INSTRUMENTS AND STATISTICAL TOOLS TO STUDY SUPERMASSIVE BLACK HOLES AT EVENT HORIZON SCALES

by

Junhan Kim

Copyright © Junhan Kim 2019

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF ASTRONOMY

In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
WITH A MAJOR IN ASTRONOMY AND ASTROPHYSICS

In the Graduate College
THE UNIVERSITY OF ARIZONA

2019
THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

As members of the Dissertation Committee, we certify that we have read the dissertation prepared by Junhan Kim, titled Instruments and Statistical Tools to Study Supermassive Black Holes at Event Horizon Scales and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

Prof. Daniel P. Marrone
Date: July 29, 2019

Prof. Erika T. Hamden
Date: July 29, 2019

Prof. Feryal Ozel
Date: July 29, 2019

Prof. Dimitrios Psaltis
Date: July 29, 2019

Prof. Christopher K. Walker
Date: July 29, 2019

Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copies of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

Dissertation Director: Dr. Daniel P. Marrone
Date: July 29, 2019
Associate Professor
Department of Astronomy
ACKNOWLEDGEMENTS

Looking back on the wonderful past six years in Steward, I want first to acknowledge my advisor Dan Marrone. We spent four summer seasons together at the South Pole, and it has been a great learning experience as an experimentalist, working closely with advisor day and night. He shared valuable lessons not only as an academic mentor but as a senior in life. I will never forget those days. I would like to thank Dimitrios Psaltis and Feryal Özel. As a Steward Event Horizon Telescope group member, I interacted with these theorists closely and could learn different ways of understanding and solving scientific problems. I am also grateful to all the Steward Event Horizon Telescope team members. This year, we enjoyed the glorious moment that does not come often.

While spending much time up at the Mountain Graham where the Submillimeter Telescope is located, and in the basement lab at the Steward Observatory building, I got tremendous help from the Arizona Radio Observatory engineers. Bob Freund taught me how to do the very-long-baseline interferometry setup and testing with his good old Hewlett-Packard pen plotter. I learned so much about the radio receiver and electronics from Gene Lauria, George Reiland, and David Forbes. I thank Submillimeter Telescope operators who spent many nights together in the telescope control room. All the Arizona Radio Observatory staffs were always kind to this graduate student who bothered them all the time. I spent several months at the South Pole during my graduate career, and I was lucky enough to work with the South Pole Telescope collaboration members. I also thank everyone who shared the workplace – fellow graduate students, faculties, staffs of the Steward Observatory.

Finally, it would not have been possible for me to successfully finish my graduate program without support from my family across the Pacific Ocean. I appreciate the support from my parents Soowon and Myoungseong, my sister Heesun, and my grandparents. I miss my grandfather Kyungho, who passed away two years ago. I still believe he waited until the last minute for me to fly out from Antarctica so that I could say my last goodbye to him in Seoul.
DEDICATION

For my family
# TABLE OF CONTENTS

LIST OF FIGURES .......................................................... 8

LIST OF TABLES .......................................................... 10

ABSTRACT ................................................................. 11

CHAPTER 1 Introduction ................................................... 13
  1.1 The Supermassive Black Hole at the Galactic Center ............. 13
  1.2 The Event Horizon Telescope .................................. 17
  1.3 Very Long Baseline Interferometry ............................ 19
  1.4 This Work ......................................................... 23

CHAPTER 2 A VLBI Receiving System for the South Pole Telescope ........................ 25
  2.1 Introduction ....................................................... 25
  2.2 Receiver System ................................................. 27
    2.2.1 Receiver .................................................. 28
    2.2.2 Frequency Reference ..................................... 37
    2.2.3 VLBI Backend ............................................. 38
    2.2.4 Calibration System and Spectrometer ..................... 38
  2.3 Optics ............................................................ 42
    2.3.1 Design ..................................................... 42
    2.3.2 Beam Measurement ....................................... 42
  2.4 Software ......................................................... 47
  2.5 Summary ........................................................ 49

CHAPTER 3 Tilted Beam Measurement of VLBI Receiver for the South Pole Telescope .... 50
  3.1 Introduction ....................................................... 51
  3.2 SPT VLBI Receiver .............................................. 51
  3.3 Vector Beam Measurements ................................... 53
  3.4 Analysis ........................................................ 58
    3.4.1 Gaussian Beam Propagation of Tilted Horn ............. 58
    3.4.2 Beam Fitting ............................................. 61
  3.5 Conclusion ....................................................... 64
TABLE OF CONTENTS – Continued

CHAPTER 4  The 1.4 mm Core of Centaurus A: First VLBI Results with the South Pole Telescope ........................................... 65
  4.1 Introduction ........................................... 66
  4.2 Observations and Data Reduction ................................. 68
    4.2.1 Observations .................................. 68
    4.2.2 Calibration .................................. 70
  4.3 Data Analysis ........................................ 72
    4.3.1 Brightness Temperature ............................. 73
    4.3.2 Core Size .................................... 80
    4.3.3 Spectrum of the Core ............................. 82
    4.3.4 Variability .................................... 84
  4.4 Conclusion ........................................... 84

CHAPTER 5  Bayesian Techniques for Comparing Time-dependent GRMHD Simulations to Variable Event Horizon Telescope Observations ............. 86
  5.1 Introduction ........................................... 86
  5.2 Bayesian Data Analysis ................................... 89
    5.2.1 Formalism ..................................... 89
    5.2.2 Model Comparison ................................. 95
  5.3 Likelihood with Mock Data .................................. 96
    5.3.1 GRMHD Simulations ................................ 96
    5.3.2 Parameter Estimation .............................. 101
    5.3.3 Effect of Future EHT Stations ....................... 104
    5.3.4 Effect of Time Averaging ........................... 107
    5.3.5 Model Selection ................................ 109
  5.4 Applications to Early EHT Data ................................ 111
  5.5 Conclusion ........................................... 118

CHAPTER 6  Summary and Conclusions .................................. 119
  6.1 South Pole Telescope VLBI receiver ........................... 119
  6.2 Black Hole Model Comparison using the Event Horizon Telescope Observations ........................................... 119

APPENDIX A  SPT VLBI Receiver: Ortho-mode Transducer Designs ....... 121

APPENDIX B  SPT VLBI Receiver: Noise Measurements .................. 126

APPENDIX C  SPT VLBI Receiver: 230 GHz Focusing Template .......... 130

APPENDIX D  SPT VLBI Receiver: First 345 GHz Observation ............ 132
TABLE OF CONTENTS – Continued

APPENDIX E  Beam Measurement of SPT VLBI Receiver with Tertiary Mirror

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>E.1 Optical Design</td>
<td>135</td>
</tr>
<tr>
<td>E.2 Beam Measurement</td>
<td>138</td>
</tr>
<tr>
<td>E.3 Analysis</td>
<td>141</td>
</tr>
<tr>
<td>E.3.1 Beam at the Cassegrain Focus</td>
<td>141</td>
</tr>
<tr>
<td>E.3.2 Primary Mirror Illumination</td>
<td>144</td>
</tr>
</tbody>
</table>

APPENDIX F  Calibration of the 2015 SPT-APEX VLBI Test Observation

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>145</td>
</tr>
<tr>
<td>F.1 Calibration</td>
<td>145</td>
</tr>
<tr>
<td>F.1.1 Amplitude Calibration</td>
<td>145</td>
</tr>
<tr>
<td>F.1.2 Aperture Efficiency and Gain</td>
<td>146</td>
</tr>
<tr>
<td>F.1.3 Planet Model</td>
<td>148</td>
</tr>
<tr>
<td>F.2 APEX</td>
<td>149</td>
</tr>
<tr>
<td>F.2.1 System Temperature</td>
<td>149</td>
</tr>
<tr>
<td>F.2.2 Aperture Efficiency and Gain</td>
<td>150</td>
</tr>
<tr>
<td>F.3 SPT</td>
<td>151</td>
</tr>
<tr>
<td>F.3.1 System Temperature</td>
<td>151</td>
</tr>
<tr>
<td>F.3.2 Aperture Efficiency and Gain</td>
<td>153</td>
</tr>
</tbody>
</table>

APPENDIX G  Principal Component Analysis Applied to the Interferometric Measurements for Bayesian Model Comparison

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.1 Introduction</td>
<td>156</td>
</tr>
<tr>
<td>G.2 Principal Component Analysis and Interferometric Data</td>
<td>158</td>
</tr>
<tr>
<td>G.3 Bayesian Analysis Formalism</td>
<td>162</td>
</tr>
</tbody>
</table>

REFERENCES                                                166
# LIST OF FIGURES

2.1 SPT VLBI receiver system diagram ........................................... 29
2.2 CAD model of the SPT receiver cabin ....................................... 31
2.3 4 K stage of the receiver and the 230 GHz receiver assembly ........ 32
2.4 Inner surface of the split block OMT ....................................... 34
2.5 230 GHz band spectral noise temperature ................................ 36
2.6 VLBI backend installed at the SPT control room ....................... 39
2.7 Tertiary mirror assembly and the calibration load ..................... 41
2.8 Spectral channel map of the protostar IRAS 16293-2422 .............. 43
2.9 Secondary and tertiary mirrors of the SPT VLBI receiver system ...... 44
2.10 Vector beam measurement of the 230 GHz receiver .................... 46
2.11 Tuning screen for the 230 GHz receiver ................................ 48
3.1 Interior view of the SPT VLBI receiver .................................... 52
3.2 Schematic of the beam measurement system ............................ 54
3.3 Amplitude and phase pattern of 230 GHz beam measurement .......... 56
3.4 Amplitude and phase pattern of 345 GHz beam measurement .......... 57
3.5 Schematic of the tilted beam measurement ............................... 59
3.6 MCMC parameter estimation result for the 230 GHz measurement ... 62
4.1 \((u, v)\) coverage of the SPT-APEX VLBI observation of Cen A .... 71
4.2 Ranges of the brightness temperature and the size of the Cen A core 75
4.3 Brightness temperatures of the Cen A core ............................. 76
4.4 Size of the Cen A core region from multiwavelength data ............ 81
4.5 Spectrum of the Cen A core region ....................................... 83
5.1 Flow chart of the algorithm .................................................. 93
5.2 Light curves of Models A to E at 1.3 mm ................................ 99
5.3 Sample VLBI visibility amplitudes of Models A to E .................. 100
5.4 Parameter estimation test with mock data .............................. 103
5.5 Parameter estimation test using identical raw overall sensitivity ... 106
5.6 Effect of time averaging in the parameter estimation test .......... 108
5.7 Normalized posterior likelihoods using EHT 2007 and 2009 data ... 115
5.8 Visibility amplitudes of Models A to E with the best-fit parameters 117
A.1 Orthomode transducer designs ............................................ 122
A.2 Simulated OMT transmission .............................................. 124
A.3 Simulated OMT isolation ................................................... 125
LIST OF FIGURES – Continued

B.1  Y-factor measurements: amplified receiver IF and fiber optics receiver  127
B.2  Y-factor measurements: fiber optics receiver and BDC  128
B.3  Y-factor measurements: fiber optics receiver and R2DBE  129

C.1  The Jupiter raster map at different focus positions  131

D.1  345 GHz Jupiter raster map  133
D.2  345 GHz Moon raster map  134

E.1  Schematic of the SPT VLBI receiver optics  136
E.2  Beam mapping setup in the lab  139
E.3  Normalized power of the band 6 beam  140
E.4  Two dimensional Gaussian fitting of the normalized beam map  142
E.5  Beam waist radius of band 6 and band 7 receivers  143

F.1  Planet raster maps for telescope calibration  154

G.1  Uniformly distributed mock visibility amplitudes  160
G.2  Sample mock visibility amplitudes  161
G.3  Normalized likelihood for two parameter models  165
**LIST OF TABLES**

1.1 Several historic Sgr A* VLBI observations ........................................... 22
2.1 SPT VLBI receiver frequency setup ......................................................... 28
3.1 The best-fit parameters for the $x$-direction tilt angle ............................ 63
4.1 Cen A observation summary ................................................................. 69
4.2 Brightness temperature, flux density and the size of Cen A core region 78
4.2 Brightness temperature, flux density and the size of Cen A core region 79
5.1 Marginal likelihoods of model estimation tests ........................................ 110
5.2 Best-fit parameters and model evidences with EHT observations in 2007 and 2009 116
E.1 SPT VLBI receiver optical design parameters ........................................ 137
G.1 Principal components derived with a three-station mock data using model D in Chan et al. (2015a) 162
Supermassive black holes are known to exist at the center of galaxies, including the one at our Milky Way galaxy, Sagittarius A* (Sgr A*). To observationally study very near environment of the black hole close to its event horizon, we need an Earth-size telescope operating at millimeter wavelengths. In this dissertation, I developed the very-long-baseline interferometry (VLBI) receiver for the South Pole Telescope (SPT). The receiver enables the Event Horizon Telescope (EHT) VLBI array to achieve the baseline that can resolve the apparent size of Sgr A*. However, understanding physics around the black hole from the Sgr A* observation is challenging due to its rapid variability. In this thesis, I also studied statistical methods to compare the variable Sgr A* observations with physically motivated, time-dependent, black hole simulations using a Bayesian framework.

In Chapter 2, I present the development of the VLBI receiving system for the SPT. Since the SPT was built for the cosmic microwave background observation, it was solely equipped with the multi-pixel, wide-field, bolometric receiver. I designed and assembled the coherent receiver working at 1.3 and 0.87 mm, and installed the system, including the VLBI recording setup at the South Pole.

In Chapter 3, I report the vector beam measurement of the SPT VLBI receiver. Due to the receiver design inside the dewar, it was essential to analyze the beam measurement accounting for the tilted geometry and characterize the beam propagation relative to the optical components.

In Chapter 4, I report the first VLBI experiment result at the SPT. We observed Centaurus A (Cen A) and successfully detected the interferometric fringe. Although the observation focused on the demonstration of the VLBI capability at the telescope, the ~7000 km baseline between the South Pole and Chile provided the highest resolution observation of Cen A published to date.
In Chapter 5, I introduce the Bayesian technique to compare VLBI observation of the black holes such as Sgr A* to time-varying general relativistic magnetohydrodynamics (GRMHD) simulation models. The method statistically takes the variabilities of both observation and the models into account and has the power to perform parameter estimation and quantitative model comparison. I show the application of the method using synthetic data generated from the simulation as well as the three-station EHT data from its early stage.

Finally, in Chapter 6, I summarize the work presented in this dissertation.
CHAPTER 1

Introduction

1.1 The Supermassive Black Hole at the Galactic Center

Most of the galaxies in the universe, if not all, are believed to harbor a supermassive black hole (SMBH) at their center (Lynden-Bell, 1969; Lynden-Bell and Rees, 1971; Kormendy and Richstone, 1995; Miyoshi et al., 1995). Recent research suggests that the physical properties of the host galaxy and the residing SMBH are correlated, implying that the formation and evolution of the galaxy are affected by the central black hole, and vice versa (Ho, 2008; Kormendy and Ho, 2013; Heckman and Best, 2014). Observations, such as the relations between the black hole mass and the galaxy velocity dispersion (e.g., Ferrarese and Merritt, 2000; Gebhardt et al., 2000) and between the black hole mass and multiwavelength properties (e.g., Merloni et al., 2003), support the idea, although the formation process of the SMBH is not well understood yet (Volonteri, 2010). In particular, the accretion onto the SMBH is one of the critical factors that govern its activity, and the physics in the inner regions of the accretion disk is drawing much attention.

Since its discovery the black hole at the center of our Milky Way galaxy, Sagittarius A* (Sgr A*, named by Brown, 1982), has been extensively studied because of its proximity (Balick and Brown, 1974). At 8 kpc away from Earth (Reid et al., 2009, 2014), the nearest SMBH provides a close look at the astrophysical processes near the black hole (e.g., for reviews, see Melia and Falcke, 2001; Falcke and Hehl, 2003; Melia, 2007; Genzel et al., 2010; Falcke and Markoff, 2013). Sagittarius A* has been studied at wavelengths from radio to gamma rays, except optical and ultraviolet wavelengths owing to the absorption by the gas and dust towards the central region of the galaxy. It was the long-term monitoring of the stellar dynamics around the galactic center in the near-infrared that proved the existence of the four million solar
mass black hole (Ghez et al., 2008; Gillessen et al., 2009b; Boehle et al., 2016). X-ray observations using multiple space telescopes helped to understand the accretion and emission mechanism very near the black hole (e.g., Baganoff et al., 2001; Ponti et al., 2015; Zhang et al., 2017). Especially, light-crossing and dynamical timescale arguments for the X-ray flare estimate that the emission occurs as close as a few Schwarzschild radii near the black hole. More recently, the new near-infrared instrument GRAVITY on the Very Large Telescope Interferometer was able to detect the gravitational redshift from the close approach of the star around Sgr A* (Gravity Collaboration et al., 2018a), and also claimed the observation of orbiting material associated with the emission from the black hole’s immediate surrounding (Gravity Collaboration et al., 2018b).

In the radio and millimeter bands, the technique of very-long-baseline interferometry (VLBI) has been a useful tool, providing a high-resolution view of the compact source. VLBI combines a network of widely separated telescopes to simulate a much larger aperture, and thus achieve very high resolution. Observations at different wavelengths ($\lambda$) showed that the size of the Sgr A* changes with the wavelength, following a $\lambda^2$ power law due to scattering from the interstellar medium. The intrinsic size of the source had to be measured by deconvolving the scattering effect (Lo et al., 1998; Doeleman et al., 2001; Bower et al., 2004, 2006; Shen et al., 2005). The frequency dependent intrinsic source size of the Sgr A* provided the mass density limits for the source, which is a piece of indirect evidence supporting that the Sgr A* is a black hole. Also, the size proportional to the observing wavelength indicated that the different wavelength probes different region of the emission structure, and this could be explained by various physical models with the synchrotron emission from the accretion flow (Bower et al., 2004, 2006; Shen et al., 2005; Doeleman et al., 2008).

Soon after its discovery, Sgr A* was shown to be a variable radio source. The 3-year monitoring of Sgr A* using the National Radio Astronomy Observatory (NRAO) interferometer at Green Bank detected the variability in its flux density at both 11 and 3.7 cm, on timescales of days to months (Brown and Lo, 1982).
Centimeter wavelength monitoring with the Very Large Array (VLA), spanning 20 years, claimed a characteristic variability timescale of ~100 days, showing more variability towards the shorter wavelengths (Zhao et al., 2001; Herrnstein et al., 2004), although Macquart and Bower (2006) inferred that the Sgr A* does not exhibit distinct periodicity in the lightcurve. At submillimeter wavelengths, the recent analysis of Dexter et al. (2014) claimed ~8 hrs of the characteristic timescale. The variability has also been reported in the infrared (e.g., Stone et al., 2016; Witzel et al., 2018), and even for its polarization, which can probe the magnetic field (Marrone et al., 2006; Bower et al., 2005; Marrone et al., 2007; Liu et al., 2016). Sometimes a rapid increase of luminosity happening in as short as few hour duration, the flare, has been observed in different wavelengths from submillimeter to X-ray (Zhao et al., 2003; Eckart et al., 2006; Yusef-Zadeh et al., 2006; Marrone et al., 2008; Yusef-Zadeh et al., 2009; Dodds-Eden et al., 2010; Zhang et al., 2017; Gravity Collaboration et al., 2018b). The simultaneous flares are often, but not always, detected at multiple wavelength windows, implying it has the synchrotron origin.

Being the closest SMBH we can observe, the rich multiwavelength data of Sgr A* is particularly useful to validate the theory of accretion. The broadband spectrum of Sgr A* shows that it is an extremely faint source, whose bolometric luminosity is a few orders of magnitude lower than its Eddington luminosity. The understanding of the physical conditions near the black hole also requires an estimate of the accretion rate, since the observed radiation is powered by accretion of the surrounding materials. The X-ray observation of Sgr A* found the emission associated with the source, and an arcsecond level spatial resolution provided information on the temperature of the X-ray gas that is gravitationally bound to the black hole thus the accretion rate (Baganoff et al., 2003). Radio polarization measurements also give an independent estimate of the accretion rate. This is because the Faraday rotation measure, calculated by the change of the polarization angle with wavelength, depends on the line-of-sight electron density (and the magnetic field) of the magnetized medium (Marrone et al., 2006). The measured accretion rates indicate that the accretion process around Sgr A* is radiatively inefficient to produce its luminosity. This radia-
tively inefficient flow is explained by the low-density plasma where the temperatures of the electrons and ions are different due to weak Coulomb coupling between them. The electron density profile, as well as the magnetic field strength and temperature prescriptions, are parameterized to model the plasma, and the physical models describe the plasma under the assumptions of the black hole properties often with general relativistic considerations (e.g., Falcke and Markoff, 2000; Özel et al., 2000; Bromley et al., 2001; Broderick and Loeb, 2006; Mościbrodzka et al., 2009; Dexter et al., 2009, 2010; Broderick and Loeb, 2006; Mościbrodzka et al., 2009; Dexter et al., 2009, 2010; Gold et al., 2017). The models that can fit the spectral energy distribution of Sgr A* generally agree that the radiation comes from the accretion flow surrounding the SMBH, and can be best explained by synchrotron emission from mildly relativistic electrons, although the synchrotron emissivity and its frequency dependence are influenced strongly by the physical parameters. With this understanding, it is predicted that at submillimeter wavelengths the view to the event horizon should be unimpeded by synchrotron opacity. General relativistic effects should lead to a bright narrow ring surrounding the relatively darker region near the black hole, caused by the gravitational light bending and the photon capture at the event horizon (Bardeen, 1973; Luminet, 1979; Falcke et al., 2000). The size of this bright photon ring linearly scales with the black hole mass and is about five times the Schwarzschild radius, depending weakly on the black hole spin and geometry (Falcke et al., 2000). Millimeter VLBI with Earth-spanning baselines should provide enough angular resolution to resolve the structure (Johannsen et al., 2012). This realization led to the global effort, called the Event Horizon Telescope (EHT), to form a millimeter VLBI array that can directly resolve the apparent shape of the emission in the vicinity of Sgr A*.

\footnote{The most recent estimate of the parameters: electron density \((2 - 5) \times 10^6\) cm\(^{-3}\), electron temperature \((1 - 3) \times 10^{11}\) K, magnetic field strength \(10 - 50\) G, is from Bower et al. (2019), employing a single component synchrotron emission model of thermal electrons.}
1.2 The Event Horizon Telescope

In this section, I review the progress EHT has made since its 2007 observation, mainly focusing on the observational efforts. In April 2007, the Submillimeter Telescope (SMT) in Arizona, Combined Array for Research in Millimeter-wave Astronomy (CARMA) in California, and the James Clerk Maxwell Telescope (JCMT) in Hawaii observed Sgr A* at 1.3 mm. Having only three baselines, a simple Gaussian model was adopted to measure the size of the emitting region. It was 37 μas, about ~4 Schwarzschild radii of the central black hole. This early EHT attempt demonstrated the feasibility of the millimeter VLBI observation to resolve the event horizon scale emission around the black hole (Doeleman et al., 2008). The three-station observations then revealed interesting physics of Sgr A* even before the larger array was formed. Fish et al. (2011) performed the VLBI using the same set of the triangle during three consecutive days in 2009 and detected the time variable correlated flux density of Sgr A*. The increase of the visibility amplitude was seen on all baselines, implying the brightening of the emission arises on Schwarzschild radius scales. In 2013, the phased Submillimeter Array (SMA) participated for the first time, measuring one polarization from Mauna Kea with the JCMT measuring the opposite polarization at the co-located site. From this polarimetric VLBI observation, Johnson et al. (2015) found the spatially resolved, ordered magnetic field structure. The emission of Sgr A* we detect using VLBI is believed to originate from the accretion disk where the magnetic field plays an important role, and interferometric polarimetry would provide a better understanding of the physics near the black hole. VLBI provides additional non-imaging information, including the closure phase, which is the sum of the phase around a triangle of baselines. This quantity is not affected by atmospheric phase corruption or most calibration errors, and so only contains the information of the observed source. Fish et al. (2016) compiled the three-station EHT data over four years and discovered that the closure phase is nonzero, indicating that the emission around Sgr A* is asymmetric, as expected from the theoretical simulation models. In addition to Sgr A*, the early
EHT reported the observations of another SMBH at the center of galaxy M87, which has the second largest apparent size on the sky, and some quasar calibrators as well (Doeleman et al., 2012; Lu et al., 2012, 2013; Akiyama et al., 2015).

Following the successful observations, the EHT has expanded to assemble a VLBI array with the millimeter facilities around the world (Doeleman et al., 2009). An Earth-diameter baseline would yield $\sim 20 \, \mu\text{as}$ angular resolution at 1.3 mm, which can resolve the apparent size of the emission surrounding Sgr A* (Johannsen et al., 2012). To achieve this, a number of telescopes were added to the array. It became possible with the developments of low-noise, the sensitive receiver using the improved mixer technology (Kerr et al., 2014), a wideband digitization of the signal with the fast VLBI recording system (Whitney et al., 2011; Vertatschitsch et al., 2015), and commercial hydrogen maser, an atomic clock, that can precisely timestamp the data. The Atacama Pathfinder Experiment (APEX) detected the first 1 mm fringe with the SMT and the SMA in 2012 (Wagner et al., 2015) and participated in the EHT observation in 2013 (Lu et al., 2018). In 2015, the South Pole Telescope (SPT; Kim et al., 2018a,b) and the Large Millimeter Telescope (LMT) successfully detected VLBI fringes with the existing VLBI facilities. The LMT inclusion happened in parallel with the 3 mm VLBI at the site that led to the observation of Sgr A* with the Very Long Baseline Array (VLBA; Ortiz-León et al., 2016). In VLBI, the sensitivity of a baseline is proportional to the geometric mean of the sensitivities of the telescopes forming the baseline. Thus the addition of the phased Atacama Large Millimeter/submillimeter Array (ALMA) dishes was also critical to leverage the overall sensitivity of the array, due to its collecting area (Matthews et al., 2018; Goddi et al., 2019; Issaoun et al., 2019). Theoretically, the phasing of $\sim 40$ 12 m ALMA dishes is equivalent of a $\sim 70$ m effective diameter, although the phasing efficiency loss needs to be considered.

In April 2017, EHT carried out the first Earth-size, full-array science observation for five days. The participating telescopes were: ALMA and APEX in Chile, LMT in Mexico, SMA and JCMT in Hawaii, Institute de Radioastronomie Millimétrique (IRAM) 30m in Spain, SMT in Arizona, and the SPT in Antarctica. The array
observed Sgr A* and the black hole at the center of M87 (M87*), as well as the other bright millimeter sources, and released the first results of M87* in April 2019 with the first-ever image of a black hole (Event Horizon Telescope Collaboration et al., 2019a,b,c,d,e,f). The M87* results showed the ring of emission from surrounding plasma with a central brightness depression as expected from general relativity, and the asymmetric brightness distribution caused by the Doppler beaming due to the geometry of the angular momentum vector. The analysis on the Sgr A* data is underway, but its dynamical time scales much shorter than that of M87 makes the source less stable, and the analysis is more challenging.

The EHT is planning to incorporate more telescopes, including the 12 m ALMA prototype antenna (Mangum et al., 2006) at the Kitt Peak, Arizona, to better fill in the coverage. The Greenland Telescope (GLT; Inoue et al., 2014) was another ALMA prototype antenna and already participated the EHT observation in 2018, following its installation at the Thule Air Base, Greenland (Chen et al., 2018). The shorter wavelength gives a better resolution with the same physical baseline, and another future EHT direction is to equip the existing sites with 0.8 mm capability. The EHT now records data at a much higher rate than was possible in its early experiments, which increases the sensitivity and enables the expansion of the array, allowing better resolution and baseline coverage. The longest baseline of Doeleman et al. (2008) was $\sim 3.5 \, \text{G}\lambda$, and the recording bandwidth was $\sim 1 \, \text{GHz}$. The EHT observation data in 2018 was recorded in dual-polarization across $\sim 8 \, \text{GHz}$ bandwidth, with the maximum baseline over $\sim 8.5 \, \text{G}\lambda$. However, the technical development for VLBI is not EHT specific and has been a continuous effort from the past.

### 1.3 Very Long Baseline Interferometry

In this section, I review the brief history of the VLBI developments from the 1970s that has enabled the progress on the VLBI observations of Sgr A*. A full review of the VLBI system development history can be found elsewhere (Kellermann and Cohen, 1988; Kellermann and Moran, 2001; Clark, 2003).
The angular resolution given by an interferometric baseline is proportional to the wavelength and inversely proportional to the baseline length. The first demonstration of the technique was made in Canada at approximately 1-meter wavelength using a 200 m baseline in the late 1960s (Broten et al., 1967). Since then, VLBI has advanced toward shorter wavelengths and more extended baselines for better resolving power. VLBI observations of Sgr A* progressed using broader recording bandwidths, which increases the measurement sensitivity. This was achieved with the development of the VLBI recording system (Table 1.1). In VLBI, a phase-sensitive heterodyne receiver system is used to detect the electromagnetic wave and the coherence pattern is measured later from the correlation with the signals recorded at different observing sites. The receiver outputs the intermediate frequency (IF) signal as a result of mixing between the incoming radiation from the sky and the reference local oscillator signal provided at the site. The frequency shifted IF band signal contains the original information from the sky signal, and it is recorded to the media for the later correlation with the other VLBI sites. Often the IF signal is mixed down again to the “baseband”, i.e., to a band starting near 0 Hz to minimize the required sampling rate of the system.

The first Sgr A* VLBI detection of Lo et al. (1975) was made in 1975 at 3.7 cm, using a ~240 km baseline between Goldstone and Owens Valley, California (note that the Sgr A* discovery of Balick and Brown 1974 was made using a linked NRAO interferometer). The recording system used for the observation was Mark II (Clark, 1973), a successor of Mark I (Bare et al., 1967), developed by NRAO, using a video cassette recorder. The recorder achieved Nyquist sampling of 2 MHz of bandwidth. Several other Sgr A* observations followed until the early 1980s relied on the same system (Lo et al., 1977, 1981; Jauncey et al., 1989). The Sgr A* size measurement continued with the next generation recording system Mark III and IIIA (Rogers et al., 1983; Clark et al., 1985). The systems used Honeywell 96 magnetic tape as a recording media and could record 28 channels simultaneously, up to 2 MHz bandwidth for each channel.

After a long history of centimeter VLBI, the VLBI observations opened a new
window in the millimeter wavelength. There are some challenges as we go to shorter wavelengths, especially (sub)millimeter, including increased noise in electronics and higher atmospheric opacity. The phase fluctuation caused by atmospheric turbulence affects the data significantly, and the integration time gets shorter than that of longer wavelength. Also, the size of the collecting area generally becomes smaller than the longer wavelength so that the integration of broader bandwidth data can compensate for the sensitivity. This means that a recording system that can handle an increasing data rate is necessary for millimeter VLBI. The development of Mark III led to the first 3.4 mm VLBI fringe detection between Owens Valley and Hat Creek in California in 1981 (Readhead et al., 1983). The first intercontinental observation followed that at 7 mm in 1986 (Marcaide et al., 1985; Bartel et al., 1988) and the first 1.4 mm VLBI in 1989 (Padin et al., 1990).

The Sgr A* observations at millimeter wavelengths also began in the late 90s (Krichbaum et al., 1998; Doeleman et al., 2001). The Mark V development made a significant change in the recording system (Whitney, 2002, 2004, 2008). The system was based on commercial off-the-shelf equipment, including the commercial hard disk drive as recording media. The sampling rate was as high as 4 gigabits/sec with a few hours of recording time. In fact, this system was used to record the data of the early EHT observation in 2007 (Doeleman et al., 2008). As of 2019, the standard recording system for the EHT millimeter VLBI is Mark VI developed by Haystack Observatory. At a single station, there are four Mark VI data recorders with 64 commercial hard drives in total (16 per Mark 6 recorder; those are grouped to 8 disk ‘modules’), and the overall system can take up to 64 Gbps data rate with the addition of digitization system (Whitney et al., 2013; Vertatschitsch et al., 2015). The sensitivity for the first Sgr A* VLBI observation using the Mark II system was 0.1 Jy for 2 MHz bandwidth and 15 min integration (Lo et al., 1975). The most recent VLBI system of the Event Horizon Telescope can achieve a few milliJansky level sensitivity on the baselines to ALMA, for 4 GHz bandwidth and 10 sec integration time (Event Horizon Telescope Collaboration et al., 2019b). It has been a significant progress during the past ~40 years.
Table 1.1. Several historic Sgr A* VLBI observations

<table>
<thead>
<tr>
<th>Year</th>
<th>Wavelength (cm)</th>
<th>Baseline$^a$(λ)</th>
<th>Bandwidth (MHz)</th>
<th>Recording System$^b$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975</td>
<td>3.7</td>
<td>$4 \times 10^6$</td>
<td>2</td>
<td>Mark II</td>
<td>Lo et al. (1975)</td>
</tr>
<tr>
<td>1977</td>
<td>6.0, 3.6, 2.8, 1.3</td>
<td>$1 \times 10^8$</td>
<td>2</td>
<td>Mark II</td>
<td>Lo et al. (1981)</td>
</tr>
<tr>
<td>1982</td>
<td>13, 3.6</td>
<td>$7 \times 10^6$</td>
<td>2</td>
<td>Mark II</td>
<td>Jauncey et al. (1989)</td>
</tr>
<tr>
<td>1983</td>
<td>3.6, 1.4</td>
<td>$2 \times 10^7$</td>
<td>56</td>
<td>Mark III</td>
<td>Lo (1984); Lo et al. (1985)</td>
</tr>
<tr>
<td>1983</td>
<td>13, 3.6, 1.3</td>
<td>$2 \times 10^8$</td>
<td>56</td>
<td>Mark III</td>
<td>Marcaide et al. (1985)</td>
</tr>
<tr>
<td>1991</td>
<td>3.6, 1.4</td>
<td>$2 \times 10^7$</td>
<td>56</td>
<td>Mark III</td>
<td>Lo et al. (1993)</td>
</tr>
<tr>
<td>1995</td>
<td>0.35, 0.14</td>
<td>$8 \times 10^8$</td>
<td>112</td>
<td>Mark IIIA</td>
<td>Krichbaum et al. (1998)</td>
</tr>
<tr>
<td>1999</td>
<td>0.35</td>
<td>$5 \times 10^8$</td>
<td>56</td>
<td>Mark III</td>
<td>Doeleman et al. (2001)</td>
</tr>
<tr>
<td>2002</td>
<td>0.35</td>
<td>$7 \times 10^8$</td>
<td>128</td>
<td>VLBA$^c$</td>
<td>Shen et al. (2005)</td>
</tr>
<tr>
<td>2007</td>
<td>0.13</td>
<td>$3 \times 10^9$</td>
<td>960</td>
<td>Mark V</td>
<td>Doeleman et al. (2008)</td>
</tr>
<tr>
<td>2013</td>
<td>0.13</td>
<td>$7 \times 10^9$</td>
<td></td>
<td>Mark V</td>
<td>Lu et al. (2018)</td>
</tr>
</tbody>
</table>

$^a$The most extended baseline during the experiment.

$^b$For more complete list of VLBI data storage system, see Section 9.6 of Thompson et al. (2001).

$^c$VLBA implemented the system based on Mark IIIA.
1.4 This Work

The successful inclusion of the SPT to the EHT network made the EHT a true Earth-sized millimeter array. The addition of the SPT also came with several advantages. The longest EHT baseline to the SPT in the 2017 array corresponds to a fringe spacing of 24 micro-arcseconds at 1.3 mm, and this is comparable to half the apparent size of Sgr A*. Also, the SPT always overlaps with any other EHT stations around the globe while looking at Sgr A*, giving the longest time series possible to search for time-domain effect around the black hole. Needless to say, it is an excellent site for millimeter observations, and the weather is usually great compared to the other sites. However, unlike the other EHT stations, the SPT did not have a coherent receiver system. The SPT is primarily an experiment designed to do a bolometric, wide-field observation of the cosmic microwave background (Carlstrom et al., 2011), and adding the VLBI observing capability at the SPT has been a many-year project since I started graduate school. It required four Summer season trips to the South Pole for deployment and testing.

Chapter 2 describes the development of the SPT VLBI receiving system, including the coherent dual-band, the dual-polarization receiver. It has a very sensitive 1 mm receiver built using ALMA Band 6 mixer-preamplifiers (Kerr et al., 2014). The chapter also describes associated software and hardware, such as the separate VLBI optics at the SPT. For efficient optical coupling of the elements, I did the careful vector beam measurements of the receiver and Gaussian optics analysis to characterize the geometry of the receiver assembly. Chapter 3 reports this measurement and analysis. The receiver installation at the SPT was followed by VLBI fringe detection with the APEX telescope. Chapter 4 presents the first VLBI test observation results of the Southern bright radio source Centaurus A. It was a 7,000 km single-baseline observation providing 40 μas resolution, detected the fringe with signal-to-noise ratios over 50 for most of the scans, and proved the functionality of the SPT as a millimeter VLBI station with the receiver system I developed.

The EHT science goal is to understand the physics around the immediate en-
environment of SMBHs. The project has made progress with parallel efforts of instrument development, observation, and theoretical modeling. We heavily rely on the simulation using general-relativistic magnetohydrodynamics (GRMHD) to describe the physics around the black hole realistically. I developed a statistical tool as a preparation for the EHT data analysis along with the receiver development. As described in the earlier Sections, Sgr A* shows rapid variability that makes it extremely difficult to assume a static physical structure. There are the ways suggested to investigate the variability using the VLBI observables (Medeiros et al., 2017, 2018a; Roelofs et al., 2017), but we still need a qualitative method to perform parameter estimation in a given physically motivated model, and also the comparison among the models using different physical prescriptions. Thus the statistical approach comparing the data with time-dependent black hole simulation models using the VLBI observables will become extremely useful for the analysis of the Sgr A* data. Chapter 5 introduces the mathematical formalism of the technique and application examples with the early three-station EHT data from 2007 and 2009, as a proof of principle. Finally, Chapter 6 summarizes the results achieved with the thesis and discusses the future directions based on the instrument development, and the statistical tool described here.
The Event Horizon Telescope (EHT) is a very-long-baseline interferometry (VLBI) experiment that aims to observe supermassive black holes with an angular resolution that is comparable to the event horizon scale. The South Pole occupies an important position in the array, greatly increasing its north-south extent and therefore its resolution.

The South Pole Telescope (SPT) is a 10-meter diameter, millimeter-wavelength telescope equipped for bolometric observations of the cosmic microwave background. To enable VLBI observations with the SPT we have constructed a coherent signal chain suitable for the South Pole environment. The dual-frequency receiver incorporates state-of-the-art SIS mixers and is installed in the SPT receiver cabin. The VLBI signal chain also includes a recording system and reference frequency generator tied to a hydrogen maser. Here we describe the SPT VLBI system design in detail and present both the lab measurements and on-sky results. ¹

2.1 Introduction

The Event Horizon Telescope (EHT) is a very-long-baseline interferometry (VLBI) experiment operating at observing frequencies of 230 and 345 GHz (Doeleman et al., 2009). The EHT aims to study the immediate environment of supermassive black holes such as Sagittarius A* (Sgr A*) at the center of our galaxy (Doeleman et al., 2009). All of the work described below was carried out by me, with help from co-authors Daniel P. Marrone, Christopher Beaudoin, John E. Carlstrom, Shepherd S. Doeelman, Thomas W. Folkers, David Forbes, Christopher H. Greer, Eugene F. Lauria, Kyle D. Massingill, Evan Mayer, Chi H. Nguyen, George Reiland, Jason SooHoo, Antony A. Stark, Laura Vertatschitsch, Jonathan Weintroub, and André Young.

¹A version of this chapter originally appeared as a published paper in the Proceedings of the SPIE (Kim et al., 2018a).
2008) and the black hole in the center of galaxy M87 (Doeleman et al., 2012) with angular resolution sufficient to resolve the event horizons of these black holes. The EHT array is composed of submillimeter telescopes and telescope arrays around the globe, each of which is outfitted with precise time standards and systems for fast digitization and recording of data. As of 2017, the EHT has performed its first 230 GHz observation with an array consisting of the Submillimeter Array (SMA) and the James Clerk Maxwell Telescope (JCMT) in Hawaii, the Submillimeter Telescope (SMT) in Arizona, the Large Millimeter Telescope (LMT) in Mexico, the Institute de Radioastronomie Millimétrique (IRAM) 30 m telescope at Pico Veleta, Spain, the Atacama Large Millimeter/submillimeter Array (ALMA; Matthews et al., 2018) and the Atacama Pathfinder Experiment (APEX; Wagner et al., 2015) in Chile, and the South Pole Telescope (SPT) in Antarctica.

The inclusion of the SPT is crucial to the EHT because of its geographic location. The South Pole is not only an outstanding place for submillimeter observations, due to its low precipitable water vapor and stable atmosphere (Radford and Peterson, 2016), but it also provides the most extended baselines in the array when the SPT is paired with the other EHT sites, most of which are located in the northern hemisphere. For example, the baseline between the SPT and the SMT is greater than 10,000 km and gives 15 micro-arcsecond (\(\sim 14 \mu \text{as}\)) angular resolution at 345 GHz. For the main target, Sgr A*, the apparent diameter of the black hole event horizon is approximately 50\(\mu\)as, and this high resolution is necessary for imaging of the innermost region around the black hole event horizon. Polarization-sensitive EHT observations also enable the study of magnetic structure in the accretion flow (Johnson et al., 2015). Finally, the SPT can serve as a continuous observing partner for any other EHT station because Sgr A* never sets at the South Pole. This allows the longest time series observations, which will be an important resource for time variability studies of this unique object.

The SPT is a 10-meter diameter, off-axis telescope built to observe the cosmic microwave background (CMB) radiation at millimeter wavelengths (Carlstrom et al., 2011). It is sited at the Dark Sector Lab (DSL) of the Amundsen-Scott South Pole
Station, Antarctica, along with the Background Imaging of Cosmic Extragalactic Polarization (BICEP) project. The SPT CMB camera has gone through several upgrades, including adding polarization sensitivity and increasing the number of detectors in the array (Chang et al., 2009; Austermann et al., 2012; Benson et al., 2014). Currently, the third generation SPT-3G is in operation (Sobrin et al., 2018). Since the camera uses a transition-edge sensor (TES) bolometer array that is insensitive to the phase of incoming radiation, a new coherent signal chain is required to perform interferometric observation in coordination with the other EHT sites. We have developed a dual-frequency VLBI receiver system to incorporate the SPT to the EHT array. Both 230 and 345 GHz receivers facilitate dual-polarization, two-single-sideband observations. We have deployed the receiver system including hydrogen maser and the VLBI recording setup to the South Pole. The 230 GHz receiver had the first on-sky test in January 2015 and successfully detected an interferometric fringe with the APEX (Kim et al. 2018b; Chapter 4). It began scientific operation with the EHT observation in April 2017. In this chapter, we present the receiver design, optics, and software, and report the lab test results of the system components.

2.2 Receiver System

The VLBI receiving system of the SPT comprises the receiver, the receiver electronics, VLBI backend, and optics as illustrated schematically in Figure 2.1. The receiver combines electromagnetic waves from the sky with a high-purity and stable reference tone (local oscillator; LO) in a superconducting mixer, downconverting the sky signal to an intermediate frequency (IF) of a few GHz. The IF signal is amplified through the receiver electronics and forwarded to the VLBI backend. The VLBI backend digitizes the analog signal and records the data to arrays of hard disk drives with accurate timestamping. The whole receiver system is synchronized with the 10 MHz reference signal from the hydrogen maser, an atomic clock. In this section, we explain each of the receiver system element. We will describe the optics
separately in the next section.

### 2.2.1 Receiver

The receiver operates at both 230 and 345 GHz frequency bands. Following ALMA terminology, we will often refer to the 230 GHz and 345 GHz portions of the receiver as band 6 (211–275 GHz) and band 7 (275–373 GHz), respectively. Telescopes in the EHT use identical LO and IF frequencies, with the IF chosen to match the ALMA receivers. The LO frequencies were chosen to avoid the rest frequencies of carbon monoxide (CO), which would absorb emission from the galactic center, and maximize atmospheric transmission in both sidebands in bands 6 and 7. Table 2.1 shows the LO frequency, IF and the corresponding sky frequency of each band. The band 7 mixer for the SPT receiver is under development as of May 2018.

The SPT VLBI receiver incorporates bands 6 and 7 in a package that fits within the confined space of the climate-controlled SPT receiver cabin. The cabin is dominated by the SPT-3G receiver, its optics, and optical path, so the VLBI receiver is positioned behind the SPT-3G tertiary mirror towards the primary (Figure 2.2) and illuminated by a separate optical system. The receiver cryostat surrounds a Sumitomo RDK-408D2P closed-cycle refrigerator, which is connected to an F-70L helium compressor. The two-stage Gifford-McMahon cold head achieves temperatures of 43 K for the first and 4 K for the second stages. Eight LakeShore DT-670 silicon diodes (Effland, 2003) monitor both stages of the cold head and the mixer block.

<table>
<thead>
<tr>
<th>Table 2.1: SPT VLBI receiver frequency setup</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band 6</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Gunn oscillator frequency</td>
</tr>
<tr>
<td>Local oscillator (LO) frequency</td>
</tr>
<tr>
<td>Intermediate frequency (IF)</td>
</tr>
<tr>
<td>Sky frequency</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
Figure 2.1: SPT VLBI receiver system diagram. The coherent receiver works at 230 and 345 GHz frequencies. Receiver electronics provide LO locked to hydrogen maser to the mixers as well as deliver the IF signal to the VLBI backend for digitization and recording. Receiver control, telescope control, and VLBI control computers run software for the system.
temperatures. The feed horns, ortho-mode transducers (OMTs), and mixers are cooled to 4 K, and the rest of the cold assembly including frequency triplers for the LOs, LO waveguide, and wiring harnesses are coupled to the first stage (Figure 2.3). Gold plated heat straps between the mixer blocks and the second stage ensure that the blocks cool efficiently. To prevent the thermal contact between the stages via the electrically conductive LO waveguide, we use waveguide thermal isolators with periodic bandgap structure (Hesler et al., 2003) supported by G-10 fiberglass. There are two sets of isolators, separating the ambient and first-stage waveguide segments, and the first and the second stage segments.
Figure 2.2: CAD model of the SPT receiver cabin. The SPT-3G and VLBI systems are indicated by white and yellow arrows, respectively. Prime and Cassegrain foci are shown with black stars. The primary is located beyond the right side of the figure. The grey box shows the location of receiver cabin roof and walls.
Figure 2.3: Left: The 4 K stage of the receiver. The left half is the 230 GHz receiving system, the right half is the 345 GHz system. Feed horns for both frequencies are tilted by 5.74 degrees with respect to the central vertical plane to share the rotatable tertiary mirror. Right: 230 GHz receiver assembly. Feed horns, OMT, and mixer blocks are attached to the second refrigeration stage and cooled to 4 K.
The 230 GHz receiver employs an ALMA band 6 corrugated feed horn and two mixer/preamplifier modules developed by the National Radio Astronomy Observatory (Kerr et al., 2014). The mixers are driven by the LO signal generated from a Spacek Labs bias-tuned Gunn oscillator and tripled by Virginia Diodes WR3.4×3 broadband triplers. The LO is waveguide-injected. The fixed-frequency Gunn oscillator reduces complexity compared to frequency/backshort tuning for broadband oscillators, which is useful for winter operation of the receiver. We fabricated a waveguide quadrature hybrid (Srikanth and Kerr, 2001) to equally divide the Gunn output for the two mixer blocks, and electronically control the LO power to each using QuinStar Technology PIN diode variable attenuators. The LO system, including a harmonic mixer and a cross-guide coupler, is attached outside the dewar, and waveguide vacuum feedthroughs (Ediss et al., 2005) bridge the vacuum shield. All the waveguide designs of the assembly follow the ALMA standard (Kerr et al., 1999). Each mixer block delivers two isolated sidebands in a single polarization, with a 4-12 GHz IF. The polarization splitting is achieved by the OMT, which is a version of the design in Dunning et al. (2009), proportionally scaled to operate in 230 GHz frequency band. The S-parameters and the polarization isolation of the scaled design were simulated using the frequency domain solver of the CST Microwave studio (see Appendix A). Figure 2.4 shows the cross-section of the OMT block. The OMT separates the input radiation from the horn to two linearly polarized signals. We add a polarization twist (Chattopadhyay et al., 2010) at the end of one OMT output simplify the geometry of the mixer blocks in the dewar. The IF output of the mixers are sent via stainless coaxial cable to the SMA feedthroughs on the bottom of the dewar, and then to the electronics rack inside the cabin through LMR-240 coaxial cables.
Figure 2.4: Left: The inner surface of the split block OMT. We have a polarization twist on one end of the OMT. Right: The T-shaped waveguide of the OMT for the polarization separation of the input signal.
The receiver assembly is surrounded by an aluminum radiation shield with Mylar super-insulation. There are separate vacuum windows above the two feed horns. The size of the window is chosen such that it is greater than five times the beam waist size. We use Z-cut quartz manufactured by Boston Piezo-Optics for the windows and bonded Teflon sheets to both sides of the window to implement anti-reflection (AR) coating (Koller et al., 2001). We estimate the insertion loss to be less than 0.05 dB in the receiver sky frequency bands, from the thickness measurement of the Teflon glued window. An AR-coated quarter-wave plate is installed on top of the window to convert circular polarization to linear polarization. We also ran mechanical finite element analysis (FEA) of the dewar. For dewar tilt angles between 0 and 90 degrees, the maximum displacement of the horn aperture is less than 80 µm, corresponding to \( \sim 2\% \) of the aperture diameter.

The receiver electronics shown in Figure 2.1 are installed in the SPT receiver cabin under the optical bench, together with the SPT-3G electronics. An external computer controls mixer and amplifier bias settings. The LO phased locked loop (PLL) incorporates a 100 MHz crystal reference oscillator that is locked to the maser 10 MHz, to which a 12.3 GHz dielectric resonator oscillator (DRO) is locked for the band 6 LO. This LO is locked to the 6th harmonic of the DRO signal with a 100 MHz offset, again referenced to the 100 MHz crystal. The band 7 LO is locked to a 12.7 GHz DRO at the 9th harmonic. The PLL box has computer-adjustable loop parameters and lock monitoring. The warm IF amplifier box encompasses four chains (two sidebands, two polarizations) of amplifiers and computer-controlled variable attenuators. Since the cabin tilts and rotates with the telescope, the IF signal is transferred via optical fiber link to the stationary SPT control room, where the VLBI backend is installed. The iBOB spectrometer is explained in a later section.
Figure 2.5: 230 GHz band spectral noise temperature for both sidebands of one polarization. The receiver temperature is $\sim 40$ K in the 5-9 GHz IF passband.
We verified the receiver performance with noise temperature measurements, tone injection tests, and the phase stability tests. We used the tone of known frequency in the receiver IF band to ensure the frequency setup and cabling of the instruments. The tone injection was also used to measure the image rejection of the mixers by measuring the tone amplitudes at two sidebands (Kerr et al., 2001). Figure 2.5 shows the noise temperature of the band 6 receiver across the IF band for one polarization. It is \(~40\) K for all four IF channels. This is measured through the Y-factor technique using a nitrogen-temperature cold load. The same test measured at the VLBI backend shows no degradation in noise temperature from the subsequent gain, fiber, and downconversion stages (see Appendix B).

2.2.2 Frequency Reference

A Hydrogen-maser manufactured by T4Science was installed at the DSL to serve as the fundamental frequency reference for VLBI observation. It provides 10 MHz references for synchronizing the receiver system with LO and time-stamping of recorded data. We set up the maser inside a temperature-controlled enclosure placed on a stack of urethane to vibrationally isolate the maser from the DSL building and keep a temperature stable environment. The maser signal is connected to the receiver cabin via a long run of coax: approximately 100 feet of LMR-400 through the interior of the SPT/DSL building and approximately 200 feet of Times Microwave Phase Track 210 (PT210) cable through the azimuth and elevation cable wraps of the SPT. These cables are selected to be particularly phase stable at 10 MHz at their operating temperatures (Rogers, 2008). The PT210 cable is run through silicone foam sponge tubing to slow thermal changes, and this is further encased in a flexible metal conduit. Both the LMR-400 and PT210 segments are composed of a pair of cables so that the round-trip phase on the 10 MHz can be monitored at the maser. A low noise distribution amplifier (LNDA) inside the receiver cabin distributes the 10 MHz coming from the maser and provides a 10 MHz loopback signal that returns on the second coax. During observations the 10 MHz round-trip phase is continuously monitored for changes induced by thermal, mechanical, or
other disturbances. The Allan deviation of the round-trip maser phase is virtually identical to what was found by the manufacturer when beating the maser against an identical model, around $1 \times 10^{-13}$ at 1 s and $1.2 \times 10^{-15}$ at 1000 s. To ensure that the maser is operating properly before observations, it is compared against an Oscilloquartz crystal oscillator to verify the frequency stability and the phase noise characteristics.

2.2.3 VLBI Backend

The VLBI backend is designed to ingest four IF bands from the receiver (two sidebands of two polarizations), each spanning 4 GHz. For band 6 these are 5-9 GHz, for band 7 they are 4-8 GHz. The block downconverter (BDC) divides each IF into two 0–2 GHz basebands, using an internal LO at 7 or 6 GHz. When all of these bands are digitized to two bits of precision at the Nyquist rate, the instantaneous recording rate is 64 Gbits per second. The digitization is done by ROACH2 (Reconfigurable Open Architecture Computing Hardware) digital backend (R2DBE) units, which have demonstrated 4096 megasamples per second sampling for two channels (Vertatschitsch et al., 2015). The 64 Gbps EHT backend system consists of four R2DBEs and four Mark 6 recorders (Whitney et al., 2011; Whitney et al., 2013). Figure 2.6 shows the VLBI backend setup at the SPT. The recorded data are correlated on the DiFX correlators (Deller et al., 2011) at the MIT Haystack Observatory and the Max Planck Institute for Radio Astronomy.

2.2.4 Calibration System and Spectrometer

We developed a calibration system\textsuperscript{2} for the receiver to keep track of the system temperature during the observation. The system temperature can be derived by the ratio of powers received from the sky and a load of known temperature. Due to the limited space inside the receiver cabin, the calibration system is installed above the receiver cryostat, along with the tertiary mirror mount (Figure 2.7). We

\textsuperscript{2}Receiver Selection and Calibration Unit for EHT-SPT (RESCUES); http://hdl.handle.net/10150/579318
Figure 2.6: VLBI backend installed at the SPT control room. These racks comprise a 64 Gbps data recording system.
use microwave absorber as an ambient load, coupled to a Schneider Electric Motion LMDCE421 motor with a rotation shaft. It covers the receiver beam as we run the calibration. The AD-590 temperature transducer is buried inside the absorber to read the load temperature. The atmospheric opacity is useful information for the calibration because the system temperature depends on a source elevation. At the telescope site, we have a 350 $\mu$m tipping radiometer installed and can convert 350 $\mu$m opacity to 225 GHz opacity by a conversion relation given in Radford and Peterson (2016). The calibration load also carries a feed horn and a harmonic mixer that can be positioned in the receiver beam to generate a coherent tone for signal path verification and coherence testing.

To aid in pointing, we installed a digital spectrometer for measurements of CO lines that lie near to the EHT observing bands. The spectrometer itself is a pair of FPGA spectrometers based on the CASPER iBOB board$^3$. They are fed by a special signal chain that converts the CO 2-1 line (230.538 GHz, IF=9.438 GHz for band 6) and CO 3-2 (345.796 GHz, IF=3.196 GHz for band 7) to approximately 750 MHz. An example of a spectral line map with this system is provided in Figure 2.8.

$^3$https://casper.berkeley.edu/wiki/1_GHz_-_1024_Channel_Wideband_Spectrometer
Figure 2.7: *Left:* The tertiary mirror assembly installed on top of the receiver dewar for the observation. *Right:* The calibration load for the system temperature measurement, on the tertiary mount. A feed horn is located on the calibration load with a harmonic mixer for a tone injection. The tone injection feed horn assembly is tilted downward so that it can illuminate the 230 GHz receiver feed horn.
2.3 Optics

The SPT has an off-axis Gregorian design with a 10-meter primary mirror to minimize blockage and scattering of incident light from the faint CMB (see Padin et al. 2008 for detail). The SPT was not initially designed to illuminate any instrument other than its CMB camera, so special optics are required to redirect the light from the primary mirror to the VLBI receiver.

2.3.1 Design

The VLBI optical system has a Cassegrain design, with hyperbolic secondary and ellipsoidal tertiary mirrors. The mirror parameters were optimized with the Zemax optical design software. The model was chosen such that the optics illuminate the 10 m dish to greater than 12 dB at both frequencies, given the beam parameters and locations of the 230 and 345 GHz feed horns. In Figure 2.2, we show the VLBI optics installed around the SPT-3G receiver and its optics. The VLBI secondary mirror blocks the 3G secondary mirror and reflects the beam from the primary to the VLBI tertiary mirror, and then to feed horns of the VLBI receiver. The tertiary mirror is mounted to the top of the dewar and, to simplify winter operation, this mirror rotates around the optical axis so that it focuses the beam towards either 230 or 345 GHz side of the receiver. During observation, we move the position of the receiver in the three-dimensional space using the optical bench for focusing (see Appendix C for the on-sky examples). The mirrors are easily removable to clear the optical path for the CMB receiver and only installed for the EHT observing campaigns. We use a SpitzLift portable crane for transport of the mirrors to the receiver cabin top. Figure 2.9 shows the secondary and tertiary mirrors installed for VLBI observation.

2.3.2 Beam Measurement

The receiver beam pattern has been measured in both frequency bands (Figure 2.10) by measuring the response to a coherent tone as it is scanned across a plane above
Figure 2.8: Spectral channel map of the protostar IRAS 16293-2422, a pointing source. The map shows 16 1 MHz channels starting from a second IF of 695 MHz. Pointing offsets can be determined from one or many channels, as appropriate for each source.
Figure 2.9: The secondary and tertiary mirrors of the SPT VLBI receiver system installed at the South Pole, viewed from the primary mirror. The mirror assemblies are covered by environmental seals to prevent cold air flowing into the receiver cabin.
the receiver (see Kim and Marrone 2018; Chapter 3). As shown in Figure 2.3, both 230 and 345 GHz receivers are inside a single dewar and the feed horns are intentionally tilted inward, toward the centerline of the receiver between the two horns, so that they both can face the shared tertiary. We model the near-field scan as a Gaussian beam propagating at an angle to the measurement plane. We characterize the model parameters, including the three-dimensional location of the feed horn phase center and its tilt angle, using the fit between the model and the data. The inferred parameters indicate that the feed horn assemblies are correctly positioned and oriented.

We also measured the beam pattern after the tertiary mirror using the same technique in planes near the Cassegrain focus. These data were used to verify the beam propagation direction between tertiary and secondary, based on the measured propagation angle and the position of the beam in several parallel planes along the propagation direction.
Figure 2.10: The vector beam measurement of the 230 GHz receiver provides both power (left) and phase (right). The blue dots and the red squares show the data and the best fit model.
2.4 Software

The SPT VLBI receiving system has three types of software: receiver control, calibration, and telescope control. The receiver control software runs on a BeagleBone Black (BBB), a single-board computer that supports the Linux environment. The software controls the receiver and related electronics until the optical fiber relay. The receiver tuning screen in Figure 2.11 is the primary control interface and was originally developed by Thomas W. Folkers for the Kitt Peak 12-m and Submillimeter Telescope (SMT) on Mount Graham, operated by the Arizona Radio Observatory (ARO). This software has been adapted for the SPT application. The primary parameters controlled/monitored by this software are the mixer and amplifier bias settings, local oscillator power levels and phase-locked loop parameters, thermometry, and the gain of the warm amplifier chain.

The calibration software obtains data for the system temperature measurement and a priori calibration of the telescope. It involves the monitoring of the calibration load temperature and the IF power, and positioning of feed horn for the tone injection. The software interacts with the telescope control software, Generic Control Program (GCP; Story et al., 2012), by which the SPT control and data acquisition are done.
Figure 2.11: The tuning screen for the 230 GHz receiver. The screen controls mixer bias, the Gunn oscillator lock, and LO injection, and monitors the temperature inside the receiver.
2.5 Summary

In this chapter, we describe the development of the VLBI receiver for the SPT. We deployed the receiver system, including a hydrogen maser and VLBI recording backend system, to the South Pole in the 2016-17 austral summer, and the SPT joined the EHT array in its first campaign in April 2017. This system samples two polarizations near LO frequencies of either 221.1 or 342.6 GHz and instantaneously digitizes 16 GHz of receiver bandwidth, yielding a 64 Gbps VLBI data rate. The clean receiver optical path, low-noise mixers, and the atmospheric environment of the South Pole combine to create a high-sensitivity VLBI station that significantly extends the baseline coverage of the EHT array.

---

4The 345 GHz receiver had the first light in January 2019 with the prototype mixer under development (Appendix D) but has not joined the VLBI observation yet.
Tilted Beam Measurement of VLBI Receiver for the South Pole Telescope

We have developed a 230 and 345 GHz very-long-baseline interferometry (VLBI) receiver for the South Pole Telescope (SPT). With the receiver installed, the SPT has joined the global Event Horizon Telescope (EHT) array. The receiver optics select the 230 or 345 GHz mixers by rotating the tertiary mirror around the optical axis, directing the chief ray from the secondary mirror to the feed horn of the selected frequency band. The tertiary is installed on top of the receiver cryostat, which contains both mixer assemblies. The feed horns are placed symmetrically across the centerline of the telescope optics and tilted inward by 5.7 degrees from the vertical plane so that their beams intersect at the chief ray intersection on the tertiary mirror.

We have performed vector beam measurements of the SPT VLBI receiver in both frequency bands. The measurements preserved the relative location of the beams, to establish the relative locations of the phase centers of the two horns. Measurements in two parallel reference planes above the cryostat were used to suppress reflected light. To model the beam, we derive a general expression of the electric field vector on the measurement plane for a tilted beam and infer the feed horn position parameters for both frequency bands by fitting models to data with a Markov chain Monte Carlo (MCMC) method. The inferred parameters such as the tilt angle of the feed horn are in good agreement with the design. We present the measurement setup, amplitude and phase pattern of the beam, and the fitting result here.  

1A version of this chapter originally appeared as a published paper in the Proceedings of the 29th International Symposium on Space Terahertz Technology (Kim and Marrone, 2018). All of the work described below was carried out by me, with help from co-author Daniel P. Marrone.
3.1 Introduction

The South Pole Telescope (SPT) is a cosmic microwave background (CMB) experiment located at the Amundsen-Scott South Pole station (Carlstrom et al., 2011; Benson et al., 2014). Its 10-meter diameter primary dish was designed with a surface accuracy suitable for submillimeter-wavelength observation. Its geographic location, far from other submillimeter telescopes, provides an opportunity to greatly increase the size millimeter wavelength very-long-baseline interferometry (VLBI) arrays, such as the Event Horizon Telescope (EHT; Doeleman et al., 2009).

We have developed a 230 and 345 GHz VLBI receiver for the SPT. In this chapter, we report the vector beam measurement of the receiver using a technique developed for submillimeter receivers (e.g., Tong et al., 1994; Chen et al., 1998). In particular, we use the measurement to characterize the location and tilt of the horn phase centers inside the receiver dewar. We also measured the beam with the tertiary mirror installed above the dewar, to trace the beam propagation between the tertiary and the secondary mirror optics (see Appendix E). In Section 2, we describe the receiver and its optical design. In Section 3, we show the beam measurement setup and the test result. In Section 4, we introduce a general expression of the electric field when the tilt angle of the beam is considered, and then infer the model parameters using Markov chain Monte Carlo (MCMC) method. We have found that the estimated model parameters from the data agree well with the design.

3.2 SPT VLBI Receiver

The SPT VLBI receiver operates in the 230 and 345 GHz frequency bands. Figure 3.1 shows the inside of the dewar. In each band, the RF components feed horns, ortho-mode transducers (OMT, a polarization separation device), and mixer blocks are cooled down to 4 K. The 230 GHz receiver uses ALMA band 6 sideband-separating SIS mixers (Kerr et al., 2014). The 345 GHz mixer is under development and expected to be installed in early 2019.

The sky signal is delivered to the VLBI receiver with removable secondary and
Figure 3.1: An interior view of the SPT VLBI receiver. The left and right are 230 GHz and 345 GHz assemblies of the receiver, respectively. Both 230 GHz and 345 GHz feed horns are designed to be 5.74 degrees tilted inward so that the receiver operates at both frequencies using a rotating tertiary mirror above the dewar. The yellow arrows show the beam propagation directions of the feedhorns, and the white dashed line indicates the vertical plane.
tertiary mirrors that are, independent of the SPT-3G optics (for details, see Kim et al. 2018a; Chapter 2). The tertiary mirror assembly sits on the receiver dewar and the mirror rotates around the optical axis to switch between 230 and 345 GHz. To couple the chief ray reflected through the tertiary mirror to each feed horn, we designed the feed horns to be tilted toward each other by 5.74 degrees.

3.3 Vector Beam Measurements

Accurate positioning of the two feedhorns is critical to the coupling of the two frequency bands to the optics in this submillimeter-wavelength receiver. We perform 230 and 345 GHz vector beam measurement of the SPT VLBI receiver to characterize their location. We set up the vector beam measurement system as shown in Figure 3.2. The test tone is generated from a Gunn oscillator followed by a frequency tripler (230 GHz: VDI WR-5.1 × 3, 345 GHz: VD1-WR2.8 × 3). We put WR-4 open waveguide probe (0.04 inch × 0.02 inch) whose cutoff frequency is ~140 GHz, on the tripler output, and the transmitter system is mounted on X-Y translation stage composed of two Parker Motion 403XE linear stages. The test tone frequency is chosen such that the intermediate frequency (IF) of the receiver is placed within the IF amplifier operating range. The IF is again mixed down to ~546 MHz to use a K&L Microwave bandpass filter with 10 MHz bandwidth to increase the signal to noise ratio. We can read the amplitude and relative phase of the second IF with the reference input from HP 8648C signal generator. During the beam scan, all the hardware components around the setup are covered with AN-72 broadband microwave absorber to reduce the reflection of the injected tone.

We perform a two-dimensional scan of a region 100 mm × 100 mm in size with a 2.5 mm step, in two parallel planes: The planes are 200 mm and 150 mm vertically above the horn phase center plane. The 230 GHz scan region is centered on \((x, y) = (25 \text{ mm}, 0 \text{ mm})\) and the 345 GHz region is centered on \((x, y) = (−25 \text{ mm}, 0 \text{ mm})\), to locate the maximum amplitude position near the center of the scanning area. After every column scan, the probe moves to the center
Figure 3.2: Schematic of the beam measurement system. Frequencies for the 230 GHz chain are listed in blue, with 345 GHz in red. The signal generators and the phase locked loop for the system are locked to a common 10 MHz signal.
position to track amplitude and phase stability. Interpolating reference measurements compensates the time-dependent phase drift during the scan. At each plane we make two maps on planes that are a quarter-wavelength apart in the z-direction. Averaging these two sets of data can further reduce the reflection. Figure 3.3 shows the amplitude and phase of 230 GHz at $z = 200$ mm plane. For 230 GHz, we recorded the beams of two different polarizations, by rotating the waveguide probe by 90 degrees. Figure 3.4 shows the amplitude and phase of 345 GHz at $z = 200$ mm plane.
Figure 3.3: (Top left) The amplitude and (bottom left) the phase pattern of 230 GHz beam measurement at \( z = 200 \) mm. The contours in the amplitude map are in dB units, and the phase map is plotted in degree units. The amplitude map shows more than 30 dB of dynamic range. The phase is not centered due to the tile of the feedhorn. (Top right) The amplitude and (bottom right) the phase pattern of the beam, using the analytic model with the best-fit parameters.
Figure 3.4: (Top left) The amplitude and (bottom left) the phase pattern of 345 GHz beam measurement at $z = 200$ mm. (Top right) The amplitude and (bottom right) the phase pattern of the beam, using the analytic model with the best-fit parameters. In this Figure, the $x$-axis is reversed from the scanning setup to use the identical model fitting procedure as 230 GHz data.
3.4 Analysis

To analyze the data, we derive a general form of the electric field distribution of the tilted Gaussian beam, when the scanning plane is not vertical to the axis of propagation. Then, we fit the model to the data to infer the feed horn position and its tilt angle.

3.4.1 Gaussian Beam Propagation of Tilted Horn

The normalized electric field distribution (Goldsmith, 1998) is

\[
E(r, z) = \frac{2}{\pi w^2} \exp\left( -\frac{r^2}{w^2} - jkz - \frac{j\pi r^2}{\lambda R} + j\phi_0 \right)
\]  

(3.1)

where \( z \) is the distance along the axis of propagation and \( r \) is the perpendicular distance from the axis of propagation. \( R, w, \) and \( \phi_0 \) are the radius of curvature, beam radius, and the Gaussian phase shift and expressed as

\[
R = z + \frac{1}{z} \left( \frac{\pi w_0^2}{\lambda} \right)^2, \\
w = w_0 \sqrt{1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2}, \\
\phi_0 = \arctan \left( \frac{\lambda z}{\pi w_0^2} \right),
\]  

(3.2), (3.3), (3.4)

where \( w_0 \) is the beam waist radius.

In Figure 3.5, we first define two planes parallel to each other: the horn phase center plane and the scanning plane where the waveguide probe mounted on the translation stage moves. We assume arbitrary horn phase center position \( P_h = (x_h, y_h, 0) \) to derive an electric field at the scan position \( P_s = (x, y, z_0) \).

The equation of the line \( l_{beam} \) is

\[
l_{beam} : (x_h, y_h, 0) + t(- \tan \theta_x, - \tan \theta_y, 1). 
\]  

(3.5)

The directional vector of axis of propagation on \( l_{beam} \) is

\[
\vec{b} = (- \tan \theta_x, - \tan \theta_y, 1),
\]  

(3.6)
Figure 3.5: Schematic of the tilted beam measurement. The feed horn is tilted by $\theta_x$ and $\theta_y$ on the vertical plane in $x$ and $y$-direction. The Gaussian beam radiates from the horn phase center $(x_h, y_h, 0)$ along its axis of beam propagation $l_{\text{beam}}$ (red arrow). The beam scanning plane and the horn phase center plane are vertically separated by $z_0$. 
and the vector between $P_h$ and $P_s$ is

$$\overrightarrow{P_hP_s} = (x - x_h, y - y_h, z_0). \tag{3.7}$$

The vertical distance from the horn phase center to the wave plane that hits $P_s$, $z_{\text{tilt}}$, is

$$z_{\text{tilt}} = \frac{\left| \vec{b} \cdot \overrightarrow{P_hP_s} \right|}{|\vec{b}|}, \tag{3.8}$$

where

$$\left| \vec{b} \cdot \overrightarrow{P_hP_s} \right| = |z_0 - (x - x_h) \tan \theta_x + (y - y_h) \tan \theta_y|, \tag{3.9}$$

and

$$\vec{b} = \sqrt{\tan^2 \theta_x + \tan^2 \theta_y + 1}. \tag{3.10}$$

The offset from the axis of propagation on the reference plane is

$$r_{\text{tilt}} = \frac{\left| \vec{b} \times \overrightarrow{P_hP_s} \right|}{|\vec{b}|}, \tag{3.11}$$

where

$$\left| \vec{b} \times \overrightarrow{P_hP_s} \right|^2 = [z_0 \tan \theta_y - (y - y_h)]^2 + [z_0 \tan \theta_x + (x - x_h)]^2 +$$

$$[(y - y_h) \tan \theta_x - (x - x_h) \tan \theta_y]^2$$

$$= z_0^2 (\tan^2 \theta_x + \tan^2 \theta_y) + (x - x_h)^2 (\tan^2 \theta_y + 1) +$$

$$(y - y_h)^2 (\tan^2 \theta_x + 1) + 2z_0 (x - x_h) \tan \theta_x +$$

$$2z_0 (y - y_h) \tan \theta_y + 2(x - x_h)(y - y_h) \tan \theta_x \tan \theta_y$$

$$= z_0^2 (\sec^2 \theta_x + \sec^2 \theta_y) + (x - x_h)^2 \sec^2 \theta_y + (y - y_h)^2 \sec^2 \theta_x$$

$$- 2z_0 [z_0 - (x - x_h) \tan \theta_x][z_0 + (y - y_h) \tan \theta_y] \tag{3.12}$$

We now have the normalized electric field distribution for the tilted beam on the scanning plane

$$E(r, z) = \sqrt{\frac{2}{\pi w_{\text{tilt}}^2}} \exp \left( -\frac{r_{\text{tilt}}^2}{w_{\text{tilt}}^2} - jkz_{\text{tilt}} - \frac{j\pi r_{\text{tilt}}^2}{\lambda R_{\text{tilt}}} + j\phi_{0,\text{tilt}} \right). \tag{3.13}$$
where

\[
R_{\text{tilt}} = z_{\text{tilt}} + \frac{1}{z_{\text{tilt}}} \left( \frac{\pi w_0^2}{\lambda} \right)^2,
\]

(3.14)

\[
w_{\text{tilt}} = w_0 \sqrt{1 + \left( \frac{\lambda z_{\text{tilt}}}{\pi w_0^2} \right)^2},
\]

(3.15)

\[
\phi_{0,\text{tilt}} = \arctan \left( \frac{\lambda z_{\text{tilt}}}{\pi w_0^2} \right).
\]

(3.16)

The center of the beam that hits the scanning plane is where the equation of line \(l_{\text{beam}}\) intersects with \(z = z_0\) plane and

\[
x_c = x_h - z_0 \tan \theta_x,
\]

(3.17)

\[
y_c = y_h + z_0 \tan \theta_x,
\]

(3.18)

\[
z = z_0
\]

(3.19)

3.4.2 Beam Fitting

We fit the beam mapping data to the tilted Gaussian beam profile described in the previous Section IV-A, maximizing the power coupling coefficient between the data and the model electric fields. The power coupling coefficient is an absolute square of the field coupling coefficient

\[
K_{ab} = \left| \iint E_a * E_b \, dS \right|^2
\]

(3.20)

where \(E_a\) and \(E_b\) are the electric field distributions of two Gaussian beams. Then, we use Markov chain Monte Carlo (MCMC) sampling with the python package emcee (Foreman-Mackey, 2016).

The corner plot (Figure 3.6) is an example of the parameter estimation for one polarization of the 230 GHz beam at \(z = 200\) mm. For all scans, the \(y\)-phase center position is less than 0.5 mm and the \(y\)-direction tilt angle is less than 0.5 degrees. The best-fit parameters for the receiver tilt angle in the \(x\)-direction are listed in Table 3.1 and an example of the model beam is shown in Figure 3.3. The 5.74 degrees \(x\)-direction tilt angle from the design is within 1-sigma uncertainty range of all the measurements.
Figure 3.6: Parameter estimation result for the 230 GHz measurement at $z = 200$ mm plane using MCMC. The inferred parameters agree well with the design.
Table 3.1. The best-fit parameters for the $x$-direction tilt angle

<table>
<thead>
<tr>
<th></th>
<th>230 GHz</th>
<th>345 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$z$ (mm)</td>
<td>$\theta_x$ (degrees)</td>
</tr>
<tr>
<td>Pol 0</td>
<td>200</td>
<td>5.62 (+0.56 / -0.56)</td>
</tr>
<tr>
<td></td>
<td>155</td>
<td>5.45 (+0.74 / -0.78)</td>
</tr>
<tr>
<td>Pol 1</td>
<td>200</td>
<td>5.50 (+0.58 / -0.54)</td>
</tr>
<tr>
<td></td>
<td>155</td>
<td>5.48 (+0.75 / -0.76)</td>
</tr>
</tbody>
</table>
3.5 Conclusion

We have performed the beam measurement of the SPT VLBI receiver at 230 and 345 GHz. We introduced the functional form of the electric field for the tilted beam. The MCMC fitting of the model to the data gives the estimation of the horn parameters, especially the tilt angle of the feed horn that is critical to the beam coupling between the horn and the optical elements.
The 1.4 mm Core of Centaurus A: First VLBI Results with the South Pole Telescope

Centaurus A (Cen A) is a bright radio source associated with the nearby galaxy NGC 5128 where high-resolution radio observations can probe the jet at scales of less than a light-day. The South Pole Telescope (SPT) and the Atacama Pathfinder Experiment (APEX) performed a single-baseline very-long-baseline interferometry (VLBI) observation of Cen A in January 2015 as part of VLBI receiver deployment for the SPT. We measure the correlated flux density of Cen A at a wavelength of 1.4 mm on a \( \sim 7000 \) km (5 G\( \lambda \)) baseline. Ascribing this correlated flux density to the core, and with the use of a contemporaneous short-baseline flux density from a Submillimeter Array observation, we infer a core brightness temperature of \( 1.4 \times 10^{11} \) K. This is close to the equipartition brightness temperature, where the magnetic and relativistic particle energy densities are equal. Under the assumption of a circular Gaussian core component, we derive an upper limit to the core size \( \phi = 34.0 \pm 1.8 \) \( \mu \)as, corresponding to 120 Schwarzschild radii for a black hole mass of \( 5.5 \times 10^7 M_\odot \).

\(^1\)A version of this chapter originally appeared as a published paper in the Astrophysical Journal (Kim et al., 2018b). All of the work described below was carried out by me, with help from co-authors Daniel P. Marrone, Alan L. Roy, Jan Wagner, Keiichi Asada, Christopher Beaudoin, Jay Blanchard, John E. Carlstrom, Ming-Tang Chen, Thomas M. Crawford, Geoffrey B. Crew, Sheperd S. Doeleman, Vincent L. Fish, Christopher H. Greer, Mark A. Gurwell, Jason W. Henning, Makoto Inoue, Ryan Keisler, Thomas P. Krichbaum, Ru-Sen Lu, Dirk Muders, Cornelia Müller, Chi H. Nguyen, Eduardo Ros, Jason SooHoo, Remo P. J. Tilanus, Michael Titus, Laura Vertatschitsch, Jonathan Weintroub, and J. Anton Zensus.
4.1 Introduction

Centaurus A (PKS 1322-428, hereafter Cen A) is the brightest radio source associated with the galaxy NGC 5128 (see Israel 1998 for a review) and located at a distance of 3.8 ± 0.1 Mpc (Rejkuba, 2004; Karachentsev et al., 2007; Harris et al., 2010, and references therein). It has a prominent double-sided jet and belongs to the Fanaroff-Riley type I class of radio galaxies (Fanaroff and Riley, 1974). Its proximity and brightness make it an especially suitable target for high-angular-resolution observations with very-long-baseline interferometry (VLBI), which can reveal the jet structure as well as the innermost region of the active galactic nucleus (AGN; Boccardi et al., 2017).

Although the Very Long Baseline Array (VLBA) has monitored the source from the northern hemisphere (e.g., Tingay and Murphy, 2001; Tingay et al., 2001) at wavelengths as short as 7 mm (Kellermann et al., 1997), observations of Cen A have been mostly limited to longer wavelength VLBI arrays located in the southern hemisphere (e.g., Tingay et al., 1998; Müller et al., 2011, 2014) due to its low declination of −43°. Previous observations have achieved angular resolutions of 0.4 mas × 0.7 mas at 3.6 cm using an array spanning Australia, Chile, and Antarctica (Müller et al., 2011) through the Tracking Active Galactic Nuclei with Austral Milliarcsecond Interferometry (TANAMI; Ojha et al., 2010; Müller et al., 2018) program and 0.6 mas at 6.1 cm with the VLBI Space Observatory Programme (VSOP; Horiuchi et al., 2006) satellite. Observations of the source would benefit from the inclusion of southern stations and measurements at short millimeter wavelengths where the radio core could be explored on smaller scales.

VLBI observations of Cen A have probed its morphology in multiple wavelengths from 13 cm to 7 mm (Tingay et al., 1998, 2001; Horiuchi et al., 2006; Müller et al., 2011, 2014). Tingay et al. (1998) and Müller et al. (2014) find that there is a compact component within the jet structure. The VLBA data of Kellermann et al. (1997) suggests that the observed structure is already dominated by a single component at 7 mm wavelength. At shorter wavelengths, the lower synchrotron opacity should
provide access to deeper regions of the stationary core.

The Event Horizon Telescope (EHT) is a VLBI network operating at 1.4 mm, and in the near future at 0.9 mm (e.g., Doeleman et al., 2008, 2009, 2012). Most of the EHT stations are located in the northern hemisphere, namely the Submillimeter Array (SMA; 8 × 6 m dishes) and the James Clerk Maxwell Telescope (JCMT; 15 m) in Hawaii, the Submillimeter Telescope (SMT; 10 m) in Arizona, the Institute de Radioastronomie Millimétrique (IRAM) 30 m telescope in Spain, and the Large Millimeter Telescope (LMT; 50 m) in Mexico. The sensitivity, imaging capability, and north-south extent of the array have recently been improved through the inclusion of southern hemisphere stations: Atacama Pathfinder Experiment (APEX; 12 m) and the Atacama Large Millimeter/submillimeter Array (ALMA; ∼37 × 12 m dishes; Matthews et al., 2018) in Chile, and the South Pole Telescope (SPT; 10 m; Carlstrom et al., 2011). These sites will enable the EHT to provide better imaging of Cen A at a much shorter wavelength than past VLBI experiments.

Cen A is powered by a supermassive black hole with a mass of 5.5 ± 3.0 × 10^7 M_☉ (Cappellari et al., 2009; Neumayer, 2010). The apparent diameter of the black hole event horizon, accounting for its own gravitational lensing, is ∼5R_{sch} where R_{sch} is the Schwarzschild radius (Bardeen, 1973). The corresponding apparent angular size of the event horizon is 1.5 μas, well below the ∼20 μas resolution of the EHT at 1.4 mm. However, the EHT still provides the resolution to observe the inner region close to the black hole. For example, the 7000 km baseline between the SPT and APEX provides a fringe spacing of 40 μas (150 au at the distance of Cen A at 1.4 mm). This is better angular resolution than any VLBI observation of Cen A published to date.

In this chapter, we report results from the VLBI observation of Cen A with the SPT and APEX at 1.4 mm during commissioning observations for the SPT VLBI system. In section 4.2, we describe the observation and the visibility amplitude calibration. In section 4.3, we present the analysis of the data to infer physical properties of the radio core of Cen A. This single-baseline observation places a lower limit on the brightness temperature of the Cen A core region, and, when used with
the zero-baseline flux density measurement, allows us to place an upper limit on the core size.

4.2 Observations and Data Reduction

4.2.1 Observations

On January 17, 2015, the SPT and APEX performed VLBI observations of several sources, including J0522-363, B1244-255, W Hya, Sagittarius A*, and Cen A. It was the first VLBI observation using the 1.4 mm SPT VLBI receiver. APEX used its 1.4 mm SHeFI receiver (Belitsky et al., 2007; Vassilev et al., 2008). APEX had previously demonstrated millimeter VLBI capability in an experiment with the SMT in Arizona (Wagner et al., 2015).

The observation of Cen A included eight scans between 07:20 UT and 08:55 UT. Each scan was a 5-minute integration. Data were recorded for frequencies between 214.138 and 216.122 GHz, a receiving bandwidth of 2 GHz, centered on 215.13 GHz, or 1.39 mm, in left circular polarization. At each station, the data were digitized by a ROACH2 (Reconfigurable Open Architecture Computing Hardware) digital back-end (R2DBE; Vertatschitsch et al., 2015) and recorded at 2-bit precision on Mark 6 recorders (Whitney et al., 2013). We correlated the data on the DiFX correlator (Deller et al., 2011) at the MIT Haystack Observatory. The fringe fitting of the correlated data was done by fourfit of Haystack Observatory Post-processing System² (HOPS). We then segmented the data down to 1 s and incoherently averaged to produce the fringe amplitude (Rogers et al., 1995). We found strong detections for all eight scans with signal-to-noise ratios (SNRs) between 36 and 69. The \((u, v)\) coverage of the scans is shown in Figure 4.1. All scans correspond to a baseline length of approximately 5 G\(\lambda\).

²http://www.haystack.mit.edu/tech/vlbi/hops.html
Table 4.1. Cen A observation summary

<table>
<thead>
<tr>
<th>Year</th>
<th>Month</th>
<th>Date</th>
<th>UT (hh:mm:ss)</th>
<th>$u$ (Mλ)</th>
<th>$v$ (Mλ)</th>
<th>SEFD$^a$ (APEX, Jy)</th>
<th>SEFD$^a$ (SPT, Jy)</th>
<th>Correlated Flux Density$^b$ (Jy)</th>
<th>SNR$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>1</td>
<td>17</td>
<td>07:20:00</td>
<td>-2857</td>
<td>4125</td>
<td>7380</td>
<td>8560</td>
<td>0.45 ± 0.04</td>
<td>36</td>
</tr>
<tr>
<td>2015</td>
<td>1</td>
<td>17</td>
<td>07:30:00</td>
<td>-2720</td>
<td>4208</td>
<td>7250</td>
<td>8560</td>
<td>0.60 ± 0.05</td>
<td>69</td>
</tr>
<tr>
<td>2015</td>
<td>1</td>
<td>17</td>
<td>07:40:00</td>
<td>-2577</td>
<td>4288</td>
<td>7210</td>
<td>8560</td>
<td>0.58 ± 0.05</td>
<td>67</td>
</tr>
<tr>
<td>2015</td>
<td>1</td>
<td>17</td>
<td>07:50:00</td>
<td>-2430</td>
<td>4362</td>
<td>7200</td>
<td>8560</td>
<td>0.59 ± 0.05</td>
<td>68</td>
</tr>
<tr>
<td>2015</td>
<td>1</td>
<td>17</td>
<td>08:20:00</td>
<td>-1961</td>
<td>4560</td>
<td>6950</td>
<td>8560</td>
<td>0.56 ± 0.05</td>
<td>66</td>
</tr>
<tr>
<td>2015</td>
<td>1</td>
<td>17</td>
<td>08:30:00</td>
<td>-1797</td>
<td>4616</td>
<td>6960</td>
<td>8560</td>
<td>0.56 ± 0.05</td>
<td>66</td>
</tr>
<tr>
<td>2015</td>
<td>1</td>
<td>17</td>
<td>08:40:00</td>
<td>-1629</td>
<td>4667</td>
<td>6990</td>
<td>8560</td>
<td>0.47 ± 0.04</td>
<td>55</td>
</tr>
<tr>
<td>2015</td>
<td>1</td>
<td>17</td>
<td>08:50:00</td>
<td>-1459</td>
<td>4713</td>
<td>6990</td>
<td>8560</td>
<td>0.49 ± 0.04</td>
<td>57</td>
</tr>
</tbody>
</table>

$^a$See Section 4.2.2.

$^b$Includes only statistical errors.

$^c$The first scan shows much lower SNR than the rest of scans. See Section 4.3.4.
4.2.2 Calibration

Visibility amplitude calibration is required to convert the correlated data to flux density units. The system equivalent flux density (SEFD), the system temperature of the telescope divided by the gain, provides this calibration. I describe the detailed calibration procedure in Appendix F.

The absolute calibration of APEX was determined from observations of Saturn. System temperatures were measured by observing an ambient temperature load and recorded during every scan. The receiver noise component of the system temperature was evaluated using an absorber cooled to \( \sim 73 \) K. The SEFD uncertainty for APEX data is 7\%, based on the quadrature sum of the 5\% scatter between calibration measurements and the additional 5\% uncertainty in the absolute calibration scale from Saturn.

The absolute calibration of the SPT was determined from observations of Saturn and Venus. In 2015, the system temperature was not continuously monitored but was estimated from the combination of ambient temperature load observations on the day after the VLBI experiment and data from a 350 \( \mu \)m tipping radiometer on site (Radford and Peterson, 2016). There are several factors that contribute to the uncertainty in the SEFD calibration. The lack of contemporaneous system temperature measurements during the observation contributes an uncertainty of 10\%, allowing for 25\% changes in the opacity between days. The translation between the VLBI signal chain and a separate power monitoring signal chain that was used for the planetary calibration observations contributes an additional 7\% uncertainty, inferred from the scatter between repeated measurements. The observed pointing drift during the observation from repeated pointing measurements suggests that there are possible 10\% changes in the telescope gain due to mis-pointing. The uncertainty due to the planet model is taken to be 5\%. In quadrature, these sum to 16\% uncertainty in the SEFD.

The mean SEFD for the Cen A scans is 7,100 Jy for APEX, while the SPT SEFD is fixed at 8,560 Jy. Table 4.1 summarizes the scans, including baseline
Figure 4.1: \((u, v)\) coverage of the SPT-APEX VLBI observation of Cen A at 1.4 mm. The color of the marker shows the strength of the correlated flux density at each \((u, v)\) coordinate and the size of the markers is irrelevant. Left is the expanded view around the coordinates.
lengths, SEFDs, visibility amplitudes in flux density units, and the detection SNRs.

4.3 Data Analysis

VLBI imaging of Cen A at wavelengths longer than 1.3 cm reveals that its inner jet structure has a number of jet components emerging from a bright core (Tingay et al., 1998, 2001; Horiuchi et al., 2006; Müller et al., 2011, 2014). Tingay and Murphy (2001) compared the VLBA observation images at 13.6, 6.0, and 3.6 cm to estimate the spectral index around the subparsec-scale jet of Cen A. They reported that the spectrum towards the nucleus was highly inverted (increasing flux density with decreasing wavelength), and the core began to dominate the jet at 3.6 cm. Tingay et al. (2001) also discovered that the jet components present in the 3.6 cm images were not observed in 1.3 cm images and only the core component was detected.

Müller et al. (2011) used a southern VLBI array at wavelengths of 1.3 and 3.6 cm to produce images of Cen A with higher resolution and image fidelity. They also found that the core region is brighter at 1.3 cm than at 3.6 cm, while the spectral index along the jet steepens away from the core, with jet components dimmer at 1.3 cm than 3.6 cm. They modeled the innermost region of the jet with two Gaussian components at 1.3 cm. Furthermore, the 7 mm data of Kellermann et al. (1997) implied that the structure is close to a single resolved component, although they were not able to form an image due to limited \((u, v)\) coverage. These observations were made over the past two decades, and the structure observed in previous epochs may differ from what would be measured now in this dynamic source. However, extrapolating this general trend with wavelength, we expect the 1.4 mm emission to be dominated by the optically thick core region. We model the core as a single circularly symmetric Gaussian component. This model is a typical choice for VLBI observations with poor \((u, v)\) coverage (e.g., Kellermann et al., 1997). Because the SPT-APEX baseline rotated little in the \((u, v)\) plane during our observation (Figure 4.1), we have little power to constrain an ellipticity parameter for the source model. Similarly, our observations do not constrain source structure, though from
the considerations above we believe the approximation of a single source is plausible. The orientation of the effectively one-dimensional \((u, v)\) coverage is minimally sensitive to the elongation of the jet, so our size constraints primarily pertain to the perpendicular direction, though we have assumed circular symmetry in the source. The following discussion should be considered with the limitations of our data in mind, including the discussion of the observed visibility amplitude variability in Section 4.3.4.

4.3.1 Brightness Temperature

The brightness temperature of a circular Gaussian source of total (zero-baseline) flux density \(V_0\) and full-width at half maximum (FWHM) \(\phi\) observed at frequency \(\nu\) is given by

\[
T_b = 2 \ln 2 \frac{c^2}{k_B \nu^2} \frac{V_0}{\pi \phi^2},
\]

(4.1)

where \(k_B\) is the Boltzmann constant and \(c\) is the speed of light. For correlated flux density \(V_q\) measured on baseline length \(B\), the implied FWHM is

\[
\phi = \frac{2\sqrt{\ln 2}}{\pi} \frac{1}{B \nu} \sqrt{\ln \left(\frac{V_0}{V_q}\right)}
\]

(4.2)

and the brightness temperature is

\[
T_b = \frac{\pi}{2k_B} \frac{B^2 V_0}{\ln(V_0/V_q)}.
\]

(4.3)

We have no measurement of the zero-spacing flux density that was obtained at the same time as our VLBI observation, so deriving a brightness temperature from Equation 4.3 requires that we fix \(V_0\) to values obtained at other times. Taking the mean observed correlated flux density, \(V_q = 0.54 \pm 0.05\), the derived brightness temperature varies by a factor of roughly 2 depending on the choice of \(V_0\) for reasonable values, as shown in Figure 4.2. Without making any assumption about \(V_0\) we can derive the minimum brightness temperature (Lobanov, 2015) by setting \(\partial T_b / \partial V_0 = 0\) in Equation (4.3), resulting in

\[
T_{b,\text{min}} = \frac{\pi e}{2k_B} B^2 V_q.
\]

(4.4)
Figure 4.3 shows the minimum brightness temperature as a function of UT, and $T_{b, \text{min}}$ from our observation is roughly $7 \times 10^{10}$ K.

We can narrow the range of plausible values for $V_0$ by looking for contemporaneous measurements of the flux density of Cen A. The SMA monitors\(^3\) the flux density of radio sources for use as gain calibrators and secondary flux standards at mm wavelengths (Gurwell et al., 2007), and has two observations at 1 mm within a week of our measurement:

- $5.9 \pm 0.3$ Jy (January 16, 2015)
- $6.1 \pm 0.3$ Jy (January 22, 2015).

The SMA has sub-arcsecond resolution, and the observations are nearly contemporaneous. The SMA data therefore provide a useful total flux density (or zero-baseline flux density) for the AGN component that should not resolve out any of the 1.4 mm emission from the AGN core but that will spatially filter the emission from the dust and star formation of NGC 5128. Unless there are distant jet hot spots at 1 mm that are not predicted by the spectra of knots seen at longer wavelengths and not seen in other arcsecond-resolution 1 mm images (e.g., McCoy et al., 2017), the SMA flux should be dominated by the core. Adopting $V_0 = 6.0 \pm 0.2$ Jy, the mean of the two SMA flux densities (marked by the green vertical band in Figure 4.2), the brightness temperature implied for the Cen A core is $(1.4 \pm 0.2) \times 10^{11}$ K.

Table 4.2 lists the brightness temperatures measured for Cen A at other wavelengths. Included in this table are simultaneous measurements from 19.0 cm to 7 mm made with the VLBA in 2013 (project code BH182B; Haga et al., 2013), which have not previously been published. Our 1.4 mm brightness lower limit is comparable to that seen at other wavelengths, while the brightness temperature estimated including the SMA zero-spacing flux density is higher than nearly all others. This is what would be expected if our observation is sensitive to emission deeper in the synchrotron core due to decreasing synchrotron optical depth at shorter wavelengths. The estimated brightness temperature is still below the $\sim 10^{12}$ K inverse Compton

\(^3\)http://sma1.sma.hawaii.edu/callist/callist.html
Figure 4.2: Ranges of the brightness temperature and the size of the Cen A core region versus the true zero-baseline flux density. The shaded regions around the red and blue curves indicate the 1σ calibration uncertainties. The flux density measured by the SMA, \( V_0 = 6.0 \pm 0.2 \text{ Jy} \), is marked with the vertical green band.
Figure 4.3: Brightness temperatures of the Cen A core as a function of UT on January 17, 2015. The lower limit using only the SPT-APEX baseline correlated flux density (blue) and the brightness temperature derived with additional SMA zero-baseline data (red) are shown.
limit (Kellermann and Pauliny-Toth, 1969) even at this shortest wavelength, and close to the equipartition limit of $\sim 10^{11}$ K (Readhead, 1994). Doppler boosting from relativistic motion of the jet does not appear to be important for this source ($\delta \sim 1$; Tingay et al., 1998; Meisenheimer et al., 2007; Müller et al., 2014), so the brightness temperature does indicate that the region is near equipartition and it is likely that the amount of energy stored in particles and the magnetic field are similar.
Table 4.2. Brightness temperature, flux density and the size of Cen A core region

<table>
<thead>
<tr>
<th>Wavelength (mm)</th>
<th>Frequency (GHz)</th>
<th>Brightness temperature (K)</th>
<th>Flux density (Jy)(^a)</th>
<th>Size (µas)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>190</td>
<td>1.6</td>
<td>4.6 \times 10^9</td>
<td>0.83</td>
<td>11000 ± 530</td>
<td>VLBA, March 2013</td>
</tr>
<tr>
<td>130</td>
<td>2.3</td>
<td>2.4 \times 10^9</td>
<td>1.02</td>
<td>11000 ± 540</td>
<td>VLBA, March 2013</td>
</tr>
<tr>
<td>61</td>
<td>4.9</td>
<td>5.7 \times 10^8</td>
<td>0.69</td>
<td>7900 ± 400</td>
<td>VLBA, March 2013</td>
</tr>
<tr>
<td>61</td>
<td>4.9</td>
<td>2.2 \times 10^{10b}</td>
<td>0.30</td>
<td>2000 ± 470</td>
<td>Horiuchi et al. (2006)</td>
</tr>
<tr>
<td>36</td>
<td>8.4</td>
<td>4.7 \times 10^9</td>
<td>3.26</td>
<td>3600 ± 180</td>
<td>VLBA, March 2013</td>
</tr>
<tr>
<td>36</td>
<td>8.4</td>
<td>5.9 \times 10^{9c}</td>
<td>2.47</td>
<td>2700 ± 470</td>
<td>Tingay et al. (1998)</td>
</tr>
<tr>
<td>36</td>
<td>8.4</td>
<td>2.1 \times 10^{9c}</td>
<td>2.25</td>
<td>4300 ± 840</td>
<td>Tingay et al. (2001)</td>
</tr>
<tr>
<td>36</td>
<td>8.4</td>
<td>1.5 \times 10^{11b}</td>
<td>0.53</td>
<td>270 ± 60</td>
<td>Müller et al. (2011)</td>
</tr>
<tr>
<td>36</td>
<td>8.4</td>
<td>6.5 \times 10^{10d}</td>
<td>1.09</td>
<td>580 ± 160</td>
<td>Müller et al. (2014)</td>
</tr>
<tr>
<td>24</td>
<td>12</td>
<td>3.1 \times 10^9</td>
<td>2.07</td>
<td>2400 ± 120</td>
<td>VLBA, March 2013</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
<td>4.9 \times 10^9</td>
<td>3.15</td>
<td>1900 ± 94</td>
<td>VLBA, March 2013</td>
</tr>
<tr>
<td>14</td>
<td>22</td>
<td>4.0 \times 10^{9c}</td>
<td>2.35</td>
<td>1200 ± 170</td>
<td>Tingay et al. (2001)</td>
</tr>
<tr>
<td>13</td>
<td>22</td>
<td>4.0 \times 10^9</td>
<td>2.34</td>
<td>1300 ± 130</td>
<td>VLBA, March 2013</td>
</tr>
<tr>
<td>13</td>
<td>22</td>
<td>3.0 \times 10^{10b}</td>
<td>1.21</td>
<td>680 ± 90</td>
<td>Müller et al. (2011)</td>
</tr>
<tr>
<td>7</td>
<td>43</td>
<td>1 \times 10^{10}</td>
<td>3.15</td>
<td>500 ± 100</td>
<td>Kellermann et al. (1997)</td>
</tr>
<tr>
<td>Wavelength (mm)</td>
<td>Frequency (GHz)</td>
<td>Brightness temperature (K)</td>
<td>Flux density (Jy)$^a$</td>
<td>Size ($\mu$as)</td>
<td>Reference</td>
</tr>
<tr>
<td>----------------</td>
<td>----------------</td>
<td>----------------------------</td>
<td>----------------------</td>
<td>----------------</td>
<td>-------------------------</td>
</tr>
<tr>
<td>7</td>
<td>43</td>
<td>$4.5 \times 10^9$</td>
<td>2.17</td>
<td>$570 \pm 57$</td>
<td>VLBA, March 2013</td>
</tr>
<tr>
<td>1.4</td>
<td>215</td>
<td>$\geq 7 \times 10^{10e}$</td>
<td>5.98</td>
<td>$34.0 \pm 1.8$</td>
<td>This work</td>
</tr>
</tbody>
</table>

$^a$The flux density uncertainty given in the literature was incorporated to derive the size uncertainty. When there were multiple measurements, we considered the standard deviation as an error. For the 2013 VLBA data, we assumed the calibration errors of 10% for 7 and 1.4 mm, and 5% for wavelengths longer than 7 mm.

$^b$Brightness temperature of the highest flux density component.

$^c$Brightness temperature calculated using mean flux density and the size of the core during the observation period.

$^d$Mean of the brightness temperatures of the core region, derived during the seven epochs of observations.

$^e$Lower limit from a single baseline measurement.

$^f$Incorporating SMA flux density in addition to the single baseline measurement.
4.3.2 Core Size

Assuming that the 1.4 mm emission primarily arises from a single circularly symmetric Gaussian component, we can estimate the size using the zero-baseline flux density. If we again use $V_0 = 6.0$ Jy, as measured by the SMA, and the uncertainties in the SMA flux density as well as the correlated flux density, the FWHM size is $\phi = 34.0 \pm 1.8 \mu$as from Equation (4.2). This corresponds to $\sim 120 R_{\text{sch}}$ or $\sim 0.7$ light-day, although the mass of the black hole is uncertain at the 50% level, and $R_{\text{sch}}$ scales linearly with the mass. Figure 4.2 plots the range of core sizes as a function of zero-baseline flux density. Because the SMA flux density may include a contribution from outside the core (Weiss et al., 2008), the size we calculated needs to be considered as an upper limit.

Figure 4.4 compiles the data from the literature as well as the archival VLBA data in Table 4.2. We used the flux density and the size of the core when the component identification is available in the original papers. Horiuchi et al. (2006) and Müller et al. (2011) employed multiple components to model the core region, and the core size in Table 4.2 employs the mean of those component sizes. We adopted parameters for the brightest component when no other information is available (see the notes in Table 4.2). The core size decreases with the increasing observing frequency (decreasing observing wavelength) and we fit the data to infer its frequency dependence. The size of the radio core should correspond to the region within which the synchrotron optical depth exceeds unity (Blandford and Königl, 1979; Königl, 1981). Referencing the size evolution with wavelength to the current 215 GHz (1.4 mm) results as

$$\frac{\phi}{\phi_{215 \text{ GHz}}} = \left(\frac{\nu}{215 \text{ GHz}}\right)^{-\alpha},$$

(4.5)

where $\nu$ is the observing frequency, the best fit gives $\alpha = 1.3 \pm 0.1$. The dependence is similar to the angular size-frequency relation found in parsec-scale jets (Yang et al., 2008) and the core shift of a jet observed in M87 (Hada et al., 2011). For a conical jet, the shift and the change in size are linearly proportional to each other, following the same frequency dependence.
Figure 4.4: Size of the core region in 19.0 cm, 13.0 cm, 6.1 cm, 3.6 cm, 2.4 cm, 2.0 cm, 1.3 cm, 7 mm, and 1.4 mm from the references in Table 4.2. We use weighted average of multiple observation results, for each wavelength. Green line is the best fit to the data with the index $\alpha = 1.3 \pm 0.1$, where the core size $\phi \propto \nu^{-\alpha}$. We use 1.4 mm data as an intercept point of the fit.
4.3.3 Spectrum of the Core

The high brightness temperature of the core, greater than $10^{11}$ K, and the frequency dependence of its size are suggestive of wavelength-dependent synchrotron self-absorption. Flux-density measurements from the literature using interferometric arrays are presented in Figure 4.5, including VLBI measurements of the core flux density between 19.0 cm and 7 mm ($3.6 \times 1.2$ milliarcsec beam size at 7 mm), arcsecond-resolution measurements at high frequency from the SMA used to set the zero-baseline flux density. The flux density range observed by the SMA over 12 years of monitoring is also shown. The spectrum of the Cen A core increases until $\sim 3$ mm, and the single-dish data in Hawarden et al. (1993) show that the core has relatively flat spectrum shortward of 2 mm (Figure 1 of Kellermann et al. 1997 and Figure 5 of Abdo et al. 2010).

The VLBI data show the core flux density increasing with decreasing wavelength, a trend that continues to less than 1 mm if the SMA data also trace the core emission. Previous analyses, based on single-dish data with significantly lower resolution (tens to hundreds of arcseconds, e.g., Meisenheimer et al., 2007; Israel et al., 2008) show a spectrum that decreases in flux density with decreasing wavelength. At centimeter wavelengths these spectra are clearly dominated by the extended radio jet emission that fades most quickly toward short wavelengths because of synchrotron cooling. Because the compilation of Figure 4.5 and Table 4.2 selects the bright central components of VLBI images (in most cases), it provides the most applicable comparison for the 1.4 mm data presented here. The core flux density spectrum between 19.0 cm and 1.4 mm follows $S_\nu \propto \nu^{0.39 \pm 0.07}$. Of course, the flux density measurements span more than 20 years and show substantial variability (small points in Figure 4.5), even when obtained at many wavelengths at once (grey dotted line in Figure 4.5), so the spectral index can only be considered as a coarse average value. Nevertheless, the spectrum appears to be inverted, which can be produced by an optically thick, non-uniform synchrotron source (de Bruyn, 1976).
Figure 4.5: Spectrum of the Cen A core region. The VLBI observations between 19.0 cm and 7 mm (blue), the SMA 1.3 mm data of January 2015 (red), and the range of 1.3 mm and 0.8 mm SMA archival data (green) are plotted. The VLBI data use weighted average of multiple observations at each wavelength. The markers of the SMA archival data indicate minimum and maximum flux densities during the 12 years of observing period. The grey dotted line shows the simultaneous VLBA flux measurement in March 2013, and the small blue dots are the observations used to estimate the average spectrum.
4.3.4 Variability

The measured flux density as a function of UT is shown in Table 4.1. The flux density fluctuates from 0.45 Jy to 0.60 Jy over 1.5 hours, a variation of 16% from the mean value, with the most significant deviation found in the first scan. The most likely explanation for variations between the scans after the first is a combination of calibration errors due to pointing shifts during SPT commissioning and atmospheric decorrelation within the scans. We note that the first scan is missing roughly 50% of its data, which suggests that there may be further undiagnosed problems that reduce the amplitude of the correlation.

The baseline length changes very little over the course of these observations (2%), which, for a circularly symmetric Gaussian source, would lead to much less variation than we observe (8%). If the 1.4 mm source is actually elliptical or composed of multiple components, the variation in visibility amplitude along the 1.5 Gλ-long arc traced by the baseline in the $(u,v)$ plane could be larger. If we assume that the visibility variation is induced by ellipticity in the Gaussian source, at a position angle aligned with the center of the $(u,v)$ track, the best-fit axis ratio would be 1.6 : 1.

The nucleus of Cen A is known to be variable on daily to yearly time-scales at different wavelengths (Wade et al., 1971; Kellermann, 1974; Meier et al., 1989; Botti and Abraham, 1993; Israel et al., 2008; Müller et al., 2014). The light crossing time of the core limits the variability to $\sim 1$ day, though Doppler effects can shorten this time scale for beamed sources.

4.4 Conclusion

The first VLBI observations from the South Pole Telescope have detected correlated emission on a 7000 km, 5 Gλ baseline to the APEX telescope. With these data, we constrain the brightness temperature of the Cen A core region at 40 $\mu$as resolution. The calculated core size is $120 \, R_{\text{sch}}$ for the $5.5 \times 10^7 M_\odot$ central black hole. The frequency dependence of the core size and its spectrum suggest that we are detecting
the self-absorbed synchrotron emission region around the black hole. Once the other stations participate, the full EHT array will yield significantly better, two-dimensional, \((u, v)\) coverage, resolution, and sensitivity, allowing imaging of the Cen A core and more detailed investigation of this source.
CHAPTER 5

Bayesian Techniques for Comparing Time-dependent GRMHD Simulations to Variable Event Horizon Telescope Observations

The Event Horizon Telescope (EHT) is a millimeter-wavelength, very-long-baseline interferometry (VLBI) experiment that is capable of observing black holes with horizon-scale resolution. Early observations have revealed variable horizon-scale emission in the Galactic Center black hole, Sagittarius A* (Sgr A*). Comparing such observations to time-dependent general relativistic magnetohydrodynamic (GRMHD) simulations requires statistical tools that explicitly consider the variability in both the data and the models. We develop here a Bayesian method to compare time-resolved simulation images to variable VLBI data, in order to infer model parameters and perform model comparisons. We use mock EHT data based on GRMHD simulations to explore the robustness of this Bayesian method and contrast it to approaches that do not consider the effects of variability. We find that time-independent models lead to offset values of the inferred parameters with artificially reduced uncertainties. Moreover, neglecting the variability in the data and the models often leads to erroneous model selections. We finally apply our method to the early EHT data on Sgr A*.

5.1 Introduction

The Event Horizon Telescope (EHT) is a global array of telescopes that performs very-long-baseline interferometry (VLBI) at 1.3 and 0.8 mm wavelengths with unprecedented angular resolution. When the full array of stations from Arizona to the

\footnote{A version of this chapter originally appeared as a published paper in the Astrophysical Journal (Kim et al., 2016). All of the work described below was carried out by me, with help from co-authors Daniel P. Marrone, Chi-Kwan Chan, Lia Medeiros, Feryal "Ozel, and Dimitrios Psaltis.}
South Pole and from Hawaii to France are incorporated, the longest baselines (14 GA at 0.8 mm) will provide angular resolution better than 20 \(\mu\)as. The main targets of the EHT are the Galactic Center black hole Sagittarius A* (hereafter Sgr A*) and the nuclear black hole in the galaxy M87, at the center of the Virgo cluster of galaxies. For both of these sources, the apparent size of the event horizon is larger than the EHT resolution, millimeter emission is expected to originate very close to the horizon, and synchrotron opacity is not expected to obscure the innermost regions, thus allowing a clear and high-resolution view of their event horizons.

The images of the accretion flows around these black holes are expected to exhibit a shadow surrounded by a ring of emission (Bardeen, 1973; Luminet, 1979; Falcke et al., 2000), with a size and shape that depends primarily on the black hole mass and only very weakly on its spin (Johannsen and Psaltis, 2010). The shadows of the nuclear black holes of the Milky Way and M87 have apparent diameters of 50 \(\mu\)as and 36 \(\mu\)as, respectively, assuming Kerr black holes (Johannsen et al., 2012). Observations of Sgr A* with three stations in Hawaii, California, and Arizona measured a source size that is comparable to the event horizon scale (Doeleman et al., 2008) and provided evidence for variability of its emission (Fish et al., 2011). The Sgr A* observations of Fish et al. (2011) reported not only the increase of the total flux but also the brightening of the correlated flux density in long baselines. Three-station VLBI observations of M87 detected structure that was identified with the base of the relativistic jet and had a size comparable to that of the black hole shadow (Doeleman et al., 2012). For the rest of this paper, we will focus on Sgr A*, although our results are very general and can be applied to interferometric observations of any target.

Sgr A* is the closest supermassive black hole to the Earth. It has long been observed at wavelengths ranging from radio to \(\gamma\)-rays to study accretion physics around the black hole (see Falcke and Markoff 2013 for a recent review). In the near-infrared, adaptive optics observations of orbiting stars provide the mass of and distance to the black hole (Ghez et al., 2008; Gillessen et al., 2009a,b; Chatzopoulos et al., 2015). The angular size of Sgr A*, set by the ratio of mass to distance, is
more accurately measured and is the largest of any known black hole.

Observations at millimeter wavelengths provide the opportunity to understand the accretion process near the event horizon of Sgr A*. At these wavelengths, the strong interstellar scattering that greatly blurs the image at longer wavelengths (e.g., Doeleman et al., 2001; Bower et al., 2006) is reduced and the spectrum of Sgr A* implies a transition from optically thick to thin emission, providing a clear view of the event horizon (e.g., Narayan et al., 1998; Özel et al., 2000; Marrone et al., 2006; Bower et al., 2015). Sgr A* is observed to vary on intraday time scales at nearly all wavelengths, including millimeter wavelengths (e.g., Marrone et al., 2008; Yusef-Zadeh et al., 2009; Dexter et al., 2014), which suggests a highly dynamic environment around the black hole.

There have been many theoretical efforts to simulate accretion onto Sgr A* (see Yuan and Narayan, 2014, for a recent review). These calculations have generally focused on reproducing its spectrum and measured emission size through both semi-analytic stationary models and time-dependent magnetohydrodynamic (MHD) and general relativistic MHD (GRMHD) simulations. Time-independent models, including Özel et al. (2000), Yuan et al. (2003), and Huang et al. (2007), have used variations in the magnetic field, electron density, and temperature profiles computed from hydrodynamic models to match observational constraints. Incorporating GR ray-tracing, these models can also be used to constrain properties such as the black hole spin and orientation (e.g., Broderick et al., 2009, 2011). Time-resolved simulations of the accretion flow can similarly be adjusted through a variety of parameters, prescriptions, and initial conditions to better match existing observations. Such simulations have the potential to capture additional observational details that are not encoded in the stationary simulations, including polarization properties (e.g., Goldston et al., 2005; Shcherbakov et al., 2012; Gold et al., 2017), and especially the variability of the total emission and source structure. Several (GR)MHD simulations that can match numerous properties of Sgr A* (e.g., Chan et al., 2009; Mościbrodzka et al., 2009; Dexter et al., 2010; Dexter et al., 2012; Shcherbakov et al., 2012; Mościbrodzka and Falcke, 2013; Chan et al., 2015a,b) now exist in the
The prospect of millimeter-wavelength EHT observations that can resolve Sgr A* in both space and time, alongside simulations and existing observations that show temporal changes, indicates a need to compare data and models in a time-dependent manner. This should help us constrain the GRMHD model and physical parameters to describe the accretion flow around Sgr A*. However, there are so far no tools with which time-dependent data can be analyzed in the context of time-variable models. For example, given the likely changes in the structure of the emission region and the stochastic nature of MHD turbulence, a snapshot by snapshot comparison is neither feasible nor meaningful. In this chapter, we develop a statistical method to compare interferometric measurements with simulated mock data from time-dependent GRMHD models. In particular, we employ Bayesian tools to compute the posterior likelihood of model parameters given the observational data. We apply this to several simulations and verify the method using mock data. Then, we use EHT observational data from 2007 and 2009 to demonstrate the application of the method on Sgr A*.

### 5.2 Bayesian Data Analysis

In this section, we develop a general Bayesian framework that allows us to compare an ensemble of data points, such as VLBI visibilities, to an ensemble of simulated data predicted by a theoretical model, when both the data and the model exhibit intrinsic variability.

#### 5.2.1 Formalism

We follow the standard Bayesian approach to calculate the posterior likelihood $P(w|\text{data})$ that a theoretical model described by a vector of parameters $w$ is in agreement with a set of data, i.e.,

$$P(w|\text{data}) = CP_{\text{pri}}(w)P(\text{data}|w).$$  \hspace{1cm} (5.1)
Here, $P_{\text{pri}}(\mathbf{w})$ is the prior likelihood over the model parameters and we calculate the proportionality constant $C$ such that the integral of the posterior likelihood over the parameter space is equal to unity.

The vector of parameters for the GRMHD models that will be compared to EHT observations includes the properties of the black hole spacetime (its mass, spin, and any deviations from the Kerr solution), geometric properties of the orientation of the observer (the orientation and inclination of the spin axis on the observer’s sky), and parameters related to the microphysical properties of the plasma (e.g., the ratio of electron-to-ion temperatures in phenomenological models). Moreover, the results of the GRMHD simulations will need to be convolved with a model that describes the image blurring caused by interstellar scattering, which itself will have a number of parameters. Even though we explore below the characteristics of this Bayesian framework using a simplified two-parameter comparison between simulations and data, our methods are general and the full parameter space will need to be considered and explored when fitting actual EHT data.

Hereafter, we will consider a small set for the black hole spin, which has little effect on the overall properties of the black hole image (see Johannsen and Psaltis, 2010; Broderick et al., 2011), and fix its inclination with respect to the observer, which can be constrained significantly by the overall size of the 1.3 mm image of the black hole (Psaltis et al., 2015). We will also consider discrete models for the plasma thermodynamics (see Chan et al., 2015b), with parameters chosen such that the simulations reproduce the broadband spectral properties of Sgr A*. This set of choices leaves us with two parameters that need to be obtained by comparing models to interferometric data, i.e., an overall multiplication factor for the flux density, $F_0$, and the angle $\xi$ between the east-west axis and the projection of the black hole spin angular momentum onto the sky plane, measured in degrees east of north (e.g., Broderick et al., 2011)\(^2\). Under these assumptions, Bayes’ theorem becomes

$$P(\xi, F_0|\text{data}) = C P_{\text{pri}}(\xi) P_{\text{pri}}(F_0) P(\text{data}|\xi, F_0). \quad (5.2)$$

\(^2\)Another often used angle to measure the same orientation is denoted by $\phi$ and is the complementary angle to $\xi$, measured in degrees north of east (e.g., Bartko et al., 2009); clearly, $\xi = 90^\circ - \phi$. 
We assume flat prior likelihoods over the two model parameters $P_{\text{pri}}(\xi)$ and $P_{\text{pri}}(F_0)$, with $-180^\circ \leq \xi < 180^\circ$ and $F_{\text{min}} \leq F_0 \leq F_{\text{max}}$, with the range of the overall flux normalization to be specified later.

The last term in Equation (5.2), $P(\text{data}|\xi, F_0)$, measures the likelihood that a particular set of data is consistent with a set of model parameters. For the case of EHT, data and simulations need to be compared directly as interferometric observables. A fundamental interferometric observable is the visibility, which is a sample of the Fourier transform of the sky image, $I(\alpha, \beta)$, at discrete spatial frequencies. The visibility, $V(u, v)$, is defined as

$$V(u, v) = \int \int I(\alpha, \beta) \exp[-2\pi i (u\alpha + v\beta)] \, d\alpha \, d\beta$$

for a projected antenna separation of $(u, v)$ wavelengths within the plane perpendicular to the line of sight.

At any given observing epoch, the EHT will generate a set of simultaneous visibility amplitudes at many baselines as well as sets of closure phases on baseline triangles and closure amplitudes. For the purposes of this initial investigation, and to simplify our notation, we will assume that we only have observations of visibility amplitudes and that each measurement has independent uncertainties from the others. In other words, we will assume that

$$P(\text{data}|\xi, F_0) \equiv \prod_{i=1}^{M} \prod_{j=1}^{N} P_{ij}(\text{data}|\xi, F_0),$$

where $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, N$ denote baselines and epochs, respectively.\(^3\)

Each observation is characterized by a likelihood $P_{\text{obs}}(V; V_{ij}, \sigma_{ij})$, centered at $V_{ij}$ and with a dispersion of $\sigma_{ij}$. If, for any pair of model parameters $(\xi, F_0)$, the model made a single prediction for the visibility amplitude for baseline $i$ and epoch $j$, say

\(^3\)In principle, the covariance among the observables needs to be taken care of in the analysis. In Appendix G, I present the improved Bayesian method that considers the correlation among the VLBI observables, applying the principal component analysis (PCA) technique to the formalism introduced in this Chapter.
If \( \mathcal{V}_{ij}^{\text{sim}}(\xi, F_0) \), then the single-baseline, single-epoch likelihood in Equation (5.4) would simply be equal to

\[
P_{ij}(\text{data}|\xi, F_0) = P_{ij}^{\text{obs}}[\mathcal{V}_{ij}^{\text{sim}}(\xi, F_0); \mathcal{V}_{ij}, \sigma_{ij}] .
\] (5.5)

However, GRMHD models are highly variable (e.g., Chan et al., 2015a) and, for each baseline, they predict a range of visibility amplitudes, with a likelihood that we denote by \( P_{ij}^{\text{sim}}(\mathcal{V}; \xi, F_0) \). In this case, we write the posterior likelihood that a particular observation is consistent with the model predictions as

\[
P_{ij}(\text{data}|\xi, F_0) = \int P_{ij}^{\text{obs}}(\mathcal{V}; \mathcal{V}_{ij}, \sigma_{ij}) P_{ij}^{\text{sim}}(\mathcal{V}; \xi, F_0) d\mathcal{V} .
\] (5.6)

Our goal is to find the set of model parameters (\( \xi \) and \( F_0 \) in the example here) that maximizes the posterior likelihood in Equation (5.4), given a set of observations. This Bayesian analysis process, presented in Figure 5.1 as a flow chart, selects models for which the distribution of predicted visibilities in each baseline matches the distribution of observed visibilities.

The first factor in Equation (5.6), \( P_{ij}^{\text{obs}}(\mathcal{V}; \mathcal{V}_{ij}, \sigma_{ij}) \), is well understood. The real and imaginary components of the complex visibility individually follow Gaussian distributions. The visibility amplitude is the magnitude of the complex vector described by those variables, and its posterior likelihood is not Gaussian. The appropriate posterior likelihood for the visibility amplitude, known as a Rice distribution, is

\[
P_{ij}^{\text{obs}}(\mathcal{V}; \mathcal{V}_{ij}, \sigma_{ij}) = \frac{\mathcal{V}}{\sigma_{ij}^2} \exp \left( -\frac{(\mathcal{V}_{ij}^2 + \mathcal{V}_{ij}^2)}{2\sigma_{ij}^2} \right) I_0 \left( \frac{\mathcal{V} \mathcal{V}_{ij}}{\sigma_{ij}^2} \right),
\] (5.7)

where \( I_0 \) is the zeroth order modified Bessel function of the first kind. This likelihood approaches a Gaussian for large signal-to-noise ratios (see Chapter 6 of Thompson et al., 2001).

In order to simplify the calculation of integral (5.6), we make use of the fact that the distribution of simulated visibility amplitudes \( P_{ij}^{\text{sim}}(\mathcal{V}; \xi, F_0) \) has discrete values rather than a continuous distribution, as it is obtained from snapshot images. For \( T \) snapshots,

\[
P_{ij}^{\text{sim}}(\mathcal{V}; \xi, F_0) = \frac{1}{T} \sum_{t=1}^{T} \delta \left[ \mathcal{V} - \mathcal{V}_{ij}^{\text{sim}}(t; \xi, F_0) \right]
\] (5.8)
given $(\xi, F_0)$:

VLBI simulations of all snapshot images with

$i = 1, 2, \ldots, M$ baselines

$j = 1, 2, \ldots, N$ epochs of observation

→ distribution of visibility amplitudes $P_{ij}^{\text{sim}}(V; \xi, F_0)$

for $i$th baseline and $j$th epoch: likelihood

$$P_{ij}(\text{data}|\xi, F_0) = \int P_{ij}^{\text{obs}}(V; V_{ij}, \sigma_{ij})P_{ij}^{\text{sim}}(V; \xi, F_0) dV$$

$$P_{ij}^{\text{obs}}(V; V_{ij}, \sigma_{ij})$$

$$P_{ij}^{\text{sim}}(V; \xi, F_0)$$

Visibility Amplitude

$$P(\text{data}|\xi, F_0) \equiv \prod_{i=1}^{M} \prod_{j=1}^{N} P_{ij}(\text{data}|\xi, F_0) \text{ for all } (\xi, F_0)$$

Figure 5.1: Flow chart of the algorithm to test the performance of the Bayesian data analysis.
where $V_{ij}^{\text{sim}}(t; \xi, F_0)$ is the visibility amplitude sampled at $(u_{ij}, v_{ij})$ coordinates from the $t$th snapshot with the given set of parameters $(\xi, F_0)$. Therefore, Equation (5.6) simplifies to

$$P_{ij}(\text{data}|\xi, F_0) = \frac{1}{T} \sum_{t=1}^{T} \frac{V_{ij}^{\text{sim}}(t; \xi, F_0)}{\sigma_{ij}^2} \exp \left[ -\frac{(V_{ij}^{\text{sim}}(t; \xi, F_0) + V_{ij})^2}{2\sigma_{ij}^2} \right] 	imes I_0 \left( \frac{V_{ij}^{\text{sim}}(t; \xi, F_0) V_{ij}}{\sigma_{ij}^2} \right)$$  (5.9)

for the likelihood $P_{ij}^{\text{sim}}(V; \xi, F_0)$, with given parameters.

In the following set of examples, we calculate the posterior likelihoods over the model parameters using Equations (5.2), (5.4), and (5.9). In real applications, however, observations on different baselines are performed simultaneously, so we must consider a joint probability distribution in visibility space. For example, when an observation is carried out with three stations (or equivalently, three baselines), the posterior likelihood for the observations, $P_{\text{obs}}(V)$, becomes a multivariate Rician. Moreover, we need to consider the posterior likelihood, $P_{\text{sim}}(V)$, that the model predicts a particular combination of simultaneous visibilities on the same baselines.

Finally, we can trivially incorporate to our Bayesian inference other interferometric observables, such as closure phases and closure amplitudes. The closure phase is the sum of visibility phases around the triangle of baselines formed by any set of three stations. The closure phase is independent of instrumental and atmospheric phase fluctuations and depends only on the visibility phase of the source. If we have $N$ stations where $N \geq 3$, $N-1C_2 = (N-1)(N-2)/2$ independent closure phases can be measured. The posterior likelihood for closure phase data becomes

$$P_{lj}(\text{data}|\xi, F_0) = \int P_{\text{obs}}(\Phi; \Phi_{lj}, \sigma_{lj}) P_{lj}^{\text{sim}}(\Phi; \xi, F_0) d\Phi$$  (5.10)

for $l = 1, 2, \ldots, N-1C_2$ sets of telescopes (closure phase “triangles”), and $j = 1, 2, \ldots, N$ observations. Here, $P_{\text{obs}}(\Phi; \Phi_{lj}, \sigma_{lj})$ and $P_{lj}^{\text{sim}}(\Phi; \xi, F_0)$ are the likelihoods for closure phases from the observation and simulations, similar to $P_{ij}^{\text{sim}}(V; \xi, F_0)$ in Equation (5.6).
5.2.2 Model Comparison

In addition to inferring the most likely parameters of a given model from observations, we are also interested in comparing the ability of different models to describe the observed distributions of visibility amplitudes and closure quantities.

Information criteria approaches such as the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) have been adopted elsewhere to evaluate the quality of astrophysical models (e.g., Liddle, 2007). Broderick et al. (2011) applied these criteria in evaluating the relative success of time-independent models of Sgr A* in fitting EHT data. The BIC is defined as

$$\text{BIC} = -2 \log \mathcal{L}_{\text{max}} + k \log N,$$

where $\mathcal{L}_{\text{max}}$ is the maximum likelihood using the parameters within the model, $k$ is the number of free parameters, and $N$ is the number of data points. The definition implies that the smaller BIC value suggests a better fit to the data. However, criteria such as the BIC consider only the maximum posterior likelihoods and do not account for the relative extent of the volumes in parameter space within which each model is consistent with the data.

In the context of our work, we will perform model comparisons in terms of an appropriately defined Bayesian evidence. If we consider each model as a single point in a discrete parameter space, then, given the data, we can use Bayes’ theorem to calculate the posterior likelihood for each model as

$$P(M_m|\text{data}) = CP_{\text{pri}}(M_m)P(\text{data}|M_m),$$

(5.12)

Here $m$ is the discrete index specifying each model $\{M_m\}$, which is governed by a set of parameters $w_m$. If we denote by $P_{\text{pri}}(m)$ the prior likelihood for each model and by $P_{\text{pri},m}(w_m)$ the prior likelihoods for each set of parameters within model $m$, then we can write

$$P(M_m|\text{data}) = CP_{\text{pri}}(m)P_{\text{pri},m}(w_m)P_m(\text{data}|w_m).$$

(5.13)

The last term in Equation (5.13) is the posterior Bayesian likelihood of the data, given a model $m$ and a set of model parameters, e.g., Equation (5.4).
In order to calculate the Bayesian evidence for model \( m \), we marginalize Equation (5.13) over the space of model parameters, i.e.,

\[
\mathcal{L}(m|\text{data}) = C P_{\text{pri}}(m) \int P_{\text{pri},m}(w_m) P_m(\text{data}|w_m) dw_m.
\]  

(5.14)

Assuming a flat prior of models and the same prior likelihood over all model parameters for each model, this integral simplifies to

\[
\mathcal{L}(m|\text{data}) = C' \int P_m(\text{data}|w_m) dw_m,
\]  

(5.15)

where \( C' \) is an appropriate normalization constant.

The integrated likelihood \( \mathcal{L}(m|\text{data}) \) provides a measure of model selection statistics. As is the standard practice in Bayesian inference, we use the Bayesian evidence only for comparison among models and not to assign a posterior likelihood to any model individually.

5.3 Likelihood with Mock Data

We now turn our attention to the application of the statistical method described in the previous section to mock and real EHT data. To this end, we make use of a set of previously studied GRMHD simulations labeled A → E, for which time-dependent spectra and images were computed and fit to the time-averaged properties of Sgr A* (Chan et al., 2015b).

5.3.1 GRMHD Simulations

In a recent study, Chan et al. (2015a) investigated the variability and millimeter/infrared flare characteristics of GRMHD simulations. The simulation of the accretion flow was performed with the 3-dimensional GRMHD code HARM (Gammie et al., 2003; McKinney, 2006; McKinney and Blandford, 2009; Narayan et al., 2012; Sałowski et al., 2013) and the ray-tracing of photon trajectories was performed with the fast GPU-based algorithm GRay (Chan et al., 2013). The simulated Sgr
A* images with horizon-scale resolution and their spectra were fit to both the observed broadband spectra and the overall 1.3 mm source size determined by EHT observations (Chan et al., 2015b).

Chan et al. (2015a,b) simulated high cadence images of the accretion flow around Sgr A* for each model. The simulation results were recorded with a timestep of $10GMc^{-3}$, which corresponds to 212 s for the mass of Sgr A*. Each snapshot contains four levels of field of view: $16M \times 16M$, $64M \times 64M$, $256M \times 256M$, and $1024M \times 1024M$, containing $512 \times 512$ pixels. Here $1GMc^{-2}$ corresponds to $5.1 \mu$as for the distance to Sgr A*. We use a composite of the multi-level resolutions for our analysis in order to encompass all the emission near and away from the black hole and to generate interferometric visibilities. It is reasonable to limit our images to $1024M \times 1024M$, because the corresponding $10.2 \mu$as pixel scale is considerably finer than the highest spatial frequency ($58.9 \mu$as fringe spacing) sampled by the EHT data we use as a comparison.

The GRMHD models of Chan et al. (2015a,b, see also Narayan et al., 2012; Sad´owski et al., 2013) were categorized into two classes based on initial magnetic field configurations: Standard And Normal Evolution (SANE, disk-dominated) and Magnetically Arrested Disk (MAD, jet-dominated) models, which use multi-loop and single-loop initial magnetic fields, respectively. The simulation images at 1.3 mm for these two classes of models show dissimilar features from each other: in SANE models, the emission originates from a crescent-like shape in the disk region, whereas it is concentrated at the footprints of the outflowing material in the MAD models. Chan et al. (2015a,b) also explored the dependence of the predictions of the GRMHD models on the black hole spin parameter $a$ and on the way in which the electron temperature was prescribed.

In some of their models, the electron temperature in the funnel region of the accretion flow was fixed to be constant (constant $T_{e,\text{funnel}}$). In other models, the ratio of the electron and ion temperatures in the funnel was fixed (constant $\theta_{\text{funnel}}$). In both cases, the electron and ion temperatures have a constant ratio in the disk. Among all the configurations they explored, they narrowed the possibilities down to
five models that account for both the broadband spectrum of Sgr A* and its overall image size at 1.3 mm:

- Model A: a=0.7, SANE, constant $T_{e,\text{funnel}}$
- Model B: a=0.9, SANE, constant $T_{e,\text{funnel}}$
- Model C: a=0.0, MAD, constant $\theta_{\text{funnel}}$
- Model D: a=0.9, MAD, constant $T_{e,\text{funnel}}$
- Model E: a=0.9, MAD, constant $\theta_{\text{funnel}}$

All models show variability over the entire length of the simulation, i.e., over $\sim 10,000 \, GMc^{-3}$ ($\sim 60$ hrs). Figure 5.2 shows the 1.3 mm light curves for the simulations. Overall, the SANE models are observed to produce more variability than the MAD models.

We generated visibilities from the simulations using our own VLBI simulation code. The algorithm calculates $(u, v)$ coordinates from the observing time and celestial coordinates of the source, performs Fourier transform of the input image, and samples complex visibilities according to the telescope array setup. We tested the results of this code against the MIT Array Performance Simulator\textsuperscript{4} (MAPS) and found excellent agreement between the simulated visibilities.

Figure 5.3 shows the range of predicted visibility amplitudes for the five GRMHD models. The SANE models show more variability in the visibility space than the MAD models, just as they do in total intensity (Figure 5.2). This variability is explored in more detail in the companion paper, Medeiros et al. (2018a).

\textsuperscript{4}http://www.haystack.mit.edu/ast/arrays/maps/index.html
Figure 5.2: Light curves of Models A to E at 1.3 mm. The total fluxes of each snapshot are plotted with a $10 \ G M c^{-3}$ timestep, over a $10,000 \ G M c^{-3}$ duration. The combination $G M c^{-3}$ corresponds to 21.2 s for the mass of Sgr A*. SANE models not only show larger variability than MAD models, but also often display sudden increases of the total intensity, i.e., flares.
Figure 5.3: Sample VLBI visibility amplitudes of Models A to E. We generated the simulated visibility amplitudes using three stations in Hawaii, California, and Arizona. We produced the visibility amplitudes from GRMHD images, using \((u, v)\) coordinates of the EHT observation data in 2007 and 2009, without rotation \(\xi = 0^\circ\) and flux scaling \(F_0 = 1\). For all \((u, v)\) coordinate pairs, we plot the mean visibility amplitudes of 1024 snapshots in solid line as well as their \(\pm 1\sigma\) range in the shaded region.
In the following sections, we describe tests of our Bayesian analysis using mock data from these simulations. These tests demonstrate that our statistical method successfully recovers model parameters (section 5.3.2) and can faithfully distinguish between models (section 5.3.5).

5.3.2 Parameter Estimation

In our first test we investigated the performance of our statistical method in inferring model parameters. We constructed the test as follows:

- We assumed a fiducial set of model parameters \((\xi, F_0) = (-30^\circ, 1)\).
- We generated data using three baselines with three stations: Hawaii - Arizona, Hawaii - California, and Arizona - California.
- We sampled \((u, v)\) coordinates during the period when three stations can perform simultaneous observations of Sgr A*.
- We generated visibilities using model D, which is a MAD model.
- We randomly selected visibility amplitudes from the distribution of simulated visibilities, \(P_{ij}^{\text{sim}}(V; -30^\circ, 1)\), for each baseline. Visibilities were taken from the same simulation frame for all three baselines. We then added random Gaussian noise to the complex visibilities to produce mock observational data from the visibility amplitude samples.
- We computed the posterior likelihood \(P(\text{data}|\xi, F_0)\) using simultaneous observations on three baselines, as described in Equation (5.4). We considered orientation \(\xi\) in the range from -90\(^\circ\) to 90\(^\circ\), as visibility amplitudes without phase information are degenerate under rotations of 180\(^\circ\). We repeated the calculation for 5000 different realizations of the mock data.

We averaged the posterior likelihoods to form 500 sets of data, where each set contains \(N = 10\) observations. For every set of data, we found the orientation and flux scale where the maximum likelihood occurs. The distributions of these maximum likelihood parameters for our input simulation are shown in Figure 5.4(a) and
(b). The red vertical lines mark the model parameters \((\xi, F_0)\) that were assumed when generating the mock visibility data. The histograms indicate that the orientation and flux scale of the highest likelihoods of all data sets are distributed around the assumed parameters. We do not see any bias in the inferred parameters.

Figure 5.4(c) shows the averaged posterior likelihoods of 500 sets of data. The red dotted lines in the parameter space again mark the combination of the assumed model parameters \((\xi, F_0) = (-30^\circ, 1)\). The lines cross very near the peak of the posterior likelihood, and their intersection is located inside the 68% credible region of the likelihood. Figure 5.4(d) shows the averaged likelihoods when the 5000 mock visibilities are grouped into 200 sets, each including \(N = 25\) observations. Compiling a larger number of observations causes the allowed region of the parameter space to contract, as expected.
Figure 5.4: Parameter estimation test with mock data generated using model D and assuming a black hole spin orientation of $\xi = -30^\circ$ and an overall flux normalization of $F_0 = 1$. The first two panels show the one-dimensional likelihoods for 500 realizations of observations, each containing 10 epochs over (a) the orientation ($\xi$) and (b) flux scale ($F_0$), evaluated at cross sections of the two-dimensional parameter space that encompasses the point of maximum likelihood. Panels (c) and (d) show the averaged likelihoods in parameter space for (c) 500 realizations each containing 10 epochs and (d) 200 realizations each containing 25 epochs. Maximum likelihoods occur at $\xi = -30^\circ$ and $F_0 = 1$ for both cases. Contours indicate the 68.3%, 95.5%, and 99.7% credible regions. Given a realistic observational setup, our Bayesian method recovers the assumed parameters with no biases. Panel (e) is similar to (d), but with more complete coverage of a six-station EHT (see section 5.3.3).
5.3.3 Effect of Future EHT Stations

In the near future, more stations will be incorporated into the EHT, such as the Atacama Large Millimeter/submillimeter Array (ALMA) and the South Pole Telescope (SPT). We performed a mock observation test to explore the effect of future EHT stations. The test is similar to that of the previous section, but assuming we have 15 baselines with 6 stations. The stations are the Submillimeter Array (SMA) in Hawaii, the Combined Array for Research in Millimeter-wave Astronomy (CARMA) in California, the Submillimeter Telescope (SMT) in Arizona, the Large Millimeter Telescope (LMT) in Mexico, ALMA in Chile, and the SPT at the South Pole. The sampled \((u, v)\) coordinate sets include non-simultaneous observations because considering only simultaneous detections for six stations greatly limits the observing period. Visibility amplitude errors are estimated from the baseline sensitivity, which is proportional to the geometric mean of the System Equivalent Flux Densities (SEFDs) of antennas.\(^5\) We assumed 10 s coherent integration time and 2 GHz bandwidth for the sensitivity.

There are three factors that affect the allowed parameter ranges in the Bayesian analysis: the overall sensitivity of the interferometric array, \((u, v)\) coverage in the visibility space, and sampling of the intrinsic variability of the source. Figure 5.4(e) shows the combined effect of these: the increased sensitivity and \((u, v)\) coverage of a six-station EHT will narrow the allowed parameter space compared to Figure 5.4(d), which has the same number of observations but fewer stations and lower sensitivity. To assess the importance of source variability in parameter estimation, we constructed three array setups as follows:

(a) The early EHT array of three stations at Hawaii, Arizona, and California, with 920 observing epochs.

(b) The full EHT array with six stations when the sensitivity of ALMA is replaced with that of the co-located Atacama Pathfinder Experiment (APEX)\(^\) SMA phased array: 4000 Jy, CARMA: 10,000 Jy, SMT: 11,000 Jy, LMT: 1400 Jy, ALMA phased array: 100 Jy, SPT: 9000 Jy.

\(^{5}\)
telescope, with 61 observing epochs.

(c) The full EHT array with six stations, including ALMA, with four observing epochs.

In these three scenarios, we have varied the number of observing epochs such that the accumulated point source sensitivity of the array is constant. That is, the total thermal noise is the same in all the arrays, after accounting for the varying number of observing epochs. The comparison of the six-station array with APEX is in interesting counterpoint to the case with ALMA because it holds the ($u, v$) coverage fixed (and the sensitivity), but varies the number of observing epochs. Figure 5.5 shows that the parameter constraints are much tighter when we average over more observing epochs, emphasizing the importance of sampling the intrinsic source variability when attempting to constrain the model parameters with EHT observations. Figure 5.5(a) has the poorest ($u, v$) coverage by far, but achieves the tightest constraint, while fixing both ($u, v$) coverage and sensitivity in Figures 5.5(b) and 5.5(c) again show that simply sampling more epochs will tighten the parameter constraints significantly.

---

6SEFD of 3600 Jy
Figure 5.5: Parameter estimation test with the same underlying parameters as Figure 5.4. The three panels show the averaged likelihoods for 20 realizations each in three separate setups. The number of observations in each setup is chosen such that the raw overall sensitivity is held fixed across the three setups. The three panels correspond to (a) 920 epochs on the SMA-CARMA-SMT baseline, where previous EHT observations were done; (b) 61 epochs on baselines including SMA, CARMA, SMT, LMT, APEX, and SPT; (c) 4 epochs including the same baselines as panel (b) but with ALMA replacing APEX. These figures show that sampling the source variability is more important for parameter estimation than the raw sensitivity of the array or the \((u,v)\) coverage.
5.3.4 Effect of Time Averaging

In the parameter estimation test, we calculated likelihoods over parameter space with both time-dependent and averaged GRMHD images to compare the effect of time averaging. Figure 5.6 contrasts the likelihoods of time-resolved (black contours) and time-independent (red contours) inputs to our Bayesian method when the same number of mock observations are analyzed. Figures 5.6(a) and 5.6(b) are the likelihoods assuming the early three-station baselines and the six-station EHT with ALMA, respectively. The figure shows that a much smaller region of parameter space is allowed when comparing the mock data to a time-averaged image. However, the best-fit parameters in the time-averaged case are offset from the true parameters, as is most clearly evident in Figure 5.6(b). This demonstrates the false certainty implied by the small allowed parameter region. The time-resolved analysis, by contrast, delivers allowed parameter ranges that are consistent with the input simulation.
Figure 5.6: Effect of time averaging in the parameter estimation test for (a) the early EHT baselines and (b) the full EHT baselines including ALMA. We use the identical parameters for the mock data as Figure 5.4. The likelihoods of the averaged GRMHD image are overplotted in red contours. The likelihood using the averaged images underestimates the uncertainties of the inferred parameters. The best-fit parameters derived from the time-dependent and the averaged images are offset from each other as well.
5.3.5 Model Selection

We now use the Bayesian method to identify the relative posterior likelihood among several models, as discussed in section 5.2.2. For this test we generated mock visibilities following the same procedure as the test of the previous section. We used \((u, v)\) coordinates identical to those in the EHT observations of 2007 and 2009. As discussed earlier, SANE models present greater variability than MAD models (Figure 5.3). Therefore, we generated two mock data sets, one from model A (SANE) and one from model E (MAD), and performed our model selection test twice (labeled test A and test E, respectively).

We present in Table 5.1 the marginal likelihoods in Equation (5.15) and the \(\Delta\text{BIC}\) values in Equation (5.11). We obtained these values by fitting the mock model A data and mock model E data with all five models. The ratio of the marginal likelihoods for two models is the Bayes factor, and it measures the relative strength of the evidence against another one. Jeffreys’s scale provides a guidance for the interpretation of the Bayes factor (Kass and Raftery, 1995). A ratio between 10 and 100 is translated as strong, and a ratio greater than 100 is regarded as decisive evidence. A similar approach using the difference of information criteria exists and \(\Delta\text{IC}\) greater than 5 and 10 are understood as strong and decisive, respectively.
Table 5.1. Marginal likelihoods of model estimation tests

<table>
<thead>
<tr>
<th>Model</th>
<th>Test A</th>
<th>Test E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mathcal{L}(m</td>
<td>\text{data})$</td>
</tr>
<tr>
<td>Model A</td>
<td>1.00</td>
<td>0.0</td>
</tr>
<tr>
<td>Model B</td>
<td>0.799</td>
<td>1.5</td>
</tr>
<tr>
<td>Model C</td>
<td>$7.20 \times 10^{-9}$</td>
<td>37.5</td>
</tr>
<tr>
<td>Model D</td>
<td>$1.81 \times 10^{-4}$</td>
<td>17.3</td>
</tr>
<tr>
<td>Model E</td>
<td>$2.98 \times 10^{-10}$</td>
<td>41.4</td>
</tr>
</tbody>
</table>

Note. — Test A uses mock data using model A, and test E uses mock data using model E. The marginalized likelihoods, $\mathcal{L}(m|\text{data})$ are normalized to the best model for both tests, as are the BIC values.
In each test, our analysis prefers the model from which the data were generated. In Test A, model B is only slightly disfavored compared to model A. This results from the similar shapes of the visibility amplitude distributions in the SANE simulations. The fact that model B has a different spin than model A is also an indication that black hole spin has a very small effect on the overall appearance of SANE models for Sgr A*. In this test, the MAD models are decisively rejected, as expected, because the smaller variability of the model visibilities in the MAD simulations is inconsistent with the mock data. Conversely, Test E decisively favors model E but rejects all four other models.

### 5.4 Applications to Early EHT Data

In this section, we apply the Bayesian method to early, limited EHT visibility amplitude data. These EHT observations of Sgr A* were carried out in 2007 and 2009 with only three stations (Doeleman et al., 2008; Fish et al., 2011): the James Clerk Maxwell Telescope in Hawaii (HI), the CARMA in California (CA), and the SMT of the Arizona Radio Observatory (ARO) in Arizona (AZ). The observations measured 18 visibility amplitudes in 2007 (HI-AZ: 4, AZ-CA: 14), and 51 (HI-AZ: 19, HI-CA: 12, AZ-CA: 20) in 2009.

In our comparison of the GRMHD models to the data, we only consider values for the orientation $\xi$ from $-90^\circ$ to $90^\circ$, as before, because visibility amplitudes have a $180^\circ$ degeneracy. Closure phase information is required to resolve the ambiguity, and the closure phases were not measured well in these early data.

In order to compare the simulations with real data, which are affected by interstellar scattering, we incorporated the effects of scattering using a simple prescription. Multiwavelength observations of Sgr A* show that its intrinsic size scales as $\lambda^2$ due to scattering by electrons along the line of sight to the Galactic Center. The scattering is described as an elliptical kernel (Bower et al., 2006) having major and minor axes

$$\text{FWHM}_{\text{major}} = 1.309(\lambda/1 \text{ cm})^2 \text{ mas},$$

(5.16)
and
\[ \text{FWHM}_{\text{minor}} = 0.64(\lambda/1 \text{ cm})^2 \text{ mas} \] (5.17)

with a position angle of the major axis \( PA = 78^\circ \) east of north. In the visibility space, this corresponds to an elliptical taper of the visibility amplitude (Fish et al., 2014). Psaltis et al. (2015) revisited the inference of the scattering kernel parameters from observations and evaluated their uncertainties. Johnson and Gwinn (2015) described the refractive scintillation in more detail, where the real effect of scattering is likely more complicated than this simple convolution prescription. For the purposes of our initial study, we will not consider further these two improvements on the modeling of the scattering kernel.

Figure 5.7 shows the normalized posterior likelihoods when comparing the EHT data to the five GRMHD models. Notably, the observational data identify different physical parameters for the five models, even though all models are physically plausible descriptions of the structure of Sgr A*. For all five models, the 2009 EHT data dominate the outcome because of the larger number of detections in that campaign. Parameters with maximum likelihoods are given in Table 5.2 with 68% errors. While there is some agreement about the orientation parameter among most of the models with limited data, it is clear that inferences about the spin orientation are model specific and not general. Of course, the analysis presented here does not explore the effect of varying other parameters, such as the inclination. Expanding the dimensionality of the parameter space for each model will most likely affect the inferred model parameters and their uncertainties.

Figure 5.7 and Table 5.2 also compare the likelihoods obtained when fitting time-dependent and time-averaged GRMHD images to the data. The time-averaged analysis is similar to the time-resolved one, except that \( P^{\text{sim}}(V) \) is composed of visibilities generated from a single image obtained by averaging all the snapshots. As with the mock data, the lack of variability in the averaged model image results in a substantial underestimation of the uncertainties in the model parameters.

Remarkably, comparing model evidences shows that the SANE Models A and B are decisively rejected in the time-dependent analysis, while they are among the
most favored in the time-averaged analysis. This happens because, even though the
images of Models A and B have an overall structure that matches the observations
(compare, e.g., the data and the predictions of the five models at baselines larger
than $\sim 3 \, G\lambda$ in Figure 5.8), they are nevertheless disfavored in the time-dependent
analysis because they show much larger variability than the current limited data
(compare the range of visibilities predicted at $\lesssim 1 \, G\lambda$ by all five models in Fig-
ure 5.8). Clearly, retaining knowledge of the variability predicted by a particular
GRMHD (or other) simulation is important for evaluating how well that simulation
matches the data.

The Bayesian evidence for the different models shown in Table 5.2 seems to sug-
gest that the existing EHT data favor MAD models over the SANE ones. However,
it is important to emphasize here a requirement that needs to be satisfied before
the Bayesian method can be applied to real data: that both the simulations and the
observations have sampled adequately the variability at each baseline so that they
cover the entire range of possibilities. This is clearly not the case for the early EHT
data that we are using here, since they comprise only $\sim 10$ hrs of total integration
time. As a result, while we present this analysis as a proof of principle, inferring
model parameters from real EHT data based on our Bayesian method will be possi-
ble only after a substantial amount of data has been collected and their variability
has been characterized.

There exist in the literature several analyses of the current EHT data based on
time-averaged models. Broderick et al. (2009, 2011) modeled the accretion flow with
a time-independent, semi-analytic RIAF models and inferred the following as the
most probable black hole parameters: viewing angle$^7 i = 50^\circ \pm 10^\circ$, and position angle
$\xi = -20^\circ \pm 31^\circ$ for the 2007 observations only, and $i = 68^\circ \pm 5^\circ$ and $\xi = -52^\circ \pm 17^\circ$
for the 2007 and 2009 observations. Dexter et al. (2010); Dexter et al. (2012)
performed three-dimensional, time-dependent GRMHD simulation, then averaged
the simulated images to fit the VLBI data. They provided $i = 50^\circ \pm 35^\circ$ and $\xi = $ $-23^\circ \pm 97^\circ$
and $\xi = -70^\circ \pm 86^\circ$ using

$^7$Note that Broderick et al. (2009, 2011) uses $\theta$ instead of $i$ for the observer’s inclination angle.
the 2007 and 2009 observations.\footnote{Dexter et al. (2010); Dexter et al. (2012) use 90\% confidence for the uncertainty of estimated parameters.}

Although these values are broadly consistent with each other and with those we inferred here for most models, their uncertainties are significantly tighter than what we found here for the SANE models (which are the only types of models considered earlier). This is not surprising because, as the above analysis suggests, fitting models that do not allow for source variability yields artificially small parameter uncertainties when the source is, in fact, variable.
Figure 5.7: Normalized posterior likelihoods over the black hole spin orientation $\xi$ and the overall flux normalization $F_0$, when the five GRMHD models A to E of Chan et al. (2015a) are compared to two epochs of EHT data. The likelihood when 2007 data, 2009 data, and the combined data set are used is shown in the three sets of panels. The parameter constraints derived from comparing the data to time-averaged images are overplotted in red contours. The parameter constraints are both offset from the constraints derived when considering the intrinsic variability of the model, and the allowed parameter range is incorrectly reduced compared to the true uncertainty reflected by the time-dependent analysis.
Table 5.2. Best-fit parameters and model evidences with EHT observations in 2007 and 2009

<table>
<thead>
<tr>
<th></th>
<th>Time-dependent analysis</th>
<th></th>
<th>Time-averaged analysis</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\xi$</td>
<td>$F_0$</td>
<td>$\mathcal{L}(m</td>
<td>\text{data})$</td>
</tr>
<tr>
<td>Model A</td>
<td>$-46^{+9}_{-6}^\circ$</td>
<td>$1.41^{+0.23}_{-0.19}$</td>
<td>$1.84 \times 10^{-8}$</td>
<td>37.1</td>
</tr>
<tr>
<td>Model B</td>
<td>$-36^{+43}_{-8}^\circ$</td>
<td>$1.23^{+0.19}_{-0.22}$</td>
<td>$5.99 \times 10^{-15}$</td>
<td>68.2</td>
</tr>
<tr>
<td>Model C</td>
<td>$50^{+6}_{-5}^\circ$</td>
<td>$1.11^{+0.06}_{-0.06}$</td>
<td>$2.71 \times 10^{-2}$</td>
<td>6.9</td>
</tr>
<tr>
<td>Model D</td>
<td>$-37^{+14}_{-6}^\circ$</td>
<td>$2.44^{+0.15}_{-0.15}$</td>
<td>$1.18 \times 10^{-3}$</td>
<td>15.7</td>
</tr>
<tr>
<td>Model E</td>
<td>$-39^{+4}_{-3}^\circ$</td>
<td>$2.50^{+0.12}_{-0.11}$</td>
<td>$1.00$</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Figure 5.8: VLBI visibility amplitudes of Models A to E with the best-fit parameters listed in Table 5.2 and EHT data. The visibility amplitudes are plotted with their $\pm 1\sigma$ range in the shaded region. The EHT data include two days of observations in 2007, and three days observations in 2009. Although the simulated visibility amplitudes and observations appear to agree well with each other for all five models, marginal likelihoods and BIC show model E is the favored one with the limited EHT data.
5.5 Conclusion

In this work, we developed a statistical tool that will allow us to interpret the variable EHT observational data in the context of time-dependent model predictions. We tested the method for parameter estimation and model selection statistics. We then applied this Bayesian statistical analysis to existing EHT observations from 2007 and 2009, as a proof of principle, and investigated its applicability and statistical power.

We find that taking into account the time variability in the data and in the models increases the uncertainties in the inferred parameters beyond what is obtained when using time-averaged emission structures. Moreover, ignoring the variable nature of the data and of the simulations may lead to erroneous model selections.

In our analysis, we only considered two parameters – the orientation of the black hole spin vector in the sky plane and the flux normalization – in this chapter. Additional parameters, such as the inclination angle of the spin vector, the black hole spin and the plasma parameters of the accretion flow, can also be investigated with this method by expanding the parameter space and injecting corresponding simulation data sets.

Finally, the method presented in this chapter is quite general and can work with any time-resolved simulation. Although only visibility amplitudes were included in our testing, other observables such as closure phase and amplitude can easily be incorporated with future observations with additional baselines.
CHAPTER 6

Summary and Conclusions

In this thesis, I described the development of the SPT VLBI receiver that was crucial to forming a global Earth-sized millimeter VLBI array EHT. I also showed how the Bayesian model-fitting method could be used to compare the VLBI observations of the variable emission around the supermassive black hole and the time-dependent simulations.

6.1 South Pole Telescope VLBI receiver

I developed the VLBI receiving system for the SPT, operating at 1.3 and 0.87 mm, and the development included radio frequency component design and simulation, Gaussian beam characterization, electronics testing and control software development (Chapter 2 and 3). The system was installed at the SPT and proved its VLBI capability with the test VLBI observation with Chile (Chapter 4). The observation constrained the brightness temperature and the core size of the radio source Cen A.

The SPT successfully participated in two years of EHT science observations in 2017 and 2018, and it now has a standard procedure for the 1.3 mm VLBI operation. The analysis of the EHT Sgr A* data containing the detections from the SPT baselines is underway. In January 2019, the SPT did the first 0.87 mm on-sky test (Appendix D) and will be ready for the dual-band science operation soon.

6.2 Black Hole Model Comparison using the Event Horizon Telescope Observations

In Chapter 5, I developed the statistical technique to compare the time variable VLBI data to the time-dependent GRMHD simulations. I found that considering
the time variability in the data and the model is necessary to infer model parameters. The method will be applied to the Sgr A* and M87 data, extract the physical parameters that best describe the observation, and help to understand the supermassive black holes at their event horizon scales with realistic black hole simulation models.
The SPT VLBI receiver design uses quarter-wave plates in front of the feed horn and an orthomode transducer (OMT). The OMT is a waveguide device that separates incoming signals into two orthogonally polarized components. The OMT is often fabricated by splitting the structure horizontally into two block halves. Therefore, a mechanical error can cause misalignment between two-split blocks when assembled, and it degrades the electromagnetic performance of the OMT from that of an ideal case. During the receiver design phase, we performed the electromagnetic simulations of three different designs of OMTs to investigate their operations according to lateral misalignment between OMT block halves. The OMT designs were scaled for both 1.3 mm (230 GHz) and 0.87 mm (345 GHz) bands and simulated with intentional shifts. The electromagnetic performances of OMTs deteriorate as the misalignments increase.

A few types of structures have been employed to construct the OMTs. We selected three kinds of OMT designs with a Bøifot junction (Kamikura et al., 2008), a backward coupler junction (Navarrini and Nesti, 2009; Navarrini et al., 2010), and a T-shaped waveguide (Dunning et al., 2009). Figure A.1 shows the designs and in this appendix, we call the designs K, N, and D, respectively. The original designs were developed for different frequencies with wideband applications, and the OMT can be scaled to operate in required frequency bands. We linearly scaled the models using the ratio between our center frequencies (230 and 345 GHz) and those of the original designs (Design K: 442.5 GHz, N: 100 GHz, D: 90 GHz), then analyzed the performance. The electromagnetic simulations of the scaled OMTs were run using frequency-domain solver of the CST Microwave studio simulation software\(^1\).

\(^1\)https://www.cst.com/
Figure A.1: Three OMT designs from Kamikura et al. (2008, K), Navarrini et al. (2010, N), and Dunning et al. (2009, D).
We simulated the OMTs without misalignment and compared the results to simulations presented in each paper to ensure our CST models. Then, OMTs were split horizontally along the center of the input square waveguide port. Simulations were performed with lateral misalignments in micrometer intervals. In Figure A.2, we show the simulated transmission of the models scaled for the 345 GHz use, and the transmission loss is $\leq 0.1$ dB across the $\sim 90$ GHz bandwidth. The polarization isolation of the models are greater than 60 dB across the frequency band.

In Figure A.3, we show the polarization isolation (the transmitted signal at the unwanted port) at one output port of the OMTs, using the models with 4 $\mu$m lateral misalignment, which is comparable to $\sim 1\%$ of the OMT input waveguide dimension. Model K and N show better isolation than D, but all the models provide isolations better than 15 dB even with the misalignment. Model D has less complexity in the design that could be easily fabricated as a split block assembly, and we used this design for the SPT VLBI receiver assembly.
Figure A.2: Simulated OMT transmissions of the three models. The OMT models are linearly scaled to operate between 300 and 390 GHz.
Figure A.3: Simulated OMT isolation using the models with 4 µm lateral misalignment.
The SPT VLBI system comprises multiple signal chains from the receiver output to the recording system (Figure 2.1), and it is crucial to use proper input power levels and amplification so that the additional noise is not injected to the data. The Y-factor method is one way to measure the sensitivity of a system using two loads of different temperatures. At the South Pole, we performed the Y-factor measurement at different stages of the signal chain and compared them to monitor whether the noise performance of the system degrades.

We measured the Y-factor at four stages, for all four IF channels (two sidebands, two polarizations) including:

(A) Receiver IF output amplification. It is the IF amplifier output of the receiver backend and electronics in Figure 2.1.

(B) Fiber optics receiver. We use the optical fiber link to transfer the amplified receiver IF signal to the SPT control room as described in Section 2.2.1. It is the signal injected to the block downconverter.

(C) Block downconverter (BDC). It is the frequency downconverted signal at two 0-2 GHz basebands, from the receiver IF bands (4-8 GHz or 5-9 GHz).

(D) R2DBE units. It is the digitized signal for the recording. We read this data electronically, not through the spectrum analyzer.

Figure B.1, Figure B.2, and Figure B.3 show the comparison between (A) and (B), (B) and (C), and (B) and (D), respectively. For the measurement, we used the ambient load and the blank sky, and the absolute Y-factor varies depending on the telescope elevation.
Figure B.1: Y-factor measurements of the amplified receiver IF (blue) and fiber optics receiver (orange) for (a) Pol 0 USB, (b) Pol 0 LSB, (c) Pol 1 USB, (d) Pol 1 LSB.
Figure B.2: Y-factor measurements of the fiber optics receiver (blue) and BDC (orange and green) for (a) Pol 0 USB, (b) Pol 0 LSB, (c) Pol 1 USB, (d) Pol 1 LSB. The BDC frequency band is 0-2 GHz, and it is plotted in the original 5-9 GHz IF band in this Figure.
Figure B.3: Y-factor measurements of the fiber optics receiver (blue) and R2DBE (orange) for (a) Pol 0 USB, (b) Pol 0 LSB, (c) Pol 1 USB, (d) Pol 1 LSB. The R2DBE frequency band is 0-2 GHz, and it is plotted in the original 5-9 GHz IF band in this Figure.
APPENDIX C

SPT VLBI Receiver: 230 GHz Focusing Template

The focus position of the telescope varies depending on weather and observing elevation, and we need to find the best focus occasionally during the observation, using bright sources in the sky. At the SPT, we could move the optics bench (Figure 2.2) to change the location of the VLBI receiver in three-dimensional space. We use planets to adjust the focus and Figure C.1 shows the Jupiter raster maps at various focus positions. As the receiver gets closer to the focus, we have rounder and more sharply peaked shape of the planet.
Figure C.1: The Jupiter raster map at different focus positions. From (a) to (d), the receiver moves along the chief ray axis, and (c) provides the best focused image.
In January 2019, the SPT VLBI receiver performed the first 345 GHz (0.87 mm) observation using the mixer under development. Figure D.1 and D.2 show the first Jupiter and Moon maps, respectively.
Figure D.1: 345 GHz Jupiter raster map. 4 arcmin × 4 arcmin field-of-view, 10 arcsec resolution.
Figure D.2: 345 GHz Moon raster map. 40 arcmin × 40 arcmin field-of-view, 30 arcsec resolution. Horizontal stripes are due to the fluctuation of the amplified IF power level caused by the temperature variation inside the receiver cabin.
We used the vector beam measurement technique in Chapter 3 to characterize the beam propagation between the secondary and tertiary mirrors. In this Appendix, we describe the measurement and analysis.

E.1 Optical Design

Figure E.1 shows the optics layout of the SPT VLBI system, and the optical design parameters are listed in Table E.1 (See Figure 2.2 for the CAD model with the optical components). The Gaussian beam waist expands as it propagates from the horn to the tertiary, and the waist at the tertiary becomes

\[ w_{\text{tert}, \text{horn}} = w_{0, \text{horn}} \sqrt{1 + \left( \frac{\lambda d_{\text{in, tert}}}{\pi w_{0, \text{horn}}^2} \right)^2}, \tag{E.1} \]

where \( \lambda \) is the wavelength of the beam, \( w_{0, \text{horn}} \) is the horn waist radius, and \( d_{\text{in, tert}} \) is the distance from the horn to the tertiary. The same principle applies to the Gaussian beam propagating from the Cassegrain focus (CF) towards the tertiary and the beam waist radius at the tertiary is

\[ w_{\text{tert}, \text{CF}} = w_{0, \text{CF}} \sqrt{1 + \left( \frac{\lambda d_{\text{out, tert}}}{\pi w_{0, \text{CF}}^2} \right)^2}, \tag{E.2} \]

where \( w_{0, \text{CF}} \) is beam waist radius at the CF, and \( d_{\text{out, tert}} \) is the distance from the CF to the tertiary. Perfect beam coupling at the tertiary makes

\[ w_{\text{tert}, \text{horn}} = w_{\text{tert}, \text{CF}}, \tag{E.3} \]

and we can solve the equation to find \( w_{0, \text{CF}} \).
Table 1. Zemax Quartz Media Sizes

<table>
<thead>
<tr>
<th>Optic</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Chief Ray</td>
<td>0 mm</td>
<td>4800 mm</td>
<td>822.857 mm</td>
</tr>
<tr>
<td>Primary Vertex</td>
<td>0 mm</td>
<td>0 mm</td>
<td>0 mm</td>
</tr>
<tr>
<td>Prime Focus Chief Ray</td>
<td>0 mm</td>
<td>0 mm</td>
<td>7000 mm</td>
</tr>
<tr>
<td>Secondary Chief Ray</td>
<td>0 mm</td>
<td>102.221 mm</td>
<td>6802.593 mm</td>
</tr>
<tr>
<td>Secondary Vertex</td>
<td>0 mm</td>
<td>55.967 mm</td>
<td>6770.666 mm</td>
</tr>
<tr>
<td>Tertiary Chief Ray</td>
<td>0 mm</td>
<td>259.181 mm</td>
<td>5937.968 mm</td>
</tr>
<tr>
<td>Tertiary Vertex</td>
<td>0 mm</td>
<td>354.576 mm</td>
<td>6072.494 mm</td>
</tr>
<tr>
<td>230 GHz Horn Chief Ray</td>
<td>45 mm</td>
<td>-126.017 mm</td>
<td>5400 mm</td>
</tr>
<tr>
<td>345 GHz Horn Chief Ray</td>
<td>44.850 mm</td>
<td>-124.518 mm</td>
<td>5400 mm</td>
</tr>
</tbody>
</table>

Figure E.1: Schematic of the SPT VLBI receiver optics (Drawing by Christopher H. Greer.) ‘PF’ and ‘CF’ indicate prime focus and the Cassegrain focus, respectively. The black dots show ‘2nd’, ‘3rd’, and ‘Horn’ mark the points where the chief ray hits the secondary mirror, tertiary mirror, and the feed horn focus, respectively. The primary mirror is located right side of the figure as it is in Figure 2.2.
### Table E.1. SPT VLBI receiver optical design parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>230 GHz</th>
<th>345 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary center</td>
<td>(0 mm, 4800 mm, 822.857 mm)</td>
<td></td>
</tr>
<tr>
<td>Prime focus</td>
<td>(0 mm, 0 mm, 7000 mm)</td>
<td></td>
</tr>
<tr>
<td>Secondary center</td>
<td>(0 mm, 153.397 mm, 6802.593 mm)</td>
<td></td>
</tr>
<tr>
<td>Cassegrain focus (CF)</td>
<td>(0 mm, 259.181 mm, 5937.968 mm)</td>
<td></td>
</tr>
<tr>
<td>Tertiary center</td>
<td>(0 mm, 325.000 mm, 5400 mm)</td>
<td></td>
</tr>
<tr>
<td>Horn</td>
<td>(45 mm, -126.017 mm, 5400 mm)</td>
<td>(-44.850 mm, -124.518 mm, 5400 mm)</td>
</tr>
</tbody>
</table>

Note. — The positions of optical elements use (X, Y, Z) coordinate that was defined at the design stage. We use the same coordinate for the beam measurement (Figure E.2).
E.2 Beam Measurement

We performed the SPT VLBI receiver beam measurement in the lab, with the tertiary mirror installed, and Figure E.2 shows the setup. The frequency source, Gunn oscillator was mounted on Parker Motion 403XE linear stages assembled to scan two-dimensional space. We used a WR-8 waveguide probe and VDI\textsuperscript{1} WR-5.1 tripler for band 6 frequency, and a WR-4 waveguide probe and VDI ALMA band 7 WR-2.8 tripler for band 7 frequency.\textsuperscript{2} The Gunn oscillator was tuned to the frequencies for mapping of the two frequency bands:

- Band 6: \( f_{B6} = 76.2564 \text{ GHz} \times 3 = 228.7692 \text{ GHz} \)
- Band 7: \( f_{B7} = 112.5010 \text{ GHz} \times 3 = 337.5030 \text{ GHz} \)

We scanned the beam on five planes having different offset distances from the tertiary in \( z \)-direction, parallel to each other. From the reference plane, the other four planes are 0.25, 0.5, 0.75, and 6 inches away. In this way, we can measure the beam around the CF, propagating between the secondary and tertiary mirrors.

Figure E.3(a) shows the normalized power of the band 6 beam at the reference plane. After collecting the data, we did two-dimensional Gaussian fitting using Markov chain Monte Carlo (MCMC) method. Figure E.4 is a corner plot showing the parameter estimation with the reference plane data. We used five parameters: center coordinates of the beam \( (x_0, y_0) \), the standard deviation in each direction \( \sigma_x, \sigma_y \), and the rotation angle of the major axis. For all the fittings, the beam shape is close to a circular shape, and the rotation angle is not well constrained. When the standard deviation of the Gaussian model is \( \sigma \), its full width at half maximum (FWHM) is \( 2\sqrt{2\ln 2} \). The radius of the beam and the FWHM have a relation:

\[
w = \frac{\text{FWHM}}{\sqrt{2\ln 2}} = 2\sigma. \tag{E.4}
\]

\textsuperscript{1}Virginia Diodes, Inc. http://www.vadiodes.com
\textsuperscript{2}Band 6: 211–275 GHz, band 7: 275–373 GHz, per ALMA terminology
Figure E.2: (a) Beam source mounted on X-Y translation stage, (b) tertiary mirror and the receiver viewed from the beam source.
Figure E.3: Normalized power of the band 6 beam at the reference plane (a) measured, and (b) model fit.
E.3 Analysis

E.3.1 Beam at the Cassegrain Focus

The beam waist radii of the feedhorns of the receiver are:

- $w_{0,\text{horn,B6}} = 2.202$ mm
- $w_{0,\text{horn,B7}} = 1.304$ mm

and we can find $w_{0,\text{CF}}$ using Equations (E.1), (E.2), and (E.3).

- $w_{0,\text{CF,B6}} = 2.633$ mm
- $w_{0,\text{CF,B7}} = 1.565$ mm

Dan Marrone also estimated that the CF position $z_{\text{offset}} = 0.624$ inch away from the scanning reference plane (red dashed line in Figure E.5) using FARO-arm\(^3\) metrology. Then, we can model the beam waist radius with respect to the offset distance and fit the beam measurement data.

Figure E.5(a) shows the model and fit to the data of the band 6 side of the receiver. The fit gives beam waist radius $w_{0,\text{CF,B6}} = 2.776$ mm at the offset distance $z_{\text{offset}} = 0.156$ inch.

\(^3\)https://www.faro.com/products/3d-manufacturing/faroarm/
Figure E.4: Two dimensional Gaussian fitting of the normalized beam map at the reference plane.
Figure E.5: Beam waist radius of (a) band 6 and (b) band 7 as a function of offset distance from the scanning reference plane. Green line is the best fit to the beam measurement data, and the red line is the model.
E.3.2 Primary Mirror Illumination

As in Section E.1, we can assume 100% beam coupling at each optical element and use beam propagation to infer the illumination on the primary. Beam propagation from the CF to the prime focus (PF) gives:

\[ w_{\text{sec, CF}} = w_{0,\text{CF}} \left[ 1 + \left( \frac{\lambda d_{\text{in,sec}}}{\pi w_{0,\text{CF}}^2} \right)^2 \right] \]  
(E.5)

\[ w_{\text{sec, PF}} = w_{0,\text{PF}} \left[ 1 + \left( \frac{\lambda d_{\text{out,sec}}}{\pi w_{0,\text{PF}}^2} \right)^2 \right] \]  
(E.6)

\[ w_{\text{sec, CF}} = w_{\text{sec, PF}} \]  
(E.7)

The primary beam waist radius \( w_{0,\text{PF}} \) is:

\[ w_{0,\text{PF}} = 0.216 \sqrt{T_e \frac{f_{\text{prim}}}{D_{\text{prim}}}} \lambda, \]  
(E.8)

where \( T_e \) is the primary edge taper in dB unit, \( f_{\text{prim}} \) is the focal length of the primary, \( D_{\text{prim}} \) is the diameter of the primary. For the SPT which has an off-axis design, \( f_{\text{prim}} = 7.823 \) m. \( T_e = 12 \) dB gives \( D_{\text{prim}} = 9.994 \) m.

If we assume that the beam waist radius around the CF follows the parameters we inferred from the fit, the beam waist radius at the secondary mirror becomes

\[ w'_{\text{sec, CF}} = w'_{0,\text{CF}} \left[ 1 + \left( \frac{\lambda d'_{\text{in,sec}}}{\pi w'_{0,\text{CF}}^2} \right)^2 \right] \]  
(E.9)

and the measurement provides

- \( w'_{0,\text{CF}} = 2.776 \) mm \( (w_{0,\text{CF}} = 2.633 \) mm)\)
- \( d'_{\text{in,sec}} = 0.882 \) m \( (d_{\text{in,sec}} = 0.870 \) m),

where the numbers in the parentheses indicate the model values.
APPENDIX F

Calibration of the 2015 SPT-APEX VLBI Test Observation Data

In this appendix, we present the absolute calibration procedure for the single baseline VLBI test observation data between the SPT and the APEX in 2015.

F.1 Calibration

F.1.1 Amplitude Calibration

The visibility amplitude in the correlator unit is converted to the physical unit by multiplying the geometric mean of system equivalent flux density (SEFD) of each station, $\text{SEFD}_i$ and $\text{SEFD}_j$. The correlator unit is assumed to be 10,000 Jy, and

$$\text{Amplitude [Jy]} = \text{Amplitude [Correlator unit]} \times \frac{\sqrt{\text{SEFD}_i \times \text{SEFD}_j}}{10,000 \text{ Jy}}.$$  \hspace{1cm} (F.1)

The SEFD, the flux density equivalent of the system temperature of a station is

$$\text{SEFD [Jy]} = \text{Gain [Jy/K]} \times T_{\text{sys}} \text{[K]},$$  \hspace{1cm} (F.2)

where $T_{\text{sys}}$ is the elevation dependent system temperature at the time of observation.

In general, we measure the system temperature by comparing the powers the receiver reads between the calibration load at a known temperature ($P_{\text{load}}$) and the blank sky ($P_{\text{sky}}$),

$$P_{\text{sky}} = G(T_{\text{rx}} + T_{\text{sky}})$$  \hspace{1cm} (F.3)

$$P_{\text{load}} = G(T_{\text{rx}} + T_{\text{load}}),$$  \hspace{1cm} (F.4)

where $T_{\text{load}}$ is the temperature of the load (eccosorb, a microwave absorbing material, is widely used), $T_{\text{rx}}$ is the receiver temperature and $G$ is the gain of the receiver.
system. We define the ratio of receiver powers when the load is on and the receiver is looking at the sky,

\[ C = \frac{P_{\text{load}} - P_{\text{sky}}}{P_{\text{sky}}}. \]  

(F.5)

The sky temperature is

\[ T_{\text{sky}} = T_{\text{atm}}(1 - \exp(-\tau)), \]  

(F.6)

where \( T_{\text{atm}} \) is the atmospheric temperature and \( \tau = \tau_0 / \sin(El) \) is the opacity towards the source. \( \tau_0 \) is the zenith opacity, and \( El \) is the elevation of the source. Assuming the atmospheric temperature is close to the ambient temperature \( (T_{\text{atm}} = T_{\text{amb}}) \), the measured system temperature when we use the ambient load for the calibration \( (T_{\text{load}} = T_{\text{amb}}) \) is

\[ T_{\text{sys}} = \frac{P_{\text{sky}}}{P_{\text{load}} - P_{\text{sky}}} T_{\text{load}} \]  

(F.7)

\[ = \frac{T_{\text{load}}}{C} \]  

(F.8)

\[ = \frac{T_{\text{rx}} + T_{\text{sky}}}{(T_{\text{rx}} + T_{\text{load}}) - (T_{\text{rx}} + T_{\text{sky}})} T_{\text{load}} \]  

(F.9)

\[ = \frac{T_{\text{rx}} + T_{\text{atm}}(1 - \exp(-\tau))}{T_{\text{load}} - T_{\text{atm}}(1 - \exp(-\tau))} T_{\text{load}} \]  

(F.10)

\[ = \frac{T_{\text{rx}} + T_{\text{atm}}(1 - \exp(-\tau))}{\exp(-\tau)} \]  

(F.11)

**F.1.2 Aperture Efficiency and Gain**

We often use planets as calibrator source to derive telescope efficiency and gain. The planets can be modeled as a uniform disk. For a uniform disk of brightness temperature \( J_\nu(T_B) \) with a diameter of \( \theta_s \), the flux density at frequency \( \nu \) is

\[ S_\nu = \int_{\text{source}} \frac{2k}{\lambda^2} J_\nu(T_B) d\Omega \]  

(F.12)

\[ = \frac{2k \pi}{\lambda^2} \theta_s^2 J_\nu(T_B). \]  

(F.13)
The antenna temperature of the telescope is a convolution of the antenna power pattern $P(\theta, \phi)$ and the source brightness distribution $J(\nu)(\theta, \phi)$. That is,

$$T_A = \frac{1}{\Omega_A} \int_{\text{source}} P(\theta - \theta', \phi - \phi') J(\nu)(\theta', \phi') d\Omega$$

(F.14)

$$= \frac{A_{\text{eff}}}{\lambda^2} \int_{\text{source}} P(\theta - \theta', \phi - \phi') J(\nu)(\theta', \phi') d\Omega$$

(F.15)

$$= \frac{S_\nu}{2k} A_{\text{eff}} \int_{\text{source}} \frac{1}{J(\nu)(\theta', \phi')} d\Omega \int_{\text{source}} P(\theta - \theta', \phi - \phi') J(\nu)(\theta', \phi') d\Omega,$$

(F.16)

where $\Omega_A$ is the solid angle of the antenna pattern, and $A_{\text{eff}}$ is the effective area of the dish. Since

$$P(\theta) = \exp \left[ -\ln 2 \left( \frac{2\theta}{\theta_b} \right)^2 \right]$$

(F.17)

for a Gaussian beam with a full-width half maximum (FWHM) of $\theta_b$, the antenna temperature is

$$T_A = \frac{S_\nu}{2k} A_{\text{eff}} \int_{\text{source}} \frac{1}{J(\nu)(\theta', \phi')} d\Omega \int_{\text{source}} P(\theta - \theta', \phi - \phi') \psi(\theta', \phi') \sin \theta' d\theta' d\phi'$$

(F.18)

$$= \frac{S_\nu}{2k} A_{\text{eff}} \int_{\text{source}} \frac{1}{J(\nu)(\theta', \phi')} d\Omega \int_{\text{source}} P(\theta - \theta', \phi - \phi') \psi(\theta', \phi') \theta' d\theta' d\phi'$$

(F.19)

$$= \frac{S_\nu}{2k} A_{\text{eff}} \int_{\text{source}} \frac{1}{J(\nu)(\theta', \phi')} d\Omega \int_{0}^{\theta_s/2} \exp \left[ -\ln 2 \left( \frac{2\theta}{\theta_b} \right)^2 \right] \theta d\theta$$

(F.20)

$$= \frac{S_\nu}{2k} A_{\text{eff}} \frac{4 \ln 2}{\pi \theta_s^2} \int_{0}^{\theta_s/2} \exp \left[ -\ln 2 \left( \frac{2\theta}{\theta_b} \right)^2 \right] \theta d\theta$$

(F.21)

$$= \frac{S_\nu}{2k} A_{\text{eff}} \left[ 1 - \exp \left( -\ln 2 \left( \frac{\theta_s}{\theta_b} \right)^2 \right) \right] / \left[ \ln 2 \left( \frac{\theta_s}{\theta_b} \right)^2 \right].$$

(F.22)

Then, we can derive the aperture efficiency with the measurement of the antenna temperature and known flux density of the calibrator source,

$$\eta_A = \frac{4 A_{\text{eff}}}{\pi D^2},$$

(F.23)

where $D$ is the diameter of the telescope dish (APEX: $D=12$ m, SPT: $D=10$ m). The telescope gain is

$$G = \frac{2k}{A_{\text{eff}}},$$

(F.24)
F.1.3 Planet Model

We need diameter and brightness temperature of the planets in millimeter wavelength to estimate the aperture efficiency of the telescopes. APEX used the Saturn, and the SPT used both the Venus and the Saturn to determine the aperture efficiency (or, gain) in January 2015.

The brightness temperature measurement of the planets in 1 mm can be found in literature (e.g. Rather et al., 1974; Hildebrand et al., 1985). However, the papers are often outdated and even the recent one using the Wilkinson Microwave Anisotropy Probe (WMAP) observation (Weiland et al., 2011) only goes up to 94 GHz. ALMA has the flux density model for the planets, but without the Saturn (Butler, 2012). The Submillimeter Array (SMA) planetary visibility function calculator\(^1\) is a useful reference to calculate the flux density of the planets, but it does not provide the accurate model for the Saturn ring. The Saturn ring contributes to the emission depending on the opening angle (Figure 7 of Weiland et al., 2011), and it is not negligible.

The planet model we use for the calibration of APEX and SPT is based on Butler 2017 (private communication, hereafter B17).

The Venus:

- Radius \( R = 6,052 \) km.
- Brightness temperature \( T_b = 291.1 \) K (MJD 57042, 01/20/2015).
- Distance \( d = 1.55517849546042 \) AU.\(^2\)
- Angular diameter \( \theta_s = 10.73 \) arcseconds.
- Frequency for the model \( \nu = 221.1 \) GHz.
- Flux density \( S_{\nu,\text{Venus}} = 930.9 \) Jy.

The Saturn:

- Equatorial radius \( R_{\text{eq}} = 60,268 \) km.

\(^1\)http://sma1.sma.hawaii.edu/planetvis.html
\(^2\)NASA HORIZONS: http://ssd.jpl.nasa.gov/?horizons
- Polar radius $R_p = 54,364$ km.
- Brightness temperature normalized by the disk size $T_b = 151.3$ K (MJD 57042, 01/20/2015).
- Ring opening angle $B = 24.8$ degrees.
- Distance $d = 9.957660463951$ AU.
- Frequency for the model $\nu = 221.1$ GHz.

For the Saturn, the projected polar radius is

$$R'_p = \sqrt{R_{eq}^2 \sin^2 B^2 + R_p^2 \cos^2 B^2}$$

$$= 55,448 \text{ km},$$

and the flux density is

$$S_{\nu, \text{Saturn}} = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT_b) - 1} \frac{\pi R_{eq}R'_p}{d^2}$$

$$= 1037.8 \text{ Jy}.$$

Also, the angular size of the planet in Equation (F.13) can be expressed as

$$\theta_s = \sqrt{\left(\frac{2R_{eq}}{d}\right)^2 + \left(\frac{2R'_p}{d}\right)^2}$$

$$= 16.009 \text{ arcseconds}.$$ 

**F.2 APEX**

The local oscillator (LO) frequency was $12,283.5 \text{ MHz} \times 18 = 221.103 \text{ GHz}$ and the system used 5-7 GHz IF of the lower sideband. The block downconverter LO was 6,997 MHz, and the sky frequency through R2DBE was between 214.106 GHz and 216.154 GHz.

**F.2.1 System Temperature**

The system temperature and precipitable water vapor (PWV) are recorded in the log file after every scan. In the file, there are two system temperature measurements
each time. They are the measurements on two sub-bands of the 4-8 GHz IF. Since
we used 5-7 GHz IF that overlapped equally on both 4-6 and 6-8 GHz bands during
the observation, we use the average of two system temperatures for the calibration.
Also, the logged system temperature already includes the opacity correction.

PWV and the zenith opacity at 225 GHz follow the linear relation,

\[ \tau_0 = \text{PWV}[\text{mm}] / 24 \text{ mm} \]  \hspace{1cm} (F.31)

(D’Addario and Holdaway, 2003). The PWV measurement between 07:20 UT and
09:00 UT gives the zenith opacity between 0.140 and 0.145, so we assume that the
opacity is stationary during the observation.

F.2.2 Aperture Efficiency and Gain

The log file also recorded the antenna temperature of the Saturn 31.9 K and 31.6 K
at 09:47 UT. The Saturn flux density and the size of B17 give

the aperture efficiency \( \eta_{A, \text{APEX}} = 0.830 \) and \hspace{1cm} (F.32)

the telescope gain \( G_{\text{APEX}} = 29.4 \pm 0.14 \text{ Jy/K} \). \hspace{1cm} (F.33)

We assumed the beam size \( \theta_b = 1.2\lambda/D \). We can find the 230 GHz gains of 39
Jy/K from the web\(^3\) and 35.5 ± 1.1 Jy/K from the EHT wiki. Our calculation is
\~20 % smaller than those values. The SEFD calculation is straightforward once we
have system temperature and gain for every scan. The estimated SEFD is 5,700 ~
6,100 Jy. Below is the summary of SEFDs for Cen A scans.

\(^3\)http://www.apex-telescope.org/telescope/efficiency/
<table>
<thead>
<tr>
<th>Scan</th>
<th>$T_{\text{sys}}$ (K)</th>
<th>SEFD (10^3 Jy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>07:20:00</td>
<td>207 ± 8</td>
<td>6.1 ± 0.2</td>
</tr>
<tr>
<td>07:30:00</td>
<td>203 ± 8</td>
<td>6.0 ± 0.3</td>
</tr>
<tr>
<td>07:40:00</td>
<td>202 ± 8</td>
<td>5.9 ± 0.2</td>
</tr>
<tr>
<td>07:50:00</td>
<td>202 ± 8</td>
<td>5.9 ± 0.2</td>
</tr>
<tr>
<td>08:20:00</td>
<td>195 ± 8</td>
<td>5.7 ± 0.2</td>
</tr>
<tr>
<td>08:30:00</td>
<td>195 ± 8</td>
<td>5.7 ± 0.2</td>
</tr>
<tr>
<td>08:40:00</td>
<td>196 ± 8</td>
<td>5.8 ± 0.2</td>
</tr>
<tr>
<td>08:50:00</td>
<td>196 ± 8</td>
<td>5.8 ± 0.2</td>
</tr>
</tbody>
</table>

F.3 SPT

F.3.1 System Temperature

The load calibration procedure for estimating the system temperature during the observation is written down in the log file. The third columns in the log show the ratio of receiver powers $C$ in Equation (F.5).

In 2015, the SPT VLBI optics had a tertiary mirror with an erroneous design, and it is possible that the receiver power had a contribution from the receiver cabin or the ground. We introduce additional spillover term to the blank sky measurement. That is,

$$P_{\text{sky}}/G = T_{\text{rx}} + T_{\text{sky}} + T_{\text{spillover}}$$

$$= T_{\text{rx}} + F_{\text{eff}}T_{\text{atm}}(1 - \exp(-\tau)) + (1 - F_{\text{eff}})T_{\text{amb}}$$

Kyle Massingill measured the receiver temperature $T_{\text{rx}}$ in the DPM lab in August 2016, through the spectrometer channel of the IF box. We could not use the normal IF channel for power reading during the 2015 observation, and we used the spectrometer channel of the IF box for iBOB instead. The hot and cold load temperatures used for the Y-factor measurements are 273.16 + 24.1 K and 77.2 K, respectively. The table below shows the result, and we use $T_{\text{rx}}$ of the Pol 0 channel.
Also, the 350 $\mu$m tipper at the South Pole recorded $\tau_0(350 \mu m) = 2.05$ at 01:11:18 on January 20 before the calibration scans. Radford and Peterson (2016) gives the conversion relation

$$\tau_0(350 \mu m) = 29 \tau_0(225 GHz) - 0.2$$  \hspace{1cm} (F.36)

for the South Pole and $\tau_0(225 GHz) = 0.078$. The tipper log also provides the temperatures $T_{amb} = 251.79$ K and $T_{atm} = 235.73$ K. We assume

$$T_{atm} = 235.73 \text{ K}$$  \hspace{1cm} (F.37)

$$T_{amb} = T_{load} = 251.79 \text{ K}$$  \hspace{1cm} (F.38)

during the calibration scan and Cen A observation at the SPT. The tipper did not take the data between Jan 14 00:23:57 and Jan 18 15:19:47 and we don’t have the data collected around the time of the observation.

We performed the load calibration before and after the planet raster scans. The elevation of the planet, the ratio of powers between the load and the blank sky, and the system temperatures of the three scans are:

- Venus 20150120.012302: 16.130 degrees elevation, $C = 0.173$, $T_{sys} = 1454$ K.
- Venus 20150120.013827: 16.126 degrees elevation, $C = 0.177$, $T_{sys} = 1426$ K.
- Saturn 20150120.024756: 18.753 degrees elevation, $C = 0.213$, $T_{sys} = 1183$ K.

Then, the ratio of the receiver power for the load calibration is

$$C = \frac{P_{load} - P_{sky}}{P_{sky}}$$  \hspace{1cm} (F.39)

$$= \frac{T_{load} - F_{eff}T_{atm}(1 - \exp(-\tau)) - (1 - F_{eff})T_{amb}}{T_{rx} + F_{eff}T_{atm}(1 - \exp(-\tau)) + (1 - F_{eff})T_{amb}}.$$  \hspace{1cm} (F.40)

The fitting gives $F_{eff} = 0.874$ and we can calculate the elevation dependent system temperature assuming similar atmospheric and ambient temperatures and $\tau_0$ on January 17. The system temperature towards Cen A at 43.0 degrees elevation is $\approx 1190$ K.
F.3.2 Aperture Efficiency and Gain

We performed a single dish calibration using the Venus and the Saturn, and Figure F.1 are the raster images of the planets. Figure F.1(a) and (b) are the Venus images, but (a) shows worse baseline (less stable background sky level) than (b) does, so we use the data 20150120_013827. The Saturn data 20150120_024756 had a bad pixel and it needed to be removed. We replaced the bad pixel with the median value of the data.
Figure F.1: (a) Venus 20150120_012302, (b) Venus 20150120_013827, (c) Saturn 20150120_024756
We estimate the antenna temperature of a planet from the maximum value of the raster scan data using the system temperature measured with the load calibration \( T_A = C T_{\text{sys}} \). We also estimate the measured FWHM of the scan \( \theta_{\text{fwhm}} \) from the two dimensional Gaussian fit to the raster map. The beam width \( \theta_b \) for a uniform disk of the diameter size \( \theta_s \) when \( \theta_s \leq \theta_b \) is,

\[
\theta_b = \sqrt{\theta_{\text{fwhm}}^2 - \frac{\ln 2}{2} \theta_s^2}.
\] (F.41)

Then we can calculate \( A_{\text{eff}} \), and then the aperture efficiency \( \eta_A \) using B17 data. The table below shows the results using Venus and Saturn to find telescope parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Venus 20150120_013827</th>
<th>Saturn 20150120_024756</th>
</tr>
</thead>
<tbody>
<tr>
<td>System temperature ( T_{\text{sys}} ) (K)</td>
<td>1426</td>
<td>1183</td>
</tr>
<tr>
<td>Antenna temperature ( T_A ) (K)</td>
<td>13.7</td>
<td>17.1</td>
</tr>
<tr>
<td>FWHM of the image ( \theta_{\text{fwhm}} ) (arcsec)</td>
<td>49.0</td>
<td>47.5</td>
</tr>
<tr>
<td>Derived beam width ( \theta_b ) (arcsec)</td>
<td>48.6</td>
<td>46.5</td>
</tr>
<tr>
<td>Effective area ( A_{\text{eff}} ) (m²)</td>
<td>41.2</td>
<td>46.8</td>
</tr>
<tr>
<td>Aperture efficiency ( \eta_A )</td>
<td>0.52</td>
<td>0.60</td>
</tr>
<tr>
<td>Gain (Jy/K)</td>
<td>67.1</td>
<td>59.1</td>
</tr>
</tbody>
</table>

The SEFD is \( \sim 8,600 \) Jy during the Cen A observation. Note that the system temperature we use for the correlated data product is through the ‘DIG’ channel of the SPT VLBI IF amplification chain, whereas the system temperature we apply for the gain calibration is through the ‘SPEC’ channel.\(^4\)

\(^4\)The amplification chain has two separate channels, one (‘DIG’) to the VLBI backend and the other (‘SPEC’) to the digital spectrometer.
APPENDIX G

Principal Component Analysis Applied to the Interferometric Measurements for Bayesian Model Comparison

Recently, the Event Horizon Telescope (EHT) has conducted successful observations of the nearby supermassive black holes (SMBHs) to observe their environment directly using the very-long-baseline interferometry (VLBI) at millimeter wavelength. The full array of the EHT in 2020 will be composed of more than ten telescope sites around the world, and it is a considerable expansion from the three station array at the beginning of the experiment. The emission from the primary science targets of the EHT, especially the SMBH at our own Milky Way Sagittarius A* (Sgr A*), is known to vary in relatively short timescales comparable to the EHT observing timescale or less. In Chapter 5 (Kim et al., 2016), we developed the technique that can statistically compare the variable EHT observables to the time-dependent simulation of the black hole models. Having the number of observables that is almost an order of magnitude more than the early stage of the EHT array, the possible covariance among the data need to be investigated and compensated in the statistical procedure of the previous work. In this Appendix, we apply the Principal component analysis (PCA) to the Bayesian technique of Chapter 5 (Kim et al., 2016). The PCA is a powerful method to find the correlation among the variables in the multidimensional space, using the orthogonal transformation. The new method, as well as the application example using the simulated data, are introduced.

G.1 Introduction

The development of very-long-baseline interferometry (VLBI) techniques has recently allowed the observation of the supermassive black holes with the angular resolution smaller than the apparent event horizon scale using the Event Horizon
Telescope (EHT; Doeleman et al., 2009) array. To generate images of the emitting structure around the black hole from the interferometric visibility data of the EHT VLBI observation, various image reconstruction algorithms have been suggested (e.g., Honma et al., 2014; Bouman et al., 2016; Chael et al., 2016; Akiyama et al., 2017). However, the sparse Fourier-plane coverage in the aperture of the array fundamentally limits the information to reconstruct the image from the visibility data, and the analysis directly in the observable space to investigate the data is required.

In Chapter 5 (Kim et al., 2016), we developed the Bayesian technique to compare VLBI data with the time-dependent black hole simulation models without image reconstruction. The method can still be used to infer physical parameters as well as the geometry that describe the structure of the accretion flow around the black hole and to assess how well the different simulation models fit the data. The statistical procedure can incorporate any interferometric measurements, including the visibility amplitude and closure quantities. However, the technique assumes independence among the observables, which could not always be the case.

Principal component analysis (PCA) is a technique often used in data mining and machine learning these days to find the correlation in the multivariate data and to reduce the dimensions that describe the data. We can use PCA to orthogonally transform the correlated data into the principal component axes, which are linearly uncorrelated variables. The analysis has been already applied in the astrophysical research, for example, to study degeneracies among cosmological parameters used to model the cosmic microwave background anisotropies (Efstathiou and Bond, 1999), and to classify the spectral data of optical galaxy survey (Folkes et al., 1999). Also, Medeiros et al. (2018b) used PCA in the image domain to characterize the variability of the general relativistic magnetohydrodynamic (GRMHD) models of black holes.

In this chapter, we further develop the Bayesian statistical method of Chapter 5 (Kim et al., 2016) by utilizing the PCA decomposition. The revised method performs PCA of the data and the models to account for the correlation among different observable, then calculate posterior to perform parameter estimation or model selection. In Section G.2, we show the PCA decomposition examples of the
VLBI data. In Section G.3, we introduce the formalism used to incorporate the PCA technique to the Bayesian framework and the application example.

### G.2 Principal Component Analysis and Interferometric Data

In this section, we describe the fundamentals of PCA decomposition to find the orthogonal eigenvectors of the data and perform coordinate transformation. We refer to other literatures (e.g., Jolliffe, 1986; Francis and Wills, 1999) for the overview of the PCA technique and mathematical derivations. The principal components are the vectors in the space that maximize the variance of the data, orthogonal to each other. If we have \( n \) observations of \( p \)-dimensional data, we can write this as an \( n \times p \) matrix \( X \). Then we define the zero-centered matrix by subtracting the mean of each variable,

\[
X' = X - \frac{1}{n} \sum_{k=1}^{n} X_k. \tag{G.1}
\]

The covariance matrix of the data \( V \) is

\[
X'^T X' = nC. \tag{G.2}
\]

Then we find the eigenvector \( w \) and the eigenvalue \( \lambda \) of the covariance matrix from

\[
Cw = \lambda w. \tag{G.3}
\]

Let \( T = [t_1, \ldots, t_p] \) a \( p \times p \) matrix where \( t_i \) are the eigenvectors corresponding to the largest eigenvalues. Then, we can transform the original data \( V \) to the new axes \( V' \) defined by the principal components using a simple matrix calculation,

\[
V' = TV. \tag{G.4}
\]

EHT has expanded the array from its early three-station array (Doeleman et al., 2008, 2012) and the number of observables grow as the new stations are added to the array. In general, if we have \( N \) stations in the array, there are

- \( N C_2 = N(N - 1)/2 \) independent baselines,
- $\mathcal{N}(\mathcal{N} - 1)/2 - (\mathcal{N} - 1) = (\mathcal{N} - 1)(\mathcal{N} - 2)/2$ independent closure phases, and

- $\mathcal{N}(\mathcal{N} - 1)/2 - \mathcal{N} = \mathcal{N}(\mathcal{N} - 3)/2$ closure amplitudes.

Then the dataset composed of the visibility amplitude and closure quantities lies on $p = (3\mathcal{N} - 1)(\mathcal{N} - 2)/2$ dimensional space. In principle, other observables such as polarimetric visibilities (e.g., Johnson et al., 2014) could be used in the analysis, but we do not consider those here.

To describe and visualize how the VLBI data looks like in the observable space, we show examples here with the data generated from the simulation models. We use one of the GRMHD models (model ‘D’) in Chan et al. (2015a,b). Let us consider three EHT VLBI stations with their codes $a$ = Sw (SMA; Hawaii), $b$ = Mg (SMT; Arizona), and $c$ = Lm (LMT; Mexico).

Figure G.1 shows the uniformly distributed visibility amplitudes sampled using the three station. To generate this dataset, we used uniformly distributed variables of polar angle $\phi$ between 0 and $\pi$, azimuthal angle $\theta$ between 0 and $2\pi$ in this three dimensional space. The mock data is centered on the mean $(\mathcal{V}_{ab}, \mathcal{V}_{bc}, \mathcal{V}_{ca})$, with the radial distance $r = 0.1$. The PCA analysis on this data gives

$$
\begin{pmatrix}
\mathcal{V}'_{ab} \\
\mathcal{V}'_{bc} \\
\mathcal{V}'_{ca}
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\mathcal{V}_{ab} \\
\mathcal{V}_{bc} \\
\mathcal{V}_{ca}
\end{pmatrix}.
$$

To generate a realistic example, we sample the scan time when all those stations can observe Sgr A* using an elevation lower limit of 15 degrees on day 100 of 2018. With 300 seconds of scan interval, the scan times are spread between 11:00 and 14:50 UT, 46 scan points in total. Then we sample the visibility amplitude on the averaged image of the snapshots with corresponding $(u, v)$ coordinates for the three baselines as mock data. Figure G.2 shows the mock visibility amplitudes in three-dimensional plot. Table G.1 shows the principal components derived from the PCA of this data and their eigenvalues.
Figure G.1: The uniformly distributed mock visibility amplitudes of three station array. The visibility amplitudes are in Jy unit. The mock data are projected onto three two-dimensional spaces in the plot. The red arrows are the principal components of the data. PCA of this visibility amplitudes data gives an orthogonal matrix, thus the posterior does not change in the transformed coordinate system.
Figure G.2: The mock visibility amplitudes sampled for the test, using model B in Chan et al. (2015a).
Table G.1. Principal components derived with a three-station mock data using model D in Chan et al. (2015a)

<table>
<thead>
<tr>
<th>PC 1</th>
<th>PC 2</th>
<th>PC 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvector</td>
<td>(0.478, -0.065, -0.876)</td>
<td>(0.515, 0.829, 0.219)</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>$3.86 \times 10^{-3}$</td>
<td>$4.35 \times 10^{-4}$</td>
</tr>
<tr>
<td>Proportion</td>
<td>0.894</td>
<td>0.101</td>
</tr>
<tr>
<td>Cumulative</td>
<td>0.894</td>
<td>0.995</td>
</tr>
</tbody>
</table>

G.3 Bayesian Analysis Formalism

We introduced a Bayesian framework in Chapter 5 (Kim et al., 2016) to calculate the posterior $P(\mathbf{w}|\mathbf{d})$ for the vector of parameters $\mathbf{w}$ given the data $\mathbf{d}$ from the likelihood

$$P(\mathbf{d}|\mathbf{w}) \equiv \prod_{i=1}^{M} \prod_{j=1}^{N} P_{ij}(\mathbf{d}|\mathbf{w}),$$

where $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, N$ denote baselines and epochs, respectively. $P_{ij}$ measures how well an observable (e.g., visibility amplitude) generated from the model with parameters $\mathbf{w}$ agrees with the observed data. To apply Equation G.6, we need an assumption that the observables from each baseline $i$ and each epoch $j$ are independent, i.e., not correlated. However, those among different baselines and epochs can have a correlation and the likelihood cannot be calculated as a simple multiplication of $P_{ij}$s.

To explore how the correlated data among different baselines and epochs affect the likelihood calculation, assume we have three VLBI stations $a$, $b$, and $c$ in the array, hence three baseline pairs $i = ab, bc, ca$ as an example. We also assume that all the baselines have simultaneous detection of correlated flux density $\mathbf{V}$ for every $j$th epoch. Three stations can form a closure phase as well as visibility amplitudes,
but let us only include visibility amplitudes as observables as a proof of concept. Following Equation 5.4, we have the posterior

$$P(d|w) \equiv \prod_{j=1}^{N} [P_{ab,j}(d|w) P_{bc,j}(d|w) P_{ca,j}(d|w)].$$  \hspace{1cm} (G.7)

The likelihood for the baseline $i = ab$ is

$$P_{ab,j}(d|w) = \int P^{\text{obs}}(V; V_{ab,j}, \sigma_{ab,j}) P^{\text{sim}}_{ab,j}(V; w) dV$$  \hspace{1cm} (G.8)

The PCA of the observed data on the visibility amplitude space $(V_{ab}, V_{bc}, V_{ca})$ provides three orthogonal principal components $V'_{ab}, V'_{bc}, V'_{ca}$. The new axes can be expressed as linear combinations of the original axes $V_{ab}, V_{bc},$ and $V_{ca}$ and the observed data are not correlated along the new axes. In the $(V'_{ab}, V'_{bc}, V'_{ca})$ coordinate system, the posterior becomes

$$P(d|w) \equiv \prod_{j=1}^{N} [P_{ab,j}(d|w) P_{bc,j}(d|w) P_{ca,j}(d|w)]$$  \hspace{1cm} (G.9)

$$= \prod_{j=1}^{N} \int P^{\text{obs}}(V'; V_{ab,j}, \sigma_{ab,j}, V_{bc,j}, \sigma_{bc,j}, V_{ca,j}, \sigma_{ca,j}) P^{\text{sim}}_{ab,j}(V'; w) dV'$$

$$\int P^{\text{obs}}(V'; V_{ab,j}, \sigma_{ab,j}, V_{bc,j}, \sigma_{bc,j}, V_{ca,j}, \sigma_{ca,j}) P^{\text{sim}}_{bc,j}(V'; w) dV'$$

$$\int P^{\text{obs}}(V'; V_{ab,j}, \sigma_{ab,j}, V_{bc,j}, \sigma_{bc,j}, V_{ca,j}, \sigma_{ca,j}) P^{\text{sim}}_{ca,j}(V'; w) dV'$$

$$= \prod_{j=1}^{N} \int P^{\text{obs}}(V'; V'_{ab,j}, \sigma'_{ab,j}) P^{\text{sim}}_{ab,j}(V'; w) dV'$$

$$\int P^{\text{obs}}(V'; V'_{bc,j}, \sigma'_{bc,j}) P^{\text{sim}}_{bc,j}(V'; w) dV'$$

$$\int P^{\text{obs}}(V'; V'_{ca,j}, \sigma'_{ca,j}) P^{\text{sim}}_{ca,j}(V'; w) dV'.$$

In this three-dimensional case, three eigenvectors: $(T_{11}, T_{12}, T_{13}), (T_{21}, T_{22}, T_{23}),$ and $(T_{31}, T_{32}, T_{33})$ are derived from the PCA analysis. Then the coordinate transformation from $(V_{ab}, V_{bc}, V_{ca})$ to $(V'_{ab}, V'_{bc}, V'_{ca})$ is performed by

$$\begin{pmatrix}
V'_{ab} \\
V'_{bc} \\
V'_{ca}
\end{pmatrix} =
\begin{pmatrix}
T_{11} & T_{12} & T_{13} \\
T_{21} & T_{22} & T_{23} \\
T_{31} & T_{32} & T_{33}
\end{pmatrix}
\begin{pmatrix}
V_{ab} \\
V_{bc} \\
V_{ca}
\end{pmatrix}$$  \hspace{1cm} (G.11)
If we assume a Gaussian distribution to describe the observed visibility amplitude \( V_{ij} \) with an uncertainty \( \sigma_{ij} \), the likelihood is

\[
P^{\text{obs}}(V; V_{ij}, \sigma_{ij}) = \frac{1}{\sqrt{2\pi \sigma_{ij}}} \exp \left[ -\frac{(V - V_{ij})^2}{2\sigma_{ij}^2} \right]. \tag{G.12}
\]

This is applicable with enough signal-to-noise ratio \( (V_{ij}/\sigma_{ij}) \) and the distribution is originally described by a Rice distribution. To incorporate VLBI observables other than visibility amplitude such as closure phase (e.g., Christian et al., in prep.; Kulkarni 1989), proper distributions need to be used. The Gaussian distribution is useful here because the linear combination of the Gaussian random variable results in Gaussian again. If \( V_{ab}, V_{bc}, \) and \( V_{ca} \) all follow Gaussian distribution, their linear transformation \( V' = \alpha V_{ab} + \beta V_{bc} + \gamma V_{ca} \) has an uncertainty \( \sigma' = \sqrt{\alpha^2 \sigma_{ab}^2 + \beta^2 \sigma_{bc}^2 + \gamma^2 \sigma_{ca}^2} \).

Then we can transform the distribution of the model visibility amplitudes using an identical coordinate transformation and calculate the likelihood in the new, same dimensional space where the observables are uncorrelated along different axes. If we perform PCA analysis to find the principal axes using the model visibility amplitude, the transformation will be model-dependent.

We then transform the mock visibility amplitude data as well as the model visibility amplitude distribution to calculate the likelihood using Equation G.11. If we apply the likelihood calculation in the parameter space to the data in Figure G.1, the posterior is identical on the PCA transformed dataset since the axes do not change with the transformation. Figure G.3 shows the posterior likelihood for the data in Figure G.2 with two parameters: flux scale \( (F_0) \) and the position angle of the image on the sky \( (\xi) \), following Chapter 5 (Kim et al., 2016). We need to use grid search for this parameter estimation since Markov-Chain Monte Carlo calculation requires generating simulation images for every move in the parameter space. The PCA transformation application shows the offset from the best fit parameter derived by the previous method.
Figure G.3: The normalized likelihood for two parameter models. The likelihood (a) in the original visibility amplitude space, and (b) along the principal axes of the mock data. The contours indicate the 68.3%, 95.5%, and 99.7% credible regions.
REFERENCES


to ALMA: (Sub)Millimeter Spectroscopy of Galaxies ASP Conference Series, 375, pp. 234–.


