

ALG-AP-009: Determination of phase functions of Bennu and natural satellites

Authors:

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History:

- o Draft – 2013-July-26
- o Baseline - 2013-Nov-05

Description:

Phase functions are a mathematical equation that describes the light scattering behavior of the surface of an asteroid at varying phase angles (the Sun-asteroid-observer angle). The phase function is fit to a dataset of disk-integrated photometric measurements of asteroid Bennu or any detected natural satellites of Bennu. These observations will be conducted during the Approach Phase in the case of Bennu and during any of the Encounter phases in the case of contingency observations of a natural satellite. Each data point will consist of the mean brightness of the target object during a single rotation (i.e., rotational variation will be removed). Phase functions will be determined in all 5 MapCam filters (pan,b',v,w,x).

Parameters:

infile – Bennu temporal and phased lightcurve photometry [data product format is in UA-SIS-9.4.4-308 - OSIRIS-REx Astrometry and Photometry Derived Products SIS]

outfile – phase function photometry and phase function models for Linear regression, IAU H-G and Muinonen H-G1G2 phase functions [data product format is in UA-SIS-9.4.4-308 - OSIRIS-REx Astrometry and Photometry Derived Products SIS]

Algorithm equations:

Linear regression

$$\text{magnitude}(1,1,\alpha) = \beta * \alpha + H$$

where α is the phase angle in degrees, β is the slope of the phase function (in magnitudes per degree phase angle), H is the absolute magnitude, and $\text{magnitude}(1,1,\alpha)$ is the apparent magnitude normalized to 1 AU from the Sun and Earth at a phase angle α

IAU H-G phase function (originally presented in Bowell et al. 1989, the text below is from Muinonen et al. 2010 which is easier to follow than Bowell et al. 1989, it also corrects an error from the original)

The H , G phase function for asteroids can be described as follows. If we call α the phase angle, and $V(\alpha)$ the V magnitude (reduced to unit distance) expected for an object characterized by given values of H and G , the following relation holds:

$$V(\alpha) = H - 2.5 \log_{10}[(1 - G)\Phi_1(\alpha) + G\Phi_2(\alpha)], \quad (2)$$

where $\Phi_1(\alpha)$ and $\Phi_2(\alpha)$ are two basis functions normalized at unity for $\alpha = 0^\circ$. According to Eq. (2), the magnitude phase curve (hereinafter, "phase curve") of an object is described as the partitioning of the Φ_1 and Φ_2 functions in the ratio $(1 - G):G$. In turn, the slope parameter G is scaled in such a way that it is close to 0 for steep phase curves, and close to 1 for shallow phase curves, but values outside this interval are not excluded *a priori*.

In the H , G magnitude phase function for asteroids, the reduced observed magnitudes $V(\alpha)$ can be obtained from

$$\begin{aligned} 10^{-0.4V(\alpha)} &= a_1 \Phi_1(\alpha) + a_2 \Phi_2(\alpha) \\ &= 10^{-0.4H} [(1 - G)\Phi_1(\alpha) + G\Phi_2(\alpha)], \end{aligned} \quad (3)$$

where the absolute magnitude H and the coefficient G are⁵

$$H = -2.5 \log_{10}(a_1 + a_2), \quad G = \frac{a_2}{a_1 + a_2}. \quad (4)$$

The coefficients a_1 and a_2 are estimated from the observations by using the linear least-squares method. Thereafter, H and G follow from Eq. (4). In the H , G phase function, the basis functions are

$$\begin{aligned} \Phi_1(\alpha) &= w \left(1 - \frac{0.986 \sin \alpha}{0.119 + 1.341 \sin \alpha - 0.754 \sin^2 \alpha} \right) \\ &\quad + (1 - w) \exp \left(-3.332 \tan^{0.631} \frac{1}{2} \alpha \right), \\ \Phi_2(\alpha) &= w \left(1 - \frac{0.238 \sin \alpha}{0.119 + 1.341 \sin \alpha - 0.754 \sin^2 \alpha} \right) \\ &\quad + (1 - w) \exp \left(-1.862 \tan^{1.218} \frac{1}{2} \alpha \right), \\ w &= \exp \left(-90.56 \tan^2 \frac{1}{2} \alpha \right). \end{aligned} \quad (5)$$

These functions are accurately approximated by

$$\begin{aligned} \Phi_1(\alpha) &= \exp \left(-3.33 \tan^{0.63} \frac{1}{2} \alpha \right), \\ \Phi_2(\alpha) &= \exp \left(-1.87 \tan^{1.22} \frac{1}{2} \alpha \right). \end{aligned} \quad (6)$$

Proposed software:

- gnuplot

Additional references:

- Bowell, E., Hapke, B., Domingue, D., Lumme, K., Peltoniemi, J., Harris, A.W., 1989. Application of photometric models to asteroids. *Asteroids II*, 524–556.
- Muinonen, K., Belskaya, I.N., Cellino, A., Delb., M., Lvasseur-Regourd, A.-C., Penttil., A., Tedesco, E.F., 2010. A three-parameter magnitude phase function for asteroids. *Icarus* 209, 542–555.