Optimality versus viability in groundwater management with environmental flows

Jean-Christophe Pereau[a,*], Alexandre Pryet[b], Tina Rambonilaza[c]

[a] *GREThA UMR-CNRS 5113, University of Bordeaux, Avenue Léon Duguit, 33608 Pessac CEDEX, France and iGLOBES, UMI-CNRS 3157, University of Arizona*
[b] *Georessources and Environment, Bordeaux INP and Univ. Bordeaux Montaigne. ENSEGID, 1 allée F. Daguin, 33607 Pessac, France.*
[c] *Irstea, UR ETBX, 50 avenue de Verdun, Cestas F-33612, France.*

Abstract

The protection of environmental flows is a main challenge pursued by water regulating agencies in their groundwater management policies. A stylised hydro-economic model with natural drainage is used to compare the outcome of the optimal control approach in which environmental flows are introduced as an externality with the viable approach in which environmental flows are modelled as a constraint to satisfy. The optimal and viable paths for the water table, water extraction for irrigation and environmental flows are analytically derived together with their long term values. We show how results are sensitive to some key parameters like the discount factor and the monetary value of the externality in the optimal control approach. We show how the value of the environmental flows target in the viable approach can be derived from the optimal control approach. Numerical simulations based on the Western La Mancha aquifer illustrate the main results of the study.

Keywords: Environmental flows, Groundwater dependent ecosystem, Groundwater management, optimal control, viability, water budget myth
JEL codes: Q15, Q25, C61.

*Corresponding author
Email address: jean-christophe.pereau@u-bordeaux.fr (Jean-Christophe Pereau)

1. Introduction

Originally introduced for rivers, environmental flows refer to the quantity of water that is necessary to maintain valued ecosystems services (*Tharme* 2003, *Sophocleous* 2007, *Korsgaard et al.* 2008, *Gopal* 2016, *Yang et al.* 2016). This definition has been extended to groundwater-dependent ecosystems (GDE hereafter) that rely, directly or indirectly, to the occurrence of groundwater (*Murray et al.* 2003, *Eamus et al.* 2006, *Tuinstra and van Wensem* 2014, *Griebler and Avramov* 2015). When the natural drainage from an aquifer supports GDEs, increasing groundwater extraction for irrigation, drinking water or other uses may threaten environmental flows (*Lohman* 1972, *Konikow and Leake* 2014). In such a context, how can groundwater managers effectively account for the protection of environmental flows in their policies and what are the implications in terms of hydro-economic modelling?

Our analysis will be based on the stylised bathtub model of *Gisser and Sanchez* (1980b) which remains didactic and useful for generic discussions on groundwater management (*Koundouri* 2004, *Katic and Grafton* 2012). Two hydro-economic modelling approaches can be distinguished depending on the way environmental flows are modelled. A first approach consists in cost-benefit optimal control models in which a regulating agency is assumed to maximise the discounted present value of a welfare function. This function accounts for the benefits of water extraction (e.g. for irrigation), minus the monetary cost of the environmental externalities (e.g. reduction of aquifer drainage or aquifer depletion) (*Esteban and Albiac* 2011, *Esteban and Dinar* 2013; 2016, *Pereau and Pryet* 2018). The second modelling approach refers to the viability (or viable control) method (*Aubin* 1990, *Bene et al.* 2001, *Bene and Doyen* 2018). As surveyed by *Oubraham and Zaccour* (2018), viability has been applied to several renewable resource management including reservoir and groundwater management (*Alais et al.* 2017, *Chu et al.* 2018, *Pereau et al.* 2018, *Pereau* 2018). Its aim is to identify the conditions that allow desirable objectives or constraints to be fulfilled over time. In our case a water agency is assumed to set constraints on the amount of environmental flows for the maintenance of ecosystems and ensure that these flows remain above a minimum bound. Such a target can be set to maintain the capacity of the ecosystem to provide valuable services and expressed in terms of water level or flow perturbation with respect to pristine conditions.

The optimal modelling approach is based on the economic valuation of the ecosystem services sustained by the groundwater flows (*Rolfe* 2010, *Momblanch et al.* 2016). This evaluation consists in a three-step process, including first the identification of the dependent ecosystems, the determination of the flow regime (scenarios) that may critically affect their functioning and species living there, and finally the estimation of values that people place on related non-market goods and services. Determining environmental flow values of ecosystem services is thus complex, data-intensive and time consuming. For these reasons, public administrations and environmental protection agencies often use benefit transfer method which consists in using existing estimates provided mostly by the academic literature in place (*Plummer* 2009, *Richardson et al.* 2005). For instance *Akter et al.* (2014) develop an integrated hydro-ecological model of a river managed by dams together with non-market economic values of wetland inundation to estimate the economic value of an additional unit of water in the environment. This method has also been used for the preservation of springs and some endemic species in *Rolfe* (2010). In the groundwater context, *Esteban and Albiac* (2011), *Esteban and Dinar* (2013; 2016) give specific focus on ecosystem services provided by wetland areas related to the Western La Mancha aquifer in Spain. They start from a value transfer of recreation and tourism benefits associated to the implementation of a national park in the Tablas de Damiel (*Judez et al.* 1998; 2000). Then, they complete their sensitivity analysis with very large interval values defined above the results of the meta-analysis of wetland valuations provided by *Brander et al.* (2006).

The viability modelling approach defines environmental flows as a hydrologic constraint and does not require to evaluate the environmental flows in monetary terms. Following *Tharme* (2003) in the case of river, environmental flow assessments are used to determine the environmental water required to reach some ecological endpoints. They provide minimum flow targets to meet critical ecological needs. Even if this concept has been critized as a myth (*Stalnaker* 1972), these minimum flow requirements appear in hydro-economic models as constraint optimization problems with thresholds that a manager has to satisfy (*Booker et al.* 2012). Streamflow constraint values can be chosen as a percentage of historic average flow as in *Mulligan et al.* (2014), as a negotiated outcome between the stakeholders or a legal constraint set by a supranational authority, like the European Water Framework Directive for instance (*Pereau* 2018).

The contributions of the paper are twofold. A first contribution is to analyse and compare the properties of the optimal control and the viable control methodologies in a single unified model. The key parameters and variables are detailed in both cases. In the optimality approach, results show that the trajectories of the water table, the water allocation for irrigation and the environmental flows strongly depend on the discount factor and the cost of the environmental externality expressed in monetary terms. This externality can be defined as the sum of damages to consumptive uses (i.e the capture (*Lohman* 1972)) and non-consumptive uses (e.g. the difference between the initial water table and its current level). In the viability approach, results depend on the value of the constraint set on the environmental flows and a parameter showing the trade-off of the manager between the preservation of the resource and the economic payoffs of the farmers. A second contribution is to highlight that for a given constraint on the environmental flows in the viability approach, equivalent long term values can be obtained with the optimal control approach through the adjustment of the cost of the environmental externality. Such a relation can provide information to managers on the existing trade-off between allocating water for irrigation and for the environmental needs in a sustainable perspective (*Alley et al.* 1999, *Zhou* 2009).

The paper is organized as follows. In section 2, we present the general dynamic hydro-economic model with natural drainage in the water budget. Section 3 distinguishes three kinds of management: myopic competition, optimal control and viable control. Section 4 illustrates the main findings of the study based on the Western La Mancha aquifer. Section 5 concludes.

2. The hydro-economic model

Our analysis is based on the stylised bathtub model of *Gisser and Sanchez* (1980b) in which farmers used water as the only input to irrigate their crops. However since our analysis focusses on environmental flows, the dynamics of the resource include the natural drainage as in *Gisser and Sanchez* (1980a) and *Pereau and Pryet* (2018).

2.1. The dynamics

The dynamics of an aquifer is described by changes in the water table, measured by $H(t) \in [0; H_{\max}]$ at time t where $H_{\max}$ stands for the maximum level of the water table. The water table increases with the constant natural

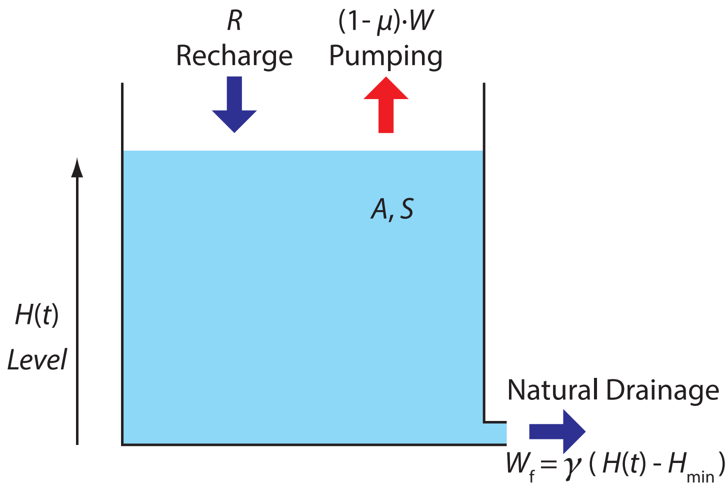

Figure 1: The stylised bathtub model of *Gisser and Sanchez* (1980a;b).

recharge $R > 0$ and is reduced by extraction $Q(t)$ dedicated to irrigated agriculture and the natural drainage $W_f(t)$. A proportion μ of the water is assumed to return to the aquifer where $0 < \mu < 1$ stands for the return-flow coefficient. Total extraction is thus $(1-\mu)\,Q(t)$. The natural drainage $W_f(t)$ provides water to GDEs. A is the area of the aquifer and S the storativity coefficient.

The water budget of the aquifer sketched in Figure 1 is:

$$AS\,(H(t+1) - H(t)) = R - (1-\mu)\,Q(t) - W_f(t), \tag{1}$$

with $H(0) = H_0$. As proposed by *Gisser and Mercado* (1972) and *Gisser and Sanchez* (1980a), aquifer drainage is defined as a linear function of the water table with respect to a critical height, $H_{\min}$, for which the natural drainage is nil:

$$W_f(t) = \gamma\,(H(t) - H_{\min})\,, \tag{2}$$

with $\gamma = \frac{R}{H_{\max} - H_{\min}} > 0$. It turns out that for $H(t) = H_{\min}$ the natural drainage is nil, $W_f(t) = 0$. This critical water table can be interpreted as a tipping point for the GDE.

Substitute (2) in the dynamics (1) gives:

$$H(t+1) = \left(1 - \frac{\gamma}{AS}\right) H(t) + \frac{R + \gamma H_{\min}}{AS} - \frac{(1-\mu)}{AS} Q(t). \tag{3}$$

At the steady-state, the water balance equation yields a negative relationship between the extraction rate, $\overline{Q}$ and the water table height, $\overline{H}$:

$$\overline{Q} = \frac{R + \gamma H_{\min}}{1-\mu} - \frac{\gamma}{1-\mu}\overline{H}. \tag{4}$$

The maximum sustainable extraction rate is obtained with $\overline{H} = H_{\min}$ and reads $\overline{Q} = R/(1-\mu)$. The maximum sustainable net extraction rate, $(1-\mu)\overline{Q}$ therefore corresponds to the recharge rate, R. Higher extraction rates at the steady-state values imply aquifer depletion and/or negative values for aquifer drainage, which is not considered in this study, but could take place through stream depletion.

2.2. The economic model

Based on the papers of *Gisser and Sanchez* (1980a;b), the linear water demand function for irrigated agriculture with $g, k > 0$ is given by:

$$Q(t) = g - kp(t), \tag{5}$$

with p the water price per unit water volume. By using the inverse demand function, the farmer revenues are given by:

$$\int_0^Q p(x)dx = \frac{g}{k}Q - \frac{1}{2k}Q^2. \tag{6}$$

The marginal pumping cost is:

$$c(H(t)) = c_0 - c_1 H(t). \tag{7}$$

From eq (6), the net benefit is:

$$NB(t) = \left(\frac{g}{k} - c_0\right) Q(t) - \frac{1}{2k}Q^2(t) + c_1 H(t) Q(t). \tag{8}$$

As pointed out by *Rubio and Casino* (2001), the marginal extraction cost of the last unit of water c_0 is assumed to be higher than the maximum value of marginal product g/k implying $c_0 \geq \frac{g}{k}$.

The next section aims at analysing the properties of several management schemes dealing with environmental flows.

3. Three water management scenarios

This section considers three management scenarios. The first one is the myopic competition in which farmers maximise their current payoff and act non cooperatively without intervention of a water agency. In the two other options, the optimal control management and the viable management, we assume that a regulating agency sets and allocates to farmers an irrigation quota, $Q(t)$. In the optimal scenario, the water agency maximises a social welfare function including the environmental externality while in the viable scenario the aim of the regulation is to avoid the water table falling below a critical threshold. The comparison between the myopic competition outcome and the optimal control case is at the origin of the famous and disturbing result known as the Gisser-Sanchez effect. *Gisser and Sanchez* (1980b) show that the inefficiency of a competitive groundwater exploitation by myopic maximizing farmers was not a sufficient condition for public intervention (*Brill and Burness* 1994, *Koundouri* 2004, *Tomini* 2014). This result can be explained by the negligible difference which exists for large aquifers between the optimal control management when farmers cooperate and internalize the pumping cost externality and the previous myopic competition. This latter differs from the open access outcome when the payoffs of the farmers are nil. To restore the need of a regulation, *Esteban and Albiac* (2011), *Esteban and Dinar* (2013; 2016) introduce an environmental externality in the welfare function of the water manager. The next subsection analyses this Gisser-Sanchez effect in the presence of environmental flows.

3.1. Myopic competition

In the myopic competition case or unregulated scenario, farmers do not internalize neither the pumping cost externality nor the environmental externality. Farmers maximise their net benefit given by (8) and extract water until their individual revenue equals individual cost extraction. Optimality conditions[1] give the agricultural water demand (9):

$$Q_D = g - k(c_0 - c_1 H(t)). \tag{9}$$

[1]The second derivative is negative implying a maximum.

Substitute (9) in the dynamics equation (3) yields a first-order difference equation:

$$H(t+1) = \left(1 - \frac{\gamma + (1-\mu)kc_1}{AS}\right) H(t) + \frac{R + \gamma H_{\min} - (1-\mu)\left(g - kc_0\right)}{AS}.$$

It gives the following proposition.

Proposition 1. *Starting from an initial value H_0, the trajectory for the water table with myopic farmers is given by*

$$H(t) = \overline{H}_{mc} + \left(H_0 - \overline{H}_{mc}\right)\left(1 - \frac{\gamma + (1-\mu)kc_1}{AS}\right)^t, \qquad (10)$$

with $\overline{H}_{mc} > 0$ the steady state water table

$$\overline{H}_{mc} = \frac{R + \gamma H_{\min} + (1-\mu)\left(kc_0 - g\right)}{\gamma + (1-\mu)kc_1}, \qquad (11)$$

The dynamics are monotonic and convergent when the root of (10) satisfies

$$c_1 < \frac{AS - \gamma}{(1-\mu)k}. \qquad (12)$$

An excessive value of c_1 decreases the marginal cost in (7) and increases the water demand in (9) in such a way that the dynamics system will be divergent.

The stabilized extraction rate $\overline{Q}_{mc}$ associated to the steady state water table height $\overline{H}_{mc}$ is:

$$\overline{Q}_{mc} = \frac{(R + \gamma H_{\min})kc_1 - \gamma(kc_0 - g)}{\gamma + (1-\mu)kc_1}.$$

where $\overline{Q}_{mc}$ is positive under the following condition

$$c_1 \geq \frac{\gamma(kc_0 - g)}{(R + \gamma H_{\min})k} \qquad (13)$$

On the opposite, to ensure a positive demand, a minimum value of the marginal cost is required since it decreases the marginal cost. The combination of the two constraints (12) and (13) on c_1 always holds for $R > 0$.

With the myopic approach, nothing ensures that environmental flows will be positive at the steady state $\overline{W_{f}}_{mc} \geq 0$. From (2), such a condition implies $\overline{H}_{mc} \geq H_{\min}$. The following corollary gives the conditions under which GDEs are preserved.

Corollary 1. *Positive environmental flows at the steady state* $\overline{W_f}_{mc} \geq 0$ *occur when*

$$\frac{R}{1-\mu} \geq g - k\left(c_0 - c_1 H_{\min}\right) \tag{14}$$

Condition (14) states that environmental flows are positive when the sustainable water extraction rate associated to $\overline{H}_{mc} = H_{\min}$ in (4) exceeds the water demand when the water table given by (9) is equal to to $H_{\min}$. However when condition (14) does not hold, the incentives for farmers to extract more water than $R/(1-\mu)$ are stronger, implying the collapse of the GDE. Such a case occurs in the numerical analysis.

3.2. Optimal approach

The program of the regulating agency is to optimize the trajectory of the quota allocated to farmers, $Q(t)$, so as to maximise the net present value of a welfare function, WF. With this approach, both the pumping cost externality and the environmental externality are internalized by the regulating agency. As pointed out by *Pereau and Pryet* (2018), the introduction of the environmental externality needs to be consistent with the aquifer water balance and his degree of renewability. The lack of consideration of the natural drainage in numerous studies have promoted the persistent water budget myth stating that net withdrawals can equal groundwater recharge on the long run without causing harmful damages (*Bredehoeft* 2002, *Devlin and Sophocleous* 2004). The maximisation program is given by:

$$\max_{Q(t)} \sum_{t=0}^{\infty} \beta^t WF\left(Q(t), H(t)\right), \tag{15}$$

under the dynamics (3) and where $0 < \beta < 1$ stands for the discount factor. The welfare function can be defined as the difference between the net benefit $NB(t)$ given by (8) and a damage function as in *Pereau and Pryet* (2018). We obtain $WF = NB(t) - D(H(t))$. The damage function is assumed to be

$$D(H(t)) = \varphi\left(R - W_f(t)\right) + \theta\left(H_{\max} - H(t)\right). \tag{16}$$

The first term of (16) measures the cost of the capture and refers to ecosystem damages associated to consumptive uses. Following *Lohman* (1972), the capture is defined as the decrease in drainage plus the increase in recharge (not considered in this study since R is constant). It differs from *Esteban and*

Albiac (2011; 2012) who consider a difference between the net pumping and the natural recharge which tends to zero in the long term due to the water budget myth. In our case the cost of capture reaches its maximum value at the steady state when the capture is equal to the sustainable net extraction rate. The second term of eq (16) measures the ecosystem damages associated to non-consumptive uses as in *Esteban and Dinar* (2013) and is equal to the difference between $H_{\max}$ and its current level $H(t)$. Using (2), (16) can be rewritten as follows:

$$D(H(t)) = d_0 - d_1 H(t), \tag{17}$$

with $d_0 = \varphi R + \gamma\varphi H_{\min} + \theta H_{\max} > 0$ and $d_1 = \gamma\varphi + \theta > 0$.

Program (15) can be rewritten as follows

$$\max_{Q(t)} \sum_{t=0}^{\infty} \beta^t \left(-d_0 + \left(\frac{g}{k} - c_0\right) Q(t) + d_1 H(t) + c_1 H(t) Q(t) + \frac{1}{2k} Q^2(t)\right), \tag{18}$$

under the dynamics

$$\begin{aligned} H(t+1) &= \left(1 - \frac{\gamma}{AS}\right) H(t) + \frac{R + \gamma H_{\min}}{AS} - \left(\frac{1-\mu}{AS}\right) Q(t), \\ H(0) &= H_0. \end{aligned}$$

It gives the following proposition.

Proposition 2. *The optimum trajectory for the level of the water table is given by*

$$H(t) = \overline{H}_{oc} + \left(H_0 - \overline{H}_{oc}\right) \lambda^t,$$

with the associated decision rule for the aggregate pumping rate

$$Q(t) = \frac{R - \gamma\left(\overline{H}_{oc} - H_{\min}\right)}{1-\mu} + \frac{AS}{1-\mu}\left(H_0 - \overline{H}_{oc}\right)\left(1 - \frac{\gamma}{AS} - \lambda\right)\lambda^t,$$

The stable root $0 < \lambda < 1$ *is given by*

$$\lambda = \left(1 + \frac{1-\beta}{2\beta} + \varepsilon\right) - \sqrt{\left(\frac{1-\beta}{2\beta} + \varepsilon\right)^2 + 2\epsilon},$$

where $\varepsilon = \frac{\frac{1}{k}\left(\frac{\gamma}{AS}\right)\left(1-\beta\left(1-\frac{\gamma}{AS}\right)\right)+\left(\frac{1-\mu}{AS}\right)c_1\left(1+\beta\left(1-2\left(1-\frac{\gamma}{AS}\right)\right)\right)}{2\beta\left(\frac{1}{k}\left(1-\frac{\gamma}{AS}\right)-\left(\frac{1-\mu}{AS}\right)c_1\right)}$ *is positive under the same condition (12) obtained in the myopic competition. The positive steady* *state water table $\overline{H}_{oc}$ is given by*

$$\overline{H}_{oc} = \frac{\frac{1}{k}\left(\frac{R+\gamma H_{\min}}{1-\mu} + kc_0 - g\right)\left(1-\beta\left(1-\frac{\gamma}{AS}\right)\right) + \beta\left(\frac{R+\gamma H_{\min}}{AS}\right)c_1 + \beta\left(\frac{1-\mu}{AS}\right)d_1}{\frac{\gamma}{(1-\mu)k}\left(1-\beta\left(1-\frac{\gamma}{AS}\right)\right) + c_1\left(1+\beta\left(1-2\left(1-\frac{\gamma}{AS}\right)\right)\right)}. \tag{19}$$

Proof. see appendix (2) ■

Corollary 2. *It can be shown that:*

- *The steady state water table $\overline{H}_{oc}$ increases with the discount factor β and the weight of the environmental externality d_1.*
- *The steady state water table $\overline{H}_{oc}$ is greater or equal to $\overline{H}_{mc}$*
- *The steady state amount of environmental flows $\overline{W_f}_{oc}$ is positive or nil when $\overline{H}_{oc} \geq H_{\min}$ which holds when*

$$\frac{R}{1-\mu} \geq g - k\left(c_0 - c_1 H_{\min}\right) - \frac{\beta k}{\beta\gamma + AS\left(1-\beta\right)}\left(c_1 R + \left(1-\mu\right)d_1\right) \tag{20}$$

Proof. See appendix (2) ■

When the regulating agency internalises the stock externality and the pumping externality, the water table at the steady state in the optimal case is always greater than in the myopic case for a positive value of the discount factor implying $\overline{H}_{oc} > \overline{H}_{mc}$. The two water table tend to be equal for values of discount factor close to zero. Moreover when the environmental externality is taken into account by the water agency, the difference between the two water table increases. Results also show that the steady state value of the optimal water table increases with the discount factor β and the weight of the environmental externality d_1. Environmental flows at the steady state are positive or nil when $\overline{H}_{oc}$ are higher than $H_{\min}$ which gives condition (20). With respect to the myopic competition outcome, condition (20) is less restrictive in the optimal case. In particular when the discount factor tends to zero, condition (20) is the same as condition (14). The maximum water demand associated to $H_{\min}$ given by (9 is now lower and it increases the likelihood that the sustainable water extraction rate $R/(1-\mu)$ exceeds the maximum water demand associated to $H_{\min}$.

However, as shown by *Gisser and Sanchez* (1980a), the case $\overline{H}_{oc} < H_{\min}$ may appear for low value of the discount factor and when the environmental externality is not internalised ($d_1 = 0$) by the water agency. This case means a higher preference for the present which implies a higher extraction and an increasing risk of GDE collapse. Such a case occurs in our numerical illustration. However, positive and increasing value of d_1 increases the likelihood that $\overline{H}_{oc} > H_{\min}$ and then $\overline{W_f}_{oc} > 0$.

3.3. Viability approach

The viability approach is a natural extension of the myopic competition case in which dynamic constraints are added. Farmers are still myopic like in the competitive groundwater exploitation regime but we consider a water agency in charge of allocating quotas to farmers at each period. Based on the evidence of several empirical works, *Guilfoos et al.* (2013; 2016) have pointed out that myopic behavior remains a reasonable assumption of economic behavior for farmers. In such a framework and as in the optimal control case, the water agency is facing a dynamic and intertemporal problem taking into account the behavior of the farmers and the constraint of keeping the water table above $H_{\min}$ for all periods. Contrary to the optimal control framework, the viability approach copes with sustainability without introducing a discount factor and does not strive to identify an optimal path but rather a corridor of feasible trajectories. Based on environmental constraint set by the water agency, we identify the viability kernel and the set of viable quotas.

3.3.1. The resource constraint

We consider that the water agency has to satisfy the following environmental constraint implying a target on the natural drainage

$$W_f(t) \geq \widetilde{W_f}. \tag{21}$$

Using eq (2), such a constraint (21) implies a new constraint on the water table:

$$H(t) \geq H_{\widetilde{W}}, \tag{22}$$

with

$$H_{\widetilde{W}} = H_{\min} + \frac{\widetilde{W_f}}{\gamma}.$$

A minima, if $\widetilde{W_f} = 0$, the environmental constraint is $H_{\widetilde{W}} = H_{\min}$ while the maximum value for $\widetilde{W_f}$ is equal to the recharge R. Using eq (4), the associated steady-state water extraction to $H_{\widetilde{W}}$ is $\overline{Q}_{\widetilde{W}}$.

3.3.2. The viability kernel

In an infinite horizon context, the viability kernel can be formally defined as the set of initial situations H_0 such that there exists water extraction $Q(t)$ and resources $H(t)$, which satisfy constraint (22), for all time $t = 0, 1, .., \infty$. It can be written as

$$Viab = \{H_0 | \exists Q(t) \text{ satisfying (22)}, \forall t = 0, 1, .., \infty\}. \tag{23}$$

Proposition 3. *The viability kernel is defined as follows:*

- *When* $Q_W > \overline{Q}_{\widetilde{W}}$, *the viability kernel is empty* $Viab = \emptyset$.
- *When* $Q_W \leq \overline{Q}_{\widetilde{W}}$, *the viability kernel is* $Viab = \left[H_{\widetilde{W}}, H_{\max}\right]$.

Proof. see appendix. ■

The intuition of this result relies on the fact that $\overline{Q}_{\widetilde{W}}$ is located on the steady-state water balance given by eq (4). Then an objective of water extraction for irrigation Q_W which is always above the steady-state water value $\overline{Q}_{\widetilde{W}}$ is not viable since the water table always decreases. The next step is to determine the amount of water which can be allocated at each period by the water agency.

3.3.3. The viable quotas

The water agency faces a dynamic problem when setting its quota supply. To ensure that the amount of water extraction $Q(t)$ can be at least allocated at every period, the water agency needs to satisfy the intertemporal viability constraint

$$H(t+1) \geq H_{\widetilde{W}}. \tag{24}$$

Using (3), condition (24) yields

$$\left(1 - \frac{\gamma}{AS}\right) H(t) + \frac{R + \gamma H_{\min}}{AS} - \frac{(1-\mu)}{AS} Q(t) \geq H_{\widetilde{W}}.$$

It implies a superior bound on the quota the water agency can allocate

$$Q(t) \leq Q_D(H(t)),$$

with

$$Q_D(H(t)) = \frac{AS - \gamma}{1 - \mu}\left(H(t) - H_{\widetilde{W}}\right) + \overline{Q}_{\widetilde{W}}.$$

This superior bound is an affine and increasing function of the water table $H(t)$. By definition, $H(t) = H_{\widetilde{W}}$ entails $Q_D(H_{\widetilde{W}}) = \overline{Q}_{\widetilde{W}}$. It gives the following proposition.

Proposition 4. *Under the environmental constraint* $W_f(t) \geq \widetilde{W_f}$*, the viable quotas are*

$$Q_{\text{Viab}}(t) = [Q_W, Q_D(H(t))] .$$

An example of viable quota policy is to consider the following policy rule

$$Q_{\text{Viab}}(t) = \rho Q_W + (1-\rho) Q_D(t), \tag{25}$$

where $0 \leq \rho \leq 1$ stands for the trade-off coefficient of the water agency. This coefficient reflects the preferences of the regulating agency between extraction and conservation. A low value of ρ favors extraction and rent while a high value favors the resource and then GDE. For $\rho = 0$, the viable policy $Q_{\text{Viab}} = Q_D$ can only be implemented in a single period t since it implies $H(1) = H_{\widetilde{W}}$ and by definition the only water extraction which maintains the water table at $H_{\widetilde{W}}$ is $\overline{Q}_{\widetilde{W}}$. In the opposite case $\rho = 1$, the amount of water extraction Q_W is implemented. When $Q_W < \overline{Q}_{\widetilde{W}}$, the choice of ρ impacts the long term values of the water table, the extraction level and the environmental flows. In the particular case $Q_W = \overline{Q}_{\widetilde{W}}$, coefficient ρ only determines the speed at which the different variables of interest move towards their long term values. In that particular case, changes in the water table decrease at a constant rate $H(t+1) - H(t) = -(\gamma/AS)(H(t) - H_{\widetilde{W}})$ until $H(t)$ tends towards its equilibrium value $H_{\widetilde{W}}$. It is also possible to consider that ρ is time-dependant with $\rho(t)$.

Corollary 3. *For every value of the environmental flows target* $\widetilde{W_f} \in [0, R]$ *corresponds a critical water table* $H_{\widetilde{W}} \in [H_{\min}, H_{\max}]$ *and a sustainable water quota* $\overline{Q}_{\widetilde{W}} \in \left[0, \frac{R}{1-\mu}\right]$.

(3) is a direct consequence of the aquifer water budget at the steady state when the constraint H_{lim} is binding in the viable case. The choice of an environmental flow target is a critical issue for policy making. It determines how much water is needed to achieved environmental objectives. Comparisons between the optimal and viable management scenarios will give insights on the trade-off between environmental flows and irrigation.

4. Results

The three management scenarios are numerically tested using data on the Western la Mancha aquifer in Spain. For a thorough description of the hydrogeologic and management context, the reader may refer to *Martinez-Santos et al.* (2008), *Closas et al.* (2017) and *Martinez-Santos et al.* (2018).

4.1. Parameter values

Parameters values given in Table 1 are mainly provided by *Esteban and Albiac* (2011; 2012), *Esteban and Dinar* (2013; 2016).

Parameters	Description	Units	Value
g	intercept of the water demand fct	€/Mm^3	4400.73
k	slope of the water demand fct	€/Mm^3	0.097
c_0	intercept of the pumping cost fct	€/Mm^3	266000
c_1	slope of the pumping cost fct	€/Mm^{3}m	400
μ	return flow coefficient	-	0.2
AS	aquifer area×storage coefficient	Mm^2	126.5
R	natural recharge	Mm^3	360
$H_{\max}$	maximum water level	m	665
$H_{\min}$	minimum water table	m	600
β	discount factor	Year^{-1}	[0.96,0.98]
γ	slope of the nat. drainage fct	€/Mm^3	5.53
φ	cost of capture	€/Mm^3	[0,30000]
θ	cost of ecosystem damage	€/m	[0,50000]

For the optimal control problem, the discount factor denoted by β in (15) is a key sensitive parameter. This point has already been highlighted by *Brill and Burness* (1994) and again more recently by *Koundouri et al.* (2017) in the groundwater literature. The discount factor favors the present since it gives a smaller value to profit or utility made in the future. A low value of the discount factor means a higher preference for the present and thus higher extraction. On the opposite, a high value of the discount factor favors the conservation of the resource. The literature often considers an interval between 0.96 and 0.98 for β or equivalently a discount rate between 2% and 4%. The second series of sensitive parameters describe the monetary value of the capture cost and the damage cost associated with non-consumptive uses, namely φ and θ in (16). φ measures the cost of damages to ecosystems for each cubic meter of depletion while θ refers to another kind of damage cost related to the difference between the maximum (initial) and the current aquifer levels. In *Esteban and Albiac* (2011; 2012), the environmental damage of depletion (φ in our case) is estimated to 0.03 €$/m^3$ and is based on the recreational value of the wetland Tablas de Daimiel obtained by *Judez et al.* (1998) using contingent valuation and travel cost methods. *Esteban and Dinar* (2016) mention a discounted value of 538 510 €, which corresponds to the fee estimated around 6 €(in 1996) that annual visitors (an average of 86 270 over the period 1992-1996) are willing to pay. The cost of 0.03 €/m^3 is obtained from the recreational value divided by the aquifer water storage ($AS.H_0$). A sensitivity analysis of this cost parameter is conducted by the authors over the large interval $[0.005; 0.5]$ €/m^3. We choose the value of 0.03 €/m^3. In *Esteban and Dinar* (2013), the value of water for the ecosystem damages has been estimated to 50 000 €per meter of aquifer depletion (θ in our case). A sensitivity analysis of the parameter θ is conducted by the authors with the values of 25 000 to 100 000 €per meter of depletion. We consider the value of 50 000 €/m.

In the viable approach, the two key parameters are given by the value of the environmental flows target and the trade-off coefficient of the water agency. Using the simulation results in the optimal case, we look at how it is possible to obtain relevant values for these parameters.

4.2. Trajectories

Figure (2) compares the myopic competition case with the optimal one when farmers internalize the pumping cost externality but not the environmental externality (the parameters φ and θ are set to 0). As expected in the myopic case, the high amount of water extraction implies a long term water table $\overline{H}_{mc} = 568$ m which is below the critical level $H_{\min} = 600$ m needed to preserve the GDE. Condition (14) in Corollary 1 is not satisfied and implies $\overline{H}_{mc} < H_{\min}$. It can be show that the steady state water table $\overline{H}_{mc}$ is higher than the water table in the open access case which ensures a nil net benefit. From eq (9), extraction is nil when $H = H_{\max} - g/(kc_1) = 551$ m.

According the value of the discount factor, the optimal outcome may preserve the ecosystem. A high value of the discount rate equal to $\beta = 0.98$ maintains the water table above $H_{\min}$ with $\overline{H}_{oc} = 604$ m contrary to a lower value $\beta = 0.96$ which gives $\overline{H}_{oc} = 598$ m (see first result of Corollary 2). It shows how sensitive are the results to this key parameter. In the first case, the water extraction and the level of environmental flows are equal to $\overline{Q}_{oc} = 421$ Mm^3 and $\overline{W_f}_{oc} = 23$ Mm^3. In the second case, a lower discount factor implies a higher amount of extraction with $\overline{Q}_{oc} = 461$ Mm^3 and the collapse of the ecosystem after 18 periods. Results also show that the difference in the water table is significant between the myopic and the optimal cases when the natural drainage both appears in the water balance and in the welfare function.

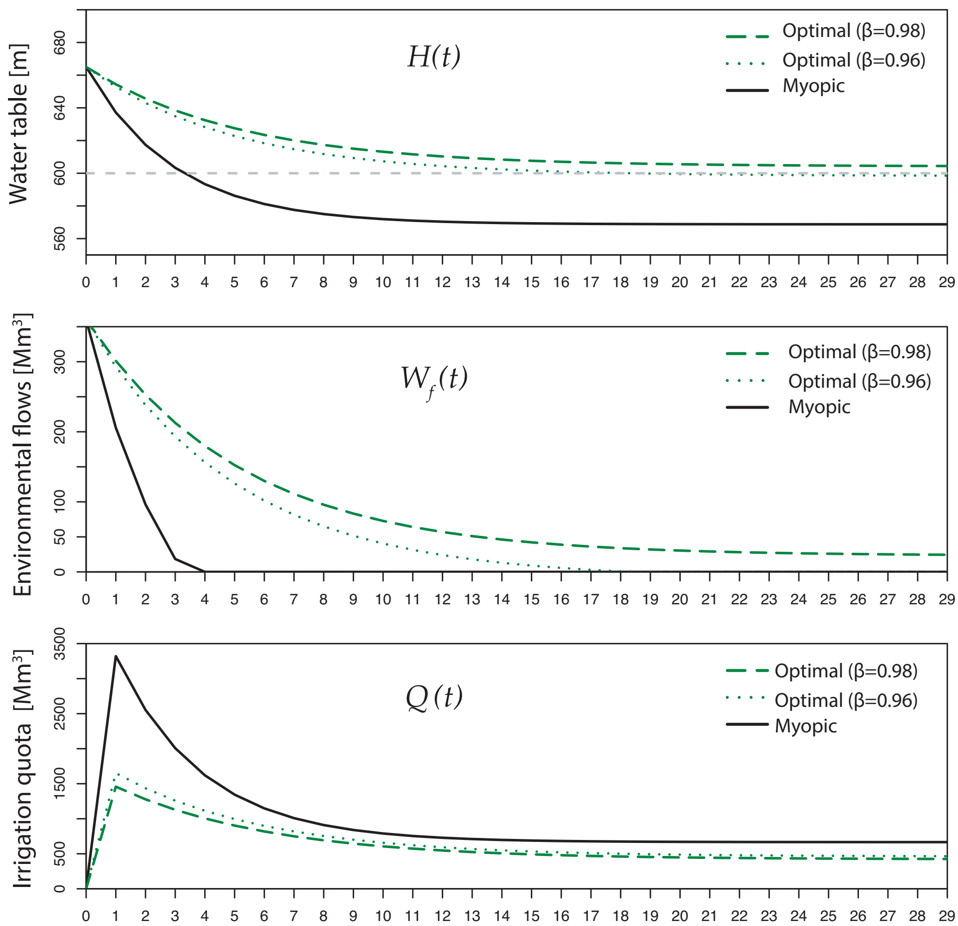

Figure 2: Comparison between the myopic competition and the optimal control model with two discount factor $\beta = 0.96$ and $\beta = 0.98$ for $\varphi = 0$ and $\theta = 0$.

Figure (3) considers the myopic case and the two management options, the optimal control case when the pumping cost and the environmental externality are internalized by the regulating agency and the viable case with a nil environmental constraint $\widetilde{W_f} = 0$ and a trade-off coefficient equal to $\rho = 0.7$ when $Q_W = \overline{Q}_{\widetilde{W}}$. The myopic case didn't satisfied the environmental constraint as before since the water table lies below its minimal value H_{min}. In the viable case, the constraint H_{min} is always satisfied by definition but environmental flows reached zero after 20 years while it was after 5 years in the myopic case. With a discount factor $\beta = 0.98$, a cost of capture equal to 30000 €/Mm^3 and a cost of damage equal to 50 000 €/m, the steady state value of the optimal water table, water extraction and environmental flows are respectively equal to $\overline{H}_{oc} = 632$ m, $\overline{Q}_{oc} = 222$ Mm^3 and $\overline{Wf}_{oc} = 182$ Mm^3. This amount of environmental flows represents 40% of the net recharge $R/(1-\mu)$. Of course considering a lower value for the discount factor and the costs of capture and ecosystem damages will give a lower amount of environmental flows (as already shown in Figure (2) for $\beta = 0.96$, $\varphi = 0$ and $\theta = 0$).

Comparing trajectories in the optimal and viable case requires to set a values to the environmental flows target $\widetilde{W_f}$ in (21). This target determines how much water is needed to achieved the environmental objectives. Figure 4 illustrates Corollary 3 and shows for different values of $\widetilde{W_f}$, from zero to the natural recharge R, the long term values of the water table and the allocated quota for farmers. A higher environmental target requires a higher water table at the expense of lower water for irrigation. When $\widetilde{W_f} = 0$, the long term water table is at its minimum $H_{\min} = 600$ m and the long term quota table at its maximum $R/(1-\mu) = 450$ Mm^3. On the opposite when $\widetilde{W_f} = R$, the long term water table is at its maximum $H_{\max} = 665$ m and the long term quota is nil.

To analyse the impact of the trade-off coefficient ρ on the long term values of the water table and the irrigation quota, we assume that the water agency chooses a specific value of $\widetilde{W_f} = R/5 = 72$ Mm^3. To this value corresponds the long term values of the water table $H_W = 613$ m and the quota $\overline{Q}_{\widetilde{W}} = 360$ Mm^3. We also assume that the water agency implements a quota $Q_W = 300$ which is lower than the long term quota $\overline{Q}_{\widetilde{W}}$ associated to $\widetilde{W_f} = R/5 = 72$ Mm^3. Figure 5 illustrates the sensitivity of the what are the long term values of the different variables to the trade-off coefficient ρ. For a nil value of ρ, the quota $\overline{Q}_{\widetilde{W}} = 360$ Mm^3 is allocated and the water table is $H_W = 613$ m. However when $\rho = 1$, the allocated quota $Q_W = 300$ Mm^3 corresponds to a

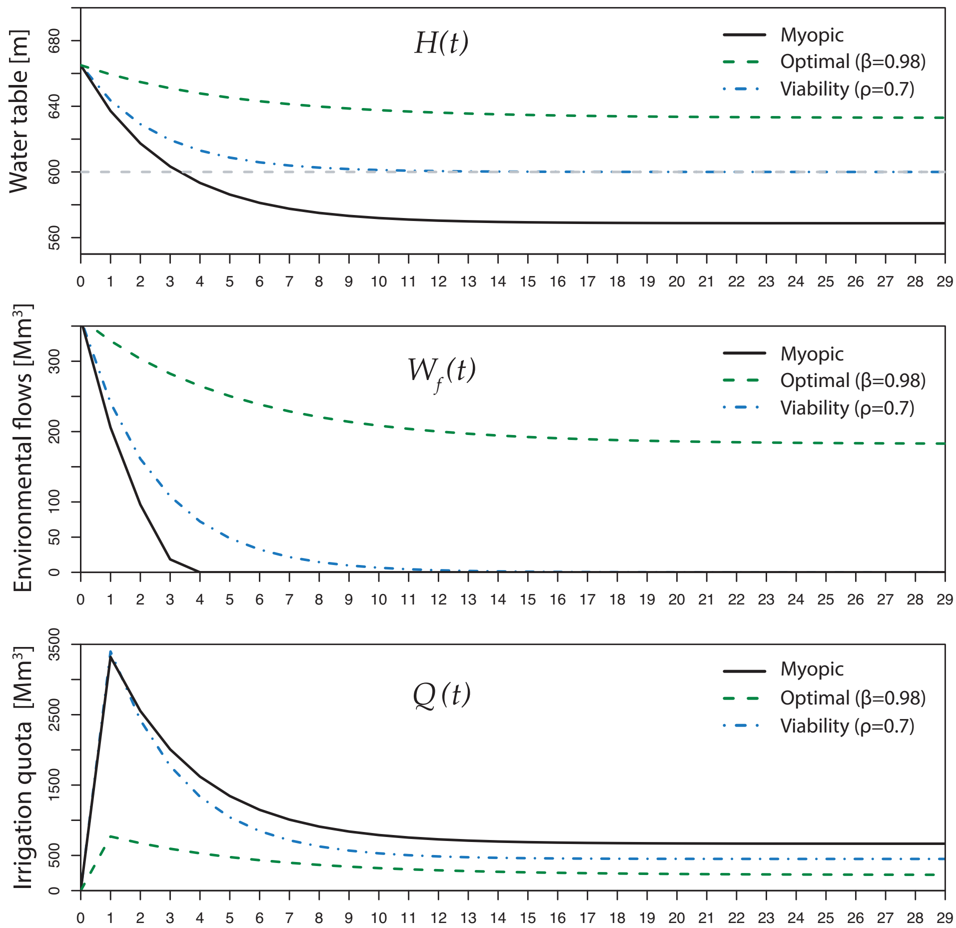

Figure 3: Comparison between the myopic competition, the optimal control model (for $\beta = 0.98$, $\varphi = 30000$ €/Mm^3 and $\theta = 50000$€/m) and the viable model with a nil environmental flow target ($\widetilde{W_f} = 0$)

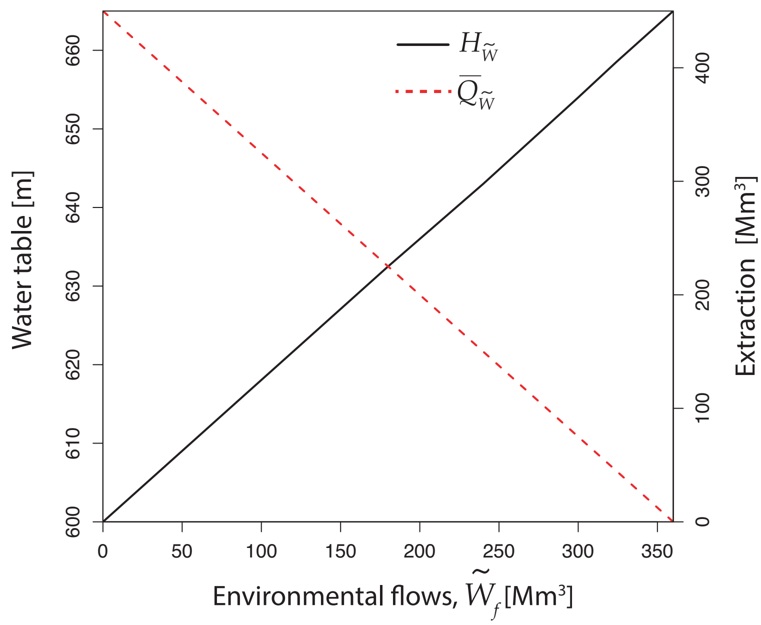

Figure 4: The long term values of the water table and the quota for different values of $\widetilde{W_f}$ illustrate the competitive interests between groundwater extraction and the preservation of environmental flows when the constraint $H_{\lim}$ is binding.

water table $H_W = 621$ m and $\widetilde{W_f} = R/3 = 120$ Mm^3. It shows that a high value ρ preserves the ecosystem at the expense of irrigated water while a low value favors extraction and rent as explained in Proposition (4).

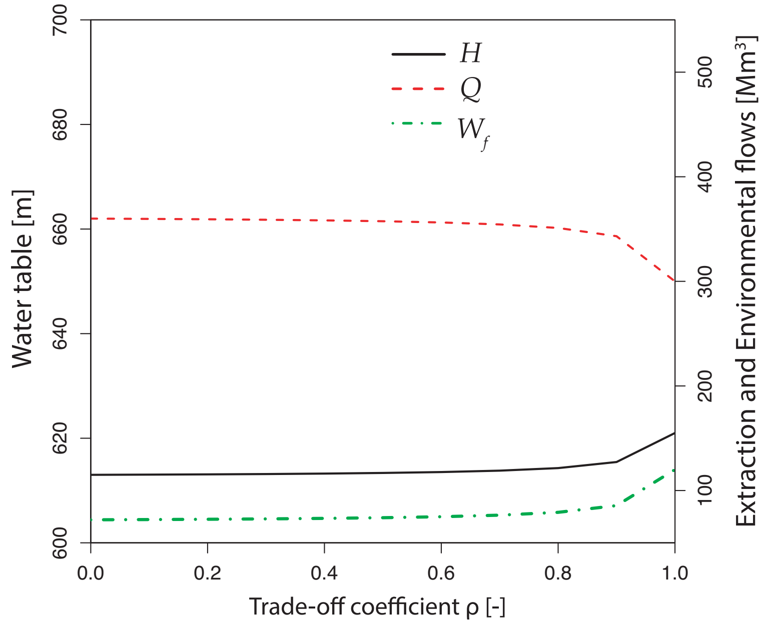

Figure 5: Impact of the trade-off coefficient on the water table, quota and environmental flows

To make comparisons between the optimal and viable outcomes, we have decided to take for the environmental flows target in the viable approach, the long term value for the environmental flow obtained in the optimal control problem with $\beta = 0.98$, $\varphi =$30 000 €/Mm^3 and $\theta =$50 000 €/m. We choose $\widetilde{W_f} = \overline{W_f}_{oc} = 182$ Mm^3. Figure (6) shows the viable corridor of feasible quota policies depending several values of the trade-off coefficient ρ in (25) in the particular case of $Q_W = \overline{Q}_{\widetilde{W}}$. The corridor lies between the two extreme values $\rho = 0$ and $\rho = 1$. In the first case extraction in the first period is such that the water table reaches its steady-state in one period while in the second case the water agency implements immediately the steady-state extraction $Q_W = \overline{Q}_{\widetilde{W}}$ which favors the resource and the ecosystem. We can show that the trajectories obtained in the control optimal case shown in Figure (3) and reproduced in Figure (6) is a particular one in the viable corridor. More precisely we can show that the optimal trajectory lies between the trade-off coefficient $\rho = 0.8$ and $\rho = 0.9$. It shows that the set of parameters we have considered in the optimal control approach favor the preservation of the resource and correspond to viable conservative policies.

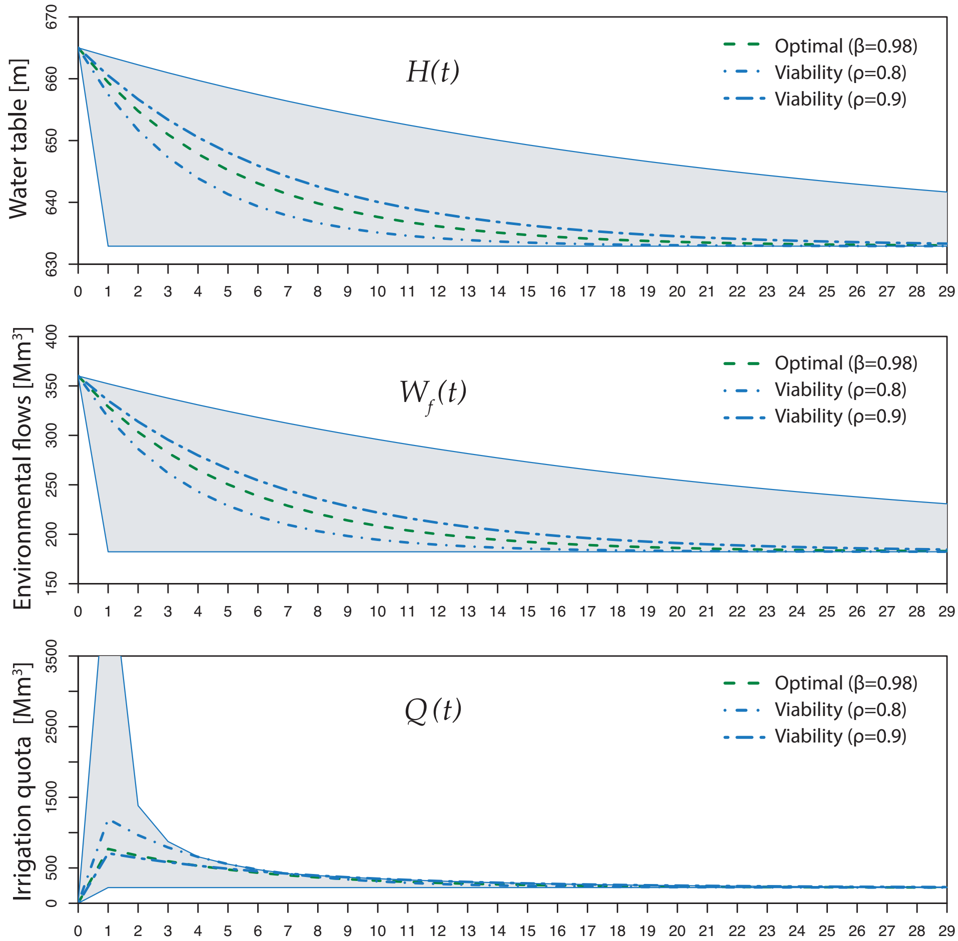

Figure 6: Comparison between the optimal control model (for $\beta = 0.98$, $\varphi = 30000$ €/Mm^3 and $\theta = 50000$€/m.) and the viable model with $\rho = 0.8$ and $\rho = 0.9$ for a same long term value of environmental flows $\widetilde{W_f} = \overline{W_f}_{oc} = 182$ Mm^3. The grey areas correspond to the corridor of all viable trajectories for ρ ranging between 0 and 1.

Figure (7) shows for two discount factors $\beta = 0.96$ and $\beta = 0.98$, the positive relationship between the cost of capture in €/Mm^3 (with no ecosystem damages) and the associated optimal long term level of water flows in Mm^3 for the ecosystem. This figure at the steady state can be interpreted in two ways. It shows that for a given value of the cost of capture obtained by state preference valuation methods, the amount of environmental flows increases with the value of the discount factor. For a cost of capture equal to 20 000 €/Mm^3, the amount of environmental flows is 104.69 Mm^3 when $\beta = 0.98$ and only 58.76 Mm^3 when $\beta = 0.96$. Another interpretation is to consider that for a given target on the environmental flows in Mm^3, the water agency is able to determine the monetary value of the environmental externality. For a target equal to $\widetilde{W_f} = \overline{W_{f_{oc}}} = 60$ Mm^3, the monetary cost will be estimated around 10000 €/Mm^3 when $\beta = 0.98$ and around 20 000 €/Mm^3 when $\beta = 0.96$. It means that when the discount factor is low, the society favors extraction at the expense of the environment. As a consequence the monetary cost of the environmental externality has to be strongly increased to protect the environment.

4.3. Discussion

The question of how to allocate water for environmental flows and irrigation is crucial to ensure a sustainable management of groundwater and the dependent ecosystems. The optimal and viable scenario differ in the way the allocation trade-off is managed. In the optimal scenario the environmental externality is introduced in the welfare function of the water agency and thus requires to evaluate the environmental flows in monetary terms while no monetary evaluation is needed in the viable approach.

Two lessons can be drawn from this methodological analysis. A first lesson is to show how the results are sensitive to the choice of some key parameters. A second lesson relies on the ability for a water agency to implement viable policies. Trajectories in the control approach appear to be highly sensitive to the values of the discount factor and the environmental externality. The evaluation of the ecosystem damages or benefits is often obtained either by state preference methods or benefit transfer methods in a context of data poor circumstances (*Plummer* 2009, *Momblanch et al.* 2016). When the result of this evaluation is used to calibrate optimal groundwater management models (*Esteban and Albiac* 2011, *Esteban and Dinar* 2013), we have shown that the amount of the long term value of the environmental flows for a given value of the cost of capture can be multiplied by around two times when the discount factor increases from $\beta = 0.96$ to $\beta = 0.98$. Finding relevant estimates of the environmental externality also appears to

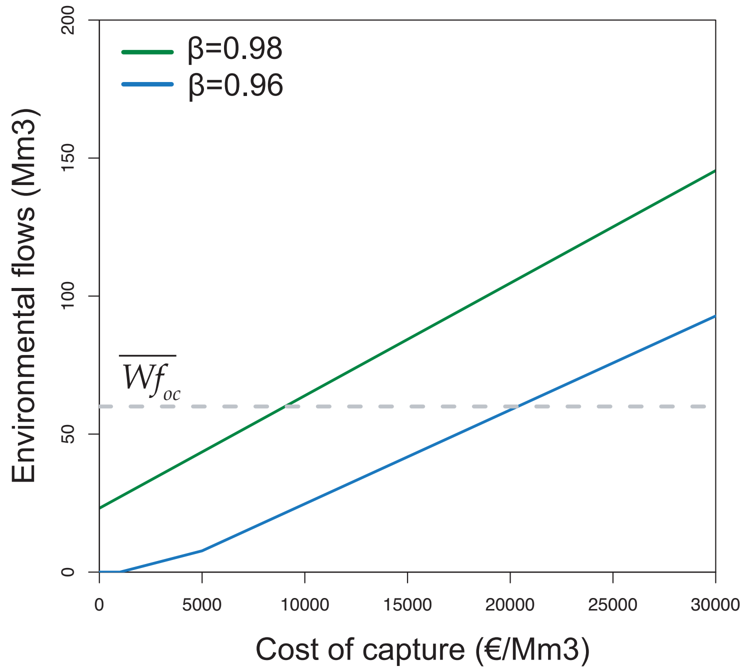

Figure 7: Relationship at the steady state between the cost of capture and value of environmental flows for a discount factor $\beta = 0.96$ and $\beta = 0.98$

be difficult. The range of the cost of the capture in the literature lies from the value of 0.005 €/m^3 to 0.5 €/m^3. We have assumed as in *Esteban and Albiac* (2011) a value of 0.03 €/m^3. However *Closas et al.* (2017) report that during the drought period between 2005-2008, the river basin authority purchased water use rights to farmers. They show that between November 2006 and November 2009, 66 millions €have been spent to purchase 29.063 Mm^3 to farmers. It shows that the price paid by the water authority to farmers for a reduction in extraction takes the value of 0.44 €/m^3. It implies an excessive water compensation price with respect to the value of the environmental externality. This purchase program stops in 2013. *Martinez-Santos et al.* (2018) add that the compensation payments received by farmers to restore the ecosystems were ultimately being used as an excuse to perpetuate aquifer overdraft and then were detrimental to ecological status of the GDEs.

A second lesson of our analysis focuses on how a regulating agency can comply with her choice of the environmental flow target when implementing her water management policy. By definition, the viability approach sustains the natural drainage and GDEs since the constraint to maintain the water table above $H_{\min}$ will always be satisfied. However, the question of how much water is needed to achieved specific environmental objectives remains open. A first issue concerns the assessment of environmental flows which require to quantify the complex linkages between hydrological processes and components and various ecological variables or ecosystem services. Several papers combining hydro-ecological-economic models have been used but they concerned the management of river by dams and tanks (*Grafton et al.* 2011, *Connor et al.* 2013, *Akter et al.* 2014). Assessment methods are numerous in river management but the effects on groundwater extraction on surface water through the decrease in water drainage to streams are difficult to determine. It requires to quantify the response of aquifer to extraction and then the response of dependent ecosystems to changes in the water table and the natural drainage. However, even if the environmental flows target can be set based on ecological and hydrogeological grounds, a second issue concerns its implementation which requires water allocation mechanisms between several consumption uses like irrigation and the environment. It can take the form on a legal right to water for the environment itself to ensure that environmental flows are not the water which remains after the consumption of the other users. A simple management rule for base flows targets can be expressed as a percentage of natural levels or historical flows implying that all the adjustment to changes in the natural recharge will be done by changes in the other uses. *Mulligan et al.* (2014) p2265 assume for instance that the stream flow constraint values are chosen as 75% of the 1990-2000 average flow. This legal right to water also means that even in a cap and trade system, water for the environment has to be excluded from the quota which will be allocated by the water agency for other purposes like irrigation (*Pereau* 2018). The logic behind the viability approach supports this objective of satisfying in priority the flow needs of the ecosystems and then allocating the remaining water on other consumptive uses. But this choice implies an implicit monetary trade-off between the different uses. We have shown that the value of environmental flows target in the viable approach corresponds to a societal value of the environmental externality in monetary terms in the control approach. It allows to investigate the trade-off between water for irrigation and environmental objectives. Moreover, the choice of the trade-off coefficient in the management rule of the water agency also allows flexibility in the allocation of water. Introducing flexibility on the environmental flow target and the weight of the policy rule may increase the compliance of the different stakeholders to the viable policy. This is a key issue since the measurement and control of private extraction rates, in particular for agricultural purposes, is a real challenge (*Dinar and Mody* 2004). Finally, it should be noted that our viability analysis has been developed in a deterministic framework. A third issue is to consider the consequences of a stochastic natural recharge availability for the regulating agency. As shown in *Doyen and Delara* (2010), stochastic viability consists in introducing uncertainty with some probability in the dynamic system. It will concern the natural recharge in our theoretical framework. The objective of the water agency is then to maximise the probability that the constraint on the water table is satisfied throughout time. Stochastic viability analysis have been applied to the management of renewable resources like fisheries (*Cisse et al.* 2015) or reservoir (*Chu et al.* 2018). In particular these authors have analysed the conditions under which a water agency can increase storage capacity in wet seasons to release water in dry seasons. Such a management can ensure the inundation of different wetlands and the ecosystem services they provide. In a robust optimal control model of groundwater management (without environmental flows), *Roseta-Palma and Xepapadeas* (2004) show that for low levels of precipitation, a water manager might decide to use less water as a precaution. Uncertainty makes more difficult the management of groundwater by the water agency. In a control optimal problem, *de Frutos Cachorro et al.* (2014) have also shown that higher uncertainty on the occurrence date of a regime shift may decrease the precautionary behavior in the short run and increase precautionary behavior in the long run.

5. Conclusion

This paper has investigated how different groundwater management scenarios deal with the protection of environmental flows. In a stylised hydro-economic model, the natural drainage which sustains these environmental flows has been explicitly introduced in the water balance equation (*Gisser and Sanchez* 1980a, *Pereau and Pryet* 2018). A critical value for the water table has be defined, $H_{\min}$, for which the environmental flows are nil. The capacity of three management policies to maintain the water table above this critical value has been studied and compared in the case-study of the Western La Mancha aquifer (*Esteban and Albiac* 2011, *Esteban and Dinar* 2013). In the myopic competition regime which refers to an unregulated case, our results show under which conditions the steady-state water table is below the critical level $H_{\min}$. When environmental flows appear as an externality in the welfare function of a water agency, the water table in the optimal control case is always greater than the myopic competition one. We also show that the water table in the optimal control increases with the discount factor and the environmental externality and this decreases the likelihood of collapse of the groundwater dependent-ecosystems. Based on the parameters used by *Esteban and Albiac* (2011), *Esteban and Dinar* (2013), our results show that the optimal control approach yields positive environmental flows in the long term. Our results also put forward the corridor of viable quota policies in the viability approach and show how the water agency deals with the trade-off between economic objectives and the maintenance of the ecosystems. The trajectories obtained in the optimal control case appear as particular paths of the viable approach. It allows us to show how the choice of the environmental flow target in the viable control approach can be associated to the environmental externality in monetary terms in the optimal control approach.

This work can be extended to several issues. The hydro-economic bathtub model with an explicit modelling of the natural discharge which sustains flows for the ecosystems remains a stylised representation of integrated groundwater-surface systems. Minimizing the impacts of groundwater extraction on streamflow requires to consider conjunctive management of groundwater and surface water resources which take into account the residence times and the fluxes between the aquifer and the ecosystems (lake, river, wetlands) (*Pulido-Velázquez et al.* 2006, *Stahn and Tomini* 2017). Moreover, the shape of the flow-ecology relationship can be linear, convex with critical thresholds from which keys elements of the ecosystems are impaired or lost. This relation can also depend on the type of the ecosystems, wetland, lakes, rivers. A challenge for future work is then to develop robust assessment which can be transferrable to other ecosystems. Of interest is also to deal with the time-varying nature of the water flows and analyse its impact on the management strategies (*Krawczyk and Tidball* 2006, *Chu et al.* 2018).

6. Appendix

6.1. Proof of Proposition (2)

The optimal maximisation program can be rewritten in a more general form

$$\max_{Q(t)} \sum_{t=0}^{\infty} \beta^t \left(e_0 + e_1 Q(t) + e_2 H(t) + e_3 H(t) Q(t) + e_4 Q^2(t)\right), \tag{26}$$

under the dynamics

$$\begin{aligned} H(t+1) &= \kappa_0 H(t) + \kappa_1 - \kappa_2 Q(t), \\ H(0) &= H_0, \end{aligned}$$

where coefficients e_i and κ_i are equal to $e_0 = -d_0$, $e_1 = \frac{g}{k} - c_0$, $e_2 = d_1$, $e_3 = c_1$, $e_4 = \frac{1}{2k}$, $\kappa_0 = \frac{R+\gamma H_{\min}}{AS}$, $\kappa_1 = \frac{1-\mu}{AS}$ and $\kappa_2 = 1 - \frac{\gamma}{AS}$. Instead of the specification in terms of ecosystem damages, the optimal control problem can be defined in terms of ecosystem health benefits as proposed by *Esteban and Dinar* (2016). The welfare function can be rewritten as $WF = NB(t) + B(H(t))$ with the following benefit function

$$B(H(t)) = \xi\left(\sigma - \eta\left(H_{\max} - H(t)\right)\right)$$

or equivalently

$$B(H(t)) = -b_0 + b_1 H(t), \tag{27}$$

with $b_0 = -\xi\left(\eta H_{\max} - \sigma\right) > 0$ and $b_1 = \xi\rho > 0$. In both specification of the environmental flows, this term refers to the indirect ecosystem value resulting from the drainage to ecosystems. With the ecosystem benefit function, only these coefficients are changed $e_0 = -b_0$ and $e_2 = b_1$ in the maximisation program (26).

Following *Burt* (1967), *Pereau and Pryet* (2018), the resolution strategy is to rewrite the maximisation problem (26) in such a way that only terms in the state variable will appear. Using the dynamics, we obtain the values of $Q(t)$ and $Q(t-1)$

$$\begin{aligned} Q(t) &= \frac{\kappa_0}{\kappa_1} + \frac{\kappa_2}{\kappa_1} H(t) - \frac{1}{\kappa_1} H(t+1) \\ Q(t-1) &= \frac{\kappa_0}{\kappa_1} + \frac{\kappa_2}{\kappa_1} H(t-1) - \frac{1}{\kappa_1} H(t) \end{aligned}$$

which can be substituted in (26). Taking the derivative wrt $H(t)$ gives the second-order equation

$$H(t+1) - 2\left(1 + \frac{1-\beta}{2\beta} + \varepsilon\right) H(t) + \frac{1}{\beta} H(t-1) \tag{28}$$

$$= -\left(\frac{\left(2e_4\frac{\kappa_0}{\kappa_1} - e_1\right)(1-\beta\kappa_2) + \beta\left(e_2\kappa_1 + e_3\kappa_0\right)}{\beta\left(2e_4\frac{\kappa_2}{\kappa_1} - e_3\right)}\right) \tag{29}$$

with

$$\varepsilon = \frac{\left(2e_4\left(1-\kappa_2\right)\left(1-\beta\kappa_2\right) + \kappa_1 e_3\left(1+\beta\left(1-2\kappa_2\right)\right)\right)}{2\beta\left(2\kappa_2 e_4 - \kappa_1 e_3\right)}$$

The term $\varepsilon > 0$ is positive for $e_3 < \frac{2\kappa_2 e_4}{\kappa_1}$ which can be rewritten as $c_1 < \frac{AS-\gamma}{(1-\mu)k}$. It corresponds to the same condition derived in the myopic competition (12). Solving (29) gives Proposition (2).

Based on the expression of the water table at the steady state (19), we obtain the following results

$$\frac{\partial \overline{H}_{oc}}{\partial d_1} = \frac{\left(\frac{1-\mu}{AS}\right)}{2\varepsilon\left(\frac{1}{k}\frac{AS-\gamma}{1-\mu} - c_1\right)} > 0$$

$$\frac{\partial H^{ss}}{\partial \beta} = \frac{ASk\left(1-\mu\right)^2\left(\left(\left(R+\gamma H_{\min}\right)kc_1 - \gamma\left(kc_0 - g\right)\right)c_1 + \left(\gamma + \left(1-\mu\right)kc_1\right)d_1\right)}{\left(\left(\beta\gamma + AS\left(1-\beta\right)\right)\gamma + \left(AS\left(1-\beta\right) + 2\beta\gamma\right)\left(1-\mu\right)kc_1\right)^2} > 0$$

and $\overline{H}_{oc} > \overline{H}_{mc}$ since

$$0 < \frac{\left(1-\mu\right)^2 k\beta\left(\left(1+\left(1-\mu\right)kc_1\right)d_1 + \left(\left(R+\gamma H_{\min}\right)kc_1 - \gamma\left(kc_0-g\right)\right)c_1\right)}{\left(\gamma + \left(1-\mu\right)kc_1\right)\left(\left(1-\beta\right)A\gamma + \beta\gamma^2 + \left(1-\beta\right)\left(1-\mu\right)Akc_1 + 2\left(1-\mu\right)k\beta\gamma c_1\right)}$$

using condition (13) to ensure that the numerator is positive even when $d_1 = 0$.

6.2. Proof of Proposition (3)

Based on the dynamics

$$H(t+1) = \left(1 - \frac{\gamma}{AS}\right) H(t) + \frac{R + \gamma H_{\min}}{AS} - \frac{1-\mu}{AS} Q(t)$$

together with the steady state water balance

$$\overline{Q}_{\widetilde{W}} = \frac{R + \gamma H_{\min}}{1-\mu} - \frac{\gamma}{1-\mu} H_{\widetilde{W}}$$

it yields

$$H(t+1) = \left(1 - \frac{\gamma}{AS}\right) H(t) + \frac{\gamma}{AS} H_{\widetilde{W}} - \frac{1-\mu}{AS}\left(Q(t) - \overline{Q}_{\widetilde{W}}\right)$$

First, we show that If $Q(t) = Q_W$ with $Q_W \leq \overline{Q}_{\widetilde{W}}$ then $Viab = \left[H_{\widetilde{W}}, H_{\max}\right]$. We consider the case $Q_W = \overline{Q}_{\widetilde{W}}$. It gives the following sequence

$$\begin{aligned} H(1) &= \left(1 - \frac{\gamma}{AS}\right) H_0 + \frac{\gamma}{AS} H_{\widetilde{W}} - \frac{1-\mu}{AS}\left(\overline{Q}_{\widetilde{W}} - \overline{Q}_{\widetilde{W}}\right) \\ &\vdots \\ H(t) &= \left(1 - \frac{\gamma}{AS}\right)^t H_0 + \frac{\gamma}{AS}\left(1 + \left(1 - \frac{\gamma}{AS}\right) + ... + \left(1 - \frac{\gamma}{AS}\right)^{t-1}\right) H_{\widetilde{W}} \end{aligned}$$

When $t \to \infty$ then $H(t) \to H_{\widetilde{W}}$.

Second, we show that If $Q(t) = Q_W$ with $Q_W > \overline{Q}_{\widetilde{W}}$ then $Viab = \emptyset$. It gives the following sequence

$$\begin{aligned} H(1) &= \left(1 - \frac{\gamma}{AS}\right) H_0 + \frac{\gamma}{AS} H_{\widetilde{W}} - \frac{1-\mu}{AS}\left(Q_W - \overline{Q}_{\widetilde{W}}\right) \\ &\vdots \\ H(t) &= \left(1 - \frac{\gamma}{AS}\right)^t H_0 + \frac{\gamma}{AS}\left(1 + \left(1 - \frac{\gamma}{AS}\right) + ... + \left(1 - \frac{\gamma}{AS}\right)^{t-1}\right) H_{\widetilde{W}} \\ &\quad - \frac{1-\mu}{AS}\frac{\gamma}{AS}\left(1 + \left(1 - \frac{\gamma}{AS}\right) + ... + \left(1 - \frac{\gamma}{AS}\right)^{t}\right)\left(Q_W - \overline{Q}_{\widetilde{W}}\right) \end{aligned}$$

When $t \to \infty$ then $H(t) = H_{\widetilde{W}} - \frac{1-\mu}{AS}\left(Q_W - \overline{Q}_{\widetilde{W}}\right) < H_{\widetilde{W}}$ since $Q_W > \overline{Q}_{\widetilde{W}}$.

7. Acknowledgements

This study has been carried out with financial support from the French National Research Agency (ANR) in the frame of the Investments for the future Program, within the Cluster of Excellence COTE (ANR-10-LABX-45). J-C Pereau thanks Chris Scott and the Udall Center for Studies in Public Policies, University of Arizona. We would like to thanks the editor and the two referees for their constructive comments.

8. References

Akter, S., R. Grafton, and W. Merritt, Integrated hydro-ecological and economic modeling of environmental flows: Macquarie Marshes, Australia, *Agricultural Water Management*, *145*, 98–109, 2014.

Alais, J.-C., P. Carpentier, and M. De Lara, Multi-usage hydrpower single dam management: Chance-constrained optimization and stochastic viability, *Energy Systems*, *8*, 7–30, 2017.

Alley, W., T. Reilly, and O. Franke, Sustainability of ground-water resources, *Tech. rep.*, US Geological Survey Circular 1186, 1999.

Aubin, J.-P., A survey of viability theory, *The SIAM Journal on Control and Optimization*, *28*(4), 749–788, 1990.

Bene, C., and L. Doyen, From Resistance to Transformation: A Generic Metric of Resilience Through Viability, *Earth's Future*, *6*, 979–996, 2018.

Bene, C., L. Doyen, and D. Gabay, A viability analysis for a bio-economic model, *Ecological Economics*, pp. 385–96, 2001.

Booker, J., R. Howitt, A. Michelsen, and R. Young, Economics and the modeling of water resources and policies, *Natural Resource Modeling*, *25*(1), 168–218, 2012.

Brander, L., J. Raymond, M. Florax, and J. Vermaat, The empirics of wetland valuation: A comprhensive summary and a meta-analysis of the literature, *Environmental and Resource economics*, *33*, 223–250, 2006.

Bredehoeft, J. D., The water budget myth revisited: why hydrogeologists model, *Ground Water*, *40*(4), 340–345, 2002.

Brill, T.-C., and H.-S. Burness, Planning versus competitive rates of groundwater pumping, *Water resources research*, *30*(6), 1873–1888, 1994.

Burt, O., Groundwater management under quadratic criterion function, *Water Resource Research*, *3*(3), 673–682, 1967.

Chu, L., Q. Grafton, and M. Stewardson, Resilience, Decision-making and Environmental Water Releases, *Earth's Future*, *6*, 777–792, 2018.

Cisse, A., L. Doyen, F. Blanchard, C. Bene, and J.-C. Pereau, Ecoviability for small-scale fisheries in the context of food security constraints, *Ecological Economics*, *119*, 39–52, 2015.

Closas, A., F. Molle, and N. Hernandez-Mora, Sticks and carrots to manage groundwater over-abstraction in la mancha, spain, *Agricultural Water Management*, *194*, 113–124, 2017.

Connor, J., B. Franklin, A. Loch, M. Kirby, and S. Wheeler, Trading water to improve environmental flow outcomes, *Water Resources Research*, *49*, 4265–4276, 2013.

de Frutos Cachorro, J., K. Erdlenbruch, and M. Tidball, Optimal adaptation strategies to face shocks on groundwater resources, *Journal of Economic Dynamics and Control*, *40*, 134–153, 2014.

Devlin, J., and M. Sophocleous, The persistence of the water budget myth and its relationship to sustainability, *Hydrogeology Journal*, *13*(4), 549–54, 2004.

Dinar, A., and J. Mody, Irrigation water management policies: allocation and pricing principles and implementation experiences, *Natural Resources Forum*, *28*(2), 2004.

Doyen, L., and M. Delara, Stochastic viability and dynamic programming, *Systems and Control Letters*, *59*(1), 629–634, 2010.

Eamus, D., R. Froend, G. Hose, and B. Murray, A functional methodology for determining the GW regime needed to maintain health of groundwater dependent vegetation, *Australian Journal of Botany*, *54*, 97–114, 2006.

Esteban, E., and J. Albiac, Groundwater and ecosystems damages: Questioning the Gisser-Sanchez effect, *Ecological Economics*, *70*, 2062–2069, 2011.

Esteban, E., and J. Albiac, The problem of sustainable groundwater management: the case of La Mancha aquifers (Spain), *Hydrogeology Journal*, *20*, 98–129, 2012.

Esteban, E., and A. Dinar, Cooperative Management of Groundwater Resources in the Presence of Environmental Externalities, *Environmental and Resource Economics*, *54*(3), 443–469, 2013.

Esteban, E., and A. Dinar, The role of groundwater-dependent ecosystems in groundwater management, *Natural Resource Modeling*, *29*(1), 2016.

Gisser, M., and A. Mercado, Integration of the agricultural demand function for water and the hydrologic model of the pecos basin, *Water Resources Research*, *8*(6), 1373–84, 1972.

Gisser, M., and D. Sanchez, Some additional economic aspects of ground water resources and replacement flows in semi-arid agricultural areas, *International Journal of Control*, *31*(2), 331–341, 1980a.

Gisser, M., and D. A. Sanchez, Competition versus optimal control in groundwater pumping, *Water resources research*, *16*(4), 638–642, 1980b.

Gopal, B., A conceptual framework for envionmental flows assessment based on ecosystem services and their economic evaluation, *Ecosystem services*, *21*, 53–58, 2016.

Grafton, Q., H. Chu, M. Stewardson, and T. Kompas, Optimal dynamic water allocation: Irrigation extractions and environmental tradeoffs in the murray river, australia, *Water Resources Research*, *47*, 1–13, 2011.

Griebler, C., and M. Avramov, Groundwater ecosystem services: a review, *Freshwater Science*, *34*(1), 355–367, 2015.

Guilfoos, T., A. Pape, N. Khanna, and K. Salvage, Groundwater management: the effect of water flows on welfare gains, *Ecological Economics*, *95*, 31–40, 2013.

Guilfoos, T., N. Khanna, and J. Peterson, Efficiency of viable groundwater management policies, *Land Economics*, *92*(4), 618–640, 2016.

Judez, L., R. de Andres, C. Perez Hulgade, E. Urzainqui, and M. Ibanez, Evaluation contingente de l'usage recreatif d'une reserve naturelle humide, *Cahiers d'economie et sociologie rurales*, *48*, 38–60, 1998.

Judez, L., R. de Andres, C. Perez Hulgade, E. Urzainqui, and M. Ibanez, Influence of bid and subsample vectors on the welfare measure estimate in dichotomous choice contingent valuation: evidence from a case study, *Journal of Environmental Management*, *60*, 253–265, 2000.

Katic, P., and Q. Grafton, Economic and spatial modelling of groundwater extraction, *Hydrogeology Journal*, *20*, 831–834, 2012.

Konikow, L., and S. Leake, Depletion and capture: revisiting the source of water derived from wells, *Groundwater*, *52*(1), 100–111, 2014.

Korsgaard, L., R. Jensen, T. Jonch-Clausen, D. Rosbjerg, and J. Schou, A service and value based approach to estimating environmental flows, *International Journal of River Basin Management*, *6*(3), 257–266, 2008.

Koundouri, P., Potential for groundwater management: Gisser-sanchez effect reconsidered, *Water resources research*, *40*(6), 2004.

Koundouri, P., C. Rosetta-Palma, and E. Englezos, Out of sight, not out of mind: Developments in economic models of groundwater management, *International Review of Environmental and Resource Economics*, *11*(1), 55–96, 2017.

Krawczyk, J., and M. Tidball, A discrete-time dynamic game of seasonal water allocation, *Journal of Optimization Theory and Applications*, *128*(2), 411–429, 2006.

Lohman, S. W., Definitions of selected ground-water terms, revisions and conceptual refinements, *Tech. rep.*, US Government Printing Office, 1972.

Martinez-Santos, P., L. de Stefano, M. Llamas, and P. Martinez-Alfaro, Wetland restoration in the mancha occidental aquifer, spain: A critical perspective on water, agricultural, and environmental policies, *Restoration Ecology*, *16*(3), 511–521, 2008.

Martinez-Santos, P., S. Castano, and A. Hernandez-Espriu, Revisiting groundwater overdraft based on the experience of the mancha occidental aquifer, spain, *Hydrogeology Journal*, *26*, 1087–1097, 2018.

Momblanch, A., J. Connor, N. Crossman, J. Paredes-Arquiola, and J. Andreu, Using ecosystem services to represent the environment in hydroeconomic models, *J Hydrol*, *538*, 293–303, 2016.

Mulligan, K., C. Brown, Y.-C. Yang, and D. Ahlfeld, Assessing groundwater policy with coupled economic-groundwater hydrologic modeling, *Water Resources Research*, *50*, 2257–2275, 2014.

Murray, B., M. Zeppel, G. Hose, and D. Eamus, Groundwater-dependent ecosystems in australia: it ' s more just water for rivers, *Ecol Manag Restor*, *4*(2), 110–113, 2003.

Oubraham, A., and G. Zaccour, A survey of applications of viability theory to the sustainable exploitation of renewable ressources, *Ecological Economics*, *15*, 346–367, 2018.

Pereau, J.-C., Conflicting objectives in groundwater management, *Water Resources and Economics*, p. In press, 2018.

Pereau, J.-C., and A. Pryet, Environmental flows in hydro-economic models, *Hydrogeology Journal*, *26*(7), 2205–2212, 2018.

Pereau, J.-C., L. Mouysset, and L. Doyen, Groundwater management in a food security context, *Environmental and Resource Economics*, *71*(2), 319–336, 2018.

Plummer, M., Assessing benefit transfer for the valuation of ecosystem services, *Frontiers in Ecology and the Environment*, *7*(1), 38–45, 2009.

Pulido-Velázquez, M., J. Andreu, and A. Sahuquillo, Economic optimization of conjunctive use of surface water and groundwater at the basin scale, *Journal of Water Resources Planning and Management*, *132*(6), 454–467, 2006.

Richardson, L., J. Loomis, T. Kroeger, and F. Casey, The role of benefit transfer in ecosystem service valuation, *Ecological Economics*, *115*, 51–58, 2005.

Rolfe, J., Valuing reductions in water extractions from groundwater basins with benefit transfer: the great artesian basin in australia, *Water Resources Research*, *46*(W06301), 1–9, 2010.

Roseta-Palma, C., and A. Xepapadeas, Robust control in water management, *The journal of Risk and Uncertainty*, *29*(1), 21–34, 2004.

Rubio, S., and B. Casino, Competitive versus efficient extraction of a common property resource: The groundwater case, *Journal of Economic Dynamics and Control*, *25*(8), 1117–1137, 2001.

Sophocleous, M., The science and practice of environmental flows and the role of hydrogeologists, *Groundwater*, *45*(4), 393–401, 2007.

Stahn, H., and A. Tomini, On conjunctive management of groundwater and rainwater, *Resource and Energy Economics*, *49*, 186–200, 2017.

Stalnaker, C., Minimum flow is a myth, *Tech. rep.*, Bilogical Report 90(5), 1972.

Tharme, R., A global perspective on environmental flow assessment: Emerging trends in the development and application of environmental flow methodologies for rivers, *River Research and Applications*, *19*, 397–441, 2003.

Tomini, A., Is the gisser and sanchez model too simple top discuss the economic relevance of groundwater management?, *Water Resources and Economics*, *6*, 18–29, 2014.

Tuinstra, J., and J. van Wensem, Ecosystem services in sustainable groundwater management, *Science of the total Environment*, *484-486*, 798–803, 2014.

Yang, W., T. Sun, and Z. Yang, Does the implementation of environmental flows improve wetland ecosystem services and biodiversity? a literature review, *Restoration Ecology*, *24*(6), 731–742, 2016.

Zhou, Y., A critical review of groundwater budget myth, safe yield and sustainability, *Journal of Hydrology*, *370*(1), 207–213, 2009.