Average Worst-Case Secrecy Rate Maximization via UAV and Base Station Resource Allocation

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Abstract—In this paper, we consider a wireless network setting where a base station (BS) employs a single unmanned aerial vehicle (UAV) mobile relay to disseminate information to multiple users in the presence of multiple adversaries. The BS, which is on the ground, has no direct link to the users or the adversaries, who are also on the ground. We optimize the joint transmit power of the BS and the UAV, and the UAV trajectory. We introduce the information causality constraint and maximize the average worst-case secrecy rate in the presence of the adversaries. The formulated average worst-case secrecy rate optimization problem is not convex and is solved sub-optimally. First, we optimize transmit power of the BS and the UAV under a given UAV trajectory. Then, we optimize the UAV trajectory under the sub-optimal UAV and BS transmit power. An efficient algorithm solves the average worst-case secrecy rate maximization problem iteratively until it converges. Finally, simulation results are provided, which demonstrate the UAV optimal track and transmit power allocation.

Keywords—resource allocation, average worst-case secrecy rate, information causality constraint, adversary, UAV, BS.

I. INTRODUCTION

The unmanned aerial vehicles (UAVs) have attracted research attention because of their many potential applications [1], [2], e.g., military surveillance, remote monitoring, and disaster recovery support. Moreover, UAV deployment is a practical approach for improving system throughput, reliability, coverage, latency, and range. This is because the UAV is mobile and often has a line of sight (LOS) advantage to the receiver. Unfortunately, the broadcast nature of the air-to-ground link is also detrimental to security. This trade-off between security and utility requires finding an optimal system configuration. However, the resulting optimization problem has many variables (including, but not limited to, UAV flight path, receivers’ and adversaries’ locations, power constraints, and information flow characteristics) and, in many cases, is not convex. Thus, one has to either simplify the model or accept a sub-optimal solution.

There are many examples of simplified models for UAV-assisted networks in the literature [3] - [5]. In [6], the authors focused on the UAV altitude optimization to serve ground users for a given area. However, alongside the altitude, UAV mobility on the horizontal plane significantly improves the system performance, as it can significantly shorten the distance between the UAV to the users. The authors in [7] provided a tractable framework in two-dimensional space to optimize the UAV trajectory. However, they did not consider service to multiple users on the ground and physical layer security.

Fig. 1: The UAV relay disseminates information from a single base station (BS) to U users on the ground in the presence of E adversaries. Direct BS-to-user links are unavailable due to the tall obstacles.

The joint optimization of transmit power of the UAV and the UAV trajectory for secrecy rate optimization in the presence of adversaries was investigated in [8], while the physical layer security was studied in [9]. However, neither [9] nor [8] investigates trajectory design considering multiple users and multiple adversaries.

The UAV-assisted mobile relaying system with multiple ground users has been studied in [10]. The authors considered a scenario where the users are located on a one-dimensional line. Though the authors achieved a significant improvement of the overall system throughput compared to the static UAV relaying system, the practical application of the model is limited.

While the papers referenced above obtain optimal solutions on simplified models, here we employ a more practical model and obtain a sub-optimal solution using an efficient algorithm. We consider a scenario depicted in Fig. 1, where a base station (BS) has to disseminate data to multiple users without a direct link. Thus, UAV assistance is required to relay the data in the presence of eavesdropping adversaries. While we seek to optimize the joint transmit power of the BS and the UAV, and the UAV trajectory to achieve the average
worst-case secrecy rate, this problem is not convex. We thus propose an efficient algorithm based on successive convex approximation (SCA) [11], and first-order Taylor expansion [12]. We find the optimal transmit power of the BS and the UAV for some UAV trajectory (at a constant altitude) under a given secrecy constraint. We then fix power and optimize the UAV trajectory solution, repeating until convergence. We introduce the information causality constraint [13] to capture the broadcast nature of communication from the BS to the users. We also consider the average and peak transmit power constraints of the UAV and the BS to achieve the average worst-case secrecy rate. In simulation results, we analyze the UAV trajectory with the change of UAV transmit power when information causality constraint is turned on/off.

The paper is organized as follows. We present our system model in Section II. We formulate the average worst-case secrecy rate maximization problem in Section III. We show a tractable approach to solve the formulated non-convex problem in Section IV. We validate the proposed algorithm via the simulation in Section V. Finally, we conclude in Section VI.

II. System Model

We consider dissemination of information from the base station (BS) to multiple users via an unmanned aerial vehicle (UAV) relay in the presence of multiple adversaries, as depicted in Fig. 1. The BS, the users, and the adversaries are on the ground, with no direct communication link from the BS to the users and the adversaries. Each of the nodes is equipped with a single antenna. The sets of users and the adversaries are $U = \{1, 2, 3, ..., U\}$, and $E = \{1, 2, 3, ..., E\}$, respectively. In the proposed system, the BS establishes a link to the UAV, and the UAV establishes the links to the legitimate users on the ground while the adversaries attempt to intercept the information on these links. The UAV flies at a fixed altitude and knows the locations of the users, the BS, and the adversaries.

A. Description of parameters

We assume that the effects of shadowing and multi-path are negligible because of the line of sight. Doppler effect from the mobility of the UAV is assumed to be compensated at the receiver. Here we describe the parameters of our proposed model.

1) Time period: The UAV serves the multiple users in $T$ seconds.

2) Node locations: Our system uses a two-dimensional coordinate system for the BS, the UAV, the adversaries, and the users. We define the locations of the BS, the UAV, the adversaries, and the users as follows: The time-varying location of the UAV is $(x(t), y(t)) \in \mathbb{R}^2$ while its altitude is fixed at $H$. The fixed location of user $u \in U$ is $(x_u, y_u) \in \mathbb{R}^2$. The location of adversary $e \in E$ is $(x_e, y_e) \in \mathbb{R}^2$. Finally, the location of the BS is $(x_b, y_b) \in \mathbb{R}^2$. We assume that the UAV knows the locations of the users, the adversaries, and the BS.

3) Data rate: Time $t$ is continuous in nature, however, to simplify the model, we use $N$ discrete time slots of duration $\rho = \frac{T}{N}$ seconds each. The initial and the final locations of the UAV are $(x[2], y[2])$ and $(x[N], y[N])$, respectively. The BS to the UAV channel gain is:

$$c_{b}[n] = \frac{\beta_0}{(x[n] - x_b)^2 + (y[n] - y_b)^2 + H^2},$$

(1)

where $\beta_0$ defines the channel power gain at the reference distance $d_0 = 1$ m [14]. The ergodic data rate between the BS and the UAV is:

$$r_{b}[n] = \log_2 \left(1 + \frac{p_{b}[n]c_{b}[n]}{\sigma^2}\right), \quad n = 1, 2, ..., N - 1.$$  

(2)

where $\sigma^2$ is the additive white Gaussian noise (AWGN) power at the receiver. $p_{b}[n] \in \mathbb{R}^+$ is the BS transmit power in the $n^{th}$ time slot.

Similarly, the channel gain between the UAV and user $u$ is:

$$c_{u}[n] = \frac{\beta_0}{(x[n] - x_u)^2 + (y[n] - y_u)^2 + H^2},$$

(3)

The data rate between the UAV and the user $u$ is:

$$r_{u}[n] = \log_2 \left(1 + \frac{p_{u}[n]c_{u}[n]}{\sigma^2}\right), \quad n = 2, 3, ..., N.$$  

(4)

where $p_{u}[n] \in \mathbb{R}^+$ is the UAV transmit power in the $n^{th}$ time slot.

Similarly, the channel gain between the UAV and the adversary $e$ is:

$$c_{e}[n] = \frac{\beta_0}{(x[n] - x_e)^2 + (y[n] - y_e)^2 + H^2},$$

(5)

The data rate between the UAV and the adversary $e$ is:

$$r_{e}[n] = \log_2 \left(1 + \frac{p_{e}[n]c_{e}[n]}{\sigma^2}\right), \quad n = 2, 3, ..., N.$$  

(6)

4) The BS and the UAV transmit power constraint: When we calculate the average worst-case secrecy rate, we consider the average and peak transmit power constraints of the BS and the UAV. The average power constraint of the BS is:

$$\sum_{n=1}^{N-1} p_{b}[n] \leq (N - 1)p_{b}^{avg},$$

(7)

where $p_{b}^{avg}$ is the average power of the BS. The peak power constraint of the BS is:

$$0 \leq p_{b}[n] \leq p_{b}^{max},$$

(8)

where $p_{b}^{max}$ is the maximum transmit power of the BS. Similarly, the average power constraint of the UAV is:

$$\sum_{n=2}^{N} p_{u}[n] \leq (N - 1)p_{uav}^{avg},$$

(9)

where $p_{uav}^{avg}$ is the average power of the UAV. The peak power constraint of the UAV is:

$$0 \leq p_{u}[n] \leq p_{uav}^{max},$$

(10)

where $p_{uav}^{max}$ is the maximum transmit power of the UAV.
5) Information causality constraint: The BS can send information to the UAV in any time slot, which is then forwarded to the user \( u \) by the UAV over its flight time. By information causality constraint in the \( n \)th time slot, we ensure that the UAV forwards only the received information from the BS to the user \( u \) in any of the remaining time slots [9, Eq. (2)] is:

\[
\begin{align*}
    r_u[1] &= 0, \\
    \sum_{i=2}^{n} r_u[i] &\leq \sum_{i=1}^{n-1} r_b[i], \quad n = 2, 3, \ldots, N. \quad (12)
\end{align*}
\]

We note that the BS does not transmit information to the UAV at the last time slot, i.e., \( p_b[N] = 0 \), and the UAV does not relay information to the nodes in the first time slot, i.e., \( p_u[1] = 0 \). Thus, \( r_u[1] = r_c[1] = r_b[N] = 0 \).

### III. Maximization Problem Formulation

The average worst-case secrecy rate optimization problem is:

\[
\begin{align*}
    \max_{x[n], y[n], p_u[n], p_b[n]} & \quad \sum_{n=2}^{N} \left[ r_u[n] - r_c[n] \right], \\
    \text{s.t.} & \quad (x[n+1] - x[n])^2 + (y[n+1] - y[n])^2 \leq (v\rho)^2, \quad (13a) \\
    & \quad (7) - (10), \quad (12).
\end{align*}
\]

where \( r_u[n] \) and \( r_c[n] \) are defined in (6) and (4), respectively.

The objective function in (13a) is the information-theoretic secrecy rate [15], used extensively as the secure information rate in the literature [13]. The UAV mobility constraint [8, Eq. (3)] is defined in (13b), where \( v \) is the UAV maximum speed. However, the optimization problem in (13) is not convex. Thus, we propose a tractable approach to solve (13) sub-optimally in the next section.

### IV. Proposed Sub-optimal Solution

First, for initially fixed UAV position \( (x[n], y[n]) \), where \( n = 2, \ldots, N \), we optimize the UAV and the BS transmit power \( p_u[n] \) and \( p_b[n] \), respectively. Second, we use \( p_u[n] \) and \( p_b[n] \) from the previous step to optimize the UAV trajectory \( (x[n], y[n]) \). We then repeat until convergence, as formalized in Algorithm 1 at the end of this section. However, first, we describe the mathematical details.

A. The UAV and the BS transmit power optimization

We first fix the UAV position to optimize transmit power of the UAV and the BS. Thus, we remove the mobility constraint from optimization problem in (13), yielding:

\[
\begin{align*}
    \max_{p_u[n], p_b[n]} & \quad \sum_{n=2}^{N} \left[ r_u[n] - r_c[n] \right], \\
    \text{s.t.} & \quad (7) - (10), \quad (12).
\end{align*}
\]

This is not yet solvable optimally because of (14) and (12) being non-convex. We modify information causality constraint in (12) by introducing a variable as follows:

\[
\sum_{i=1}^{n-1} r_b[i] \geq \sum_{i=2}^{n} j[i], \quad (15)
\]

where \( j[n] \) is newly introduced variable. However, (15) - (16) are equivalent to (12). Thus, we reformulate (14) as follows:

\[
\begin{align*}
    \max_{p_u[n], p_b[n]} & \quad \sum_{n=2}^{N} \left[ j[n] - r_c[n] \right], \\
    \text{s.t.} & \quad (15) - (16), \quad (7) - (10).
\end{align*}
\]

The UAV to the user \( u \) data rate is replaced with \( j[n] \) in (17). Now we reformulate \( r_u[n] \) using the first order Taylor series expansion at fixed feasible point as follows:

\[
\begin{align*}
    \max_{p_u[n], p_b[n], j[n]} & \quad \sum_{n=2}^{N} \left[ j[n] - c_e[n](p_u[n] - p_u[n]) - r_e[n] \right], \\
    \text{s.t.} & \quad (15) - (16), \quad (7) - (10).
\end{align*}
\]

where

\[
\begin{align*}
    r_e[n] = \log_2 \left( 1 + \frac{p_u[n] c_e[n]}{\sigma^2} \right). \quad (19)
\end{align*}
\]

Here (14) and (18) with its related constraints are equivalent and share the same sub-optimal solution because after several transformations of (14), we achieve (18) and its related constraints. However, optimization problem in (14) is now solvable using the standard convex optimization software, such as CVX [16].

B. The UAV trajectory design

Now we determine the UAV trajectory using the solution of the UAV and the BS transmit power from Section IV-A. The formulated UAV trajectory optimization problem from (13) is:

\[
\begin{align*}
    \max_{x[n], y[n]} & \quad \sum_{n=2}^{N} \left[ r_u[n] - r_c[n] \right], \\
    \text{s.t.} & \quad (12), \quad (13b).
\end{align*}
\]

where \( r_u[n] \) and \( r_c[n] \) are defined in (4) and (6), respectively. However, (20) with its related constraints is not a convex problem because of (20) and (12). We can tackle the non-convexity of the UAV trajectory design in (20) by introducing the slack variables. Thus, it is reformulated as follows:

\[
\begin{align*}
    \max_{x[n], y[n]} & \quad \sum_{n=2}^{N} \left[ r_u^g[n] - r_c^g[n] \right], \\
    \text{s.t.} & \quad (x[n] - x_u)^2 + (y[n] - y_u)^2 + H^2 - g[n] \leq 0, \quad (21b) \\
    & \quad (x[n] - x_c)^2 + (y[n] - y_c)^2 + H^2 - z[n] \geq 0, \quad (12c), \quad (13b).
\end{align*}
\]

where \( \gamma = \frac{\sigma^2}{\gamma} \) and

\[
\begin{align*}
    r_u^g[n] &= \log_2 \left( 1 + \gamma p_u[n] \right), \quad (22) \\
    r_c^g[n] &= \log_2 \left( 1 + \gamma p_u[n] \right), \quad (23) \\
    g[n] \text{ and } z[n] \text{ are the slack variables:}
\end{align*}
\]

\[
\begin{align*}
    g \Delta [g[1], g[2], g[3], \ldots, g[N]]^t, \quad (24)
\end{align*}
\]
where $z \triangleq [z[1], z[2], z[3], \ldots, z[N]]^\dagger$.  

Now we use (15) - (16) to reformulated (12) as follows:

$$
\sum_{i=1}^{n-1} r_b[i] \geq \sum_{i=2}^{n} j[i],
$$

(26)

$$
\gamma_p h[i] = j[i],
$$

(27)

where $j[i]$ is the newly added variable. However, (26) - (27) are not convex problem. Introducing the slack variable, (26) can be reformulated as:

$$
\sum_{i=1}^{n-1} r_b[i] \geq \sum_{i=2}^{n} j[i],
$$

(28)

$$
(x[i] - x_h)^2 + (y[i] - y_h)^2 + H^2 - h[n] \leq 0.
$$

(29)

where

$$
r_b[i] = \log_2 \left( 1 + \gamma_p h[i] / h[i] \right). 
$$

(30)

where $h[n]$ is newly added slack variable. Slack variable, $h[n]$ can be expressed as follows:

$$
h \triangleq [h[1], h[2], h[3], \ldots, h[N]]^\dagger.
$$

(31)

Now let’s reformulate (28) using the first order Taylor series expansion at a feasible point $h_f[i]$:

$$
\frac{\gamma_p h[i](h_f[i] - h[i])}{h_f[i](h_f[i] + \gamma_p h[i])} + r_b[i] \leq r_b[i].
$$

(32)

Thus, using (32), (26) can be reformulated as:

$$
\sum_{i=2}^{n} j[i] \leq \sum_{i=1}^{n-1} \left[ \frac{\gamma_p h[i](h_f[i] - h[i])}{h_f[i](h_f[i] + \gamma_p h[i])} + r_b[i] \right].
$$

(33)

Similarly, we address the non-convexity of (27). First, we add a new slack variable $m[n]$:  

$$
r_m h[n] \geq j[n],
$$

(34)

$$
(x[n] - x_h)^2 + (y[n] - y_h)^2 + H^2 - m[n] \leq 0.
$$

(35)

where

$$
r_m h[n] = \log_2 \left( 1 + \gamma_p h[n] / m[n] \right). 
$$

(36)

Now let’s reformulate (39) using the first order Taylor series expansion at a feasible point $m_f[n]$:

$$
\frac{\gamma_p h[n](m_f[n] - m[n])}{m_f[n](m_f[n] + \gamma_p h[n])} + r_m [n] \leq r_m [n],
$$

(37)

Thus, using (38), (27) can be reformulated as follows:

$$
\gamma_p h[n] \geq \gamma_p h[n] (m_f[n] - m[n]) / m_f[n] \gamma_p h[n] + \gamma_p h[n] \ln 2 + r_m [n].
$$

(39)

\begin{align*}
\text{We reformulate (21) as follows:} \\
\max_{x[n], y[n], j[n], m[n], z[n], h[n]} \sum_{n=2}^{N} \left[ j[n] - r_e[n] \right],
\end{align*}

(40)

\begin{align*}
\text{s.t. (13b), (21c), (29), (34), (36), (40).}
\end{align*}

Using the first order Taylor series expansion, (41) is reformulated as follows:

$$
\max_{x[n], y[n], j[n], m[n], z[n], h[n]} \sum_{n=2}^{N} \left[ j[n] - a[n] - r_e[n] \right],
$$

(42)

\begin{align*}
\text{s.t. (13b), (21c), (29), (34), (36), (40).}
\end{align*}

where

$$
a[n] = \frac{\gamma_p h[n] - z[n]}{\gamma_p h[n] + \gamma_p h[n] \ln 2},
$$

(43)

$$
r_e[n] = \log_2 \left( 1 + \gamma_p h[n] / z[n] \right).
$$

(44)

$z[n]$ defines the slack variable $z[n]$ at feasible point. Thus, (42) is a convex problem [17, Section 3] and shares the same solution with (20). Moreover, (42) can also be solved using standard optimization software, such as CVX.

\section{Proposed efficient algorithm}

We encode the approach in Sections IV-A and IV-B in an efficient algorithm to solve the average worst-case secrecy rate optimization problem in (13) sub-optimally. We employ successive convex optimization (SCA).

\begin{algorithm}
1: \textbf{Input} : Define $(x_u, y_u)$, $(x_h, y_h)$, and $\gamma$. Set an initial UAV position $(x[n], y[n])$.
2: \textbf{Set iteration number, } $m \leftarrow m + 1$. Thus, the UAV initial location is updated from the previous value.
3: \textbf{Initialize} : $x[n] = x_{(m-1)[n]}$, $y[n] = y_{(m-1)[n]}$,
4: Compute $r_b[n]$, where $n = 1, \ldots, N - 1$; $r_u[n]$, where $n = 2, \ldots, N$; and $r_e[n]$, where $n = 2, \ldots, N$.
5: \textbf{Optimization}:
6: \textbf{repeat}:
7: \textbf{Find optimal} $p[n]$, $n = 1, \ldots, N - 1$ and $p_b[n]$, $n = 2, \ldots, N$ using (18) for UAV trajectory $(x[n], y[n])$.
8: \textbf{Find optimal} UAV trajectory $(x[n], y[n])$, $n = 2, \ldots, N$ using (42) and $p_u[n]$ and $p_b[n]$.
9: \textbf{until} convergence
\end{algorithm}

\section{Simulation Results}

Here we present our simulation results. We consider a scenario with two users and two adversaries depicted in Fig. 2. We compare the secrecy rate achieved by Algorithm 1 to that of the simplified scheme when the information causality constraint is turned off. In our investigation, the UAV acceleration/deceleration process is negligible. We choose $T = 15$ s. We assume that the UAV hovers from an initial location and returns to the same location during $T$ time. The location of the BS is (600, 50) m. The UAV altitude and
The optimal UAV power allocation between the UAV and the users for different time slots, considering the UAV maximum transmit power $1$ W, is illustrated in Fig. 3a. During the initial and final time, the required UAV optimal power for Algorithm 1 is less than the UAV optimal power in the event of no information causality constraint. However, the UAV transmit power reaches the same maximum point, the UAV optimal power for both Algorithm 1 and when the information causality constraint is out.

When the UAV maximum transmit power is $3$ W, the optimal UAV power allocation to serve the users is illustrated in Fig. 3b. The UAV optimal transmit power increases with time and becomes stable at some maximum points, and finally, the optimal power drops for Algorithm 1. On the other hand, the required UAV transmit power for no information causality constraint is less than Algorithm 1. This is because the UAV does not depend on the BS once the information causality constraint is turned off.

The optimal UAV power allocation between the UAV and the users for different time slots, when the UAV maximum transmit power is $4$ W, is illustrated in Fig. 3c. The UAV optimal transmit power is higher for the proposed Algorithm 1 than the optimal transmit power when the information causality constraint is turned off. This is because the required UAV transmit power for the Algorithm 1 depends on the distance between BS and the users. However, the BS is far away from the users. Thus, the UAV requires higher power to send the received information from the BS to the users.

VI. CONCLUSIONS

We consider the UAV-assisted wireless networks that optimize the UAV and the BS transmit power, and the UAV trajectory in the presence of multiple adversaries. In our scenario, the UAV is a mobile relay that disseminates the data from the BS to the users. We enforce the information causality constraint to ensure that the UAV can only forward the previously decoded information from the BS to the users. The UAV knows the exact locations of the BS, the adversaries, and the users. We propose an iterative algorithm to maximize the average worst-case secrecy rate over the joint transmit power of the UAV and the BS, and the UAV trajectory in the presence of the adversaries. While our algorithm is sub-optimal, it is computationally tractable. First, transmit power of the UAV and the BS is optimized for the given UAV trajectory. Then, using the BS and UAV transmit power, the UAV trajectory is optimized. Then, this process is repeated until it converges. We present the simulation results based on the proposed Algorithm 1 that shows the UAV hovers between the users and the BS to guarantee the information causality constraint is applied. Moreover, UAV avoids adversaries when it flies. The simulation result also shows the allocation of the UAV transmit power.

REFERENCES

Fig. 3: UAV optimal transmit power changes with $T$ due to the distance between UAV and users. Both for algorithm 1 and no information causality constraint, the UAV transmit power level reaches to the maximum point when the UAV is in the middle of the flight time. (a) the power allocation of the UAV when maximum transmit power is 1 W, (b) the power allocation of the UAV when maximum transmit power is 3 W, and (c) the power allocation of the UAV when maximum transmit power is 4 W.


