HOLOGRAPHIC OPTICAL ELEMENTS AND THEIR APPLICATIONS IN WAVEGUIDE DISPLAY TECHNOLOGIES

by

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Dedication

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Abstract

The work documented in this dissertation presents a number of research efforts that utilize holographic techniques to bring next-generation technologies closer to present-day implementation.

A head-up display system that diffracts incident light from a source beyond the critical angle of a waveguide through the use of holographic optical elements is presented. The waveguide design presented can achieve both pupil expansion and image magnification with an increased field of view.

An approach to achieving two-dimensional pupil expansion and field of view increase is then presented. It uses a surface relief grating to achieve in-line vertical field of view expansion while utilizing the waveguide propagation techniques described in Chapter 2 for horizontal field of view and pupil expansion.

Finally, a system is presented for increasing the directed angle of light from non-mechanical beam-steering devices. Two successive holograms are paired together to diffract light from a small range of incident angles (1°) to cover a theoretical maximum of 4π sr. A time of flight experiment demonstrates the feasibility of using the system in remote sensing applications.
Chapter 1

Introduction

Holography traces its origins to the work of Dr. Dennis Gabor, who sought to develop a technique to increase the resolution of electron microscopes at the end of the 1940’s. At the time, scientists using electron microscopy were faced with a theoretical resolution limit of 4 Å and a practical limit of 12 Å, both of which leave the ability to resolve the atomic lattice frustratingly out of reach. This resolution limit was due to the trade-off between diffraction error and spherical aberration in managing the size of the electron aperture [4]. Dr. Gabor reasoned that it might be possible to record a “bad” picture of an atomic structure that still contained all the relevant information and correct it optically. He called this picture a “hologram”, as it contained the “whole” information of an object. In his first paper on holography [5], Dr. Gabor showed that recording the interference pattern between two wavefronts (object and reference) will create a “mask” which, when illuminated by one of the two wavefronts (reconstruction) used in its creation, recreates the other wavefront (image). Figure 1.1 shows the recording and reading geometries discussed above.
Since the discovery of holography, it has become a field of research all its own, with such notable contributors as Yuri Denisyuk [6], Emmett Leith and Juris Upatnieks [7], and Steve Benton [8]. Denisyuk created an in-line reflection recording configuration where the reference wavefront passes through the holographic recording medium before it is reflected from the object back toward the holographic plate as the object wavefront. Leith and Upatnieks utilized the long coherence length and narrow spectral bandwidth of lasers to create an off-axis recording configuration. Benton created rainbow holograms, which, when viewed under a white-light source, changed color according to the viewing angle.

Because of its unique ability to reconstruct entire wavefronts, holography has many applications. These uses range from the straightforward, where holograms act as a display medium [9] or an optical element [10] to the more unexpected,
where they are used in anti-counterfeiting applications for currency and credit cards [11], nondestructive testing of composites [12], medical imaging [13], and even data storage [14].

This dissertation will present research efforts undertaken at the University of Arizona’s James C. Wyant College of Optical Sciences that utilize holographic technology and techniques to improve upon the usefulness of a variety of next-generation technologies for consumer use.

The term head-up display (HUD) describes a technology where relevant vehicle operation information, such as speed, heading, and altitude, are displayed within the operator’s view while they are looking outside of the vehicle. This method of displaying information offer users many advantages over traditional head-down display (HDD) systems, both in automotive and avionic applications. Despite the fact that HUD offers improved user safety and comfort [15], it has yet to be widely implemented in consumer automobiles. This delay is primarily due to the fact that, for the current HUD display technique, the size of the displayed image is directly related to the size of the projection optics [16–18]. Thus, to increase the size of the HUD image, the size of the projector must be increased which, in automotive and aeronautic applications, quickly becomes untenable due to available space constraints.

Chapter 2 will present a method of using holographic optical elements (HOEs) affixed to a waveguide surface to couple light from a small projector into the waveguide and extract it over an expanded eyebox (also called an exit pupil) while magnifying the image for a larger field of view (FOV). First, a method of one-dimensional (1D) pupil expansion and image magnification using holograms affixed to a planar waveguide is presented. This work demonstrates an improvement over other waveguide-coupled HUDs in that the in-coupling element, which is here referred to
as the injection hologram, is encoded with optical power to achieve longitudinal image magnification.

Building upon the success of the 1D pupil expansion effort, Chapter 3 presents a method to achieve two-dimensional (2D) pupil expansion using an in-line geometry. In this design, an in-line surface relief grating (SRG) is affixed along the beam path to achieve vertical FOV expansion in addition to the horizontal expansion demonstrated in Chapter 2. This configuration offers an alternative approach to 2D pupil expansion that utilizes small form factor and works for wide spectral-bandwidth systems.

HUD is not the only next-generation technology promising an improved consumer experience that can benefit from holographic techniques. Light detection and ranging (LIDAR) offers the capability to capture a depth map of a scene which can be used to construct an accurate computer model of the surrounding area [19]. For example, having such a tool is a prerequisite of developing safe and reliable self-driving cars. While there are many different beam-steering technologies, each with their own advantages and disadvantages, Chapter 4 presents a method to passively increase the steering angle from a diffractive, non-mechanical device such as a spatial light modulator (SLM) to cover a theoretical maximum of $4\pi$ sr.

Chapter 5 will serve as an appendix to further explore how system parameters affect the FOV of a waveguide HUD system and the technique for recording and illuminating holograms with different wavelengths.

The work presented in this dissertation has been published or submitted for publication in three separate manuscripts: “Holographic Waveguide Head-Up Display for Longitudinal Image Magnification and Pupil Expansion” [1], “Holographic Head-Up Display with In-line Pupil and 2D Field of View Expansion” [2], and
“Holographic Amplification of the Diffraction Angle from Optical Phase Array for Optical Beam Steering” [3]. Other author publications include “Fast and continuous recording of refreshable holographic stereograms” [20] and “Holographic waveguide head-up display with 2-D pupil expansion and longitudinal image magnification” [21]. Conference proceedings include “Holography for automotive applications, from HUD to lidar” [22] and “Improving head-up display with waveguide and holographic optical elements” [23].
Chapter 2

Holographic Waveguide Head-Up Display for Longitudinal Image Magnification and Pupil Expansion

The first project presented in this dissertation is a method of using holograms applied to a planar waveguide surface to achieve longitudinal image magnification and 1D eyebox expansion for an HUD system.

2.1 Background

For applications requiring out-of-window viewing, HUDs provide major advantages over traditional HDDs, including shorter accommodation time, increased eyes-forward time, improved situational awareness, faster reaction time, and ease of use [15, 24]. By overlaying relevant diagnostic and operational data on the real-world scene outside the plane or car, pilots and drivers are saved from having to redirect their gaze from the exterior of their vehicle to see the information on the HDDs [25–27]. HUDs designed for aviation locate their projected flight symbols in the far-field (optical infinity) [28, 29], which means that the observer can simultaneously observe their environment and the information presented by the HUD. HUD
systems in aircraft or other vehicles frequently use a projection system similar to that shown in Figure 2.1 where the source is imaged by a collection of projection optics onto the combiner and redirected from there to the pilot’s eyes [30–35].

![Figure 2.1: Conventional HUD system schematic where the projected image is propagated to the combiner. From there it is reflected into the user’s eyes. The viewer sees the image located in the far-field in front of them. [1]](image)

One of the primary limitations of the system shown in Figure 2.1 is that the maximum extent of the projected image is determined by the projection optics encompassed in the “Packaged Volume” [16–18]. To increase the size of the perceived image, the size of these optical elements must be increased. Because most vehicles have a limitation on the space they can allocate to housing an HUD system, increasing the size of the projection optics quickly becomes an unreasonable proposition. Additionally, because of the projection mechanism used in this configuration, pilots have a very small eyebox from which they can observe the entire HUD image. The
“eyebox” refers to the area where an observer can situate their head and see the entire projected image. When the observer moves their eyes outside of the eyebox, the image starts to be clipped by the edge of the aperture. Ultimately, larger head movements cause the image to disappear completely.

To overcome these limitations, recent research has proposed the use of diffractive optics in combination with waveguides to increase the eyebox of HUD systems [36–40]. By using holograms to couple the light into a waveguide, the image is propagated within the waveguide and extracted multiple times, allowing for an increased eyebox. Figure 2.2 shows one example of this configuration, where several holograms and waveguides are used in conjunction to achieve 2D (both lateral and vertical) pupil expansion.

Figure 2.2: Propagation through waveguides and interaction with successive holograms allows for two-dimensional pupil expansion. [1]

The holographic elements shown in Figure 2.2 function as partially reflective elements. By modulating the diffraction efficiency (DE), it is possible to achieve
uniform intensity of the light observed across an expanded pupil. Additionally, recording holograms with optical focusing power allows for image magnification. Both of these techniques are used in this chapter to achieve concurrent longitudinal image magnification and pupil expansion in an HUD system.

2.2 Materials and Methods

This research worked to design a waveguide HUD for implementation in civilian aircraft, however the principle can be applied in a wide variety of applications, including automobiles. The system presented here is comprised of an injection and extraction hologram pair attached to a planar waveguide surface. The injection hologram diffracts incident light from a source plane inside of the waveguide beyond the waveguide’s critical angle. This diffraction angle causes the beam to reflect within the waveguide surface, due to total internal reflection (TIR). In this design effort, the waveguide and photopolymer have refractive indexes of 1.526 and 1.485, respectively. The critical angle of the light is thus 40.9°. After propagating within the waveguide, the beam reaches the extraction hologram, where it is diffracted out of the waveguide and toward the observer. Figure 2.3 shows the beam path within the waveguide. Light from a source point is incident on the hologram surface and is diffracted within the glass at TIR. This beam propagates until the extraction hologram redirects it out of the waveguide. The modulated DE of the extraction means that some light is not diffracted toward the observer by the first interaction with the extraction hologram. The light remaining in the waveguide will, instead, continue to propagate within the waveguide to interact with the extraction hologram further along the waveguide length. This work differed from other holographic
waveguide HUD approaches [41–43] because the injection hologram is recorded with optical power, a process that achieves longitudinal magnification of the injected image so that it is located in the observer’s far field.

![Schematic of the waveguide configuration](image)

Figure 2.3: Schematic of the waveguide configuration. An injection hologram diffracts light from the source at TIR along the length of the waveguide. The extraction hologram diffracts the internally propagating light toward the observer. Due to the extraction hologram’s modulated DE, some light is not diffracted by the extraction hologram during the initial interaction. This light continues propagating within the waveguide, before it is again incident on the HOE. Having multiple interactions between the internally propagating beam and the extraction hologram extends the system’s eyebox.

2.2.1 Computer Modeling

The waveguide HUD system was modeled in the ray-tracing software Zemax OpticStudio. The model allows for alteration and optimization of the system parameters while analyzing the expected output. The model has an injection hologram written from the interference between the light from a point source and a collimated beam propagating within the waveguide at TIR. Following the injection, the
light beams propagate within the waveguide until they are extracted by the second hologram.

Figure 2.4: Isometric view of the Zemax model for the HUD system. The light comes from the same location as the source point used to record the hologram (789 mm away). Light from the design source point is collimated and diffracted along the length of the waveguide before the extraction hologram diffracts the internally propagating beam perpendicular to the waveguide surface.

Figure 2.4 shows the system with an initial source located at the point used to record the injection hologram. The injection hologram is 88.9 mm × 88.9 mm, while the extraction is 152.4 mm × 88.9 mm. The system was constructed with a design wavelength of 532 nm (350 nm = $\frac{532 \text{ nm}}{n}$ used in Hologram Lens function recording parameters due to the wavelength reduction from the refractive index of the BK-7 glass ($n_{BK-7}(532 \text{ nm}) = 1.5195$)). Similarly, the 789 mm optical path length (OPL) in air from the source point to the injection hologram corresponds to an OPL of 1200 mm inside of the glass (789 mm / $n_{air} = 1200 \text{ mm} / n_{BK-7}$). Thus, a Hologram Lens function that is recorded to diffract 350 nm light coming from a source located 1200 mm away in air to 67° from the surface normal will diffract 532 nm light from
a source located 789mm away in BK-7 glass at the same angle. Similarly, the extraction hologram was recorded to diffract 532nm light incident at 67° in BK-7 to the surface normal, because the Hologram Lens function’s parameters expect 350nm light incident from the air.

To display a full image, light from an extended source is diffracted through the system. Figure 2.5 shows how light from an extended source is diffracted into the system, creating the full FOV of the image. Note that there exists a region on the source plane from which incident light will be diffracted beyond the critical angle, and outside that region, light is diffracted such that it will couple out of the waveguide. If the source object is located within a spatial region that will diffract incident light beyond the critical angle, TIR is achieved, and the light from these points is propagated within the waveguide. In Figure 2.5, light from the red source area will not diffract beyond the critical angle, but the light from the green region will couple into the waveguide by TIR and contribute to the FOV of the extracted image.
Figure 2.5: The injection hologram diffracts light at different angles depending on the incident angle. Because of this, some light will not be diffracted beyond the critical angle and will not be trapped within the waveguide. This light does not contribute to the FOV visible from the extraction.

Figure 2.6 shows how the light from an extended source propagates through the optical system and is diffracted by the extraction hologram out of the waveguide and toward the viewer’s eyes. This ray tracing shows the actual angular radiance on a human pupil-sized detector located 25cm behind the extraction hologram. The initial object occupies a 110mm square on the source plane, and all light is 532nm.
In the simulation presented in Figure 2.6, the extraction hologram is 1.7× the size of the injection hologram, which corresponds to two distinct extraction segments. Each extraction segment will diffract light toward the observer, but the light does not create a replicated image, which is a common misinterpretation of the system’s method of operation. Instead, it is the eyebox which is increased. The calculated FOV of the simulated systems is $12^\circ \times 8^\circ$ and the eyebox is $90mm \times 150mm$ with a depth over $80cm$.

### 2.2.2 Injection Hologram

The injection hologram is designed to receive the light from a source point located at a finite distance from the waveguide surface. The hologram collimates the beam and redirects the light within the waveguide at an angle which is chosen such that, after one reflection, the left edge of the beam coincides with the right edge...
of the beam at the injection surface, as presented in Figure 2.7. If this condition is not met, the beam will either overlap with itself while propagating within the waveguide or leave dark spaces between the different extractions.

Figure 2.7: Light from a source point is collimated and redirected within the waveguide at an angle beyond the critical angle, such that successive reflections are directly adjacent to each other. The width of the injection hologram (w) and the distance from the source point to the injection hologram (d) are marked on the figure.

One significant difference between the system shown in Figures 2.3 and 2.7 is the location of the source point for the injection. In Figure 2.3, the incident beam is collimated, which corresponds to a source point located in the far-field. Figure 2.7, by contrast, shows a source point located a finite distance from the injection hologram. Encoding the injection hologram with optical power allows the source to be located at a finite distance from the waveguide while it appears to be located at optical infinity. Because the image is visible in the far-field, this system produces an infinite longitudinal image magnification. Conversely, for a system whose injection hologram is recorded from the interference pattern between two collimated beams, the projection optics need to be set up to locate the source in the far-field. These projection optics require additional lenses in the projection setup, which increases
the footprint, weight, cost, and manufacturing complexity of the system.

One other advantage of using a hologram recorded to diffract light from a short image distance compared to an injection recorded using collimated light is an increased FOV. A FOV of $2^\circ \times 2^\circ$ was demonstrated with a collimated system. In contrast, the system in Figure 2.7 has a FOV of $12^\circ \times 8^\circ$.

### 2.2.3 Extraction Hologram

After propagating through the waveguide, the light is diffracted out of the TIR condition and directed toward the viewer by the extraction hologram, as presented in Figure 2.8.

![Figure 2.8: Collimated light propagating within the waveguide is extracted at the surface normal, presenting a far-field image to the viewer. [1] (Image)](image)

To achieve pupil expansion and ensure uniform intensity along the length of the combiner, the extraction hologram needs to have a modulated DE along its length and must be larger than the size of the injection hologram. This modulated DE means that some of the light is recirculated within the waveguide because it does not interact with the extraction hologram.
For observer comfort the image should maintain uniform intensity across the entire FOV. However, by extracting some light from the waveguide, there is less light remaining for later viewing positions. Thus, each successive portion of the extraction must be more efficient, as less light remains inside the waveguide to be extracted. Thus, the furthest segment of the extraction from the injection hologram must have the maximum DE, as any light remaining in the waveguide after that point is not presented to the observer and is effectively wasted. Similarly, the section of the hologram preceding the last extraction must demonstrate 50% DE, such that, of the light in the waveguide at that portion of the extraction, 50% is extracted and redirected to the viewer, while the remaining 50% propagates to the final portion of the extraction and is fully extracted. Thus, DE of each segment ($\eta_i$) follows

$$\eta_i = \frac{1}{N_{tot} - N_i + 1}$$

with the specific segment number ($N_i$), and the total number of extraction sections ($N_{tot}$).

The limitations on the factor of pupil expansion in this system are determined by the desired viewing intensity and the brightness of the source object. Hypothetically, this method could be used to infinitely expand the eyebox, but the observed intensity would decrease for each additional extraction segment added. Thus, a system with 10× pupil expansion will have, at most, 10% of the incident light visible from each extraction. Figure 2.9 shows how, for an ideal system with five extractions, the intensity of the light contained within the waveguide decreases, while the required DE increases according to Equation 2.1 to maintain uniform intensity along the entire extraction.
Figure 2.9: Ideal DE of the extraction hologram, light intensity inside the waveguide, and display intensity according to location on the combiner. The modulation of the DE compensates for decreasing intensity and maintains uniform image intensity along the entire extraction.

2.3 Results

After optimizing the parameters for the system using the ray-tracing software, a physical demonstrator was built following the same design. The waveguide is made of Diamant Glass from Saint Gobain, with dimensions of $216\,mm \times 280\,mm \times 12\,mm$. This particular glass type was chosen for its low absorption around the $532\,nm$ wavelength which can be seen by the low green glare on the edges of the waveguide. The injection and extraction holograms were recorded using Bayfol HX104 photopolymer, an ultra-violet (UV) curing photopolymer that is easy to apply and has maximum sensitivity around $532\,nm$ recording wavelength.
2.3.1 Diffraction Efficiency

As discussed in Section 2.2.3, the DE of the holograms used for the injection and extraction recordings needs to be controlled to maintain uniform intensity over the entirety of the extraction. To that end, the manufacturer’s suggested dosage of 5 \text{mJ/cm}^2 was used to record the holograms. This dosage reliably created gratings with a DE of \approx 85\%, which served as the maximum DE from which all other DEs were scaled. To decrease the diffraction efficiency of the hologram, a method of pre-exposure was used, where the photopolymer is exposed to a single beam of light before the hologram is recorded. This pre-exposure starts the polymerization of the photopolymer before the interference between two beams is recorded and leads to a lower DE of the hologram. Figure 2.10 shows how the DE of the holograms responded to the pre-exposure energy used. Relative DE refers to the DE compared to a sample recorded without pre-exposure. The relative pre-exposure diffraction efficiency of the photopolymer was measured. The best fit line is given by \frac{2.6}{E} - 0.26, where \( E \) is the pre-exposure energy, which was used to determine the necessary pre-exposure energy for the system. Thus, a 4 mJ pre-exposure was chosen for the extraction setup.
2.3.2 Recording

The recording geometry of the injection hologram follows the incidence angles prescribed by the analysis presented in Section 2.2.2. The light from the point source is simultaneously collimated and redirected at TIR within the waveguide by the the hologram. The hologram is recorded from the interference of two mutually coherent beams from a frequency-doubled YAG laser operating at 532\textit{nm}. The reference beam is coming from a diverging point source at normal incidence to the waveguide. The distance of the point source from the injection hologram was chosen so that the diverging beam will fill the injection hologram while keeping the source point as close to the hologram surface as possible. The object beam is collimated and
incident on the waveguide to achieve the TIR angle.

Since the TIR condition implies that the external angle should be $90^\circ$, a prism coupler was used to achieve the correct internal angle. However, this prism gets in the way of the reference beam, so a pair of $45^\circ$ prisms were used in the configuration presented in Figure 2.11 to ensure the overlap of the reference and object beams.

![Figure 2.11: Injection recording setup. In order to make this recording geometry possible, a combination of $45^\circ$ prisms was used to couple the collimated object beam into the waveguide beyond the critical angle while the diverging reference beam is normally incident on the waveguide surface. [1]](image)

Once the injection hologram was recorded, the hologram was illuminated with a point source and the resulting collimated, internally-propagating beam served as the reference beam for the extraction hologram. The object beam was a diverging beam whose source point was offset from the center of the hologram. Figure 2.12
shows the extraction hologram recording geometry.

Figure 2.12: The extraction hologram is recorded from a normally incident object beam and an internally propagating reference beam. The extraction hologram needs to be recorded with modulated DE to maintain uniform intensity across the entire FOV. [1]

As discussed in Section 2.2.3, the DE of the extraction hologram must be modulated according to the number of sections in the extraction. In the case of the demonstrator presented here, the extraction has two segments, so the first part of the hologram has 50% DE, while the second part has 100% DE. To achieve this different DE, the first portion of the injection hologram, which needed lower DE, was pre-exposed. In this system, a $1.7 \times$ pupil expansion was demonstrated by having two segments of the extraction. The first segment was $0.7 \times$ the size of the injection hologram and the second was $1 \times$ the injection hologram.

### 2.3.3 Testing

In order to test the HUD setup, the projection system shown in Figure 2.13 was constructed. The light from a projector is focused onto the diffuse plate by a collection lens while the diffuse plate is located at the source plane for the injection
hologram. The injection hologram diffracts the image through the waveguide where it is diffracted by the extraction hologram and presented to an external observer. It bears noting that a collection lens and a diffuse plate were used only to be able to precisely locate the image plane of the system, which is not possible with the projector by itself. In the case of an application specific system, which would be built from the ground up, the projector can be properly designed to take this image plane location directly into account.

Figure 2.13: Picture of the HUD setup, with the waveguide where the injection and extraction holograms are mounted in the foreground. The projection system can be seen in the background. [1]
Figure 2.14: Picture taken in front of the extraction hologram portion of the HUD. The green flight information is projected through the waveguide system. The hills and sky are a far-field background image displayed on a television set. [1]

Figure 2.14 is a picture of the image visible through the HUD system. The green flight information is projected through the waveguide. The hills and sky are a background image displayed on a television located several yards away. The entire image is visible, despite the fact that the background is located $\approx 10m$ behind the hologram plane. This demonstrates that the projected information is also located in the far-field and the viewer does not need to re-accommodate his eyes to look at it. There are two sources for the lack of sharpness in the projected image. First is the quality of the holographic backing medium which is a polymer film that induces aberration into the projected image. Using higher quality holographic recording medium, such as dichromated gelatin (DCG), will reduce the image aberration from the hologram material. The second source of aberration is the difference in the locations of the background image and the projected flight symbology. This HUD system projects the symbology into the far-field, but the background image is only located $\approx 10$ m behind the hologram. Figure 2.14 is focused on the background image, as opposed to the hologram symbology. This defocus between the image
and background can be addressed by increasing the optical power of the projection system and extending the distance between the hologram plane and the background image.

Additionally, aberrations in the diffracted image from the holographic gratings can be caused by mismatch between the reference and reading beams. A general equation for the position of the image point generated by a source point can be found by manipulating Equation 2.2 [44]

\[
\frac{1}{\lambda_{\text{recon}}} \left[ \sin \left( \tan^{-1} \frac{h}{z_{\text{img}}} \right) - \sin \left( \tan^{-1} \frac{h}{z_{\text{recon}}} \right) \right] = \\
\frac{1}{\lambda_{\text{write}}} \left[ \sin \left( \tan^{-1} \frac{h}{z_{\text{obj}}} \right) - \sin \left( \tan^{-1} \frac{h}{z_{\text{ref}}} \right) \right]
\]

(2.2)

where \( h \) is the height away from the optical axis, \( \lambda_{\text{recon}} \) and \( \lambda_{\text{write}} \) are the reconstruction and writing wavelengths, respectively, and \( z_{\text{img}}, z_{\text{recon}}, z_{\text{obj}}, \) and \( z_{\text{ref}} \) are the distances of the image, reconstruction, object, and reference source points from the hologram surface. Equation 2.2 shows that, even if the reference and reading source points are the same distance from the hologram, a wavelength shift can cause the image to appear at a different location than the object was located. Similarly, if the reconstruction source point has some physical extent, there will be blurring in the diffracted image. Finally, the Bayfol HX104 material experiences approximately 5% shrinkage after development, which changes the spacing and the slant angle of the grating recorded in the photopolymer. This change will cause Bragg mismatch along the length of the injection hologram, as the grating spacing and slant angle changes as a function of position within the hologram. These changing parameters will cause incident light to be diffracted slightly away from the design angle, introducing blur into the system.
2.4 Conclusion

In this chapter, the possibility to achieve both pupil expansion and longitudinal image magnification simultaneously, using holograms and waveguide optics, was presented. The magnification is provided by the optical power implemented in the injection hologram, whereas the pupil expansion is due to the multiple extractions of the light out of the waveguide by the extraction hologram. A ray tracing model was developed in Zemax to optimize the geometry of the holograms with respect to the size and location of the image source. The proof of concept system presented has an infinite longitudinal magnification and a pupil expansion of $1.7\times$. This FOV can be expanded by increasing the size of the injection and extraction hologram elements. Also, increasing the divergence of the source point beam used for the injection recording can minimize the form factor of the projection system. Appropriate design of the projection system that would eliminate the need for a separate intermediate image plane can also reduce the size of the projection system.

This work was published in OSA’s Applied Optics Journal [1].
Chapter 3

Holographic Head-Up Display with In-line Pupil and 2D Field of View Expansion

The research presented here builds upon the work from Chapter 2 to achieve 2D pupil expansion of an HUD using an in-line waveguide geometry.

3.1 Background

HUD is a technology that shows instrument readings for operation of a vehicle in such a way that the readings can be seen without the operator taking their focus away from the surrounding environment. The information is typically projected onto a windscreen or visor, where it is overlaid onto the vehicle’s surroundings. This means that a vehicle using an HUD system can display relevant safety, status, and control information so that this data is visible as the user looks at the vehicle’s operating environment. Displaying information in this way offers a number of advantages over traditional HDDs, including shorter eye accommodation times and increased eyes-forward time, both of which translate to enhanced situational awareness and faster reaction time by users [15, 24–27, 45–48].

Figure 3.1 shows the traditional HUD layout, where an image is projected
through a collection of optics to a transparent, partially-reflective combiner. From the combiner, the image is reflected back to the vehicle operator, where it appears that the image is located at optical infinity and overlaid on the external scene [28–35]. In this way, relevant control information is presented to the vehicle’s operator at the same viewing distance and in the same space as the vehicle’s surroundings.

Figure 3.1: In a traditional HUD, light from the source is encoded with the desired image before it passes through a collection of relay optics that cause the image to be located at optical infinity. The light is then projected onto a partially reflective, transparent combiner so that the image is reflected to the observer while still allowing them to see outside the vehicle. An orthogonal view of the imaging device, the image plane, and the perceived image are shown to demonstrate how an image propagates through the system. [2]

Despite the advantages that HUD offers to vehicle operators, it has yet to be widely implemented in automotive applications. This is due, in part, to the limita-
tions of the traditional projection system, chief among which is the small FOV of the projected image [16–18]. In the traditional projection geometry, the size of the relay optics needs to be increased to project a larger image. Unfortunately, increasing the size of the relay optics, and thus the packaged volume, quickly becomes unfeasible for many vehicular applications, where space is at a premium. Additionally, the traditional projection system suffers from a small eyebox, which is the area within which the observer can move their head and still observe an unvignetted image.

Researchers have begun to explore the use of diffractive optics and HOEs applied to planar waveguides as a means to achieve image magnification and pupil expansion for HUDs [1,21,36–40]. In these systems, incident light is coupled into a waveguide and diffracted beyond the critical angle by an HOE. This diffracted light propagates along the length of the waveguide due to total TIR until it interacts with other HOEs, which modify the beam profile or diffract it to another propagation direction. In previous research efforts, 2D pupil expansion was demonstrated using an “L-shaped” hologram configuration [21]. Figure 3.2 shows this L-shaped design: light from a source image is diffracted to propagate down the length of the planar waveguide by an “injection” HOE. Light incident upon the “redirection” HOE is steered laterally along the length of the waveguide while the exit pupil is expanded vertically. The DE of the redirection HOE is controlled to achieve uniform intensity across the entire beam profile as it propagates to the “extraction” [1]. The “extraction” HOE expands the eyebox horizontally using the same modulated DE technique and diffracts the light perpendicular to the waveguide surface, where it is visible to the observer.
Figure 3.2: Light from an outside source is projected through the waveguide HOE system to present an image to the observer over an expanded eyebox. Successive HOEs increase the exit pupil size, first vertically, then horizontally. [2]

The disadvantage of the L-shaped configuration shown in Figure 3.2 is the spectral and angular selectivity of the redirection hologram, which limits the spectral bandwidth and range of angles that can propagate through the entire system [21]. This selectivity may be useful for a system with monochromatic light and a narrow angular bandwidth, but wide-angle, white-light projectors are limited in what they can achieve with this design. Misalignment of the wavelength or incidence angle on the redirection hologram will reduce the intensity of diffracted light and steer the diffraction angle away from the lateral propagation necessary for the image to propagate to the extraction. This spectral and angular bandwidth is determined
by the material properties of the redirection hologram and the hologram recording geometry. It can be overcome by multiplexing different gratings into the redirection hologram, but a different method can eliminate the need for such a heavily multiplexed hologram.

In this chapter, a system is presented where a small injection hologram diffracts the incident light horizontally along the length of the waveguide to the extraction hologram. An SRG affixed to the waveguide surface along the propagation path causes vertical beam spreading, which yields an expanded FOV at the extraction. The SRG diffracts all visible wavelengths, avoiding the angular and spectral selectivity issues encountered with the L-shaped system. It also allows for a more compact system than the L-shaped design, which is helpful for space-constrained applications. A Zemax OpticStudio ray-tracing model was developed to demonstrate this design. Additionally, a physical system was built to support the concept. Both the model and the demonstrator achieved a FOV of $16^\circ \times 14.25^\circ$. Using the SRG, the vertical FOV was expanded from $1.25^\circ$ to $14.25^\circ$.

3.2 Methods

In this new in-line configuration, the incident, collimated light is diffracted along the length of the waveguide by the injection hologram. While light is reflecting within the waveguide due to TIR, interactions with the SRG expand the physical extent of the propagating beam. The extraction hologram diffracts the light perpendicular to the waveguide surface so that the projected image is visible to the user.

The internal propagation angle of the small injection hologram is chosen such
that normally incident light is diffracted at the angle bisector of the critical angle and a maximum angle determined by the dimensions of the waveguide and the injection hologram. This angle is defined such that the left-most edge of the image that has reflected once within the waveguide is incident at the right edge of the injection hologram and is calculated according to

$$\theta_{\text{max}} = \tan^{-1}\left(\frac{w/2}{t}\right)$$

(3.1)

with $w$, the width of the injection hologram, and $t$, the thickness of the waveguide. Light diffracted beyond this maximum angle will lead to a discontinuous image where a stitching gap is visible between the different extraction regions. Figure 3.3 shows these angles.

Figure 3.3: The propagation angle is chosen to bisect the critical angle and the angle defined by Equation 3.1. [2]

The SRG diffracts the internally propagating light into +1, -1, and 0 diffraction orders, which reflect within the waveguide due to TIR before they again interact with the SRG. At the second interaction, light that remained in the 0 order is diffracted into its own +1, -1, and 0 orders. Light in the ±1 orders either is diffracted by the SRG back to the 0 order propagation direction or continues on in the direction
of the first diffraction, according to the DE of the grating. In this way, continued interaction with the SRG replicates the beams traveling in the +1, -1, and 0 order diffraction directions several times, much like a branching tree. The replicated beam propagating along the length of the waveguide (the 0 order diffraction direction from the SRG) causes vertical pupil expansion. Only light in the 0 order diffraction direction is diffracted by the SRG toward the observer. Light propagating in the ±1 order diffraction directions is ignored for the purposes of this HUD system. Figure 3.4 shows this branching pattern.

Figure 3.4: Cross-sectional splitting of incident light by SRG. After the first interaction with the grating, light is diffracted into +1, -1, and 0 diffraction orders. Light in the 0 order is again split into the +1, -1, and 0 orders, while light diffracted into the ±1 orders is not diffracted or diffracted back to the 0 order direction. [2]

The extraction hologram was recorded to diffract a beam incident from inside the waveguide at the same angle determined by Equation 3.1 to the hologram plane’s surface normal. The extraction grating was also recorded with its DE modulated
for low efficiency on the side of the grating nearest the injection hologram and the SRG and maximum efficiency on the far edge. This modulated DE was for uniform intensity of the image when viewed across the length of the extraction and was discussed in a previous publication [1]. The multiple interactions across the width of the extraction hologram causes horizontal pupil expansion.

3.3 Computer Modeling

Modeling this system in Zemax OpticStudio’s Non-Sequential mode demonstrated the feasibility of the design discussed in Section 3.2. The system was constructed with a design wavelength of 532nm (350nm used in the Hologram Lens recording parameters due to the wavelength reduction from the refractive index of the glass). The waveguide used in this model was a Rectangular Volume made of BK-7 glass (215mm × 305mm × 3.175mm), while the injection and extraction holograms were Hologram Lens elements with dimensions of 12.7mm × 12.7mm × 16µm and 63.5mm × 63.5mm × 16µm, respectively. From Equation 3.1, the maximum propagation angle for this design is 63.4°, and the critical angle for 532nm light propagating through BK-7 glass is 41.8°. Thus, the injection hologram was designed to diffract collimated, normally-incident 532nm light at 52.6° within the waveguide. Because the light travelled within the waveguide at 52.6°, the extraction hologram was designed to diffract this light perpendicular to the hologram surface, toward the observer. The modulated DE of the grating was modeled by a series of eight glass plates with reflectivity increasing according to

\[
R_i = \frac{1}{N_{tot} - N_i + 1}
\]  

(3.2)
where $R_i$ is the reflectivity of the $i^{th}$ glass plate, $N_{tot}$ is the total number of segments (in this case eight), and $N_i$ is the number of the plate.

The SRG was modeled using a Lenslet Array 1 function in Zemax. The material was BK-7 glass and dimensions were $50mm \times 50mm \times 16\mu m$ with a grating spacing of 1200 lines/mm. This grating spacing was chosen because it gives a diffraction angle of 24.8°, which corresponds to a $46\,mm$ vertical extent of the replicated 0 order when propagated $50\,mm$ across the length of the SRG. At a viewing distance of $150\,mm$, this $46\,mm$ 0 order will offer a maximum vertical FOV of 17.5° to the observer. The DE of the grating in the Lenslet Array 1 function was set under the Diffraction tab of the Object Properties menu so that 33% of the light is reflected into each of the +1, -1, and 0 diffraction orders.

To simulate the projection system, a collimated source was projected through an absorbing image filter. This collimated source object was imaged by a paraxial lens to locate the image at optical infinity. A 4mm detector situated behind the extraction allowed for analysis of the system output as might be seen by the human eye. Figure 3.5 shows the completed OpticStudio design.
Figure 3.5: Light incident from the source is diffracted at the design angle by the injection hologram. Light incident on the SRG is split into the +1,-1, and 0 diffraction orders. Subsequent interactions with the SRG continue diffraction into one of the three propagation directions. The variable DE extraction hologram diffracts the expanded “0 order” toward the observer. The image inset provides an enlarged view of the branching diffraction that is caused by the SRG. [2]

Light that remained in the +1 or -1 order diffraction direction did not contribute to the final image visible in the extraction. In this design, the light propagating in these directions remained within the waveguide before it was either out-coupled or scattered at the waveguide edges. The amount of light that is lost in these directions is determined by the DE and size of the SRG, as higher efficiency gratings diffract more light into the +1 and -1 orders, but subsequent interactions diffract the light in those non-zero orders back to the initial propagation direction. Figure 3.6 shows how the grating parameters of the SRG were optimized in this ray-tracing model. It was found that 33% diffraction into each of the +1, -1, and 0
diffraction orders created the most uniform intensity distribution while maintaining image quality. At higher DE, image quality decreases, and at lower DE, the 0 order is drastically brighter than the rest of the image. Similarly, the SRG was optimized so that, with physical dimensions of $50\text{mm} \times 50\text{mm}$, the full image is visible at the extraction.

Figure 3.6: (Top) 5%, 50%, and 33% DE to ±1 diffraction orders. 33% DE maintains image quality without sacrificing uniform intensity. (Bottom) No SRG, $25\text{mm} \times 25\text{mm}$ SRG, and $50\text{mm} \times 50\text{mm}$ SRG. Increasing grating size allows for more of the projected image to be visible. All images are generated on a $3\text{mm}$ square detector located $150\text{mm}$ from the waveguide surface. [2]
3.4 Physical Demonstrator

A physical demonstrator of this design configuration was created based on the parameters laid out in Section 3.3. The waveguide has dimensions of $215\,mm \times 305\,mm \times 3.175\,mm$ and is made of a low-iron glass to decrease the absorption of light as it propagates inside the medium. The injection and extraction holograms were recorded with the same dimensions as in Section 3.3 using Bayfol HX200 photopolymer, a holographic recording film that is sensitive across the visible spectrum. Bayfol HX200 is easy to apply and cures with UV exposure. Additionally, the photopolymer is $16\,\mu m$ thick, allowing for a wide angular bandwidth of injected light.

The injection hologram was recorded to diffract a beam incident at the surface normal to $52.6^\circ$ and the extraction was recorded to diffract a beam incident at $52.6^\circ$ to the hologram surface normal, just as in the computer model described in Section 3.3. The SRG is a Thorlabs reflective holographic grating (G50-12V) and was coupled to the waveguide using index-matching oil. A 3M MP220 incoherent, white-light mobile projector was set up so that the projector was flush with the injection hologram and focused as far away as possible to provide accommodation at infinity. In the demonstrator the extraction was adjacent to the SRG, which kept all relevant optical components confined within a cube with side lengths of $200\,mm$. Figure 3.7 shows the demonstrator setup described here.
Figure 3.7: Physical demonstrator of in-line pupil expansion HUD setup. Injection and extraction holograms are recorded in Bayfol HX200, which is attached to the waveguide surface. The SRG is coupled to the waveguide with index-matching fluid. [2]

Figure 3.8 shows the physical demonstrator in operation with a projected image overlaid on a background scene. Figure 3.8a shows the scene visible when an observer focuses on the HUD itself: both the projected image and the background scene are out of focus. However, when the observer shifts their focus to the far field (Figure 3.8b), both the background and the projected image come into focus. The FOV of this HUD demonstrator is $16^\circ \times 14.25^\circ$, which agrees with the results predicted by the computer model.
(a) Focused at the waveguide plane.  

(b) Focused in at optical infinity.

Figure 3.8: The demonstrator shown in Figure 3.7 was set up with an image projected in the background. When focusing on the waveguide plane the image was not clear. Only when focus changes to the far field does the image become clear. Though the image visible in this scene is green, changing the observer’s viewing position from left to right changes the color of the displayed image from violet to red. [2]

The image visible in Figure 3.8b demonstrates some blur, which is visible both with and without the SRG attached and is thus due to scattering from repeated interactions with the injection and extraction holograms. Bayfol material has demonstrated this property in previous research efforts [1], but its ease-of-use continues to make it a valuable tool for exploratory holographic research.

Another consideration of using the Bayfol holographic recording material is the material absorption, whose effect is compounded by repeated interactions with the internally propagating beam. Marín-Sáez et al. have demonstrated that multiple types of Bayfol photopolymer achieve 90 – 99% transmittance of light across the visible spectrum, with most absorption taking place at lower wavelengths [49]. This
absorption should be taken into account when planning the extraction hologram’s modulated DE and the final intensity of the extracted image.

3.5 Conclusion

This chapter presented an HUD display system that used HOEs and an SRG affixed to a planar waveguide to achieve vertical and horizontal pupil expansion with an in-line geometry. This geometry has an advantage over the previously demonstrated L-shaped 2D pupil expansion because the SRG demonstrated 33±5% DE into the ±1 diffraction orders from 450nm – 600nm with unpolarized incident light [50] and an angular bandwidth of 70° according to the grating equation [44]. Both a Zemax OpticsStudio computer model and a physical demonstrator of the system were created that achieved pupil expansion and a FOV of 16° × 14.25°. The physical demonstrator could be fit into a 200mm cube by removing unused portions of the waveguide, which demonstrates the compact nature of this system.

A future research direction for this technology would be to explore using different materials for the injection and extraction holograms. Using a holographic material with low scattering, such as DCG, will improve the contrast and resolution of the display system. While a thicker hologram might reduce the acceptance angle, multiplexing several angles into one hologram would expand the angular acceptance of the injection and extraction gratings, supporting a larger system FOV or a full color display. Section 5.1 explores how the parameters of a waveguide HUD system affect the final FOV and eyebox.

Beyond HUD applications, this technology could also be applied to augmented reality glasses to achieve an expanded FOV with a small form factor.
This work was submitted for publication in OSA’s Applied Optics journal [2].
Chapter 4

Holographic Amplification of the Diffraction Angle from Optical Phase Array for Optical Beam Steering

Chapters 2 and 3 demonstrated that holograms have tremendous potential for directing light energy. In the work presented here, HOEs are used to magnify the diffraction angle of a non-mechanical beam steering device for remote sensing and LIDAR applications.

4.1 Background

LIDAR is a technique typically used in remote sensing applications where pulsed laser light is shined on a target and the reflected pulses are measured by a sensor. Because of the delay between returned pulses, LIDAR allows users to generate a depth map of a scene. Among a number of applications, this mapping is useful for navigation within a three-dimensional (3D) environment, which is necessary for the development of self-driving cars [51].

Beam steering ensures the angular coverage of the LIDAR system. Approaches to beam steering generally fall into three categories: mechanical steering, micro-
electro-mechanical systems (MEMS), and non-mechanical steering [52].

Mechanical steering refers to systems with physical optics that are rotated or positioned to steer the beam to the desired position. These systems include gimbals, fast-steering mirrors, wedge-prism pairs, and rotating mirrors and gratings [53–55]. While they are very efficient at directing the energy in the desired direction and can cover a maximum angular range of \(4\pi\) sr, mechanical steering solutions are hindered by their inertia and high energy cost. Additionally, many mechanical beam steering systems have relatively greater size, weight, and cost than their non-mechanical or MEMS counterparts, as well as increased sensitivity to environmental motion and vibration.

MEMS beam steering efforts seek to accomplish beam steering with drastically lower inertia and power costs than their traditional mechanical counterparts [56–59]. Though there is actual mechanical movement with MEMS, the rotation is that of a collection of small mirrors rotated on an optical axis, which allows for small size and weight, in addition to the lower inertia and power costs previously discussed. These systems are limited, however, as there is a trade-off between the diffraction angle and the size of the beam that can be diffracted. Ongoing research efforts seek to expand the range of diffraction angles from a MEMS device while maintaining a large beam size [52].

Non-mechanical steering systems use phase control across diffractive surfaces to steer the beam into the desired direction [22, 60–62]. Frequently, this steering is accomplished with a photonic chip or SLM. While this technology is quickly re-configurable and operates without the high inertia and energy cost of their mechanical counterparts, the limitation of non-mechanical steering systems is that they have relatively narrow steering angles (\(\approx 1^\circ\)) [22], as the light is diffracted according
to Bragg’s Law. [63, 64]

$$\sin \theta_B = m \frac{\lambda_0 / n}{2 \Lambda}$$  \hspace{1cm} (4.1)

where $\theta_B$ is the Bragg angle, $m$ is the diffraction order, $\lambda_0$ is the vacuum wavelength, $n$ is the refractive index, and $\Lambda$ is the distance between Bragg planes. For instance, with the HOLOEYE LC-R720 SLM, which was used in the demonstrator discussed below, pixel pitch is $20 \mu m$. For an example grating which repeats every four pixels, working at $532 nm$ in air, $\theta_B = 0.191^\circ$ for the first diffraction order.

4.2 Materials and Methods

The system presented in this chapter uses holograms to amplify the steering angle from a non-mechanical steering system to cover a solid angle of nearly $4 \pi$ sr. A multiplexed, thick volume hologram diffracts incident light into a $2 \pi$ sr hemisphere. The material characteristics of this hologram are chosen to minimize crosstalk between different gratings by increasing the angular selectivity of each individual grating. After interacting with the thick volume hologram, the diffracted light is incident on the hemispheric surface, which is coated with holograms to counteract the optical power of the hemisphere and further redirect the beams up to $4 \pi$ sr. Angular selectivity is determined by the effective grating thickness and refractive index modulation of the substrate, as dictated by Kogelnik’s coupled wave theory (CWT) [64, 65]. Increasing the grating thickness while decreasing the refractive index modulation increases the angular selectivity of the material. For a transmission phase hologram being read with the ideal reconstruction angle and wavelength, the dispersion equation of DE as a function of thickness is

$$\frac{d \eta_{TE}}{d d} = \frac{2 \pi \Delta n}{\lambda_0 \sqrt{C_R C_S}} \sin \left( \frac{\pi \Delta n d}{\lambda_0 \sqrt{C_R C_S}} \right) \cos \left( \frac{\pi \Delta n d}{\lambda_0 \sqrt{C_R C_S}} \right)$$ \hspace{1cm} (4.2)
where $\Delta n$ is the refractive index modulation, $\lambda_0$ is the free-space wavelength, $C_R = \cos \theta_i$, $C_S = C_R - \frac{\lambda_0}{\Lambda} n \cos \theta_{\text{slant}}$, $\theta_i$ is the incident angle, $\theta_{\text{slant}}$ is the grating slant angle, $n$ is the material refractive index, $\Lambda$ is the grating period, and $d$ is the grating thickness [64]. Figure 4.1 shows how the angular bandwidth of an example system changes based on the thickness of the holographic material.

Figure 4.1: Increasing the thickness of the holographic film increases the angular selectivity of the recorded holograms when paired with a proportional decrease in refractive index modulation. [3]

Using this method, it is possible to accomplish beam steering that takes advantage of the low cost, form-factor, energy efficiency, and refresh rate of non-mechanical systems without being restricted to a narrow angular range. Figure 4.2 shows a schematic of how the system will operate.
Figure 4.2: System layout for diffraction angle amplification. Incident beam is split into a reference and signal beam. Signal beam is steered by SLM through successive diffractive elements to diffract a collimated signal at the design direction. Returning signal beam is passed to the detector for time-of-flight (TOF) measurement. [3]

This angular amplification system is composed of two distinct elements: a multiplexed grating that can direct the beam across $2\pi$ sr, which is recorded in a thick holographic recording medium to achieve high angular selectivity, and a shell hologram affixed to the surface of a hemispherical lens, whose purpose is to increase the diffraction angle from the initial multiplexed grating up to $4\pi$ sr while cancelling the focusing power of the hemisphere’s surface.

A demonstration system was built with five distinct angle combinations, as shown in Table 4.1. Note that Hologram 0 does not diffract the light from the multiplexed grating, but it does at the hemisphere surface. Due to the high angular selectivity of the gratings recorded in the multiplexed sample, light will not be diffracted by the multiplexed hologram, except at very specific angles. Hologram 0
demonstrates that even the light that is not diffracted by the volume hologram can be diffracted at the hemisphere surface for further beam steering purposes.

<table>
<thead>
<tr>
<th>Grating #</th>
<th>Incident Angle</th>
<th>Diffracted Angle (Multiplexed)</th>
<th>Diffracted Angle (Hemisphere)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0°</td>
<td>0°</td>
<td>20°</td>
</tr>
<tr>
<td>1</td>
<td>0.25°</td>
<td>20°</td>
<td>45°</td>
</tr>
<tr>
<td>2</td>
<td>0.5°</td>
<td>40°</td>
<td>90°</td>
</tr>
<tr>
<td>3</td>
<td>0.75°</td>
<td>60°</td>
<td>135°</td>
</tr>
<tr>
<td>4</td>
<td>1.0°</td>
<td>80°</td>
<td>150°</td>
</tr>
</tbody>
</table>

Table 4.1: System diffraction angles. Light with a given angle of incidence is diffracted by the multiplexed sample into the angle described. From there, it propagates to the hemisphere surface and is diffracted to the final angle.

4.2.1 Multiplexed Injection

The multiplexed injection hologram is designed to diffract a number of incident beams with relatively small inter-beam angles (< 1°) such that the diffracted inter-beam angle becomes much larger (≈ 90°).

The multiplexed hologram was recorded in a holographic photopolymer called phenanthrene quinone doped polymethyl methacrylate (PQ/PMMA). This material was chosen because the material properties and manufacturing process are well understood [66–73] and the thickness can be customized through the fabrication process to achieve the desired angular bandwidth of the encoded gratings. Additionally, the high refractive index modulation of the PQ/PMMA when recording holographic
gratings is well-suited to multiplexed recording [67], as it allows for high DE of all recorded gratings.

PQ/PMMA samples were prepared in the manner described by Luo et al. in their work characterizing PQ/PMMA for holographic recording [74]. A recording geometry like that shown in Figure 4.3a was assembled to record four distinct gratings with the reference beams near normal incidence and the object beams changing their incidence angle from 20° to 80° by steps of 20° for a multiplexed thick volume hologram. The beams are turned on sequentially, in pairs. Prisms are coupled to the front of the photosensitive sample to allow for injection of reference beams near normal incidence and object beams beyond the critical angle of 41.8°. An out-coupling prism prevents light from reflecting back within the material and creating secondary holograms.
(a) Recording configuration for multiplexed injection recording. All beams are shown at once, but each grating was recorded individually.

(b) Real table layout for the multiplexed injection recording. Between recordings, the rotation stage was turned by $0.375^\circ$ and the mirrors were removed, in order.

Figure 4.3: Recording configuration for multiplexed injection hologram. Gratings are recorded into the PQ/PMMA with 10 second dark-delay time between exposures.

[3]

4.2.2 Hemispheric Lens

The light diffracted by the PQ/PMMA sample can cover up to a $2\pi$ sr solid angle. Coupling this light into a hemispheric lens and applying holograms along the surface of that lens allows for that $2\pi$ sr solid angle to be expanded up to $4\pi$ sr. This angular magnification was achieved by coating a hemispheric lens with HOEs, which simultaneously counteract the focusing effect of the convex surface to maintain beam collimation and diffract the beam beyond its original propagation angle for greater angular coverage. Figure 4.4 shows the recording configuration selected to create these HOEs. A diverging beam whose source point is at the focal point of the hemisphere acts as the reference beam for recording these HOEs, while a collimated beam coming from the desired diffraction direction acts as the object.
beam.

Figure 4.4: Holograms are recorded on the hemisphere surface. The reference beam is a diverging beam that propagates toward the hemisphere surface where the focal point is the same distance as a collimated beam exiting the hemispheric lens. The object beam is collimated and counter-propagates from the final diffraction direction.

Appropriate hologram recording geometries were modeled in Zemax OpticStudio to calculate the source point location of the reference beam for collimated diffraction from the hemisphere surface. Figure 4.5 shows collimated light incident at each hologram position and a collimated beam being diffracted by the hologram on the hemisphere surface.
4.3 Results

A multiplexed grating was successfully recorded while minimizing signal cross-talk between the four diffraction angles presented in Table 4.1: $20^\circ$, $40^\circ$, $60^\circ$, and $80^\circ$. These angles were chosen for even spacing between the gratings on the hologram surface. Figure 4.6 shows the measured relative intensity values at those observation positions when the incidence angle is varied from $0^\circ$ to $1.25^\circ$. 

Figure 4.5: Holograms recorded with the geometry shown in Figure 4.4 will diffract collimated light from the hemisphere surface. [3]
Kogelnik’s CWT calculates an effective thickness and refractive index modulation for each of the gratings recorded in the multiplexed sample. Figure 4.7 shows the CWT curves overlaid on the DE curves and table 4.2 supplies the thickness and refractive index modulation parameters.
Figure 4.7: CWT curve fitting of observed DE from multiplexed PQ/PMMA. [3]
<table>
<thead>
<tr>
<th>Grating #</th>
<th>Thickness (µm)</th>
<th>Δn</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1900</td>
<td>3.9e-5</td>
</tr>
<tr>
<td>2</td>
<td>625</td>
<td>1.3e-4</td>
</tr>
<tr>
<td>3</td>
<td>310</td>
<td>1.7e-4</td>
</tr>
<tr>
<td>4</td>
<td>160</td>
<td>4.6e-5</td>
</tr>
</tbody>
</table>

Table 4.2: Effective grating thickness and refractive index modulation of each multiplexed gratings in the thick volume HOE.

The intensity at the final diffraction angles (45°, 90°, 135°, 150°) were also measured after the light had propagated through the multiplexed hologram and the hemispheric shell. Figure 4.8 shows the observed intensities as a function of incident angle.

Figure 4.8: Relative DE values for light diffracted by the multiplexed hologram and hemispheric shell holograms. Observation angles are 45°, 90°, 135°, and 150°. [3]
Note that there is relatively less light in the 45° beam compared to the 90° beam in Figure 4.8 than between the 20° and the 40° beam in Figure 4.6. This decreased relative power is because the DE of successive elements must be considered. Table 4.3 shows the power budget for this demonstrator system.

<table>
<thead>
<tr>
<th>Element</th>
<th>DE</th>
<th>Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLM</td>
<td>37.3% at ±0.381° to 92% at 0°</td>
<td>Effective DE follows a $sinc^2(1/N)$ response to the number of pixels (N) in one period of the blazed grating</td>
</tr>
<tr>
<td>Multiplexed Grating 1</td>
<td>20.6%</td>
<td>DE depends on grating thickness</td>
</tr>
<tr>
<td>Multiplexed Grating 2</td>
<td>35.7%</td>
<td>DE depends on grating thickness and refractive index modulation</td>
</tr>
<tr>
<td>Multiplexed Grating 3</td>
<td>28.7%</td>
<td></td>
</tr>
<tr>
<td>Multiplexed Grating 4</td>
<td>5.8%</td>
<td></td>
</tr>
<tr>
<td>Hemisphere Grating 1</td>
<td>6.6%</td>
<td>DE depends on recorded efficiency, and agreement between design angle and diffraction angle</td>
</tr>
<tr>
<td>Hemisphere Grating 2</td>
<td>37.0%</td>
<td></td>
</tr>
<tr>
<td>Hemisphere Grating 3</td>
<td>2.4%</td>
<td></td>
</tr>
<tr>
<td>Hemisphere Grating 4</td>
<td>4.1%</td>
<td>from multiplexed grating</td>
</tr>
</tbody>
</table>

Table 4.3: Power budget for demonstrator. DE of Multiplexed grating combines with DE of hemisphere grating to determine overall efficiency of the system.

After each individual component was tested, the entire system was assembled to demonstrate its feasibility with a LIDAR type TOF measurement. Figure 4.9 shows the final system layout, which is based on the design in Figure 4.2.
Figure 4.9: Final system layout for diffraction angle amplification. System behaves as described in Figure 4.2. [3]

Light from a pulsed laser (6\text{ns} pulse width) is expanded and passed through a polarizing beam splitter (PBS), which sends most of the light to the SLM, while a low power reflection is used as a reference signal and sent directly to the detector. The beam incident on the SLM is double-passed through a half-wave plate (HWP), which rotates the polarization state 45° on each pass, so that it is reflected from the plane bisecting the PBS and exits to the beam steering setup where it interacts with another HWP set to rotate the polarization state by 45°. Light is diffracted by the multiplexed hologram to one of the hemispheric shell holograms. From there, the beam propagates a long distance (in this case, \( \approx 10\text{m} \), but potentially up to thousands of meters) before it is reflected back along the same path to the hemisphere. Interactions with the shell holograms and the multiplexed hologram diffract the returned beam back to the PBS. Bragg selectivity of the holograms recorded on the hemisphere’s surface prevents stray light or unwanted diffraction orders from being diffracted back to the detector. Because the main beam has double-passed
through the HWP, its polarization is again rotated 90° so that it passes through the PBS to the detector.

A high speed silicon photodetector connected to a 3GHz oscilloscope was used to measure the signal. Figure 4.10 shows the trace of the difference between the system with and without the signal beam.

![Photodetector Difference Signal](image)

Figure 4.10: Taking the difference between the detector signal with and without the beam steering arm blocked shows that there is a return signal from the beam steering system. [3]

The measured time difference between the reference peak and the TOF signal was 32.4 ± 0.5ns, and the total path length difference between the two arms of the beam steering arm was measured to be 9.8 ± 0.05m, which corresponds to a flight time of 32.6 ± 0.2ns. This agreement between the measured and theoretical values demonstrates that this holographic, non-mechanical, beam steering system
functions as intended and, with continued refinement, will be a viable tool for use in LIDAR and other remote sensing applications.

4.4 Conclusion

Successive diffractive elements are combined to demonstrate a non-mechanical beam steering method that can achieve a theoretical maximum of $4\pi \text{ sr}$ redirection for LIDAR applications. This angular range will aid in the design and implementation of systems that quickly and efficiently perform complete environmental scans. Commercial realization of this technology will require continued refinement of the hologram recording process to eliminate aberrations from the hemispheric lens, as well as a specialized application process to easily and uniformly affix a holographic film to the convex surface.

Commercial implementation of this process will also require changing the design wavelength of the system. While the demonstrator discussed here is designed to work at $532\text{ nm}$, LIDAR applications use $905\text{ nm}$ or $1.55\mu\text{m}$, both of which fall into the infrared (IR) spectrum. This wavelength discrepancy could be addressed by recording the relevant holograms with a laser of the same wavelength as the final use case but may be made difficult by decreased material sensitivity to IR wavelengths during recording. Another approach is to calculate grating spacing, refractive index modulation, and slant angle necessary to diffract the IR light. With these calculated parameters, gratings could be recorded with a visible wavelength laser for IR reading, as discussed by Bjornson et al [75]. Section 5.2 further explores the idea of recording and reading holograms with different wavelengths.

Future research efforts will seek to increase the angular selectivity and max-
imum DE of each recorded grating in the multiplexed hologram by increasing the exposure energy. Improving these parameters can be achieved by increasing the effective grating thickness and the refractive index modulation within the hologram. By making these adjustments, the multiplexed hologram will also be able to support a greatly increased number of gratings for higher precision beam steering. The maximum number of gratings that can be multiplexed into a system are determined by a material parameter called the $M/\#$, which depends on responsiveness and the maximum refractive index modulation available in the holographic recording medium [76]. The average DE of a number of multiplexed gratings in a holographic medium is given by Equation 4.3

$$\eta_{avg} = \left( \frac{M/\#}{N_{tot}} \right)^2$$

(4.3)

where $M/\#$ depends on the material parameters and $N_{tot}$ is the total number of gratings multiplexed into the material. The PQ/PMMA samples used in this work have an $M/\#$ of 6.18, so the maximum number of gratings that can be multiplexed into the hologram and maintain 100% DE is 6. Figure 4.11 shows how quickly the average DE of multiplexed gratings falls off as a function of total number of gratings.
Figure 4.11: Average DE of multiplexed gratings falls off as $1/N_{Tot}^2$. Gratings drop to average efficiency of $< 1\%$ after 61 recorded gratings.

As it has been demonstrated that thousands of holograms can be multiplexed in a thick volume hologram for purposes of data storage [77], it is also possible to record a similar number for purposes of beam steering, provided the appropriate material.

At reading angles between the design angular values, the light will be partially diffracted to multiple angles, though with greatly reduced intensity. One feature of this system is that it becomes possible to change the wavelength propagating through the beam steering system. A slightly altered wavelength will diffract between the design angles and will increase the number of holograms that can be recorded on the hemisphere surface and increase the total number of angles at which the system can scan.

This work was submitted for publication in OSA’s Applied Optics journal [3].
5.1 FOV and Eyebox Limitations in Waveguide HUD

Chapters 2 and 3 presented waveguide HUD systems that achieve expanded FOV and image eyebox to the observer. An injection hologram coupled incident light into the waveguide and an extraction hologram diffracted the trapped light toward the observer. The injection hologram was recorded with optical power in Chapter 2 to achieve longitudinal image magnification, while in Chapter 3 the image magnification took place due to the optics of the projection system.

In both systems, the extraction hologram was notably larger than the injection hologram and was recorded with modulated DE to facilitate uniform image intensity across the length of the extraction. Angular and spectral selectivity of the extraction hologram determine the FOV of that the extraction will be able to present to the
observer. The angular selectivity of the extraction hologram can be calculated using Kogelnik’s CWT [64]:

$$\eta_{TE} = \sin^2 \left( \frac{\sqrt{\nu^2 + \xi^2}}{1 + \xi^2 / \nu^2} \right)$$

(5.1)

where $\nu = \frac{\pi \Delta n \cdot t}{\lambda \sqrt{C_R \cdot C_S}}$ is a parameter describing diffraction under ideal conditions and $\xi = -\frac{\Delta t}{C_S}$ describes detuning of either the incidence angle or wavelength from the ideal conditions. The above parameters are calculated using $\Delta = \Delta \theta \frac{2\pi}{\Lambda} \sin (\theta_{slant} - \theta_B)$, $C_R = \cos \theta_R$, and $C_S = C_R - \frac{\lambda/n}{\Lambda} \cos \theta_{slant}$ where $\Delta n$ is the system’s refractive index modulation, $t$ is the hologram thickness, $\Delta \theta$ is the angular detuning of the system, $\Delta \lambda$ is the spectral detuning, $\Lambda$ is the grating pitch, $\theta_{slant}$ is the slant angle of the grating, $\theta_B$ is the Bragg angle, and $\theta_R$ is the angle drawn between the reference beam and the hologram surface normal. By determining an acceptable DE for the extraction, holographers can use the specific parameters of their system to determine a theoretical maximum FOV that can be diffracted by the extraction hologram. This calculation can also be applied to the injection hologram to determine the maximum angular extent that can be coupled into the waveguide.

Using the DE calculations described above, it is possible to calculate the super-blaze profile for a given grating, which describes the maximum DE of each wavelength at its ideal Bragg angle. The Bragg angle for any given wavelength is calculated according to

$$\theta_B = \theta_{slant} - \cos^{-1} \left( \frac{K}{2 \beta} \right)$$

(5.2)

where $\beta = \frac{2\pi}{\lambda/n}$. Using the specific Bragg angle for each wavelength in Equation 5.1 above, it becomes possible to calculate the super-blaze profile. Figure 5.1 shows how the super-blaze profile predicts the DE of a grating over large spectral and angular bandwidths.
Figure 5.1: Combining the blaze curve diffraction profiles for multiple wavelengths interacting with a specific grating yields the super-blaze profile, describing the maximum DE capable with that grating.

The waveguide geometry can also contribute to the FOV limitations of the system. The angular range from the critical angle to the maximum propagation angle defined in Equation 3.1 defines the angular bandwidth at which light can propagate within the system and not leave dark stitching segments in the extracted image. This means that decreasing the waveguide thickness or increasing the refractive index will increase the FOV to be diffracted from the system. If the extraction hologram is recorded to diffract the design angle normal to the waveguide surface and the angular bandwidth of the extraction hologram is larger than the angular range of the system, the FOV of the system should be given by Equation 5.3

$$\text{FOV}_{WG} = 2 \sin^{-1} \left( n_{WG} \sin \left( \frac{\tan^{-1} \left( \frac{w/2}{t} \right) - \sin^{-1} \left( \frac{1}{n_{WG}} \right) }{2} \right) \right)$$  \hspace{1cm} (5.3)$$

where $n_{WG}$ is the refractive index of the waveguide, $w$ is the width of the injection
The hologram, and $t$ is the waveguide thickness.

The physical dimensions of the holograms or SRG play an important role in determining the dimensions of the system’s eyebox. In Chapters 2 and 3, the eyebox expansion was achieved by the increased size and modulated DE of the extraction hologram. Increasing the length of the extraction and modulating the DE to accommodate the increases hologram size expanded the eyebox by the same amount as the hologram extends.

Additionally, the intended viewing distance will affect the system’s eyebox, as the extraction hologram will occupy a FOV that changes depending on the viewing distance. An extraction hologram that is $400\text{mm} \times 300\text{mm}$ and projects an image with a FOV of $40^\circ \times 30^\circ$ should have an eyebox of $327\text{mm} \times 246\text{mm}$ at a viewing distance of $100\text{mm}$. However, the same system will have an eyebox of $36\text{mm} \times 32\text{mm}$ at a viewing distance of $500\text{mm}$, as more of the extraction hologram surface is used to generate the full FOV image.

5.2 Recording and Reading Holograms with Different Wavelengths

The holograms discussed throughout the rest of this dissertation were recorded with the same wavelength of light that was used in their reconstruction. This methodology offers many advantages, as the hologram can be recorded with the object and reference beams directly mimicking the reconstruction and image beams. Sometimes, though, as is the case of the PQ/PMMA used to record the multiplexed hologram in Chapter 4, the holographic recording material is not sensitive to the wavelengths of light at which it will be used. In this case, holographers can record the necessary gratings into the recording material with a different wavelength than
will be used for reading.

5.2.1 K-vector Closure

The first consideration when designing a grating is a process called “K-vector closure” wherein the designer calculates the ideal grating slant angle and pitch for the desired diffraction conditions. The reconstruction beam and image beams are known and assigned a “k-vector” that contains the relevant wavelength and steering information, as shown in Equation 5.4.

\[
\vec{k} = \frac{2\pi}{\lambda_0/n} \left( \sin \theta \hat{x} + \cos \theta \hat{z} \right) \tag{5.4}
\]

where \(\lambda_0\) is the vacuum wavelength, \(n\) is the refractive index, and \(\theta\) is the angle of beam propagation with respect to the hologram surface. Taking two k-vectors that contain the information for the desired reference and image beams allows the user to calculate the slant angle and spacing of the grating formed by the interference between the two, as shown in Equation 5.5.

\[
\vec{K} = \vec{k}_{\text{img}} - \vec{k}_{\text{recon}} = \frac{2\pi}{\lambda_{\text{recon}}/n} \left( (\sin \theta_{\text{img}} - \sin \theta_{\text{recon}}) \hat{x} + (\cos \theta_{\text{img}} - \cos \theta_{\text{recon}}) \hat{z} \right) \tag{5.5}
\]

It is possible to construct the resultant K-vector for the desired reconstruction and image beams with a different set of k-vectors, as shown in Figure 5.2. For a specific wavelength of light to record a desired grating, the resultant K-vector must be able to fit between two points on the circumference of that wavelength’s Bragg circle, whose radius is given by \(\frac{2\pi}{\lambda_0/n}\). Detuning away from the Bragg circle will decrease DE.
Figure 5.2: Different wavelengths of light can be used to produce the same resultant K-vector. Angles of incidence change according to the wavelength.

In an example system, a grating will be designed to diffract normally-incident 1.55$\mu m$ light at 45° in a medium with refractive index $n = 1.30$. Equation 5.5 gives

\[
\vec{K} = \frac{2\pi}{1.55/1.3} \left( (\sin 45° - \sin 0°)\hat{x} + (\cos 45° - \cos 0°)\hat{z} \right) \\
= 3.726\mu m^{-1}\hat{x} - 1.543\mu m^{-1}\hat{z}
\]

Now, using a writing wavelength of 532$nm$, for which the hypothetical system has a refractive index of $n = 1.45$, the necessary recording k-vector angles can be calculated.

\[
\frac{2\pi}{0.532/1.45} \left( (\sin \theta_{ref} - \sin \theta_{obj})\hat{x} + (\cos \theta_{ref} - \cos \theta_{obj})\hat{z} \right) \\
= 3.726\mu m^{-1}\hat{x} - 1.543\mu m^{-1}\hat{z}
\]

The necessary angles are 15.73° and 29.26° when recording with 532$nm$ light in this hypothetical system. Because the reconstruction and object beams do not
correspond to the any specific wavefront, the terms are treated interchangeably here.

Holographers may also need to design their system with the understanding that the grating slant angle and pitch will shift if the hologram shrinks during the curing process. Once again, the system designer will need to construct the desired grating K-vector from the desired reconstruction and image wavefronts. In this case, however, the grating z-component will be reduced proportional to the material shrinkage. The solution method remains the same, using a set of coupled equations to solve for the object and reference beam angles.

K-vector closure is the method holographers need to utilize to make sure that the grating pitch and slant angle match the desired grating when recording with a different wavelength than will be used for reading.

### 5.2.2 Refractive Index Modulation

Beyond matching the desired grating pitch and slant angle, systems that are recorded with a different wavelength than will be used for reading also should plan to achieve appropriate grating strength for the desired DE. The DE of a phase-modulating thick volume hologram follows

\[
\eta_{TE} = \sin^2 \left( \frac{\pi \Delta n d}{\lambda \cos \theta_i} \right)
\]  

(5.6)

with the variable definitions the same as those given by Equation 4.2 [44]. Thus, as the wavelength increases, the refractive index modulation will need to increase, all else held equal. Figure 5.3 shows how, for a system with \( \theta_i = 0 \) and \( d = 100\mu m \), nearly three times as much refractive index modulation is needed to achieve 100% DE for 1.55\( \mu m \) light as for 532\( nm \) light.
Figure 5.3: Different wavelengths need different amounts of refractive index modulation to achieve 100% DE. [3]

The necessary refractive index modulation can be found by determining the appropriate incidence angle change from the K-vector closure efforts described in Section 5.2.1 and applying that to Equation 5.6. The process of recording a hologram with more refractive index modulation than necessary for its writing wavelength is called over-modulation and is a necessary part of grating design and recording for systems recorded with a different wavelength than will be used for reading.

5.3 Conclusion

This chapter presented additional conceptual work to explore how system parameters affect the FOV of a waveguide HUD system. It also discussed recording methods to maximize DE when recording in a holographic medium with a different wavelength than will be used for reconstruction.
Bibliography


