Spectral Reflectance Factor

Description

The Spectral Reflectance Factor is the ratio of the reflectance of the surface to that of a perfectly diffuse surface under the same conditions of illumination. This quantity will be calculated for two different viewing geometries: (a) zero incidence, zero emission, and zero phase, and (b) 30 degrees incidence, 0 degrees emission, and 30 degrees phase angle. These quantities are most helpful in comparing Bennu spectra with laboratory spectra of meteorites as they will both be in the same REFF units.

1. Convert each spectrum from units of I/F to units of Reflectance Factor (REFF) using the equation REFF = I/F/(u0), where u0 (mu-naught) is the cosine of the incidence angle. I/F is a function of the viewing angles of the observations, i, e, and alpha (incidence, emission, and phase respectively), thus REFF is also a function of i,e, and alpha.

2. Obtain the two different viewing geometries like so:
   (a) Obtain normal reflectance from Detailed Survey Phase observations (12:30 pm station) for each OVIRS spot on the surface of Bennu by using the Photometric Model to Photometrically Correct the measurements of REFF to REFF at zero degrees incidence, zero degrees emission, and zero degrees phase angle.
   (b) Obtain RELAB-like reflectance from Detailed Survey Phase observations (10:30 am station) for each OVIRS spot on the surface of Bennu by using the Photometric Model to Photometrically Correct the measurements of REFF to REFF at 30 degrees incidence, 0 degrees emission, and 30 degrees phase angle.

References

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Installation, Program Files, User's Guide, and Test Data.

Output(s) from this algorithm to SPOC:

Fits files (for each model) that include the Reflectance Factor (REFF) values and their uncertainties (e.g., Bennu_REFF_Minnaert_data.fits)

Reflectance and Albedo Document:

Compiled by: Driss Takir and Beth Clark. 5/14/2015.V30.
With inputs from Christian Drouet d'Aubigny, Jian-Yang Li, Carl Hergenrother, and Josh Emery, Bonnie Buratti

BIDIRECTIONAL REFLECTANCE AND ALBEDO QUANTITIES

Introduction: Most reflectance and albedo quantities are unitless quantities and are frequently misused or confused. Here, we review these quantities and summarize how viewing conditions vary for each quantity, relevant to OSIRIS-REx. Caution: our target asteroid is very dark, and multiple scattering is assumed to be relatively unimportant, but this assumption may not hold for brighter asteroids.

I. Bidirectional Reflectance Quantities

1. Radiance Factor (RADF)

Hapke:

Radiance Factor (RADF) is the **ratio** of the bidirectional reflectance of a surface **to** that of a perfectly diffuse surface illuminated at *i = 0* (the Sun!), rather than at the same angle of illumination.

(Hapke 1993)

$$RADF(i,e,\alpha) = \pi r(i,e,\alpha),$$

where

$$r(i,e,\alpha) = \frac{\varpi_o}{4\pi}\frac{\mu_o}{\mu_o + \mu}\{[1 + B(\alpha)]\,p(\alpha) + H(\mu_o)H(\mu) - 1\}\,S(i,e,\alpha),$$

(The factor of π here is from the normalization and integration of the $p(\alpha)$).

(Hapke 1993 eq. 8.89)

$\mu_o = \cos(i)$, $\mu = \cos(e)$, *i* is the incidence angle (degrees), *e* is the emission angle (degrees), ϖ_o is the average particle single scattering albedo, $1 + B(\alpha)$ is the opposition effect, $p(\alpha)$ is the average particle single-scattering phase function, $(H(\mu_o)H(\mu) - 1)$ describes the isotropic multiple scattering of light, and $S(i,e,\alpha)$ is the macroscopic roughness.

Thus,

$$RADF(i,e,\alpha) = \frac{\varpi_o}{4}\frac{\mu_o}{\mu_o + \mu}\{[1 + B(\alpha)]\,p(\alpha) + H(\mu_o)H(\mu) - 1\}S(i,e,\alpha)$$
$$= \pi r(i,e,\alpha) = [I/\mathcal{F}](i,e,\alpha),$$

I is the radiance and has units of W/m^2/nm/steradian. $J = \pi\mathcal{F}$ is the collimated light (irradiance) and has units of W/m^2/nm. Strictly speaking $I/\mathcal{F}$ is a dimensionless quantity ($\mathcal{F}$ has units of W/m^2/nm/steradian and π here has units of steradian).

(Hapke 1993)

(*RADF [I/F] is* what is measured by the spacecraft)

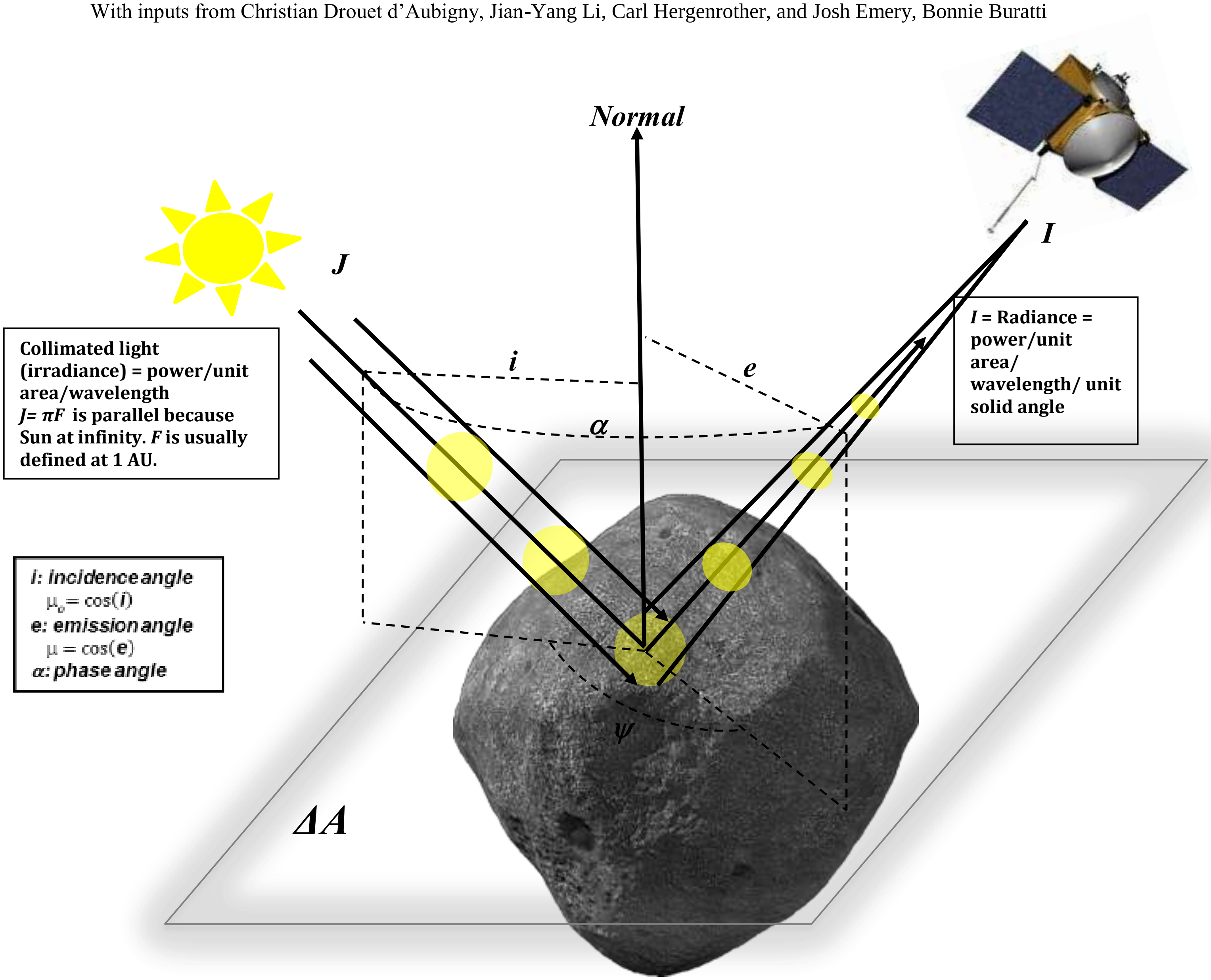

Figure 1. Schematic diagram of bidirectional reflectance from a surface element ΔA, showing the various angles. The plane containing ***J*** and ***I*** is the scattering plane. If the scattering plane also contains ***N***, it is called the principal plane. ψ is the azimuthal angle between the planes of incidence and emission [$\cos(\alpha)=\cos(i)\cos(e)+\sin(i)\sin(e)\cos(\psi)$.]
[Adopted from Hapke (1993)]

Lambert Model:

$$RADF(i,e,\alpha) = A_L\mu_o = [I/\mathcal{F}](i,e,\alpha),$$

where A_L is the Lambert albedo.

(Lambert 1759)

The Lambert model is a disk function that accounts only for limb darkening. However, A_L could be a function of phase angle (α).

Lambert Model with the Phase Function:

$$RADF(i,e,\alpha) = A_L f(\alpha)\mu_o = [I/\mathcal{F}](i,e,\alpha),$$

where $f(\alpha) = 10^{-\frac{(\beta\alpha+\gamma\alpha^2+\delta\alpha^3)}{2.5}}$ is a 3rd order polynomial phase function.

(d'Aubigny 2011)

This Lambert model accounts for limb darkening *and* the surface phase function.

Minnaert Model Disk Function:

$$RADF(i,e,\alpha) = \pi A_M \mu_o^k\, \mu^{k-1} = [I/\mathcal{F}](i,e,\alpha),$$

(Minnaert 1941)

where A_M and *k* are model parameters that characterize the Minnaert albedo and limb-darkening behavior of the surface, respectively.
The Minnaert disk function accounts only for limb darkening. However, both πA_M and *k* could be functions of phase angle(α).

Minnaert Model with the Phase Function:

$$RADF(i,e,\alpha) = \pi A_M f(\alpha)\mu_o^{k(\alpha)}\, \mu^{k(\alpha)-1} = [I/\mathcal{F}](i,e,\alpha),$$

where $\mathrm{f}(\alpha) = 10^{-\beta\alpha/2.5}$, β is the phase slope, and $k(\alpha) = k_o + b\alpha$ characterizes the limb-darkening behavior of the surface and *b* captures the linear relationship between *k* and phase angle (α). k_o is the value of k at zero degrees phase angle.

(Li et al. 2009)

Or

$f(\alpha) = 10^{-\frac{(\beta\alpha+\gamma\alpha^2+\delta\alpha^3)}{2.5}}$ (d'Aubigny 2011)

This Minnaert model includes the effects of limb darkening *and* the surface phase function.
Note: when $i = 0$ $\mu_o \rightarrow 1$ and $e = 0$ $\mu \rightarrow 1$, $RADF(0,0,0) = \pi A_M\, f(\alpha)$, which is consistent with normal reflectance when $f(\alpha)$ is normalized to 1 at $\alpha = 0$.

Lommel-Seeliger Disk Function:

$$RADF(i,e,\alpha) = \frac{\varpi_o}{4}\frac{\mu_o}{\mu_o + \mu} = [I/\mathcal{F}](i,e,\alpha)\,,$$

(Seeliger 1884)

where ϖ_o is the average particle single scattering albedo and $f(\alpha)$ is an arbitrary function that describes the variation in surface reflectance with phase angle.
The Lommel-Seeliger disk function accounts only for limb darkening. However, ϖ_o could be a function of phase angle(α).

Lommel-Seeliger Model with the Phase Function:

$$RADF(i,e,\alpha) = \frac{\varpi_o}{4}\frac{\mu_o}{\mu_o + \mu} f(\alpha) = [I/\mathcal{F}](i,e,\alpha)\,,$$

(Helfenstein and Veverka 1989)

where $f(\alpha)$ is an arbitrary function that describes the variation in surface reflectance with phase angle (α), and $A_{LS} = \frac{\varpi_o}{4\pi}$ is Lommel-Seeliger albedo.

$$f(\alpha) = e^{\beta\alpha + \gamma\alpha^2 + \delta\alpha^3}$$

This Lommel-Seeliger/Veverka model includes the effects of limb darkening *and* the surface phase function.

ROLO Model:

$$RADF(i,e,\alpha) = \underbrace{\frac{\mu_o}{\mu_o + \mu}}_{\text{Describes limb darkening}} f(\alpha) = [I/\mathcal{F}](i,e,\alpha),$$

Describes limb darkening

Where $f(\alpha) = \underbrace{C_0 e^{-C_1\alpha}}_{\text{opposition surge}} + \underbrace{A_0 + A_1\alpha + A_2\alpha^2 + A_3\alpha^3 + A_4\alpha^4}_{\text{phase function}}$. (Buratti et al. 2011)

Describes opposition surge — 4th order polynomial that describes phase function

The ROLO model includes the effects of limb darkening *and* the surface phase function.

(These are the RADFs we will use to model Bennu's surface)

2. Reflectance Factor (REFF)

Reflectance Factor (or reflectance coefficient) (REFF) is the **ratio** of the reflectance of the surface **to** that of a perfectly diffuse (Lambert) surface under the same conditions of illumination.

(Hapke 1993)

$$REFF(i,e,\alpha) = \frac{r(i,e,\alpha)}{\frac{\mu_o}{\pi}} = \frac{\pi r(i,e,\alpha)}{\mu_o},$$

Thus

$$REFF(i,e,\alpha) = \frac{\varpi_o}{4}\frac{1}{\mu_o+\mu}\{[1+B(\alpha)]\,p(\alpha)+H(\mu_o)H(\mu)-1\}-1\}S(i,e,\alpha) = \frac{[I/\mathcal{F}]\,(i,e,\alpha)}{\mu_o}$$

(This reflectance quantity is what is measured in the laboratory. For OVIRS spectral indices, the OVIRS data will be in these units.)

$$REFF\ (i{=}0,\ e{=}0,\ \alpha{=}0) = \frac{I/\mathcal{F}(i=0,e=0,\alpha=0)}{\mu_o} = \textit{Normal Reflectance.}$$

(This is the most reasonable quantity to use when mapping the "albedo" of the surface.)

3. Hapke Bidirectional Reflectance Distribution Function (BRDF)

Bidirectional Reflectance Distribution Function (*BRDF*) is the **ratio** of the radiance scattered by a surface into a given direction **to** the collimated power incident on a unit area of the surface.

(Hapke 1993)

$$BRDF(i,e,\alpha) = \frac{Jr(i,e,\alpha)}{J\mu_o} = \frac{r(i,\ e,\alpha)}{\mu_o},$$

$J\mu_o$ is the incident radiant power per unit area of surface and $Jr(i,e,\alpha)$ is the scattered radiance.

Thus

$$BRDF(i,e,\alpha) = \frac{\varpi_o}{4\pi}\frac{1}{\mu_o+\mu}\{[1+B(\alpha)]\,p(\alpha)+H(\mu_o)H(\mu)-1\}S(i,e,\alpha)$$
$$= [I/(\mu_o\pi\mathcal{F})](i,e,\alpha).$$

(Functions of this form are requested by OSIRIS Instrument Teams to be used to predict Bennu's brightness. See conclusion Table 2 showing relationship between RADF and BRDF.)

II. Albedo Quantities

1. Lambertian Albedo

Lambertian albedo(A_L) is the **ratio** of the total power scattered per unit area of a Lambert surface to the incident power per unit area.

(Hapke 1993)

$$A_L = \frac{P_L}{J\mu_o},$$

$P_L = \int_{2\boldsymbol{\pi}} I(i,e,\ \alpha)\,\mu\,d\alpha = \int_{e=0}^{\pi/2}\int_{\alpha=0}^{2\pi} JK_L \cos i \cos e \sin e\; de\; d\alpha = \pi JK_L\mu_o$ is the total power scattered per unit area of Lambert surface into all directions of the upper hemisphere.
Where $K_L = r_L(i,e,\alpha)/\mu_o$ is a constant (*Lambert's law*). When K_L is constant then we have a Lambertian surface.

Thus the *Lambert reflectance* is $\pi r_L(i,e,\alpha) = A_L\,\mu_o = RADF(i,e,\alpha) = [I/\mathcal{F}](i,e,\alpha)$.

Therefore $$A_L = [I/(\mu_o\mathcal{F})](i,e,\alpha).$$

2. Geometric Albedo

Physical albedo (a.k.a, Geometric albedo)(A_{geo}) is the **ratio** of the brightness of a body at zero phase angle $\alpha = 0$ **to** the brightness of a perfect *Lambert disk* of the same radius and at the same distance as the body, but illuminated and observed perpendicularly.

(Hapke 1993)

$$A_{geo} = \int_{2\pi} r(e,e,0)\,\mu d\Omega,$$

where $d\Omega = 2\,\pi\,\sin(e)de = -2\,\pi\,d\mu$.

Geometric Albedo is usually presented at one wavelength, the V passband, or 0.55 µm.

2.1 Lommel_Seeliger

$$A_{geo} = \frac{A_{LS}}{2}\,\pi\, f(0)\int_0^{\pi/2}\cos(e)\sin(e)\,de = \frac{A_{LS}}{2}\pi\, f(0)$$

2.2 ROLO

$$A_{geo} = f(0)\int_0^{\frac{\pi}{2}}\cos(e)\sin(e)\,de = \frac{f(0)}{2}$$

2.3 Minnaert:

$$A_{geo} = 2\pi\, A_M\, f(0)\int_0^{\pi/2}[\cos(e)]^{2k_o}\sin(e)\,de = A_M\ \frac{2\pi}{2k+1}f(0)$$

k_o is the value of $k(\alpha)$ at zero degrees phase angle.

3. Normal Albedo

The *normal albedo* A_n is the ratio of the brightness of a surface observed at zero phase angle from an arbitrary direction to the brightness of a perfectly diffuse surface located at the same position, but illuminated and observed perpendicularly.

$$A_n = \frac{[Jr(e,e,0)]}{[\frac{J}{\pi}]} = \pi r(e,e,0)$$

4. Spherical Bond Albedo

Spherical bond albedo (a.k.a., Bond albedo) (A_{sph}) is the total fraction of incident irradiance at one wavelength (usually 0.55 µm) scattered by the body into all directions.

(Hapke 1993)

$$A_{sph} = \frac{1}{\pi}\int_{2\pi}\int_{2\pi} r(i,e,\alpha)\ \mu\, d\,\Omega_i\, d\,\Omega_e,$$

where $d\Omega_i$ = sin(i) $di\, d\psi$ *and* $d\Omega_e$ = sin(e) $de\, d\psi$ with ψ is the azimuth.

The *spherical bond* albedo can also be expressed as $q\ A_{geo}$, where q is the phase integral, defined as:

$$q = 2\int_0^{\pi} \Phi(\alpha)\sin(\alpha)\, d\alpha$$

where $\Phi(\alpha) \equiv \frac{F(\alpha)}{F(0^o)}$ is the disk-integrated brightness at phase angle α, assuming a spherical body (Buratti and Veverka 1983). $F(\alpha)$ is the phase dependence of the disk-integrated flux defined as:

$$F(\alpha) = \frac{R^2}{r^2}\int_{\alpha-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{\alpha-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{I}{F}(i,e,\alpha)\cos(w)\cos^2(\psi)dw\, d\psi\ ,$$

where w = photometric longitude, ψ = photometric latitude, R = radius of the satellite, and r = observer-satellite distance.

(This is the albedo quantity shown in Table 1 that requires observations covering phase angles from 0^o→180^o.)

4. Bolometric Bond Albedo

Bolometric bond albedo (A_{bolo}) is the average of the spherical *Bond albedo* $A_{sph}(\lambda)$ weighted by spectral irradiance of the Sun $J_s(\lambda)$. This integrates *Spherical albedo* over all λ.

$$A_{bolo} = \frac{\int_0^\infty A_{sph}(\lambda)\, J_S(\lambda) d\lambda}{\int_0^\infty J_S(\lambda) d\lambda},$$

where $J_S(\lambda)$ is the solar flux spectrum (λ).

(The OSIRIS-REx Science Team has adopted the solar flux model of Reike et al. 2008)

(This is the quantity required for Yarkovsky and thermal inertia measurements for OSIRIS-REx)

III. Examples of Reflectance and Albedo Quantities:

Table 1a. Comparison of reflectance and albedo quantities for different asteroids:

	Ceres[0]	Ida[1]	Eros[2]	Eros[3]	Dactyl[1]	Gaspra[1]	Mathilde[4]	Vesta[5]	Bennu[6,7]	Phobos[8]	Deimos[8]
Geometric Albedo	0.088	0.206	0.290	0.23	0.198	0.23	0.047	0.38±0.01	0.045	0.071	0.068
Spherical Bond Albedo	0.020	0.081	0.12	0.093	0.073	0.12	--	0.20±0.02	0.016	0.021	0.027
Normal Reflectance	--	0.207	--	--	0.198	0.23	0.047	--	--	0.071	0.068

[0]Li et al. (2006). [1]Helfenstein et al. 1994, [2]Domingue et al. 2002, [3]Li et al. 2004, [4]Clark et al. 1999, [5]Li et al. 2013, [6]Hergenrother et al. 2013. [7]Emery et al. (2014). The geometric albedo and normal reflectance values are for 0.55 µm. Note that Helfenstein and Domingue do not seem to agree on the meaning of Bond albedo terms.[8]Simonelli et al. (1998) and Thomas et al. (1996).

Table 1b. Comparison of reflectance and albedo quantities for different comets:

	9P/Tempel 1[1]	19P/Borrelly[2]	81P/Wild 2[3]	28P/Neujmin 1[4]	2P/Encke[5]
Geometric Albedo	0.059±0.009	0.080±0.020	0.059	0.026	0.047
Spherical Bond Albedo	0.014±0.002	0.018	0.0093	--	--

[1]Li et al. (2013).[2]Li et al. (2007).[3]Li et al. (2009).[4]Campins et al. (1987). [5]Fernandez et al. (2000)

IV. Conclusions

- The value of albedo measured with an integrating sphere (in the laboratory) can be comparable to the *spherical bond* albedo of a body covered with the same material (Barucci et al. 2012).
- The *bolometric bond* albedo is *not* equal to the *spherical bond* albedo. The bolometric albedo is the average of the spectral bond albedo weighted by spectral irradiance of the Sun.
- It is generally assumed that the *spherical bond* albedo in the V passband (~0.55 μm) is a good representation of the *bolometric bond* albedo. This is because (a) most of the Sun's energy is in the visible and (b) most spectra of Solar System bodies do not change drastically over the UV/Vis (Emery, personal communication).
- For disk-resolved observations and Lommel-Seeliger surfaces, the value of $I/(\mu_o\mathcal{F})$ is close to the value of the *geometric* albedo at wavelength λ when observed at a phase angle $\alpha = 0$. $I/\mathcal{F} = 1$ is for a flat *Lambertian* surface when viewed at normal incidence.
- For disk-integrated observations and Lommel-Seeliger surfaces, the *geometric* albedo, which is similar to the *normal* albedo, is a measure of a surface's brightness relative to a perfectly scattering Lambertian disk.
- The Lambert and Minnaert functions are disk functions with no dependence on phase angle and account only for limb darkening. However, the Hapke, Minnaert, the Lommel-Seeliger/Veverka, and ROLO functions include surface phase functions and limb darkening.
- For Bennu and other dark objects, the normal reflectance value is close to (1/π) times the geometric albedo value for Lommel-Seeliger surfaces. The normal reflectance is the most reasonable quantity to use when mapping the "albedo" of the surface.
- Lester et al. (1979), who called the normal albedo (p_n), found that the geometric albedo (p) is equivalent to the normal albedo (p_n) for Lommel-Seeliger surfaces.
- **Table 2.** In this table we show various models converted from RADF to BRDF by division of $\pi\mu_o$. Note that in our equations, $I/\mathcal{F}$ is unitless, where I = measured radiance from the surface in W/m^2/sr/nm and $J = \pi\mathcal{F}$ = solar irradiance (flux) in W/m^2/nm. π has units of steradian and $\mathcal{F}$ units of W/m^2/nm/steradian.

	RADF	***BRDF***
	$[I/\mathcal{F}](i,e,\alpha) =$	$\frac{[I/\mathcal{F}](i,e,\alpha)}{\pi\mu_o} =$
Lambert Model	$A_L f(\alpha)\mu_o$	$\frac{A_L f(\alpha)}{\pi}$
Minnaert Model	$\pi A_M f(\alpha)\mu_o^{k(\alpha)}\ \mu^{k(\alpha)-1}$	$A_M f(\alpha)\mu_o^{k(\alpha)-1}\ \mu^{k(\alpha)-1}$
Lommel Seeliger Model	$\frac{\varpi_o}{4}\frac{\mu_o}{\mu_o+\mu} f(\alpha)$	$\frac{\varpi_o}{4\pi}\ \frac{1}{\mu_o+\mu}\ f(\alpha)$
ROLO Model	$\frac{\mu_o}{\mu_o+\mu} f(\alpha)$	$\frac{1}{\pi}\ \frac{1}{\mu_o+\mu}\ f(\alpha)$

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