DESIGN OF PROBING WAVEFORMS IN SOFTWARE-DEFINED SENSING AND IMAGING SYSTEMS

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ABSTRACT

The probing waveforms play a crucial role in the performance of software-defined sensing-imaging systems. The characteristics of the probing waveforms govern both the computation complexity and accuracy of the estimation. This paper describes the concepts of the design and utilization of the probing waveforms for sensing and imaging applications.

INTRODUCTION

Active sensing and imaging are two interesting and important branches of telemetry applications [1]. Both areas share similar mathematical structures and estimation objectives, and both involve advanced signal processing algorithms. Time-delay estimation is a widely utilized time-domain application. This technology also propagates into the applications to range estimation and bearing-angle estimation. With physical transceiver arrays, the space-domain sensing with microwave and acoustic illumination include applications such as radar, sonar, and medical ultrasound imaging. These two sub-areas traditionally function in the time and space domains separately. And the key common component of these two areas is the probing waveforms. As the operating modality elevates to the level of software-defined sensing, the design of the probing waveforms takes the center stage.

The coherent wave equation provides the mathematical foundation and structural interconnections of these two operating modalities. Examining the two components of the wave equations enables us to analyze the time-domain and space-domain functionalities with a unified framework. Through the characteristics of the probing waveforms, we are able to establish the equivalence of the accuracy and resolution limit in both time and space domain. This leads to the optimization of the design procedures of the probing waveforms in software-defined sensing and imaging in an organized and thorough manner.

In this paper, we first partition the coherent wave equations into two components. The time-domain component is analyzed for time-delay estimation, with extension to range estimation. The space-domain component is examined for displacement estimation and image formation. Subsequently, the characteristics of the probing waveforms illustrate the performance, accuracy, and resolution limit of both time and space domain approaches in an integrated format. This allows us to visualize the interconnections and equivalence of these applications.

WAVE PROPAGATION

In many applications in telemetry, especially in sensing and imaging, wave propagation is associated with the data-acquisition process. For an in-depth analysis of the performance of sensing and imaging systems, we return to the fundamental structure of the wave equations. The space-time formulation of coherent wave-propagation kernel is commonly written in the form

$$h(r,t) = A\ exp(j(\beta r - \omega t))$$

where the propagation distance in *3D* is $r = (x^2 + y^2 + z^2)^{1/2}$. And A is the complex amplitude

$$A = \frac{1}{j\lambda r}$$

where λ is the wavelength [2]. The amplitude is a normalization term for the purpose of conservation of energy of the waves in the propagation process, which is often characterized as propagation loss. This term is slightly different for the *2D* case

$$A = \frac{1}{\sqrt{j\lambda r}}$$

The term β is known as the wave number, which is defined as

$$\beta = \frac{2\pi}{\lambda}$$

The wave-propagation kernel can then be partitioned into two components,

$$h(r,t) = h_a(r) \cdot h_b(t) = A\ exp(j\beta r) \cdot exp(-j\omega t)$$

The first term is the space-domain component, which is a function of the propagation distance,

$$h_a(r) = A \cdot exp(j\beta r)$$

This formula allows us to observe the wave-field patterns in space at a particular time instant. The second component is a time-domain function

$$h_b(t) = exp(-j\omega t)$$

This term enables us to model the propagation as a linear and time-invariant system for signal analysis and processing in the time domain.

These two terms represent the two perspectives of the identical wave-propagation process. The partition allows us to formulate the direct relationship between the time and space domain analysis and clearly illustrate link between the *temporal frequency* and *spatial frequency.*

TIME-DOMAIN ESTIMATION

The time-domain component of the wave propagation kernel is in the simple form

$$h_b(t) = \exp(-j\omega t)$$

This component is useful to characterize the functionalities of many time-domain estimation procedures. In telemetry, the applications include time-delay estimation, bearing angle estimation, and range estimation. The basic concept is that the estimation objective such as temporal offset, angle of arrival, and range distance, is a function of the relative time delay

$$\Delta t = t_1 - t_2$$

given two tracks of time signals

$$h_1(t) = h_b(t - t_1)$$

and

$$h_2(t) = h_b(t - t_2)$$

The most direct approach to time-delay estimation is the correlation method. The autocorrelation function of $h_b(t)$, which is often denoted as $R(t)$,

$$R(t) = \mathcal{F}^{-1}\{ \mid H_b(j\omega) \mid^2 \}$$

is the time-domain representation of the power spectrum $|H_b(j\omega)|^2$ [1]. The autocorrelation function $R(t)$ has a peak at $t = 0$. Because of linearity and time invariance, the cross-correlation function is a shifted version of $R(t)$,

$$R_{12}(t) = R(t - (t_1 - t_2)) = R(t - \Delta t)$$

with a peak at

$$t = \Delta t = t_1 - t_2$$

Thus the location of the peak of the cross-correlation function indicates the relative time offset.

According to the scaling property of Fourier transformation, the resolving capability of the time-delay estimation by cross-correlation method is governed by the bandwidth of the power spectrum. Thus, the resolution limit is in the form

$$\delta = 1/B$$

where B is the bandwidth of the function $h_b(t)$. This means wider bandwidth produces more accurate time-delay estimates. That also implies coherent or narrow-band waveforms are not suitable to time-delay estimation operations.

THE LINEAR FMCW WAVEFORMS

In order to achieve high-resolution time-delay estimation, the bandwidth of the probing waveforms becomes an important parameter. To produce an adequate bandwidth within a defined time period, one example of probing waveforms is the linear *FMCW* signal,

$$h_b(t) = exp(-j\omega t) = exp(-j(2\pi f_o t + \pi B t^2/T))$$

with the waveform period *T*,

$$-T/2 \leq t \leq +T/2$$

The frequency f_o is the center frequency and *B* is the designed bandwidth [1]. It is a waveform of constant amplitude and quadratic phase variation

$$\varphi = -(2\pi f_o t + \pi B t^2/T)$$

The frequency variation of the waveform can be observed from the change of the phase

$$\frac{\partial \varphi}{\partial t} = -2\pi(f_o + Bt/T)$$

It now can be seen that the frequency of the waveform is a linear function of time, spread over the spectral band from f_{min} to f_{max}, with bandwidth *B.* The slope of the spectral ramp is thus *B/T*.

$$f_{min} = f_o - B/2$$

$$f_{max} = f_o + B/2$$

and

$$\Delta f = f_{max} - f_{min} = B$$

The quadratic phase term of the waveform allows us to produce a bandwidth *B* within a finite duration *T*. If this waveform is applied to time-delay estimation with the correlation method, the temporal resolution limit of the estimation is

$$\delta = 1/B$$

which translates into the space-domain resolution for range estimation

$$\Delta r = v/B$$

where *v* is the propagation velocity. In bearing-angle estimation, the angle of arrival θ is in the form

$$sin(\theta) = \Delta r/D = v/BD$$

where *D* is the separation of the receiver pair.

FOURIER TRANSFORM TECHNIQUE

When the linear *FMCW* waveforms are applied to time-delay estimation, the signal processing procedures will not be limited to the correlation method. Consider the linear *FMCW* waveform,

$$h_b(t) = exp(-j\omega t) = exp(-j(2\pi f_o t + \pi B t^2/T))$$

the delayed version can be written as

$$h_b(t - \Delta t) = exp(-j\omega(t - \Delta t))$$

$$= exp(-j(2\pi f_o(t - \Delta t) + \pi B(t - \Delta t)^2/T))$$

The time offset Δt is due the propagation delay, which is the main objective parameter in range estimation. If we mix the returned signal with the transmitted waveform, the product becomes

$$h_b(t - \Delta t) \cdot h_b^*(t) = exp(j(2\pi f_o \Delta t) \cdot exp(-j\pi B(\Delta t)^2/T) \cdot exp(j2\pi B(\Delta t)t/T)$$

with complex amplitude *C*,

$$C = exp(j(2\pi f_o \Delta t)\ exp(-j\pi B(\Delta t)^2/T)$$

The sole time-varying component is in the form of a coherent waveform with one single temporal frequency

$$h_b(t - \Delta t) \cdot h_b^*(t) = C \cdot exp(j2\pi B(\Delta t)t/T)$$

The frequency of the coherent waveform is linearly related to the time-delay variable Δt,

$$\Omega_o = 2\pi B(\Delta t)/T$$

with the scaling factor *B/T*

$$f = (B/T) \cdot \Delta t$$

It means the Fourier spectrum gives one single peak at $(B/T) \cdot \Delta t$. Thus, alternatively, the time-delay estimation can be achieved with a Fourier transformation operation and the spectrum is a scaled version of the time-delay profile.

This approach provides an alternative technique to the time-delay estimation problems. It replaces the correlation method, which is a convolution procedure, with a combination of a multiplication and a Fourier-transform operation. And similarly, this technique has been applied to range estimation and bearing-angle estimation.

STEP-FREQUENCY FMCW TECHNIQUE

For digital processing, the linear *FMCW* modality can be modified into the discrete format. The discrete version of the continuous-time linear *FMCW* waveforms is to utilize a collection of coherent frequencies for the purpose of establishing a substantial bandwidth sequentially in order to produce estimates of adequate accuracy. Instead of one single wideband waveform, the alternative is a sequential coherent waveform in the form

$$h_b(t) = exp(-j\omega t) = exp(-j\omega_n t)$$

For simplicity, the sequence is organized with constant increment $\Delta\omega$,

$$\omega_n = \omega_o + n\Delta\omega$$

where $n = 0, 1, 2, .. N\text{-}1$. For each frequency step, the product produces one complex scalar,

$$h_b(t-\Delta t) \cdot h_b^*(t) = exp(j\omega_n \Delta t)$$

Thus, for N sequential frequencies, an N-point sequence is generated,

$$g(n) = exp(j\omega_n \Delta t) = exp(j(\omega_o + n\Delta\omega)\Delta t)$$

$$= exp(j\omega_o\Delta t) \cdot exp(jn\Delta\omega\Delta t)$$

$$= exp(j2\pi f_o\Delta t) \cdot exp(j2\pi n\Delta f\Delta t)$$

The N-point sequence has a complex amplitude $exp(j2\pi f_o\Delta t)$. The component $exp(j2\pi n\Delta f\Delta t)$ is a function of the index n. If we apply an N-point *FFT* to the sequence, the phase variation is multiplied to the *FFT* kernel for matching,

$$2\pi n\Delta f\Delta t = 2\pi nk/N$$

The match produces a peak at

$$k = k_o = (N\Delta f)\, \Delta t = B \cdot \Delta t$$

Thus the *FFT* spectrum is a scaled version of the time-delay profile with the scaling factor B,

$$\Delta t = k_o / B$$

Because the *FFT* spectrum index is integer, the resolution limit in time is $1/B$, which is consistent to the result of the linear *FMCW* modality [1]. This technique is commonly applied to time-delay estimation for the simplicity of waveform generation and transmission, as well as computation procedures due to the availability of *FFT*.

THE GREEN'S FUNCTION

Now we move the focus of the analysis to the space domain. The space-domain component of the wave propagation kernel is in the form

$$h_a(r) = A \exp(j2\pi r/\lambda)$$

This term is commonly known as the *Green's function* [1]. For simplicity, we denote the propagation distance in *2D* as

$$r = (x^2 + z^2)^{1/2}.$$

Suppose a finite-size linear aperture of length L is arranged along the x-axis,

$$-L/2 \leq x \leq L/2$$

The phase variation along the aperture is in the form,

$$\varphi = 2\pi r/\lambda = 2\pi(x^2 + z^2)^{1/2}/\lambda$$

Hence, the angular frequency of the kernel in the x-direction is [3],

$$\frac{\partial\varphi}{\partial x} = 2\pi\, (x/(x^2 + z^2)^{1/2}\lambda) = 2\pi \sin\theta /\lambda$$

The spatial frequency over the aperture is a function of the position

$$f_x = x/(x^2 + z^2)^{1/2}\lambda = \sin\theta /\lambda$$

For the finite-size aperture, the spatial frequency covers the spectral interval of (f_{min}, f_{max}), where

$$f_{min} = -(L/2)/((L/2)^2 + z^2)^{1/2}\lambda = \sin\theta_{min} /\lambda$$

$$f_{max} = +(L/2)/((L/2)^2 + z^2)^{1/2}\lambda = \sin\theta_{max} /\lambda$$

Thus, the wavefield pattern over the aperture provides a spatial-frequency bandwidth [4],

$$B_x = L/((L/2)^2 + z^2)^{1/2}\lambda = (\sin\theta_{max} - \sin\theta_{min})/\lambda$$

Thus, if we apply the correlation method for the estimation process, for the available spatial-frequency bandwidth, the resolution limit in the x-direction is

$$\delta_x = \lambda((L/2)^2 + z^2)^{1/2})/L = \lambda/2\sin(\Delta\theta)$$

where

$$\sin(\Delta\theta) = (L/2)/((L/2)^2 + z^2)^{1/2})$$

This result is exactly the same as that formulated in classical Fourier optics, which is widely known as the *Rayleigh resolution limit* [2].

If the far-field approximation is applied, the spatial frequency at a position over the aperture can be approximated in the form

$$f_x = \sin\theta / \lambda \approx x/z\lambda$$

The spatial-frequency bandwidth provided by the aperture becomes a linear function of the aperture size

$$B_x = L/z\lambda$$

The resolution limit of the estimation procedure in the x-direction is thus

$$\delta_x = z\lambda/L$$

One interesting feature is the equivalence and correspondences between the time-domain and space-domain analysis, which is documented in the table.

Time-domain analysis	*Space-domain analysis*
Variable: time (t)	*Variable: space (r)*
$h_b(t) = \exp(-j(2\pi f_o t + \pi B t^2/T))$	$h_a(r) = A \exp(j2\pi r/\lambda)$
observation period (T)	*aperture size (L)*
slope $= B/T$	*slope* $= 1/z\lambda$
bandwidth B	*bandwidth* $L/z\lambda$
resolution $\delta = 1/B$	*resolution* $\delta_x = z\lambda/L$

Table (1): Equivalence and Correspondences of Parameters

DISPLACEMENT ESTIMATION

The resolving capability of an imaging system is governed by the capability of displacement estimation in the space domain. For a small displacement in both the *x*- (cross-range) direction and the *z*- (range) direction, the offset distance becomes

$$r' = ((x - \Delta x)^2 + (z - \Delta z)^2)^{1/2}$$

The product term can then be written in a simple form

$$h_a(r') \, h_a^*(r) = |A|^2 \, exp(j2\pi(r'-r)/\lambda)$$

When the range distance z is large, the offset between the range distances can be approximated by the low-order terms

$$r' - r = ((x - \Delta x)^2 + (z - \Delta z)^2)^{1/2} - (x^2 + z^2)^{1/2} \approx -\Delta z - \Delta x \cdot x/z$$

Accordingly, the product term becomes

$$h(r') \, h^*(r) = |A|^2 \, exp(-j2\pi\Delta z/\lambda) \cdot exp(-j2\pi\Delta x \cdot x/\lambda z)$$

with the amplitude term $|A|^2 exp(-j2\pi\Delta z/\lambda)$ and phase offset $exp(-j2\pi\Delta x \cdot x/\lambda z)$ [5.6]. The displacement in the range direction introduces a constant phase term $exp(-j2\pi\Delta z/\lambda)$. The spatial frequency of this waveform is

$$\omega_o = 2\pi\Delta x/\lambda z$$

and the corresponding spatial-frequency is thus linearly related to the spatial displacement with a scaling factor λz,

$$f_x = \Delta x/\lambda z$$

This means the displacement in the x-direction can be estimated by a Fourier transform operation followed by a scaling process by the factor of λz.

This indicates the time-domain linear *FMCW* waveforms are modified versions of the space-domain Green's function, with similar characteristics and properties. For this reason, the Fourier transform method is applicable in far-field analysis, which is commonly known as Fresnel and Frounhofer approximation. That also implies displacement estimation in space is equivalent to time-delay estimation in the time domain.

When the linear *FMCW* waveforms are employed as probing signals in far-field imaging applications, image formation in both range and cross-range direction can be approximated by Fourier transformation. This explains the utilization of multi-dimensional Fourier transformation has been commonly applied to synthetic-aperture radar and sonar imaging.

It is important to note that if the system is designed to achieve identical resolving capability in both range and cross-range direction, it leads to the relationship

$$v/2B = z\lambda_o/L$$

where λ_o is the wavelength corresponding to the center frequency f_o. The extra factor of two is due to round-trip propagation. If we simplify the equation with the relationship $v = f_o \lambda_o$, the relationship becomes,

$$f_o \lambda_o/2B = z\lambda_o/L$$

It translates into the relationship

$$2B/f_o = L/z$$

This suggests the bandwidth B in probing waveforms is equivalent to the aperture size L of an array system, and the center frequency f_o is equivalent to the range distance to the target region. It can be seen the resolution is governed by the bandwidth B in the range direction and aperture size L in the cross-range direction.

DESIGN OF PROBING WAVEFORMS

In the previous sections, the analysis shows the common component of the system performance in both the range and cross-range directions of sensing and imaging applications is the probing waveforms. It suggests the selection of probing waveforms is the crucial to the design and performance of the systems. So, in this section, the steps for the design of probing waveforms are compiled to formulate a procedure in a robust and systematic manner.

Channel estimation and selection: Prior to the sensing and imaging procedures, one key task is channel estimation. The task is conducted through passive detection, which is to estimate the spectral activities near the transceivers. The spectral profile of the undesired signals can then be constructed and the magnitude of noise floor can also be estimated. The segments of frequency band with noise floor lower than the designated threshold will be selected as candidates of the sensing or imaging exercises.

Utilization of the bandwidth: If the linear FMCW waveforms are employed as the probing signal, the center frequency f_o and bandwidth B of the waveforms will be determined accordingly. Then the waveform period T can be selected subsequently. The concept can be extended to the step-frequency FMCW modality, by selecting N operating coherent frequencies within the available band with equal spacing $\Delta\omega$.

We can also generate a new class of probing waveforms by converting the space-domain Green's function to the time domain in the form

$$h(t) = A\, exp(-j2\pi(f_o t + (t^2 + \alpha^2)^{1/2}))$$

This is to replace the quadratic phase term $(\pi B t^2/T)$ by $(2\pi(t^2+\alpha^2)^{1/2})$, where

$$\alpha = T/B$$

This new class of waveforms provides the same level of performance and accuracy, without exhibiting the predictable characteristics of the conventional FMCW waveforms.

The amplitude and weighting: For the optimal performance of the probing waveforms, a frequency-dependent weighting is necessary. There are three important factors involved in the formulation of the amplitude weighting. The first is a linear weighting term. This is to compensate the larger propagation loss for higher frequencies. The second weighting term is determined based on the noise floor through the channel estimation. Higher weighting is given to the frequency components with higher noise floor, in order to maintain the same level of signal-to-noise ratio *(S/N).* In combination, a weighting profile can be formulated.

For both the traditional or modified FMCW signal format, the frequency distribution within the waveform structure is governed by the time variable, in the form

$$f = f_o + Bt/T$$

$$f = f_o + t/(t^2 + \alpha^2)^{1/2}$$

respectively, where α is a real and positive scalar. The frequency-dependent weighting can then formulated in the form of a time-domain amplitude component in the form of *A(t).* Since the frequency weighting and *S/N* normalization are real and positive, the amplitude term *A(t)* is a real and positive function. Thus, we can generalize the formula of the probing waveforms with a time-varying amplitude term,

$$h(t) = A(t) \cdot exp(-j(2\pi f_o t + \pi Bt^2/T))$$

and

$$h(t) = A(t) \cdot exp(-j(2\pi(f_o t + (t^2 + \alpha^2)^{1/2})))$$

The implementation of frequency weighting for the step-frequency FMCW modality is simpler. Because the probing signal of the modality is a sequence of coherent waveforms. Thus the amplitude weighting can be individually in the form

$$h(t) = A(n) \cdot exp(-j\omega_n t)$$

Hilbert transform pairs: In practice, the probing waveforms are in the form of real signals. Thus, the complex probing waveforms can be decomposed into the real and imaginary components,

$$h_I(t) = A(t) \cdot cos(2\pi f_o\, t + \pi Bt^2/T)$$

$$h_Q(t) = A(t) \cdot sin(2\pi f_o\, t + \pi Bt^2/T)$$

or for the modified case,

$$h_I(t) = A(t) \cdot cos(2\pi(f_o t + (t^2 + \alpha^2)^{1/2}))$$

$$h_Q(t) = A(t) \cdot sin(2\pi(f_o t + (t^2 + \alpha^2)^{1/2}))$$

The Hilbert transform pairs are real functions, in the same frequency band, with the same power spectra, and mutually orthogonal.

CONCLUSION

In the field of sensing and imaging, the estimation procedures are commonly formulated in time and space domain. The time-domain tasks are typically for time-delay estimation, responsible for the reconstruction in the range direction. And the space-domain analysis is in general for the image formation in the cross-range direction. For the active systems, the probing waveforms play the key role in the resolving capability in both range and cross-range direction. Thus, the objective of this study is to develop a robust and systematic process for the formulation of the probing waveforms for software-defined sensing and imaging systems.

First, the mathematical structure is partitioned into two components, corresponding to the time and space domains respectively. The time-domain component is used to analyze the time-delay estimation, as well as the related applications such as range estimation and bearing-angle estimation. And the space-domain component governs the displacement estimation and image reconstruction.

The paper provided the structure of a unified framework for the performance evaluation of sensing and imaging systems and examined the equivalence and correspondences of the two components. This enabled us to formulate an organized procedure for the design of the probing waveforms for optimal performance and accuracy of the sensing and imaging operations. It is especially important for software-defined systems where the probing waveforms are modified and updated dynamically according to the operational environment.

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