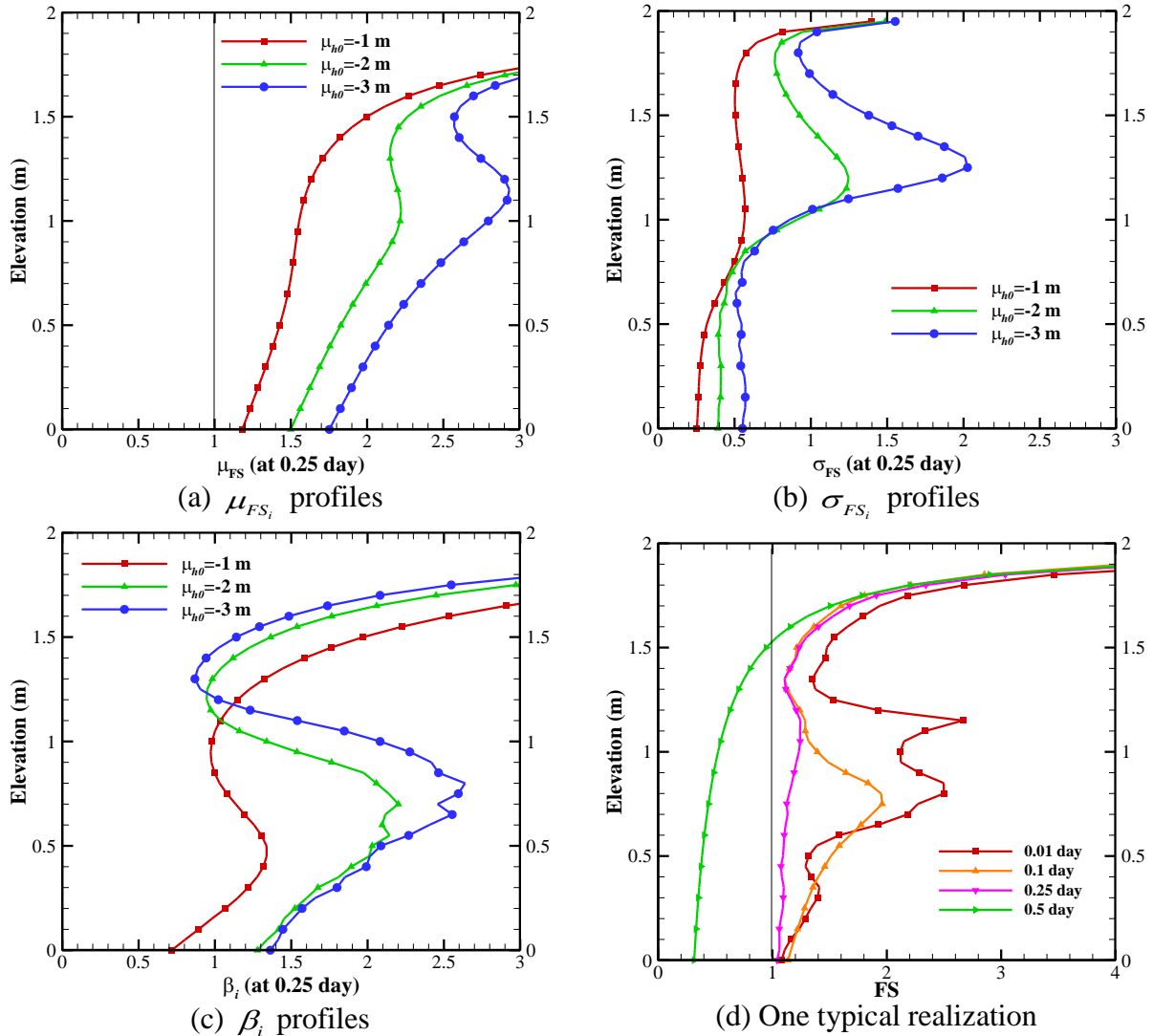
of K_s

637 Fig. 3 σ_{FS_i} 's and β_i 's at different potential slip surfaces at the selected times for the base set.

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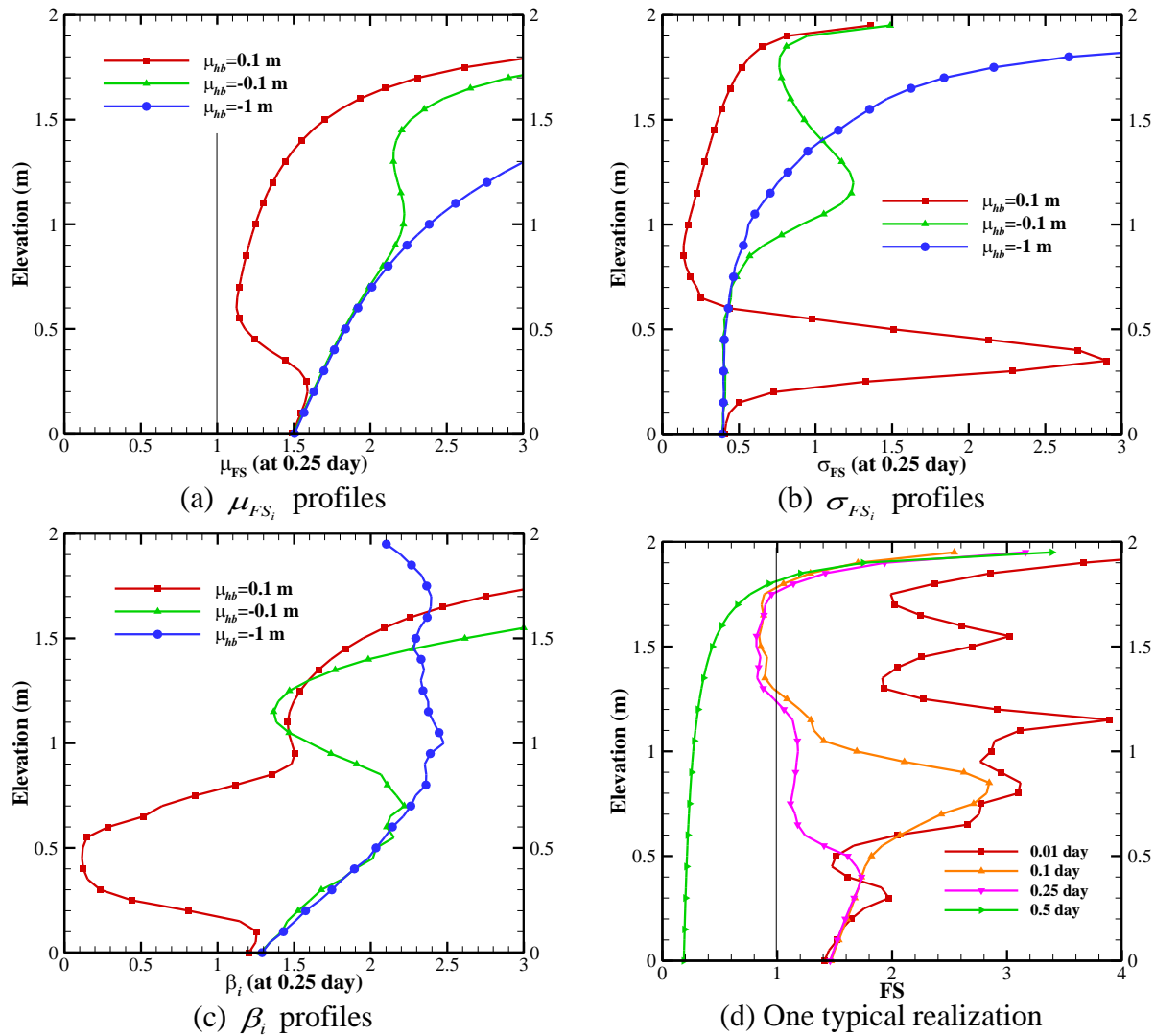
641 Fig. 4 μ_{FS_i} 's, σ_{FS_i} 's and β_i 's at different potential slip surfaces at the selected time ($t=0.25$ day)

642 with different mean initial pore water pressure distribution.

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646 Fig. 5 μ_{FS_i} 's, σ_{FS_i} 's and β_i 's at different potential slip surfaces at the selected time($t=0.25$ day)

647 under different rainfall intensities.

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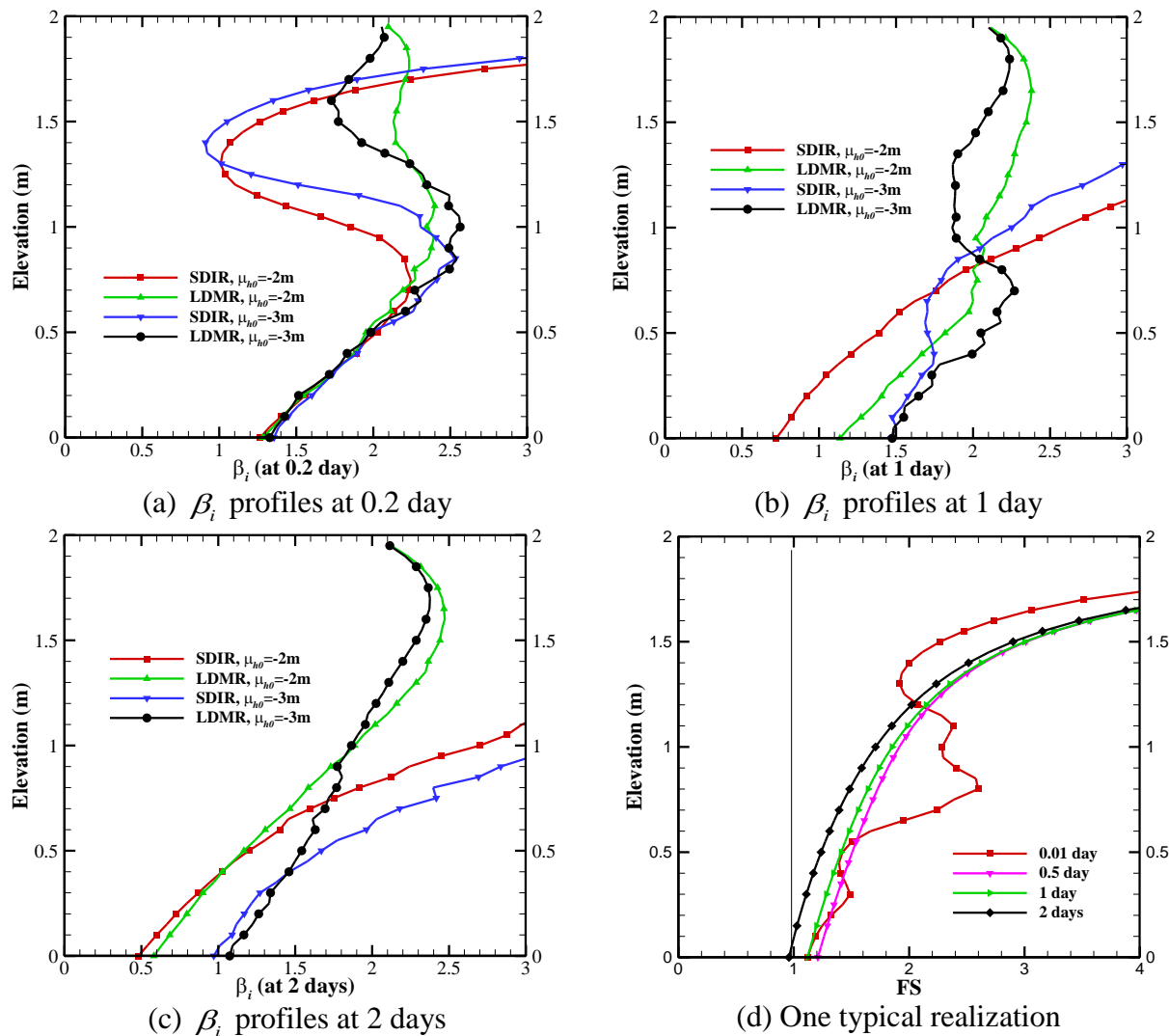
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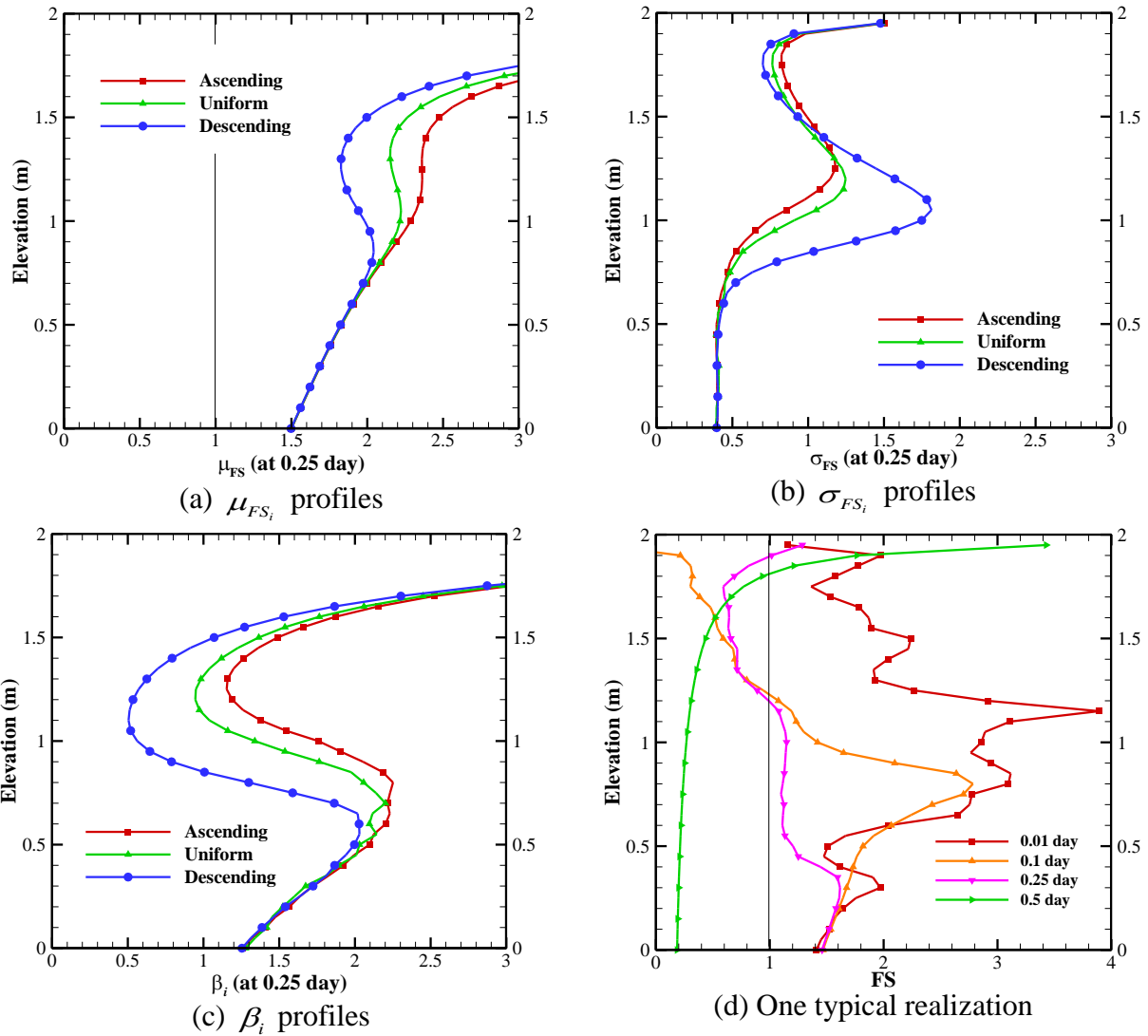
654 Fig. 6 β_i 's at different potential slip surfaces at different times with different rainfall durations.

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659 Fig. 7 μ_{FS_i} 's, σ_{FS_i} 's and β_i 's at different potential slip surfaces at the selected time($t=0.25$ day)

660 under different rainfall patterns.

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663 **List of Table Captions:**664 **Table 1** Statistics of h_0 , h_b and K_s , related parameters and slope geometrical parameters.665 **Table 2** Scenarios in which the uncertainties are very likely to threaten the reliability of slope stability
666 evaluations.

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Table 1 Statistics of h_0 , h_b and K_s , related parameters and slope geometrical parameters.

Parameters	Values
Mean of h_0 , μ_{h_0}	-2 m
Mean of h_b , μ_{h_b}	-0.1 m
Mean of K_s , μ_{K_s}	0.2592 m/d
COVs of h_0 , h_b and K_s	1
Correlation scale of h_0 in space, λ_{h_0}	0.3 m
Correlation scale of h_b in time, λ_{h_b}	0.3 day
Correlation scale of K_s in time, λ_{K_s}	0.3 m
Effective cohesion c' , $\mu_{c'}$	5.0 kN/m ²
Effective friction angle ϕ' , $\mu_{\phi'}$	32°
Specific storage, S_s	0.001 m ⁻¹
Saturated volumetric moisture content, θ_s	0.358
Residual volumetric moisture content, θ_r	3.58×10 ⁻⁴
Coefficient 1 in VG model, α_1	0.5 m ⁻¹
Coefficient 2 in VG model, α_2	1.289
Slope height, H	2 m
Slope angle, β	40°
Dry unit weight, γ_d	16 kN/m ³
Unit weight of water, γ_w	9.8 kN/m ³

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675 Table 2 Situations in which the uncertainties are likely to impact the reliability of slope stability
 676 evaluations.

Number	Conditions	Critical time	Low-reliability zone
1	Close to saturation	Early times	Any elevation of the slope
2	Unsaturated slope under heavy rainfalls	Early times	Shallow parts of the slope
3	Rainfall persists	Late times	near the impermeable bedrock
4	Rainfall has a descending trend	Early times	Shallow parts of the slope

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680 **List of Figure Captions for Appendix:**

681

682 **Fig. A.1** Normalized σ_{FS_i} 's due to each parameter (h_0 , h_b and K_s) at different potential slip
683 surfaces at the selected time($t=0.25$ day) with different mean initial pore water pressure
684 distribution.

685 **Fig. A.2** Normalized σ_{FS_i} 's due to each parameter (h_0 , h_b and K_s) at different potential slip
686 surfaces at the selected time($t=0.25$ day) under different rainfall intensities.

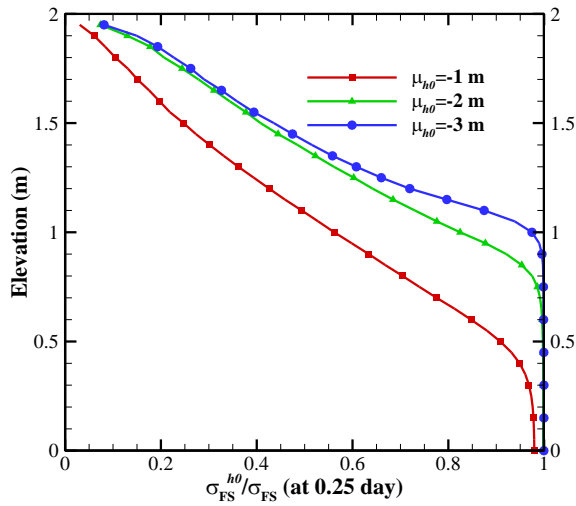
687 **Fig. A.3** Normalized σ_{FS_i} 's due to each parameter (h_0 , h_b and K_s) at different potential slip
688 surfaces at the selected times under different rainfall patterns.

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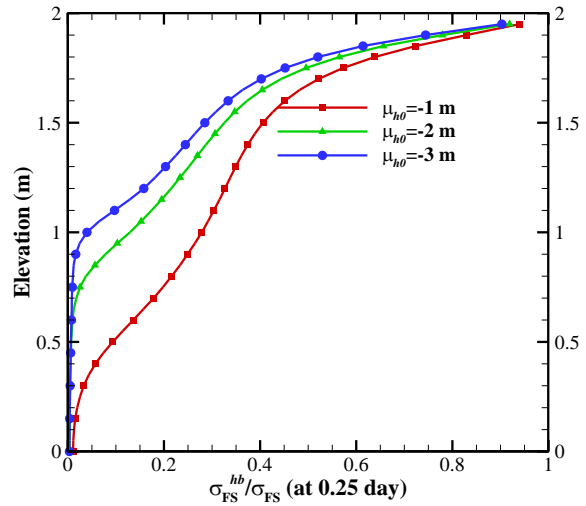
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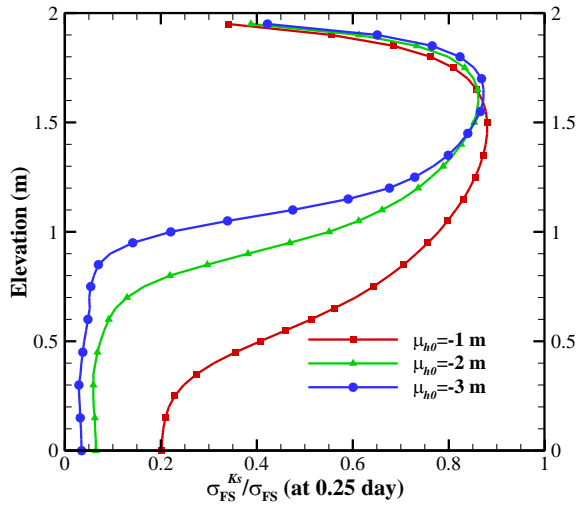
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(a) σ_{FS_i} profiles due to variation of h_0



(b) σ_{FS_i} profiles due to variation of h_b



(c) σ_{FS_i} profiles due to variation of K_s

693 Fig. A.1 Normalized σ_{FS_i} 's due to each parameter (h_0 , h_b and K_s) at different potential slip

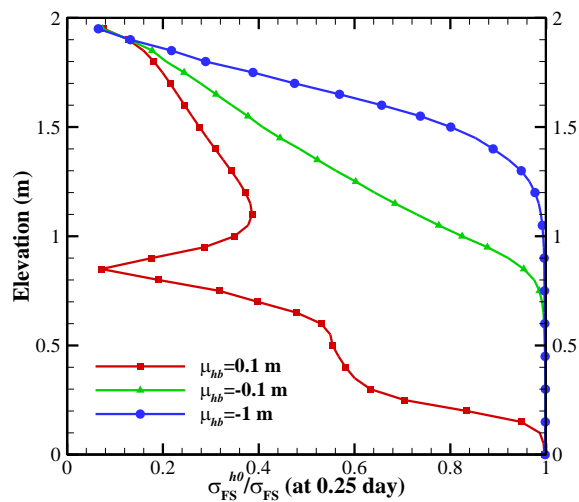
694 surfaces at the selected time ($t=0.25$ day) with different mean initial pore water pressure

695 distribution.

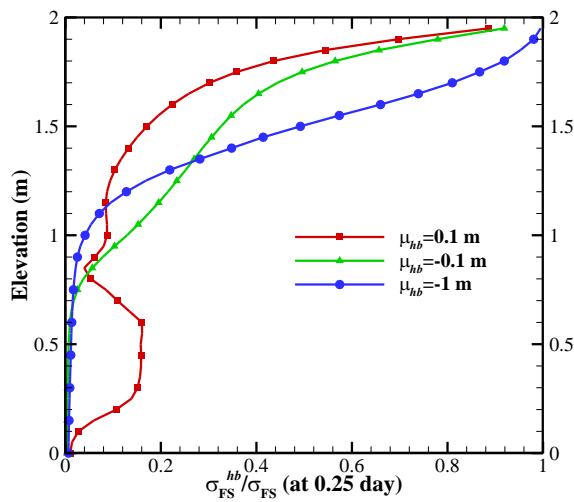
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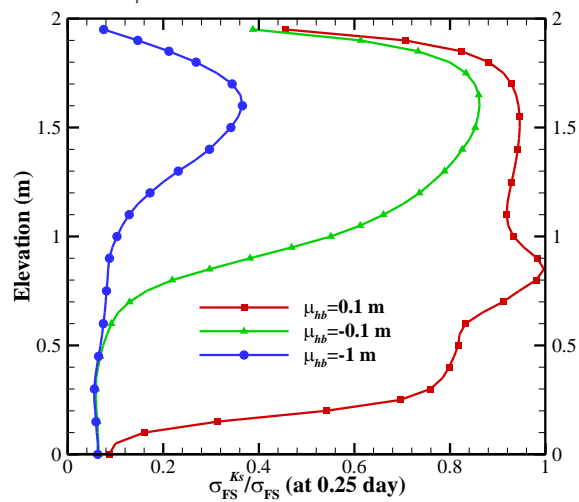
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(a) σ_{FS_i} profiles due to variation of h_0



(b) σ_{FS_i} profiles due to variation of h_b



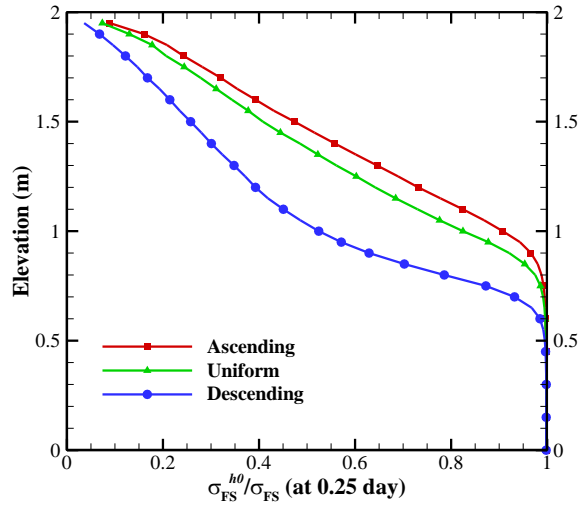
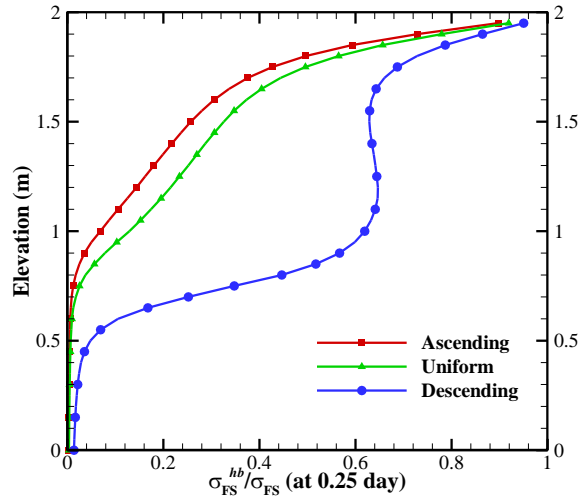
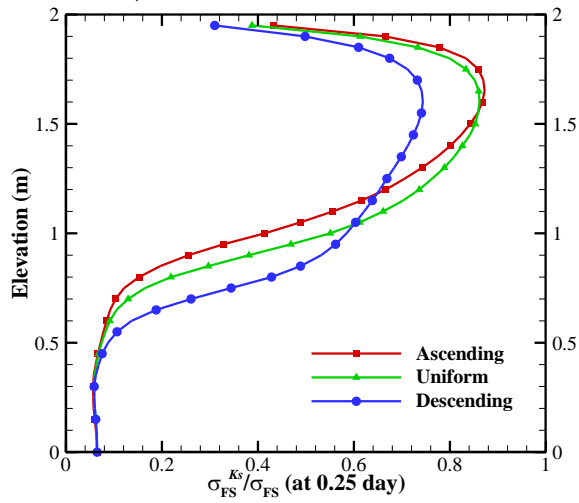
(c) σ_{FS_i} profiles due to variation of K_s

701 Fig. A.2 Normalized σ_{FS_i} 's due to each parameter (h_0 , h_b and K_s) at different potential slip
702 surfaces at the selected time($t=0.25$ day) under different rainfall intensities.

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(a) σ_{FS_i} profiles due to variation of h_0 (b) σ_{FS_i} profiles due to variation of h_b (c) σ_{FS_i} profiles due to variation of K_s

706 Fig. A.3 Normalized σ_{FS_i} 's due to each parameter (h_0 , h_b and K_s) at different potential slip

707 surfaces at the selected times under different rainfall patterns.

708