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Non-Markovian dynamics of collective atomic states coupled to a waveguide

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ABSTRACT

When atoms are optically coupled to a one dimensional waveguide, they can interact through macroscopic distances. The retardation effects inherent to field propagation and the associated delay in information backflow between the atoms result in a departure from the familiar Markovian dynamics. We study the case of two two-level atoms coupled along a waveguide. One remarkable feature of the dynamics in this regime is the formation of long-lived bound states in the continuum (BIC),[30] that refer to a hybrid diatomic molecule bound together by propagating modes of a field. In particular, we study the probability of reaching such a bound states of the system starting in an initially anti-symmetric state of the emitters.

Keywords: Waveguide Quantum Electrodynamics (QED), Non-Markovian dynamics, Time-delayed feedback, Dicke superradiance

1. INTRODUCTION

Collections of emitters coupled to waveguides offer a promising platform for scalable quantum information processing.[1–4] The applications of such systems range from building long-ranged quantum networks,[5,6] quantum memory devices,[7–9] and metrology,[10] to accessing new parameter regimes with exotic light-matter interactions,[11,12] oftentimes governed by non-Markovian effects.

Non-Markovian dynamics is far richer than the more widely studied Markovian regime. Its different physical origins encompass strong coupling between system and bath, initial system bath correlations, structured reservoirs, and time-delayed coherent feedback, among others.[16,17,19–22] Non-Markovian collective dynamics due to structured bath and strong coupling has been previously studied in Refs.[23–28]

Another example is the situation where the separations between emitters become comparable to the coherence length of the photons mediating their interaction. Interference effects associated with the phase properties of the electromagnetic (EM) field can then be modified by group dispersion of the photon wavepackets. In such cases, the retarded backaction of the EM field on the atoms leads to a coherent time-delayed feedback on the system dynamics,[13,14] making it non-Markovian.[15–18] This is the situation that we consider here.

We consider specifically a geometry where two two-state atoms are coupled to an optical waveguide, as depicted in Fig. 1. In such a system where the distance between the atoms can be made quite large, one can consider the situation where the atoms are far enough apart that the retardation effects and finite coherence time scales of the photons mediating their interaction become significant. As a remarkable feature of the dynamics, it has been shown that after the field propagates back and forth between the two atoms it 'learns' that they act as an optical cavity in which it then remains trapped, as in the case of a bound state in continuum (BIC) discussed in Ref.[30,31] We study here the formation of such a state starting from an initial antisymmetric state of the two atoms.

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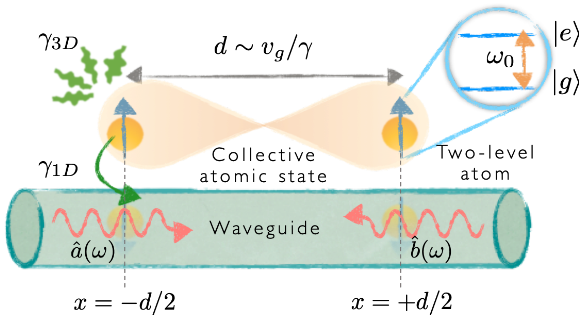

Figure 1. Schematic representation of two atoms coupled to the waveguide. We consider the parameter regime where the interatomic separation is comparable to the coherence length of a spontaneously emitted photon into the waveguide. The atoms are located at positions $x_1, 2 = \pm d/2$ along the waveguide such that $d \sim v_g/\gamma$, where v_g is the group velocity of the field in the waveguide and γ is the total spontaneous emission.

2. MODEL

To analyze this situation in detail we assume that the two atoms are located at positions $x_1 = d/2$ and $x_2 = -d/2$ along the length of the waveguide, and share the same radial and azimuthal coordinates for simplicity. In the interaction picture we can write the atoms-field interaction Hamiltonian as

$$H_{\text{int}} = \hbar \sum_{m=1,2} \int_0^\infty \mathrm{d}\omega\, g(\omega) \left[\hat{\sigma}_+^{(m)} \left\{\hat{a}(\omega) e^{i\omega x_m/v_g} + \hat{b}(\omega) e^{-i\omega x_m/v_g}\right\} e^{-i(\omega-\omega_0)t} + \text{h.c.}\right], \tag{1}$$

where $\hat{a}(\omega)$ and $\hat{b}(\omega)$ are the annihilation operators for the right- and left-propagating field modes in the waveguide, and v_g stands for the group velocity of light in the waveguide.

We assume that the field is initially in the vacuum state and that the atoms share only one quantum of excitation. Specifically, we consider the initial state to be an anti-symmetric state of the two atoms, the so-called subradiant state, with the field in vacuum state, such that $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|e, g, \{0\}\rangle - |g, e, \{0\}\rangle)$. The state of the atom + field system at time $t > 0$ is then

$$|\Psi(t)\rangle = \sum_{m=1,2} c_m(t)\, \hat{\sigma}_+^{(m)} |g, g, \{0\}\rangle + \int_0^\infty \mathrm{d}\omega \left[c_a(\omega, t)\, \hat{a}^\dagger(\omega) + c_b(\omega, t)\, \hat{b}^\dagger(\omega)\right] |g, g, \{0\}\rangle, \tag{2}$$

where $|g, g, \{0\}\rangle$ refers to the state where both the atoms are in the ground state and the field is in vacuum. Considering that the total Hamiltonian preserves the overall number of excitations in the atom+field system, the above state describes the most general state that the total system can evolve to when starting in a single excitation manifold.

From the Schrödinger equation one can then write the time evolution of the probability amplitudes appearing in (2) as

$$\dot{c}_a(\omega, t) = -i \sum_{m=1,2} c_m(t)\, g(\omega)\, e^{-i\omega x_m/v_g} e^{i(\omega-\omega_0)t}, \tag{3}$$

$$\dot{c}_b(\omega, t) = -i \sum_{m=1,2} c_m(t)\, g(\omega)\, e^{i\omega x_m/v_g} e^{i(\omega-\omega_0)t}, \tag{4}$$

$$\dot{c}_m(t) = -i \int_0^\infty \mathrm{d}\omega\, g(\omega)\, e^{-i(\omega-\omega_0)t} \left[c_a(\omega, t)\, e^{i\omega x_m/v_g} + c_b(\omega, t)\, e^{-i\omega x_m/v_g}\right]. \tag{5}$$

The formal integration of (3) and (4) yields

$$c_a(\omega, t) = -i \int_0^t \mathrm{d}\tau \sum_{m=1,2} g^*(\omega)\, c_m(\tau)\, e^{-i\omega x_m/v_g} e^{i(\omega-\omega_0)\tau}, c_b(\omega, t) \quad = -i \int_0^t \mathrm{d}\tau \sum_{m=1,2} g^*(\omega)\, c_m(\tau)\, e^{i\omega x_m/v_g} e^{i(\omega-\omega_0)\tau}, \tag{6}$$

and substituting these expressions into (5) gives

$$\dot{c}_m(t) = -\int_0^\infty d\omega\ |g(\omega)|^2 \int_0^t d\tau \sum_{n=1,2} c_n(\tau)\, e^{-i(\omega-\omega_0)(t-\tau)} \left[e^{i\omega(x_m-x_n)/v_g} + e^{-i\omega(x_m-x_n)/v_g} \right],$$
$$= -\frac{\gamma}{2} \sum_{n=1,2} c_n\left(t - \frac{|x_m-x_n|}{v_g}\right) e^{i\omega_0|x_m-x_n|/v_g}. \tag{7}$$

Here we have assumed that the density of modes is flat around the atomic resonance such that $g(\omega) \approx g(\omega_0)$ and have introduced the one-dimensional spontaneous decay rate along the direction of the waveguide $\gamma \equiv 4\pi\, |g(\omega_0)|^2$. This gives the equations of motion for the excitation amplitudes of the two atoms as

$$\dot{c}_1(t) = -\frac{\gamma}{2}\left[c_1(t) + c_2(t - d/v_g)\,\Theta(t - d/v_g)\right], \tag{8}$$
$$\dot{c}_2(t) = -\frac{\gamma}{2}\left[c_2(t) + c_1(t - d/v_g)\,\Theta(t - d/v_g)\right], \tag{9}$$

where we have assumed that the phase $\phi_p \equiv d\omega_0/v_g = 2n\pi$ and a perfect coupling between the atoms and the waveguide. We note that the above equations are symmetric under the exchange of the two atoms.

3. INITIAL SUBRADIANT STATE

For an initial atomic state $|\Psi_{\text{atoms}}(0)\rangle = \frac{1}{\sqrt{2}}(|e,g\rangle - |g,e\rangle)$ we have $c_{1,2}(0^+) = \pm\frac{1}{\sqrt{2}}$. Let us then consider the Laplace transform of Eqs. (8),

$$s\tilde{c}_1(s) = \frac{1}{\sqrt{2}} - \frac{\gamma}{2}\left[\tilde{c}_1(s) + e^{-ds/v_g}\tilde{c}_2(s)\right] \tag{10}$$
$$s\tilde{c}_2(s) = -\frac{1}{\sqrt{2}} - \frac{\gamma}{2}\left[\tilde{c}_2(s) + e^{-ds/v_g}\tilde{c}_1(s)\right]. \tag{11}$$

This yields readily

$$\tilde{c}_1(s) = -\tilde{c}_2(s) = \frac{1}{\sqrt{2}\left[s + \frac{\gamma}{2} - \frac{\gamma}{2}e^{-ds/v_g}\right]} \tag{12}$$
$$= \frac{1}{\sqrt{2}\gamma\left[\tilde{s} + 1/2 - e^{-\eta\tilde{s}}/2\right]}. \tag{13}$$

Consider first the limiting case of $d \to 0$. In this limit, we also note that $\phi_0 \to 0$. This allows us to write $\tilde{c}_{1,2}(s) \to \frac{1}{\sqrt{2}(s)}$, which corresponds to $c_{1,2}(t) \to \frac{1}{\sqrt{2}}$, illustrating that the two atoms are perfectly subradiant when coincident with each other. We also see that if the atoms are far apart, $d \to \infty$ one obtains $\tilde{c}_{1,2}(s) = \pm\frac{1}{\sqrt{2}[s+\gamma/2]}$. In that limit, the atoms therefore decay at the spontaneous emission rate γ for both atoms, a signature of the fact the atoms radiate independently of each other as expected.

For an arbitrary interatomic separation we can write the time dependent amplitudes as

$$c_{1,2}(t) = \pm\frac{1}{2\pi i}\int_{-i\infty+\epsilon}^{+i\infty+\epsilon} \frac{ds}{\sqrt{2}} \frac{e^{st}}{s + \frac{\gamma}{2} - \frac{\gamma}{2}e^{-ds/v_g}}, \tag{14}$$
$$= \pm\frac{1}{2\pi i}\int_{-\infty-i\epsilon}^{+\infty-i\epsilon} \frac{d\tilde{s}}{\sqrt{2}\gamma} \frac{e^{\tilde{s}\gamma t}}{\tilde{s} + \frac{1}{2} - \frac{1}{2}e^{-\eta\tilde{s}}}. \tag{15}$$

The pole of the denominator is determined by the characteristic equation

$$\tilde{s} + \frac{1}{2} - \frac{1}{2}e^{-\eta\tilde{s}} = 0 \implies (\eta\tilde{s})\, e^{\eta\tilde{s}} = \frac{\eta}{2}e^{\eta/2} \implies \tilde{s}_n = \frac{1}{2}\left[1 - \frac{W_n\left(\frac{\eta}{2}e^{\eta/2}\right)}{\eta/2}\right], \tag{16}$$

where we have introduced $\bar{s} \equiv \tilde{s}+1/2$. Here $W(z)$ is the Lambert W-function, or more precisely a set of functions $W_n(z)$ comprising the n branches of the inverse relation of the function $f(z) = ze^z$, where z is a complex number. In other words $z = f^{-1}(ze^z) \equiv W(ze^z)$, $W(z) = f^{-1}(z)$, and $W_n(x)$ is its n-th branch.[29]

We can now write the Laurent series expansion of the denominator of the integrand

$$\frac{1}{\tilde{s}+\frac{1}{2}-\frac{1}{2}e^{-\eta\tilde{s}}} = \sum_{n\in\mathbb{Z}} \frac{\alpha_n}{\tilde{s}-\tilde{s}_n}, \tag{17}$$

such that

$$\alpha_n = \lim_{\tilde{s}\to\tilde{s}_n} \frac{\tilde{s}-\tilde{s}_n}{\tilde{s}+\frac{1}{2}-\frac{1}{2}e^{-\eta\tilde{s}}} = \frac{1}{1+W_n\left(\frac{\eta}{2}e^{\eta/2}\right)}, \tag{18}$$

where we have used the property of the W-function that $W(z_0)e^{W(z_0)} = z_0$. It can be seen from the definition of the W-function that given $W(ze^z) = z$, if we substitute $y \equiv ze^z$ in this definition, one obtains $W(y) = z$. Substituting $W(y) = z$ back into $y \equiv ze^z$ yields $y = W(y)e^{W(y)}$. This gives

$$\frac{1}{\tilde{s}+\frac{1}{2}-\frac{1}{2}e^{-\eta\tilde{s}}} = \sum_{n\in\mathbb{Z}} \frac{1}{\left\{1+W_n\left(\frac{\eta}{2}e^{\eta/2}\right)\right\}(\tilde{s}-\tilde{s}_n)}, \tag{19}$$

so that we can write the inverse Laplace transform as

$$c_{1,2}(t) = \pm\frac{1}{\sqrt{2}}\sum_{n\in\mathbb{Z}} \alpha_n e^{-\gamma_n t/2}, \tag{20}$$

where we have defined

$$\alpha_n \equiv \frac{1}{1+W_n\left(\frac{\eta}{2}e^{\eta/2}\right)} \tag{21}$$

$$\gamma_n \equiv \gamma\left[1-\frac{W_n\left(\frac{\eta}{2}e^{\eta/2}\right)}{\eta/2}\right]. \tag{22}$$

Observing that $\gamma_0 = 0$ and that for $n \neq 0$, $\mathrm{Re}\,\gamma_n > 0$, it follows that as $t \to \infty$, there is a steady state term corresponding to $n = 0$ that is given as

$$c_{1,2}(\infty) \to \pm\frac{1}{\sqrt{2}(1+\eta/2)}. \tag{23}$$

One can also express the dynamics of the field amplitudes from Eqs. (6) as

$$c_a(\omega,t) = -i\sqrt{\frac{\gamma}{4\pi}}\int_0^t \mathrm{d}\tau \sum_{m=1,2} c_m(\tau)\, e^{-i\omega x_m/v_g} e^{i(\omega-\omega_0)\tau}, \tag{24}$$

$$c_b(\omega,t) = -i\sqrt{\frac{\gamma}{4\pi}}\int_0^t \mathrm{d}\tau \sum_{m=1,2} c_m(\tau)\, e^{i\omega x_m/v_g} e^{i(\omega-\omega_0)\tau}. \tag{25}$$

We note that for an initial antisymmetric state, the fields emitted into the left and right going modes have opposite phases such that

$$c_a(\omega,t) = -c_b(\omega,t) = -\sqrt{\frac{\gamma}{2\pi}}\sum_{n\in\mathbb{Z}} \alpha_n \sin\left(\frac{kd}{2}\right)\frac{e^{[i(\omega-\omega_0)-\gamma_n/2]t}-1}{i(\omega-\omega_0)-\gamma_n/2}. \tag{26}$$

For $t \to \infty$ this reduces to

$$c_a(\omega,t) = -c_b(\omega,t) = \sqrt{\frac{\gamma}{2\pi}}\frac{1}{(1+\eta/2)}\sin\left(\frac{kd}{2}\right)\frac{1}{i(\omega-\omega_0)}. \tag{27}$$

so that can thus write the asymptotic state of the system for $t \to \infty$ as

$$|\Psi_\infty\rangle = \frac{1}{\sqrt{2}\,(1+\eta/2)}\left[(|eg\rangle - |ge\rangle)\otimes|\{0\}\rangle + \sqrt{\frac{\gamma}{\pi}}\int \mathrm{d}\omega\,\sin\left(\frac{kd}{2}\right)\frac{1}{i\,(\omega-\omega_0)}\left[\hat{a}^\dagger(\omega) - \hat{b}^\dagger(\omega)\right]|gg\rangle\otimes|\{0\}\rangle\right] \tag{28}$$

4. BOUND STATE IN THE CONTINUUM

The bound state in the continuum (BIC) of two separated atoms trapped along a waveguide is given by[30]

$$\begin{aligned}|\Psi_{\mathrm{BIC}}\rangle = \frac{1}{\sqrt{1+\eta/2}}&\left[\frac{1}{\sqrt{2}}\{|eg\rangle + |ge\rangle\}\otimes|\{0\}\rangle\right.\\ &\left.-i\sqrt{\frac{\gamma}{4v_g}}\int_{-d/2}^{d/2}\mathrm{d}x\left\{e^{ik_0(x+d/2)}\hat{a}_R^\dagger(x) - e^{-ik_0(x+d/2)}\hat{a}_L^\dagger(x)\right\}\right]|gg\rangle\,|\{0\}\rangle,\end{aligned} \tag{29}$$

where the creation operators in the position basis can be written in terms of the momentum basis expressions as

$$\hat{a}_R^\dagger(x) = \sqrt{\frac{v_g}{\gamma}}\int_0^\infty \mathrm{d}k\,e^{-ikx}\hat{a}^\dagger(\omega) \tag{30}$$

$$\hat{a}_L^\dagger(x) = \sqrt{\frac{v_g}{\gamma}}\int_0^\infty \mathrm{d}k\,e^{ikx}\hat{b}^\dagger(\omega) \tag{31}$$

Substituting these forms in the BIC expression (29) gives

$$\begin{aligned}|\Psi_{\mathrm{BIC}}\rangle =&\frac{1}{\sqrt{1+\eta/2}}\left[\frac{1}{\sqrt{2}}\{|eg\rangle - |ge\rangle\}\otimes|\{0\}\rangle\right.\\ &\left.-\frac{i}{2}\int_0^\infty \mathrm{d}k\int_{-d/2}^{d/2}\mathrm{d}x\left\{e^{-i(k-k_0)x}e^{ik_0d/2}\hat{a}^\dagger(\omega) - e^{i(k-k_0)x}e^{-ik_0d/2}\hat{b}^\dagger(\omega)\right\}\right]|gg\rangle\,|\{0\}\rangle \qquad (32)\\ =&\frac{1}{\sqrt{1+\eta/2}}\left[\frac{1}{\sqrt{2}}\{|eg\rangle + |ge\rangle\}\otimes|\{0\}\rangle\right.\\ &\left.-\frac{i}{2}\int_0^\infty \mathrm{d}k\left[\frac{e^{i(k-k_0)d/2} - e^{-i(k-k_0)d/2}}{i\,(k-k_0)}\right]\left\{e^{ik_0d/2}\hat{a}^\dagger(\omega) - e^{-ik_0d/2}\hat{b}^\dagger(\omega)\right\}\right]|gg\rangle\,|\{0\}\rangle \qquad (33)\\ =&\frac{1}{\sqrt{1+\eta/2}}\left[\frac{1}{\sqrt{2}}\{|eg\rangle + |ge\rangle\}\otimes|\{0\}\rangle - \frac{i}{2}\int_0^\infty \mathrm{d}\omega\,\sin\left(\frac{kd}{2}\right)\frac{1}{i\,(\omega-\omega_0)}\left\{\hat{a}^\dagger(\omega) - \hat{b}^\dagger(\omega)\right\}\right]|gg\rangle\,|\{0\}\rangle \qquad (34)\end{aligned}$$

The overlap between the BIC and the asymptotic state (28) of the atom-field system is therefore

$$|\langle\Psi_\infty|\Psi_{\mathrm{BIC}}\rangle|^2 = \frac{1}{2\,(1+\eta/2)^3}\left[\frac{1}{2} + \frac{1}{2} + \frac{\gamma}{\pi}\int_0^\infty \frac{\mathrm{d}\omega}{(\omega-\omega_0)^2}\sin^2\left(\frac{\omega d}{2v_g}\right)\right]^2 \tag{35}$$

$$\implies |\langle\Psi_\infty|\Psi_{\mathrm{BIC}}\rangle|^2 = \frac{1}{1+\eta/2}, \tag{36}$$

which is the result recently referred to in Ref.[31]

5. CONCLUSION

We have thus calculated the probability of exciting the BIC state starting from an initial subradiant state of two macroscopically separated atoms. We find that for atoms prepared in an appropriate collective internal state, they can behave as effective mirrors that evolve collectively into a Fabry-Pérot-like resonator, with a bound state stored in the cavity formed by the atoms. This behavior depends strongly on both the collective atomic state and on the atomic separation and suggests an intriguing method to store single photons in macroscopic resonators with dynamically controllable mirror characteristics.

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