AN EXPLORATION OF PARENT-CHILD CONVERSATIONS AT A MATHEMATICS EXHIBIT

by

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As members of the Master's Committee, we certify that we have read the thesis prepared by Brian Bennett, titled An Exploration of Parent-Child Conversations at a Mathematics Exhibit and recommend that it be accepted as fulfilling the thesis requirement for the Master's Degree.

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Final approval and acceptance of this thesis is contingent upon the candidate's submission of the final copies of the thesis to the Graduate College.

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Abstract

Although there is increasing interest in mathematics learning in informal settings, there is still limited research regarding mathematics in science centers and museums, and even less research on the conversations that parents and children have in these settings. This study explores how parent-child conversations at three hands-on mathematics exhibits in a science center affect an informal notion of learning. Six families were videotaped while engaging with mathematics puzzles, and their conversations were coded and analyzed based on the types of learning talk that occurred. Most types of learning talk were observed among all family conversations, however families infrequently made connections to prior knowledge or life experiences. Analyzing conversations revealed that time spent at an exhibit did not have an influence on the types of learning talk that occurred. Additionally, in contrast to prior research, explanations were prevalent amongst the families, and explanations originated from both parents and children.

Chapter 1: Introduction

The research contained within this study on mathematics learning in a science center was first born out of a general interest in informal mathematical learning. What began as an interest in ethnomathematics, the mathematics practiced among cultural groups as expressed through their art, games, etc. (D'Ambrosio, 1985), quickly pivoted towards an interest in informal mathematics learning at home and the role that parents play (Civil, 2002; Moll, Amanti, Neff, and González, 1992). As I began reading and learning about the role that parents play in their children's mathematics learning outside of school, I was presented with the unique opportunity to conduct research at a science center that had a mathematics exhibition opening in partnership with the University of Arizona Mathematics Department.

This mathematics exhibition was designed with the intention of fostering creative thinking and problem solving amongst visitors. It began as a small set of exhibits focused on geometry, but soon expanded into a large exhibition with various logic puzzles and games for visitors to engage with and solve. These exhibits were also designed with input from local mathematics teachers to tie all the activities to educational standards. With my previous reading in parental involvement of their children's mathematics learning, the goal of my study then became to better understand how parents and children interact in this informal mathematics environment. More specifically, my research question became: how do parent-child conversations and interactions affect learning at a mathematics focused science center exhibit? Note that by learning I do not mean learning as measured by formal assessment but measured by the conversations taking place.

As a form of mini pilot study, I began by conducting preliminary observations to determine what age range of children I wanted to focus on and what exhibits prompted the most conversation. I then recruited families for the present study, videotaped their interactions at three specific puzzles, and analyzed their conversations and interactions. The subsequent findings presented in this thesis address my research question via an adapted coding framework for learning talk. In the next chapter, I will present literature related to research in museums and science centers with a focus on family interactions and conversations and exhibit design. First, I discuss why conducting research in these settings makes sense; second, I discuss what research has been conducted in relation to science-focused and mathematics-focused exhibits in museums and science centers.

Chapter 2: Review of the Literature

Why Conduct Research in Museums and Science Centers?

Science centers and museums, along with places like zoos, aquariums, and nature centers, offer a unique setting where people of every age and background can expand their understanding of culture, science, and mathematics without the constraints and formal structure of schools (Haden, 2010). These environments also are conducive to broadening visitors' perceptions and understanding of science and mathematics in a collaborative, engaging, hands-on way that helps foster quality family time (Mokros, 2006). In other words, these informal environments are places where mathematics and science can come alive. With an increasing emphasis in society to bolster Science, Technology, Engineering, and Mathematics (STEM) education, any environment where mathematics and science can come alive is ripe for theoretical and practical research opportunity (Callanan, 2012; Nemirovsky, Felton, & Civil, 2017). In fact, "in recent years there has been an obvious increase in research aimed at describing the nature of parent-child conversations about science in museum settings" (Haden 2010, p 65). Haden suggests that because of this, there is increased opportunity for research to explore how conversations affect learning outcomes in museums and science centers.

My review of the literature demonstrates that research conducted in museums, science centers, and other informal learning environments can generally be placed into one of three categories based on the subject matter of the exhibits: science exhibits, mathematics within science exhibits, and stand-alone mathematics exhibits. From a content perspective, Falk and Dierking (2000) proposed a now widely cited Contextual Model of Learning in which learning within museum settings is situated in one of three contexts: the personal (i.e., visitor motivations), the sociocultural (i.e., family conversations/interactions), and the physical (i.e., exhibit design). Due to the nature of my data collection, the personal context is not as relevant to my study, so I will focus primarily on the latter two contexts, but will make mention of the personal context where appropriate. In the sections to follow I will give an overview of research done in each of these three exhibit categories and what the research says about family conversations/interactions and exhibit design. I begin with research focused on science exhibits and work toward mathematics exhibits, since the latter is the focus of my study.

Research on Science Exhibits

In research conducted at the highly influential Exploratorium in San Francisco, Humphrey and Gutwill (2005) designed and advocated for exhibits that encourage what they have termed Active Prolonged Engagement (APE). These types of exhibits are characterized as: posing challenges that can be solved by doing the activity, encouraging visitors to ask and address their own “what if” questions, ensuring that there are several interesting outcomes of the exhibit activity, and trying to delight visitors rather than confuse them. Szechter and Carey (2009) looked at how parent-child interactions differed at APE and non-APE exhibits. They found that parents and children spent more time and engaged in more learning talk at APE exhibits than non-APE exhibits. Parents elicited more predictions at APE exhibits and children described evidence and gave direction more often at APE exhibits.

In another study looking at family conversations in science centers and museums, Ash (2004) specifically examined how parents' questioning affects children's sense making at dioramas in a natural history museum. The parents in all three families of the study used some form of questioning to help their children construct meaning. Some parents used open-ended questions that did not demand immediate answers, some parents would ask questions based on their children's past experiences, some parents asked questions to rapidly allow for answers, and some parents left questions unanswered. As a result of her study, Ash called for exhibits to be designed with multiple points of entry for learners with different levels of expertise. She argued that this design for exhibits would allow for open-ended questioning to occur, as this type of questioning encourages more conversational opportunities for learning.

The counterpart to questioning is answering or explaining, and in their study about parent-child conversations at a zoo and science museum, Geerdts, Van de Walle, and Lobue (2015) found that parents very infrequently offered up explanations to their children. Furthermore, when children tried to explain, parents never explored their explanations further. This observation aligned with the authors' survey of families that found that fun or practicality (e.g., the children were off from school) were the primary motivations for visiting the research sites; education was rarely ever a motivating factor. On the other hand, Falk, Moussouri, and Coulson (1998) found that visitors to museums who had educational motivations showed increased conceptual understanding. So it is very possible that the types of conversations that take place between families in museums and science centers are affected by the motivations for visiting (Geerdts et al., 2015).

In a study focused more generally on learning in informal settings, Borun, Chambers, and Cleghorn (1996) explored what learning in museums and science centers looks like. In their study of 129 families at a zoo, an aquarium, a nature museum, and the Franklin Institute in Philadelphia, the authors identified three levels of learning: identifying (e.g., one-word statements or answers), describing (e.g., multiple word answers or statements establishing connections), and interpreting and applying (e.g., statements about the concepts behind an exhibit). They found that there were specific behaviors that distinguished between these levels of learning, such as asking questions, answering questions, or reading labels and signs aloud. They concluded that the relationship between learning and observable behaviors is a crucial connection to understand and research further

In a more exhibit design focused study, Povis and Crowley (2015) explored the notion of joint attention (i.e., two people knowingly focusing on the same object). They wanted to know how the physical signage and design of an exhibit helps establish joint attention, and whether joint attention, once established, affected the conversations and learning talk between families. In the study of 54 families, half were given one flashlight per family and asked to walk through darkened diorama areas at a natural history museum; the other half were asked to explore the diorama area while fully illuminated. Of the 27 families given flashlights, half were given additional signage prompts. The same was true for the families in the fully illuminated exhibit. The authors found that the additional signage prompts had no effect on establishing joint attention and did not lead to increased time spent at the dioramas. The families that were given a flashlight spent 50% more time viewing the dioramas than the families without flashlights, suggesting that light or other visual effects can influence joint attention and time spent at exhibits. However, there was not a statistically significant difference in the amount of learning talk and conversations that occurred between the families with flashlights and the families without. The results of this study emphasized that more time and attention given to exhibits does not necessarily lead to an increased number of conversations among the visitors. Povis and Crowley called for future research to explore other potential focusing tools (i.e., cooperative tasks and goal-based challenges) and their effectiveness for diverse groups of visitors.

In sum, studies on science exhibits in informal learning spaces have explored the motivations and content of interactions between families as well as considerations for exhibit design. This research on science centers dates to the late 1980s (e.g. Diamond, 1986; Hilke, 1989), however, similar research on exhibits including mathematics only dates to the late 1990s (e.g. Guberman, Flexer, Flexer, & Topping, 1999), despite the *prominent Mathematica: A World of Numbers and Beyond* existing since the 1960s. In the next section, I will describe studies that have explored mathematics in science centers and museums.

Research on Science Exhibits with a Mathematics Focus

As part of the *Math Momentum* project, Mokros (2006) suggests that there are three main areas where incorporating mathematics into science exhibits have the greatest potential: the areas of data, measurement, and the mathematics of change (patterns, algebra, and calculus). Since data analysis and measurement are such natural components of science, the foundation is there for mathematics to be emphasized in science exhibits; the difficulty is to bring out the mathematics in an engaging way. Similarly, since scientists use graphs, tables, and the principles of algebra and calculus to describe patterns, growth, and changes over time, science exhibits offer a good opportunity to highlight the underlying math. The difficulty here is getting visitors to see past any negative memories and feelings towards algebra and calculus to see the relevance and beauty of these topics.

As a means of addressing some of the difficulties of incorporating mathematics into science-based exhibits, Mokros (2006) offered six ingredients for a successful, engaging exhibit. Such an exhibit should start with a motivating question that can interest a wide range of people, prompt inquiry into questions that do not have an immediate answer, encourage discussion and collaboration, connect to a larger scientific phenomenon, engage visitors with substantial mathematics, and integrate a physical, kinesthetic activity with the mathematical challenge. As a further means of addressing these difficulties, Mokros and Nemirovsky (2006) suggest following the principles of APE exhibits put forth by Humphrey and Gutwill (2005), mentioned previously.

Going further than some of these general exhibit design principles for science exhibits with a mathematics focus, there have been a few studies that look at family interactions and exhibit design outside of the *Math Momentum* project (Cooper, 2011; de Freitas & Bentley, 2012; Vandermaas-Peeler, Massey, & Kendall, 2015), but the body of research in this area is still rather limited. Cooper (2011) looked at exhibits at a children's museum, a history museum, and a zoo that were not part of a larger "Mathematics in Museums" type of project. These exhibits were not specifically designed to engage visitors in mathematical conversations. Cooper was trying to determine the current practice for including mathematics for young children in informal learning exhibits and how these exhibits prompted visitors to discuss mathematics. She found that overall the potential for mathematics in informal learning exhibits was there, but it was underutilized. Additionally, there were very few mathematical conversations taking place, but when there were conversations, they were most often prompted by parents.

Vandermaas-Peeler et al. (2015) explored the effect of giving parents explicit instructions about parental guidance and children’s reasoning before entering the *Flip it, Fold It, Figure It Out!* Exhibit at the North Carolina Museum of Life and Science. Perhaps surprisingly, the authors found that the control group and the group of families given instruction showed no significant difference in total mathematics-related conversations that occurred when interacting with the exhibits. Also, across both control and experimental groups very few families read any of the posted signage around the exhibits and even fewer made connections to their children’s prior knowledge, although the authors were not sure if the issue of signage was related to their specific exhibit or if the finding was generalizable. However, the authors did find that parents that were given instructions asked more conceptual questions (i.e., why and how questions) of their children, and as a result, their children more frequently gave correct responses to explaining and reasoning prompts.

De Freitas and Bentley (2012) explored an alternate high school mathematics-physics curriculum where a series of lessons were held at *The Cradle of Aviation* flight museum in New York. This study is rather unique in that it focused on high school students in museum settings and showed a direct partnership with between schools and museums. The students in the study, teens from under-resourced schools, were first shown videos in the museum about bird flight and wing design. Then, they had the opportunity to measure and interact with actual planes in the museum. Finally, they were asked to design a wing that would maximize the distance flown by a model foam plane. The authors found that the students reflected positively on the hands-on learning experience and were able to qualitatively describe the mathematical concepts they encountered; however, the students continued to describe mathematics as a purely symbolic thing, despite developing a more material, concrete sense of mathematics through the project.

Research on Mathematics Exhibits

The previous studies have all explored exhibits with some focus on demonstrating and helping visitors understand concepts related to science. The following studies explore exhibits that focus primarily on mathematics.

Wright and Parkes (2015) used the Partner Motion exhibit, part of the *Math Moves* project (see Selinda Research Associates, 2016), where visitors use motion detectors that graphically display their rate of change, to explore how visitors understand the mathematical principles behind what they were creating with their movement. Similar to de Freitas and Bentley's (2012) study, where students were able to verbally articulate the mathematics concepts they encountered in museum exhibits, Wright and Parkes (2015) found that the Partner Motion exhibit was more successful at developing a qualitative understanding of slope, graphs, rate of change, and other mathematics concepts of motion (e.g., saying things like "the faster I move, the faster the graph moves") than a rigorous, symbolic understanding. Despite this apparent lack of rigorous understanding, the authors concluded that visitors were developing mathematical understanding, even if visitors struggled to articulate that understanding. The engaging, hands-on nature of some activities in the *Math Moves!* exhibits led young visitors to believe that they were not doing math; that exhibits were too "fun" to be related to the mathematics they encountered in school settings (Nemirovsky et al., 2017). The disconnect between informal and formal mathematics prompted Nemirovsky and colleagues to question how researchers and educators define and assess mathematics experiences.

In another study, Guberman et al. (1999) identified four different varieties of mathematics exhibits. The first is an "exploration exhibit" (p. 294) where visitors are presented with manipulatives like cubes or tangram pieces but are not given any task and are free to explore the manipulatives. The second type is a “multiple solution exhibit” (p. 294) where the challenge presented to the visitor can be solved in multiple ways. The third variety of exhibit is “single solution, self-confirming,” (p. 294) where the visitor makes a prediction and uses the exhibit materials to test if their solution is correct. The final variety of exhibit is “single solution requiring external validation” (p. 294) where the visitor comes up with a solution to the exhibit task, but there is no way to confirm or refute their solution. Guberman et al. aimed to design mathematics exhibits under Project Math-Muse that were inexpensive and helped preschoolers through third graders develop mathematics concepts with NCTM standards in mind. They designed a “single solution, self-confirming” exhibit and a “single solution requiring external validation” exhibit and explored how children engaged with these exhibits. Despite the task designers’ clear intentions, the children either ignored the exhibits entirely, interacted with the exhibits but used the materials for some other purpose, interacted with the exhibit but came up with their own challenge, or attempted to solve the challenge presented to them. Ultimately, the study concluded that the challenge of an exhibit comes from the attitudes and prior knowledge of the children, the interactions amongst the children, and the physical exhibit itself, in alignment with the Contextual Model of Learning (Falk and Dierking, 2000).

In an often cited study at the *Handling Calculus* exhibit in the Science Museum of Minnesota (Gyllenhaal, 2006; Nemirovsky & Gyllenhaal, 2006), visitors engaged with a series of exhibits explicitly focusing on calculus concepts such as limits, differentiation, and integration. The researchers found that visitors that were math adverse or had negative feelings towards calculus did not necessarily run away from the exhibits, but would engage with the aspects of the exhibits that they were comfortable with (e.g., ignoring labels that mentioned things like limits and instead focusing on relatively simple challenges presented by the exhibit, such as using lasers to trace the slope of a graph). Other mathematics adverse visitors were drawn deeper into the exhibits by their companions because the exhibits were seen as fun social experiences. Furthermore, visitors who were currently students in calculus or pre-calculus courses were often excited to see the concepts they were studying in school show up elsewhere. Additionally, people who lacked any calculus experience rarely identified the exhibits as being about calculus, but still found ways to engage with exhibits. The researchers concluded that for a science center exhibit on higher level math to be successful, it should turn abstract concepts into physical, kinesthetic experiences, address the complex (often negative) emotions that people have toward math, tap into visitors' fragmented memories by using keywords related to calculus they are likely to recall, and use multiple points of entry so that a young child can engage with aspects of the exhibit just as well as someone with higher level mathematics experience.

Many studies examine learning at informal learning centers and museums, but few have explored the practices of museum facilitators. Pattison et al. (2017) recorded unstructured interactions and unscripted conversations between museum educators and visitors, particularly with families, at exhibits. Relying on the theory of interactional sociolinguistics, this study focused on the negotiations of rules, roles, and customs of conversations (e.g., turn-taking) within human interactions. These negotiations are particularly complex in the context of families. Furthermore, this study was influenced by an asset-based perspective of learning (e.g. Calabrese Barton, Drake, Perez, St. Louis, & George, 2004) which built on funds of knowledge research in informal STEM spaces (e.g. González, Andrade, Civil, & Moll, 2005).

Data were collected from visitors to three mathematics exhibits at the Oregon Museum of Science and Industry in Portland, Oregon. Families engaged in exhibits where mathematics and science concepts guided designs for racecars, a collaborative slider game within a coordinate plane, and an artistic mobile based on balancing weights; meanwhile, facilitators took turns engaging with families or letting them work and converse on their own. Since this was a design-based study, visitors' experiences with exhibits informed how museum facilitators responded, which then shaped the satisfaction and interactions of visitors, in an iterative process. Pattison et al. (2017) found that there were several factors that influenced visitor experience, such as the size of the group, the members' social goals, the ages of the children, and the roles that the adults played during the interactions. They concluded that their iterative model for successful facilitation hinges on an asset-based approach of observing, supporting, and reflecting as educators interact with families to promote mathematical reasoning. The authors suggested that schools and other formal learning settings could learn from their facilitation model, particularly how to balance traditional learning goals (e.g., mathematical reasoning) with goals that are often valued by families (e.g., intergenerational learning and enjoyment).

Later, Pattison et al. (2018) described quantitative results of this study based on measuring certain observed constructs, including engagement time, mathematical reasoning, mathematical enjoyment, general satisfaction, intergenerational communication, and mathematical awareness. Overall, the presence of a trained facilitator significantly increased the amount of time that families spent at exhibits. However, they found that engagement time influenced only some variable outcomes; in other words, the amount of time that families spent at exhibits had an effect on mathematical reasoning, general satisfaction, and mathematics enjoyment (at one exhibit), but did not influence other variables, such as intergenerational communication and mathematical awareness. Additionally, Pattison and colleagues found that the content of the exhibit influenced the outcomes of some variables as a result of increased facilitation, likely due to some exhibits being more mathematically challenging or more accessible than others. They concluded that future research should investigate the diversity of family interactions and relationships at mathematics exhibits, particularly to examine verbal and non-verbal communication from a culturally-responsive, inclusive approach.

There have been other studies that have occurred within the setting of a mathematics focused science center exhibit, but the research questions were slightly off-topic from the focus of this paper. Dancu, Gutwill, and Hido (2011) conducted a study at the *Geometry Playground* exhibit at the Exploratorium in San Francisco, but were focused on child psychology and how the exhibit supported the notion of "play" rather than family interactions and exhibit design. In a similarly more developmental psychology leaning study, Nemirovsky, Kelton, and Rhodehamel (2013) studied how a collaborative mathematics exhibit called *Drawing in Motion* influenced a concept called perceptuomotor integration – "the intertwining of perceptual and motor aspects of tool use" (p. 373). An example of this would be being able to play a note on guitar that one hears/perceives. Neither of these studies attended to parent-child conversations.

A more recent example of the direction that mathematics focused exhibits are heading can be found in the Museum of Mathematics (MoMath) in New York City. MoMath opened in late 2012, devoted to revealing the rich diversity of mathematics and promoting an interest in the subject for its visitors (Henebry, 2012). While a few other countries have similar museums, this museum, completely centered on mathematics, is unique in the U.S. Prior to the creation of MoMath, there was another mathematics museum, The Goudreau Museum, but this museum permanently closed in 2006 (Rosmarin, 2015). From the moment that visitors step inside, the MoMath is inclusive and considerate of many who have anxiety or negative feelings toward mathematics by creating individualized badges that each person uses to interact with exhibits. The hands-on exhibits in the MoMath often connect to higher-level mathematics in a way that is accessible to many (similar to the *Handling Calculus* exhibit in Gyllenhall, 2006), such as shapes and puzzles based on hyperbolic geometry, cars driving along a Möbius strip, and a number grid where random pairs "magically" sum to 111 (Henebry, 2012). A popular exhibit lets visitors ride a tricycle with square wheels along a non-flat surface, resulting in a smooth yet "bizarre ride" (Rosmarin, 2015, p. 106). These engaging activities are created to counter the belief that mathematics is simply "a set of rote practices for solving problems" (Henebry, 2012, p. 17), but is instead "a form of reasoned discovery motivated by curiosity and insight" (p. 17). Other MoMath contributions include the cable series Math Encounters, a series of fun, family-oriented talks given by research mathematicians to highlight "real math" (Lawrence, 2014, p. 417), as opposed to the "pencil-and-paper conventions of school mathematics" (Rosmarin, 2015, p. 104). Overall, the goal of the Museum of Mathematics was to convey the beauty, variety, and playfulness of mathematics in a way that no museum had done before. Although MoMath is an exciting new locus of informal mathematic activity, there has been very little formal research conducted there.

In sum, there is significantly less research focused on museum and science center exhibits focusing solely on mathematics. The studies cited in this review relating to science exhibits are only a handful of the total body of research on science exhibits, however the studies cited about mathematics exhibits is somewhat exhaustive of the research done to date. The research that does exist extends beyond the bounds of K-12 mathematics, ranging from concepts meant to be accessible for preschoolers to calculus concepts. However, the exhibit design principles for purely mathematics focused exhibits are much the same as for the science focused or mathematics within science focused exhibits.

Apart from Cooper (2011), Pattinson et al. (2017; 2018), and Vandermaas-Peeler et al. (2015), very little research on mathematics in science centers and museums has focused on parent-child interactions and their effect on learning. Going further, there is an apparent lack of research on parent-child interactions in entirely mathematics focused exhibits. The study presented in this paper attempts to fill this gap in the body of research. In the next section I will discuss the location and exhibits that offered a unique opportunity to study parent-child interactions and the framework used for analyzing these interactions.

Chapter 3: Methods

Location and Puzzle Descriptions

The study was conducted at a science center in the Southwest United States. This science center had an exhibition that challenged visitors to directly interact with mathematics and logic while trying to solve various puzzles. The puzzles and exhibits were similar to the ones found at MoMath in that the puzzles and exhibits sought to promote the exploratory, whimsical side of mathematics without rooting the mathematics in any real world or scientific processes. While the exhibit contained roughly twenty different puzzles, the data collected for this study was at a trio of arithmetic based puzzles named Odds & Evens, Magic Cross, and Uranium Pile. These puzzles were chosen because preliminary observations showed that visitors tended to vocalize their thought process with these puzzles more than others. I also believe that the relatively simple mathematics involved in the puzzles caused people to stick with the puzzles longer after initial attempts to solve them failed. I believed this would lead to more interactions to observe. To point, there was another very popular puzzle at the science center called a Soma Cube that tasked users to construct a cube from smaller, oddly shaped pieces. While there was no doubt lots of interesting thinking and strategy involved in this puzzle, it was largely a silent, solitary activity, and as such, would not have made for a good puzzle for this study.

The Odds & Evens puzzle board consisted of seven slots to place numbered pieces one through fourteen: one slot in the center and three lines going through the center with a slot at each end of each line (see Figure 1). The families were given three similar puzzle options. They could place pieces one through seven on the board so that each line of three pieces added up to 12, they could place just the odd numbered pieces on the board so that each line of three had the same unspecified total, or they could place just the even numbered pieces on the board so that each line of three had the same unspecified total.

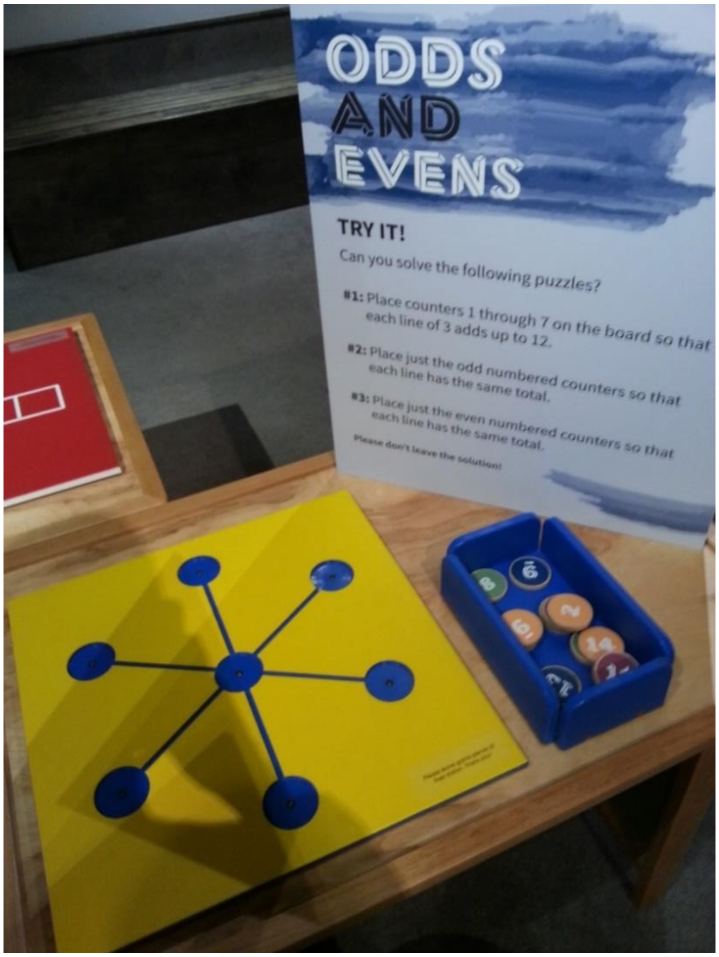

Figure 1. Odds and Evens

The Magic Cross puzzle board consisted of five horizontal slots and five vertical slots to place numbered pieces with the horizontal and vertical lines intersecting in the middle (see Figure 2). Like the Odds & Evens puzzle, the families were given three similar puzzle options. They were tasked with arranging the numbers one through nine so that horizontally and vertically the lines both added up to 27, 25, or 23.

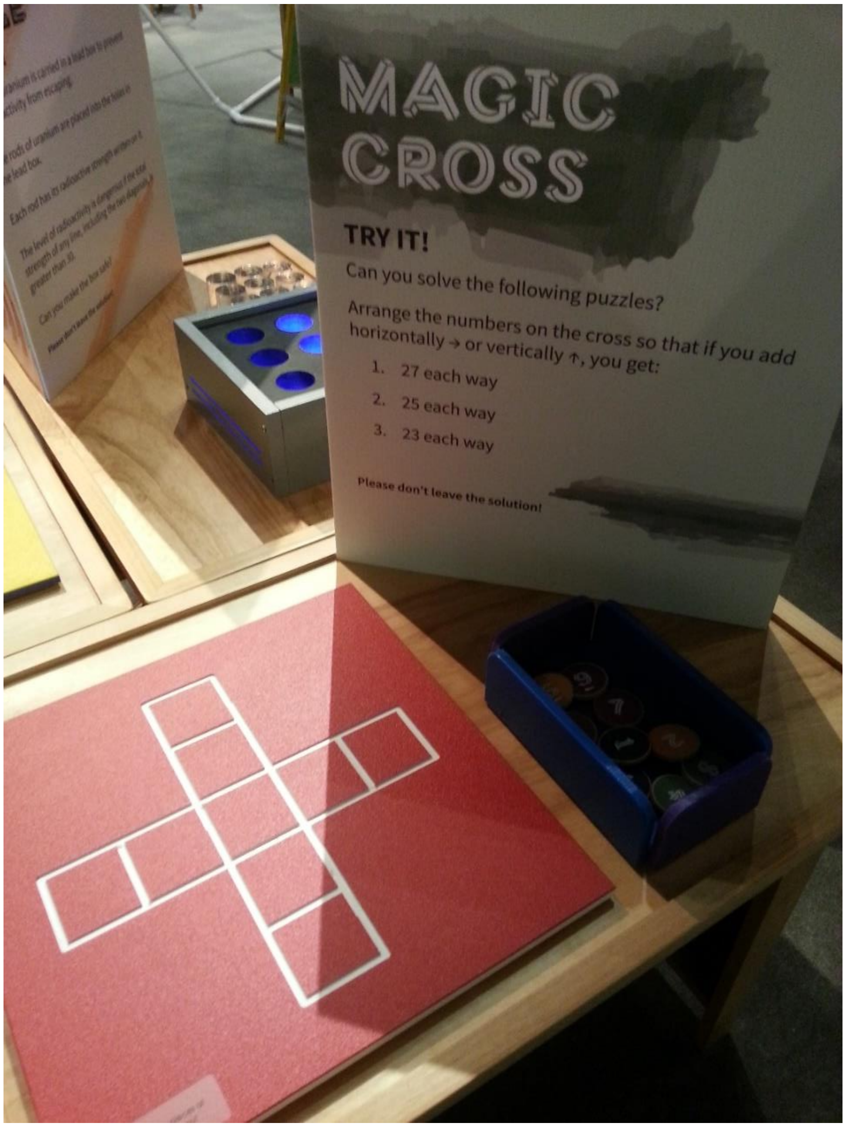

Figure 2. Magic Cross

The third puzzle was The Uranium Pile. It consisted of a square “lead” box with nine cylindrical holes and nine numbered cylinders of “uranium” with radioactive strengths of: 3, 6, 7, 9, 10, 11, 13, 14, 17 (see Figure 3). The families were told that the level of radioactivity is dangerous if any line of three in the square, including diagonals, has a sum greater than 30.


URANIUM PILE

TRY IT!

Active uranium is carried in a lead box to prevent radioactivity from escaping.

The rods of uranium are placed into the holes in the lead box.

Each rod has its radioactive strength written on it.

The level of radioactivity is dangerous if the total strength of any line, including the two diagonals, is greater than 30.

Can you make the box safe?

Please don't leave the solution!

Figure 3. Uranium Pile

All three puzzles consist of adding three numbers up to a common value, or in the case of the Uranium pile, less than a specific number, so the mathematics involved relies upon addition and logical thinking. The difficulty in the Odds & Evens and Magic Cross puzzles come from figuring out what number needs to be in the middle slot, as that number is shared amongst all of the lines that need to be summed. While trial and error is generally enough to solve these two puzzles, a more logical approach can be applied. In the case of the Odds & Evens puzzle, this involves identifying complementary number pairs (e.g. 1+13 = 14, 3+11 = 14, and 5+9 = 14, so 7 must be in the middle slot). In the Magic Cross puzzle, one way of going about solving the puzzle logically is realizing that 1+2+…+9 = 45, and if the sum of the horizontal and vertical lines are each to be 25, then a total sum is 50. However, the middle piece is counted twice, so the middle piece must be 50-45= 5. Similarly, if each line is to total 23, the middle piece must be 46-45 = 1, or if each line is to total 27, the middle piece must be 54-45 = 9.

The Uranium Pile puzzle is a little more complex than the other two puzzles, in that the user of the puzzle needs to keep track of the sum of eight different lines as opposed to the two or three required for the Magic Cross and Odds & Ends respectively. Additionally, every line in the Uranium Pile puzzle shares numbers with multiple other lines, instead of a single number shared amongst all the lines as in the other two puzzles. So swapping any two pieces in the puzzle does not just affect the sum of two lines, but three or four lines, making a trial and error approach to solving the puzzle much more difficult than the Magic Cross or Odds & Evens. And while a detailed logical solution to the Uranium Pile is rather cumbersome to include in this paper, one of the strategic keys to solving the puzzle is to realize that the 17 piece (the highest numbered piece) needs to be included in as few lines as possible, i.e., placing the 17 piece in the middle will mean that it is included in the sum of four different lines, thus limiting the options one has for where to place the 11, 13, and 14 pieces.

Participants:

Six families consented to be a part of this study. Here, a family is defined to be at least one parent and one child, though two of the families had more than one adult and/or child. It has been noted that socioeconomic diversity among museum study participants is a cause for concern (Callanan, 2012), so two of the families were recruited at an unrelated family math event at a local area school that involved primarily working-class, Latinx families. The other four families were recruited from the visitors to the science center.

The age range of the children in this study was 10-14 years old. This age range was chosen based off preliminary observations. Children that were estimated to be younger than 10 years old were observed to not have lengthy engagement times with the exhibits, and children that were estimated to be older than 14 were observed to be more independent and less likely to interact with their parents.

Data Collection and Analysis

The families were observed and videotaped at the target exhibits, although the families were not required to attempt all puzzles. These videos were then transcribed and coded according to the concept driven codes based on the categories in the framework detailed in the following section. I had these codes in mind when I read through the transcripts of the recorded interactions

Chapter 4: Coding Framework

With the goal of this study being to explore the nature of parent-child, I wanted to choose a theoretical framework that defined learning in terms of the conversations taking place and not in terms of formal assessment. With that in mind, I chose to adapt the framework used by Allen (2002), which I will refer to henceforth as the Allen Framework of Learning (AFL).

In her study, Allen (2002) studied visitor interactions (not specifically parents and children) at a science exhibit about frogs as the Exploratorium in San Francisco. She used a specific coding framework to analyze the frequency of the types of learning-talk taking place amongst visitors. In her framework, Allen defined learning from the following principles:

- There is a cognitive science perspective on learning in line with Bloom's Taxonomy
- There is a sociocultural perspective on learning as a group process instead of an individual outcome. As such, learning of individuals at exhibits is not analyzed but rather the whole group is considered.
- Since the interactions that take place are happening in an informal learning setting, no formal learning assessment should take place.
- Learning should refer only to the exhibits and the topic area of those exhibits and all talk that referred to other areas of the museum or science center visit should be excluded.
- Learning does "not require intentionality on the part of the speaker or listener…" nor does it "require explicit expression of awareness or acknowledgement" (p. 263).

Within the AFL, transcripts of conversations at exhibits are coded into five categories and sixteen subcategories of learning-talk. The categories and subcategories are as follows:

Perceptual Talk – This category captures any sort of conversation or action that draws attention to the objects and text surrounding an exhibit. These are considered evidence of learning since they are “an act of identifying and sharing what is significant in a complex environment” (p. 274). There are four subcategories of perceptual talk:

- Identification – pointing to objects or interesting parts of an exhibit.
- Naming – naming an object in an exhibit
- Feature – pointing out a concrete property of an exhibit
- Quotation – pointing to or reading aloud text associated with an exhibit

Conceptual Talk – This category classifies cognitive interpretations of what was happening at an exhibit. There are four subcategories:

- Simple inference – a single interpretive statement about part of an exhibit
- Complex inference – a hypothesis or generalization of exhibit information beyond interpreting what is explicitly displayed at an exhibit
- Prediction – any verbalized expectation of what will happen at an exhibit
- Metacognition – verbalized reflection of a visitor’s own knowledge

Connecting Talk – This category includes any stated connections between exhibits and previous knowledge and experience. There are three subcategories:

- Life connection – any connection of exhibit elements to something familiar
- Knowledge connection – any statements connecting to knowledge gained prior to exhibit interaction

- Inter/Intra-exhibit connection – any connection between elements within an exhibit or between elements at different exhibits

Strategic Talk – This category includes explicit discussion of how to use and interact with exhibits. There are two subcategories:

- Use – any statement about how to use an exhibit
- Metaperformance – any statement that evaluates a visitor's own or family member's performance, action, or abilities

Affective Talk – This category includes any expressions of feeling. There are three subcategories:

- Pleasure - expressions of positive feelings or appreciation of aspects of an exhibit.
- Displeasure -expressions of negative feelings or dislike towards aspects of an exhibit, including sadness or sympathy.
- Intrigue - expressions of fascination or surprise.

It should be noted that after an initial pass through the conversation data, I made a couple of slight changes to the categories from Allen's (2002) original coding framework. One such change is leaving off the subcategory of "Feature" in the Perceptual Talk category. I chose to leave this out because the description of this subcategory was indistinguishable from that of Identification in my mind. Additionally, I expanded the descriptions of the Affective Talk subcategories to include the additional expressions of laughter in the Pleasure subcategory, frustration in the Displeasure subcategory, and excitement in the Intrigue subcategory. I felt that the original coding framework was not as descriptive in the types of feelings that could arise at the exhibits. I also allowed for all of these instances of affective talk to apply towards a family member, as the original definitions specifically say that the expressed feelings should apply towards aspects of an exhibit, and I wanted to be sure to include any expressions of feeling between family members.

Where my application of this framework differs from that of Allen (2002) is that she used these categories in a quantitative way to compare the frequency of certain types of learning-talk occurring. Since I had a small number of participants in the study, I wanted to focus on these categories in a more qualitative way, paying attention to *how* these categories appeared and what happened because of their occurrence. I noted how certain categories were absent from my data and whether families used one form of talk more than another.

I allowed for data-based codes that emerged from patterns or themes that I noticed in the data. For instance, I did not go through the video transcripts looking for the someone to say, "I predict…" or "I use...". I looked for interactions that met the description of the Prediction or Use subcategories and coded those interactions accordingly.

Below, Table 1 shows an example statement made by a parent or child for each Learning Talk subcategory and a description of why each statement fits into that subcategory.

		Example	Description
Perceptual Talk	**Identification**	"So those go there and there (pointing)"	The parent is identifying to the child where the puzzle pieces should go
	Naming	"We can put the five in"	The child here is specifically referring to and naming *the* five piece
	Quotation	"But we have to add more numbers. It says to add all of one through seven"	"It" here is the puzzle directions of the Magic Cross Puzzle
Conceptual Talk	**Simple Inference**	"We're off by a number"	The mother is identifying simply that they are off by a number. There is no generalization of information or hypothesizing.
	Complex Inference	"You gotta change this (pointing to the five piece) because that'll be 15 (pointing to the line)"	The child is identifying a clear cause and effect relationship between one of the puzzle pieces and one of the lines of numbers in the Magic Cross Puzzle.
	Prediction	"I think if we left the three in the center it would reduce the numbers	The father is predicting what will happen if the three piece in the Uranium Box puzzle is left in the center. This statement was also coded as a Complex Inference, showing that often these two codes were coded for the same statement.
	Metacognition	"We know this one is three too many"	The child here is reflecting on his or her current knowledge of the puzzle
Connecting Talk	**Life Connection**	None	None
	Knowledge Connection	"Don't we have to use partial product?"	The child here was referencing math knowledge from school
	Inter/Intra-exhibit Connection	"Do you want to try that one over there?	The mother is making a connection to a puzzle that the child had
Strategic Talk	**Use**	"So maybe if you just switch those two would it work?"	The mother is suggesting to her daughter how to use the exhibit.
	Metaperformance	"No, we were closer the other way"	The father is evaluating the family's attempted solution of the puzzle.
Affective Talk	**Pleasure**	"Yay! Finally! It's not impossible after all"	The daughter is showing her joy upon solving the Uranium Pile puzzle.
	Displeasure	"Don't be moving them"	The mother is scolding her son for moving pieces of the puzzle.
	Intrigue	"So this needs to be a four!"	The daughter is expressing excitement at potentially finding a solution to the puzzle

Table 1. Example Statements and Descriptions

Chapter 5: General Findings

As a reminder, the goal of this study was to explore how the parent-child interactions that take place at math focused exhibits affect learning. In this section I will broadly outline some of the general findings.

Below, Table 2 shows the makeup of each family that participated in this study, the exhibits where each family were recorded, and the length of time spent at each exhibit. Table 3 shows, with respect to my theoretical framework, which categories and subcategories of learning-talk appeared amongst the families.

As Table 2 shows, only families A and B had multiple parents or multiple children present. These families also happened to be the families that were recruited outside of the science center at a family math event, as noted in the methods section. One thing to note about families A and B is that they were friends and came to the exhibit together. This led to difficulties in recording as both families were interacting with the puzzles at the same time, meaning that I had to make the choice of when to record each family and at what exhibit. So, while Table 2 shows that family A only worked with the Uranium Pile puzzle and family B only worked with the Odds and Evens puzzle, the families worked with multiple puzzles, but only one puzzle could be recorded for each.

Table 2 also shows that families C and D were the only families recorded interacting at multiple puzzles. Number of puzzles does not however corelate to more interactions. For instance, family C spent only three minutes at the Uranium Pile puzzle (since it proved to be too difficult for the daughter to solve), five minutes at the Magic Cross puzzle, and five minutes at the Odds and Evens puzzle. Whereas, family E spent 31 minutes at the Uranium Pile puzzle.

However, time alone at a puzzle is also not indicative of the interactions and learning-talk that took place. Family A spent six minutes at the Uranium Pile and family F spent six minutes at the Odds and Evens puzzle, but as Table 3 shows, family A had more categories of learning-talk observed amongst the children than family F. There was not enough evidence to determine if the puzzles visited by the participants played a role in the amount of learning talk that was observed.

Family	Members	Exhibits Visited and Time Spent at Exhibits
A	Mother, Father, Son, Daughter	Uranium Pile - 6 minutes
B	Father, Daughter 1, Daughter 2	Odds and Evens - 8 minutes
C	Mother, Daughter	Magic Cross - 5 minutes, Uranium Pile - 3 minutes, Odds and Evens - 5 minutes
D	Mother, Son	Magic Cross - 9 minutes, Uranium Pile - 5 minutes, Odds and Evens - 3 minutes
E	Mother, Daughter	Uranium Pile - 31 minutes
F	Mother, Son	Odds and Evens - 6 minutes

Table 2. Family Composition and Exhibits Visited

Table 3 shows that four of the larger categories of learning talk were present among all families; the only one that was practically not present is Connecting Talk. Among the four types that were present, Family E had the most subcategories of learning talk show up while Family F had the least. The child in Family F also exhibited the least number of subcategories when compared to the children of the other families.

		Family A		Family B		Family C		Family D		Family E		Family F	
		Parents	Children	Parents	Children	Parents	Children	Parents	Children	Parents	Children	Parents	Children
Perceptual Talk	Identification	X	X	X	X	X		X	X	X	X	X	
	Naming	X		X	X	X		X		X	X	X	
	Quotation	X		X	X	X		X		X		X	
Conceptual Talk	Simple Inference	X	X	X	X	X	X	X		X	X	X	X
	Complex Inference	X	X	X	X	X		X		X	X	X	
	Prediction	X	X		X	X	X		X	X	X	X	
	Metacognition				X					X	X		
Connecting Talk	Life Connection												
	Knowledge Connection								X				
	Inter/Intra-exhibit Connection						X	X					
Strategic Talk	Use	X	X	X	X	X		X	X	X	X	X	X
	Metaperformance	X	X	X	X	X		X	X	X	X	X	
Affective Talk	Pleasure	X	X	X		X	X	X	X	X	X	X	
	Displeasure		X	X	X		X	X	X	X	X	X	
	Intrigue		X		X		X	X		X	X		X

Table 3. Presence or Absence of Learning Talk Among Families

Chapter 6: Interpretation of Findings with Respect to Learning Talk Categories

In the following sections I give examples, summarize data, and note trends for each learning-talk category and subcategory.

Perceptual Talk

Perceptual Talk appeared amongst all the families, with the majority of this form of learning talk coming from the parents. Identification talk most often took the form of parents or children tracing rows of numbers with their fingers or pointing to specific numbers as they were trying to solve the puzzles, e.g. the mother in family E asking her daughter “What do I have here?” while tracing the middle row of the uranium pile puzzle with her finger. Due to the nature of the exhibits analyzed, Naming learning talk almost always took the form of referring to number pieces or rows/columns of the puzzles, e.g. the mother in family F saying, “Let’s try the three in the middle.”

It was interesting to note that Quotation learning talk and Naming learning talk occurred amongst both children in family B, while each subcategory was almost non-existent among children in other families. This could be due to the collaborative family dynamic that was apparent in family B. Contrasted with the other families, the father in family B was putting himself more on the same level as his children. It was apparent that they were solving the puzzle together, as opposed to him trying to lead his children to the solution or him trying to solve the puzzle independently while his children observed, as was the case with the other five families. Taking that into consideration, it only seems natural that Quotation and Naming learning talk would also come from the children in that family.

Conceptual Talk

When it comes to Conceptual Talk, the six families fall into what I saw as three distinct tiers in regards to frequency of learning talk between parents and children: family E had the most frequent use of Conceptual Talk, followed by families A and B in the second tier, and families C, D, and F in the third tier. Starting with families C, D, and F we can see from Table 3 no instances of Complex Inference were observed amongst the children. In families D and F, I attribute this to the fact that the mothers in those families were clearly trying to solve the puzzles for their own benefit, and as such, were the ones that were verbalizing their conceptual thinking. In the case of family C, it became apparent to me and the mother that the puzzles were too difficult for the daughter to solve, and as such the exhibit experience became more about the mother trying to solve the puzzles on her own while also trying to include her daughter. So, while the children in these families were present and making occasional Predictions or Simple Inferences, most of the Conceptual Talk was coming from the parents.

Interestingly, all the parental Conceptual Talk from family A came from the father, despite the fact that the mother was the one trying to steer the children towards a solution. The father would say things like, “I think if we left the number three in the center, it would reduce the numbers (reduce the totals in other rows and columns),” and then his comments would go ignored by the other family members, as the children would then keep listening to the mother’s directions.

Lastly, family E had the most frequent use of conceptual talk, although that is perhaps to be expected given that family E spent over half an hour trying to solve the uranium pile puzzle (family D spent a little over 16 minutes across all three puzzles for comparison). Having spent that amount of time at arguably the most difficult puzzle of the three, the video shows a lot of back and forth between the mother and daughter predicting what will happen if a number X is moved to position Y. The following excerpt is illustrative of the type of Conceptual Talk that was taking place in this family:

> Mother – Mm-hmm. Ok. I think the 17 is going to be our big trouble maker. What do you think?
>
> Daughter – Yeah
>
> Mother – Uh huh.
>
> Daughter – Um (*counting out loud)*
>
> Mother – What if you swap these two? *(points to top right and bottom right)*
>
> Daughter – *(swaps 11 in top right and six in bottom right)* It doesn't make any difference in this *(traces right column)* but it doesn't matter *(traces top row, suggesting the top row still won't add up correctly)*
>
> Mother – So does it work now? *(points to top row)*
>
> Daughter – *(counts out loud)* 30. Yes

Here we see the mother making a Prediction and Complex Inference when stating that the 17 is the big trouble maker, and the daughter responding to the mother's suggestion of swapping two pieces with a complex inference of her own.

Notably, the subcategory of Prediction showed up amongst all families. In Allen's (2002) original study, this was the least prevalent subcategory. This contrast is likely due to differences in exhibit design. The exhibit in Allen's study was a science exhibit about frogs with different kinds of hands on and visual elements that did not necessarily lend themselves to verbal expressions of prediction. The exhibits in this study being puzzles naturally lead to families trying solutions, evaluating, retrying solutions, and re-evaluating – behavior that unsurprisingly leads towards verbal expressions of prediction.

Finally, I should mention the subcategory of Metacognition. As Table 3 shows, only two of the families made any statements that would be categorized as Metacognition with a total of four statements made among them. All of those statements were of the variety where one of the family members was stating that they already figured something out or already knew the answer, e.g. the daughter in family E saying “no, I already know that’s wrong!”

Strategic Talk

Across all six families, Strategic Talk showed up in roughly the same fashion. Metaperformance Talk was limited to evaluating whether or not a given configuration in the puzzles was correct or not (e.g., “No, we were closer the other way” or “So this one is ok too, huh?”). This was consistent across all families. Use Talk also showed up in all families, but with a greater variety than Metaperformance Talk. Some of the Use Talk was more limited and direct in nature (e.g., “We need a four over there,” or “So you just want the odd numbers”), whereas other Use Talk appeared to have more strategic depth to it (e.g., “I don’t think that’s going to work because seven is going to have to be right here and it’s going to be too much. So, I think seven should probably stay I the middle”). Most of this more in-depth Use Talk was also cross-coded as Complex Inference Talk. As in the previous example, the parent was saying that the “seven” piece needed to be used in a particular way to solve the puzzle, indicating Use Talk, but the parent went further and justified why the seven piece needed to be used in that way, indicating a Complex Inference.

As noted previously, in family C, it became apparent that the puzzles were a little too challenging for the daughter. As a result, the daughter did not contribute to any sort of Strategic Talk. She was manipulating the exhibits at the mother’s direction, but not offering up her own suggestions of what pieces should be placed where or whether a certain a configuration of the puzzle was correct or not. Additionally, in family F, the son did not have any verbal statements of Metaperformance Talk, although there were several instances in the video where it was apparent that he was internally evaluating what was not working with their attempted solution. Family F also had the shortest video of all the families, so given additional time at the exhibits it is possible that Metaperformance Talk would have appeared from the son.

Affective Talk

As Table 3 shows, all three subcategories of Affective Talk appeared in all six families to some degree. Pleasure Talk showed up in the same ways across families and typically took the form of exclamations of happiness when a puzzle was solved, laughter, or in the case of Family A, expressions of positive reinforcement from the mother (e.g., “I know you guys can do this”). Similarly, Intrigue Talk looked the same across families with most Intrigue Talk showing up as excitement from the children at the prospect of a good idea (e.g., “So we need a three!”) or surprise at an attempt that worked or did not work (e.g., “Wait! This is safe!”).

The most interesting form of Affective Talk was Displeasure Talk. Across the six families, parents and children, Displeasure Talk typically manifested as expressions of frustration with an attempted solution that did not work or frustration that a certain puzzle was too difficult. A representative example of this Displeasure Talk can be seen in family B below:

Daughter 1 – Oh, is that the hugest number? (*pointing to the 5 piece)* Let’s trade this to 7 (*swapping the 5 and 7 piece)*

Father – No, we were closer the other way. (*long pause)* Ahhh! Ok, I give up

Daughter 1 – I don’t know. I can’t do this one.

Daughter 2 – Unless we use another 6 (*trying to use the 9 piece upside down as a 6)*

Here the father is expressing frustration and giving up after another attempt at a solution fails. One of the daughters in the family is similarly frustrated. In an amusing response to this, the second daughter tries to get creative and turns a nine piece upside down to look like a six as a last-ditch effort to solve the puzzle.

A notable instance of Displeasure Talk occurred in family E where the daughter repeatedly expressed frustration that the Uranium Pile puzzle was too difficult, but similarly to the positive reinforcement shown in family A mentioned earlier (“I know you guys can do this”), the mother in family E would not let her give up. This interaction took place several times during family E’s time with the uranium pile puzzle. None of the other parents expressed this kind of determination and encouragement in solving the puzzles.

Connecting Talk

Connecting Talk was the least occurring form of learning talk amongst the six families. Only families C and D had any form of connecting talk, and even then, it was rather minimal. The daughter in family C stated that she “tried to do this,” referring to a previous attempt at the Magic Cross puzzle before filming began. The mother in family D referred to the uranium box puzzle as being “like the other one…but with numbers,” while the son in family D had a Knowledge Connection to his schooling when he asked, “don’t we have to use partial product?” None of the families had any Life Connections while interacting with the exhibits.

One possible reason for the lack of Connecting Talk in the families could be due to the nature of the puzzles. Since the puzzles are arithmetic based and lacking any stimuli outside of the puzzle implements and directions, there is very little to latch onto in terms of Knowledge Connections or Life Connections (unless one of the parents happens to work with uranium on a regular basis). A conceivable Life Connection would be if one of the family members recalled a similar puzzle they may have seen elsewhere, but none of the families in this study expressed any such recollections.

In the following section I will discuss these findings and trends in the data with respect to prior research.

Chapter 7: Discussion

Connections to the Literature

As this study shows, there is merit in designing math exhibits about puzzles. These types of puzzles help to establish the type of “joint attention” described by Povis and Crowley (2015) that leads to increased learning talk. The downside to these types of puzzle exhibits is that the difficulty level can be hard to balance, and difficult puzzles can lead to frustration and subsequent disinterest amongst exhibit goers. The Uranium Pile puzzle arguably was the most difficult to solve puzzle at the science center (evidenced by the 30 plus minutes it took for family E to solve it), and as such, it led to some frustration in family C, who quickly stopped attempting the puzzle when it became clear that the puzzle was too difficult for the daughter. Considering the whole exhibition of 20 puzzles, it is likely good to have a few very difficult puzzles, as a visitor can just move on to a new puzzle. In the context of a study looking at a small subset of exhibits however, it would be wise to consider the mixture of puzzle difficulties beforehand.

The puzzles at this exhibit were all of the “single solution, self-confirming variety” (Guberman et al., 1999, p. 294). As Guberman et al. point out, exhibits of this variety can give “the impression that mathematics is simply about doing given tasks in a fixed, predetermined manner and arriving at a single ‘correct’ solution” (p. 294). The solutions to the puzzles are also not provided to visitors, leading to situations where visitors can walk away discouraged and unsatisfied with their experience.

As noted previously, it became apparent that the puzzles in the present study were too difficult for the daughter in family C and that she was uncomfortable with attempting to solve them. Rather than completely withdrawing from the exhibits, she took more of an observational role and was content to manipulate the puzzle pieces at the mother's direction. This is similar to Gyllenhaal (2006) who found that math adverse visitors would often engage with aspects of an exhibit that they were comfortable with; in this case, the daughter in family C was comfortable with manipulating the puzzle pieces. Gyllenhaal called for an increase in mathematics exhibits that allow for multiple points of entry for learners of different levels of expertise and address the negative emotions visitors may have towards mathematics. While the exhibits in the present study did not seem to manifest these two aspects, which perhaps limited the open-ended questioning discussed by Ash (2004), the experience of Family C showed that there are still ways for visitors with a mathematics aversion to engage and have fun with a puzzle-like mathematics exhibit.

Pattison et al.'s (2017) discussion of balancing traditional learning goals with goals that are valued by families connects to some family interactions in this study. For example, this is the case of the father and two daughters (intergenerational aspect) in Family B who struggled to solve the Odds and Evens puzzle as intended (attending to mathematical reasoning) but still spent time at that exhibit laughing and having a great time together (goal of family enjoyment of mathematics). They joked about turning the nine into a six, which suggests that they were thinking critically about the puzzle and analyzing other possible strategies to solve it. Of course, only one solution was acceptable by the given parameters of the exhibit, but from an asset-based approach, this family demonstrated mathematics traits of creativity and problem-solving, as well as positivity.

Additionally, Pattison et al. (2017) found that family members' social goals and the roles of the adults were factors that influenced visitor experience. In this study, the distribution of learning talk (i.e., the equitability of learning talk across family members, or lack thereof) was mostly controlled by the adults in the family. For example, the father in Family B played a laid-back role and emphasized having fun as a goal for his two daughters. Perhaps due to his role, there was an equal distribution (i.e., high equitability) of learning talk between the three of them, and their visitor experience was (seemingly) enjoyable. On the other hand, the mother in Family D played a role of taskmaster for her son and sometimes took pieces away from him toward the goal of solving the task perfectly. This led to a skewed distribution of learning talk and apparent feelings of frustration, potentially resulting in a less positive visitor experience.

Consistent with Pattison et al. (2018), engagement time at exhibits in this study did not influence the types of learning talk that occurred within family interactions. Some families spent the same amounts of time at exhibits but varied greatly in the number of learning talk categories they attended to. However, as Pattison and colleagues noted, this could be due to the differences between exhibits, such as the accessibility of the mathematics.

Findings from other studies connected to the various learning talk categories in this study. Although Geerdts et al. (2015) found that in a science exhibit setting parents infrequently offered up explanations, the findings of this study suggest that in a mathematics focused exhibit, this is not the case. While there is not explicitly an Explanations Talk learning category in the coding framework, most of the interactions that could be labeled as an explanation would fall under the Conceptual and Strategic Talk categories. Geerdts et al. attributed the lack of explanations to visitor motivations. While I did not look at motivation in this study, the data collected suggest that the puzzles naturally lend themselves to offering explanations regardless of visitor motivation.

Similarly, Cooper (2011) found that very few mathematical conversations took place in science centers, museums, and other informal settings, and those that did take place were usually prompted by parents. It is difficult to compare the present study to Cooper's since her study was specifically in exhibits that were not designed to engage visitors in mathematical conversations and the exhibits in the present study were. However, the findings of the present study indicate that in an exhibit focused on mathematics, there are many mathematical conversations taking place. Moreover, as Table 3 shows, those conversations come from *both* parents and children.

Vandermaas-Peeler et al. (2015) found that families rarely read any posted signage or made connections to prior knowledge. The authors were uncertain if families not reading signage was an issue with their specific exhibit or if it was a generalizable finding. Considering that all six of the families in the present study had instances of Quotation Talk, this suggests that the problems Vandermaas-Peeler et al. saw were indeed specific to their exhibit. However, as shown in Table 3, there were almost no instances of Connection Talk among the six families – in alignment with Vandermass-Peeler and colleagues' finding. This suggests that making connections to prior knowledge and experience is one of the harder aspects of learning for mathematics exhibits to bring out.

Related to the difficulty and need for exhibits to bring out connections to prior knowledge, Dancu et al. (2011) noted in their study that, "Adult visitors struggled to find a motivational purpose at the exhibit. When asked if there was anything confusing about the exhibit, several adults requested a connection to real-world motivations and responsibilities" (p. 352). None of the visitors in the present study expressed lack of motivation to engage in the exhibits, suggesting that the puzzle nature of the exhibits seemed to be motivation enough; however, participants of the present study did demonstrate a lack of Connection Talk and connections to prior knowledge.

Revised Coding Framework

In the course of my coding and analysis I found that there were several notable and interesting interactions in the data that did not seem to quite fit into any of the categories of my adapted AFL. For example, Family D had their own unique way of expressing displeasure at the exhibits. As previously noted, the mother in family D was in control at the exhibits, trying to solve them for her own benefit rather than collaboratively with her son. She was so in control that she refused to let her son move the pieces of the puzzles once she had placed them:

> Mother – No, but I think this is the number that we need to change (*pointing to the eight piece).*
>
> Son – (*swaps the nine piece for the eight piece)*
>
> Mother – Don’t be moving them
>
> Son – (*swaps the pieces back)*

Also, the mother in family D on several occasions physically took pieces that the son was holding in his hands and would push his hands away when he would start to try and manipulate the puzzles. To me, it seems like this this interaction is somewhere in between Strategic and Affective Talk where the family members are battling with each other on what strategy to use in solving a puzzle and the interaction could be labeled something like Confrontation Talk.

Another change I would make to the Affective Talk category would be the addition of a category named Encouraging Talk. As mentioned previously, this category of learning talk would be particularly applicable to Family E in their persistent pursuit of solving the Uranium

Pile puzzle. The encouraging statements that took place could not quite be placed as pleasure or displeasure, thus the need for another category.

Additionally, in a revised coding scheme, I would eliminate the subcategory of Naming Talk. In the context of these puzzle exhibits, Naming Talk seemed to always take the form of just naming the puzzle pieces, (e.g. "we switch *the* three and *the* seven") which to me was not really "an act of identifying and sharing what is significant in a complex environment (p 274)" as Allen (2002) had put it. Moreover, this type of Naming Talk interaction almost always occurred in the context of an interaction or statement that was also coded as Strategic Talk. So, the significance of any Naming Talk interactions would not be lost if this subcategory was eliminated.

Another aspect of the coding framework that seemed lacking in the context of these puzzles was that the framework as constructed did not seem to capture the nonverbal communication that was taking place. The puzzles naturally lend themselves to causing a lot of pointing and gesturing. I tried to capture a lot of this communication in the Identification and Use Talk categories, but in a revised coding framework, I would want to give more consideration for breaking out nonverbal communication into its own category. There would likely be a lot of cross-coding, but I think the nonverbal communication is fundamentally different than the verbal.

Finally, an aspect of the learning talk that became apparent despite not being specifically part of the coding scheme, was the equitability of the learning talk that was taking place, or in other words, in a given family how much of the learning talk is coming from a parent versus from a child. While it was not quantified in this study, in the course of observation it was very clear that families A and B had more equitability of learning talk compared to the other families, with the children in families A and B taking on a more prominent role in the solving of the puzzles.

This is not to say that families A and B were more successful or had “better” conversations taking place. Family E had arguably the richest conversations amongst the families despite not being as equitable as families A and B. This notion of equitability of learning talk may not be considered learning talk per se, but rather is a dynamic unique to each family that influences the types of learning talk that arise.

Chapter 8: Limitations and Implications

Limitations

Perhaps the most limiting aspect of this study was the study population size. It proved to be rather difficult to recruit families for this study. The foot traffic at the science center was rather limited, even on weekends when I was there to recruit families. This was exacerbated by the fact the main attraction for families at the science center was a planetarium show that was run at regular intervals. So, most families that were seen interacting with the puzzles were merely idly trying to solve the puzzles while waiting for the show to start. This put a time constraint on how long families could be observed and whether they wanted to participate in the study. There was more than one instance of prospective families opting not to participate because their planetarium show was about to start.

This recruitment issue was somewhat alleviated by the two families (Families A and B) that were recruited outside of the planetarium, as mentioned in the Methods section. However, an added complication from these two families was that they were friends and coordinated their visits to occur at the same time. As a result, it was impossible to film both families at the same time. So, while Family A was only filmed at the Uranium Pile Puzzle and Family B was only filmed at the Odds and Evens Puzzle, both families attempted to solve the other puzzles. Unfortunately, those interactions could not be recorded.

On one hand I only focused on three of the roughly 20 exhibits which could be seen as a limitation. On the other hand, this allowed me to do a more in-depth exploration of interactions at these three exhibits, similar to the study design of Pattinson et al. (2017). Still, if a different set of three puzzles had been chosen for this study, the interactions observed among the families may have been very different.

Implications for Research

There are some implications for future research based on the findings of this study. As mentioned previously the equitability of learning talk and the overall family roles and dynamics were interesting to observe, but the study scope and methodology did not allow for an adequate way to analyze these things. Future research should explore the relationship between family roles and dynamics and learning talk in museums and science centers, echoing a similar call to action by Pattison et al. (2018).

While I did not collect any demographic data of the participants, I did learn while conversing with Family A that the mother was a kindergarten teacher. After reviewing the video, the mother clearly knew how to elicit information from her children, likely as a result of her teacher preparation and experience. The children also clearly saw her as the authority, since the father’s often helpful suggestions were largely ignored by the children. Future research may also explore the role that parent background and education play in the types of learning talk that occur in mathematics focused exhibits.

Based on the experiences and reactions of young visitors to the *Math Moves!* exhibits (see Selinda Research Associates, 2016; Wright & Parkes, 2015), Nemirovsky et al. (2017)

called for future research to examine how mathematics is made recognizable in various settings, particularly in informal settings such as science centers and museums. Pattinson et al. (2018) began to explore this idea by measuring mathematical awareness while families engaged in mathematics exhibits. Arguably, my study briefly attended to this topic by exploring the conversations of families and categorizing the conversations into more traditionally accepted ways of talking about mathematics (e.g., Conceptual Talk and Strategic Talk) and informal ways of talking about mathematics that may be more valued by various families and cultures (e.g., Connecting Talk and Affective Talk). However, future research should continue research of this kind, where family interactions are studied qualitatively and in-depth, to better understand how families recognize and learn mathematics in informal settings.

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