Deriving the PCK parameters for Bennu

Steve Chesley – Sept. 22, 2014

Sept. 22, 2014: Revisions

- Item 1) An unimportant cut and paste error
- Item 5) Corrected the seconds past J2000 for the JD 2453642.0 epoch. (Need to use TDB not UTC for PCK.)
- Item 5) Restored spin rate to the period in DRA Rev 9
- Validation needed to correct for TDB-UTC time difference

Sept. 10, 2014: Revision due to further inputs from Mike Nolan. Specifically, updated values for body-fixed axis orientations, spin rate, and a mapping to epoch of J2000.0 for validation.

From SPICE Required reading: "Let RA and DEC represent the right ascension and declination of a body's north pole as expressed in the J2000 frame, and let W be the prime meridian location, measured in the counterclockwise direction, from the direction defined by the cross product of the Z direction in the J2000 frame (the Earth's ``mean'' North pole at the J2000 epoch) and BODY's North pole at ET, to BODY's prime meridian at ET." See the documentation for more.

1) From the Mike Nolan and the radar model we have the body-fixed axes at a given time. Note that the +x-axis intersects the prime meridian at the equator.

```
# 23_00 JD 2453642.00000 (UT 2005 Sep 28 12:00:00)
#       ecliptic coords of body-fixed +x:   0.650102  0.759050  0.034796
#       ecliptic coords of body-fixed +y:   0.759446 -0.650565  0.002689
#       ecliptic coords of body-fixed +z:   0.024678  0.024678 -0.999391
#       Euler angles for body-fixed axes:  135.000000 178.000000 85.581674 deg
```

2) Converting to the equatorial J2000 frame gives body-fixed unit vectors:

```
x_bennu_eq = 0.650101910767018   0.682573611605758   0.333857410216613
z_bennu_eq = 0.024677670778336   0.420176161108871  -0.907106943089274
```

Note that I have used the Euler angles to compute the z-axis, rather than the less precise Cartesian orientation above. From z_bennu_eq we can also derive the PCK values

RA0 = +86.6388 deg
DEC0 = -65.1086 deg

3) Define the equinox frame with x given by X_ref, z given by Z_Bennu, and Y to complete. The reference direction is given by the equinox X_ref = Z_equatorial × Z_Bennu.

```
>> x_eqx_eq = cross([0 0 1], z_eq); x_eqx_eq = x_eqx_eq/norm(x_eqx_eq);
>> y_eqx_eq = cross(z_bennu_eq, x_eqx_eq);
>> z_eqx_eq = z_eq;
>> rot_eq2eqx = [x_eqx_eq;y_eqx_eq;z_eqx_eq]

rot_eq2eqx =
  -0.998279741372758   0.058630691320672                   0
   0.053184307175106   0.905546484544594   0.420900218340680
   0.024677670778336   0.420176161108871  -0.907106943089274
```

The rows in the rotation matrix give the equatorial frame directions of the equinox frame axes. Columns give the equinox frame directions of the equatorial axes.

4) The orientation of the Bennu +x-axis in the equinox frame and the associated value W(t):

```
>> x_bennu_eqx = rot_eq2eqx * x_bennu_eq'

x_bennu_eqx = -0.608963804620741  0.793198011004684 -0.000000214084293

>> W = atan2(x_bennu_eqx(2), x_bennu_eqx(1)) * 180/pi

W = 127.5146
```

5) Finally we can derive W0 from W(t) = W0 + dW/dt * (t-t0), where t0 = epoch of J2000.0.

For t = JD 2453642.0 UT and t0 = JD 2451545 TDB, we have (t-t0) = 181180864.182350 sec

For rotation period we have 4.297461 h (from DRA Rev 9), which yields a rotation rate dW/dt = 0.023269553813286 deg/sec.

Taking W(t) from step 4 and solving for W0 (mod 360), we obtain

W0 = 89.6456 deg.

Validation

1) Mike Nolan also provided a Bennu +x-axis orientation at 12UT on 01-JAN-2000, which is the J2000 epoch. Thus the W(t) from this +x direction should match the W0 calculated above.

```
# 22_00 JD 2451545.00000 (UT 2000 Jan 01 12:00:00)
#       ecliptic coords of body-fixed +x:   0.070992  0.997128  0.026375
#       ecliptic coords of body-fixed +y:   0.997172 -0.071599  0.022855
#       ecliptic coords of body-fixed +z:   0.024678  0.024678 -0.999391
```

2)

```
x_bennu_eq = 0.0709920087649223  0.904355792754153  0.420833381284717
```

3) Rotation matrix same as above.

4)

```
>> x_bennu_eqx = rot_eq2eqx * x_bennu_eq'

x_bennu_eqx =
   -0.0178468788203486 0.999840731774978 2.19350899444137e-07

>> W = atan2(x_bennu_eqx(2), x_bennu_eqx(1)) * 180/pi

W = 91.0226
```

5) For t = JD 2451545 UT and t0 = JD 2451545 TDB, we have (t-t0) = 64.183927sec

For rotation period we have 4.297461 h (from DRA Rev 9), which yields a rotation rate dW/dt = 0.023269553813286 deg/sec.

W0 = W(t) - dW/dT * (t-t0) =

Taking W(t) from step 4 and solving for W0 (mod 360), we obtain

W0 = 89.5291 deg.

Conclusion: Validation confirms correct values because W(J2000) = W0 to within 0.1 deg. Difference is due to rounding-off errors between DRA rotation period and spin rate used in Nolan printouts.