Estimating Grazingland Yield from Commonly Available Data

KARIN WISIOL

Abstract

Range managers must often estimate or predict annual forage yield at a distance, from minimal data, or for a variety of sites. This study compared and modified 6 simple formulas potentially useful for this purpose. The grazingland data that were used represented 44 sites on 5 continents. Soil texture at site affected accuracy of all formulas. Shrubbiness affected accuracy of formulas based on evapotranspiration. Some formulas modified to include past-year yield as a variable were fairly accurate over a variety of grazinglands. An equation based only on past-year yield predicted yield within an average of 34% at new sites. Equations that incorporated past-year yield, used a measure of current effective moisture, and had a limiting-factor approach estimated current yield within an average of 18-19% at new sites.

Attempts to estimate forage production of particular grazinglands have resulted in hundreds of mathematical models (Van Dyne et al. 1977). The general relation between forage yield and climate has also occupied researchers (Coupland 1979, Lauenroth 1979, Singh et al. 1980). However, range managers and resource planners concerned about carrying capacity must often estimate a year's production for an unmodeled site or for a variety of sites. They may have available only such data as precipitation, temperature, and possibly past yield. But differences in the performance of yield models that claim some generality and require only readily available data have not been investigated systematically for a variety of rangeland sites.

In this study the ability of 6 sample formulas to estimate grazingland yield for specific years for a variety of sites was tested, and the formulas were modified. This paper compares the equations and suggests that some modified versions could be used to estimate a year's aboveground dry-matter production on grazinglands where detailed data are scarce.

Formulas, Data, and Methods

Six published formulas (Table 1) that were intended to be large-area plant production models and required minimal climate data were chosen for testing. Each related annual yield to precipitation, temperature, or variables derived from these, through coefficients estimated by regression. Two formulas had the same structure; they were designed to estimate aboveground standing dry matter on rangelands of 2 regions, the Sahel-Sudan and the Mediterranean Basin (Le Houerou and Hoste 1977). The other 4 formulas (by Czarnowski 1973; Lieth 1973, 1975; Lieth and Box 1972; and Rosenzweig 1968) were designed to estimate net primary productivity of any vegetation type. Estimates of the Czarnowski, Lieth, and Lieth-Box formulas, all for total plant production, were halved to apply to aboveground production, a relation common for rough conversion (e.g., Lieth 1973).

Data on climate and concurrent aboveground dry matter production came from grazinglands at 28 locations in North America, Asia, Africa, Australia-New Zealand, and Europe. Sites differing in physical characteristics or sampling method were available for some locations and consecutive data years were available for most, for a total of 194 years of climate and yield data at 44 sites (Table 2). Nearly all sites were exclosures on native grazed range.

Basic variables chosen were those thought likely to affect accuracy of simple rangeland yield models. Variables included 3 qualitative ones derived from site descriptions:

(1) dominant vegetation type—perennial herb using the "C4" photosynthetic pathway (Downton 1975, Moore 1977, Waller and Lewis 1979), perennial herb using the "C3" pathway, annual herb, or shrub;

(2) soil texture—sand, loam, clay, or silt, based on particle size and proportion (Soils Survey Staff 1951);

(3) method of sampling yield—peak standing crop, end-of-season crop, sum of peak crops by species, or sum of crop increments (Kelly et al. 1974, Singh et al. 1975).

Numeric variables totaled 104. Basic ones were latitude, elevation, precipitation, temperature, growing-season length and temperature, water-balance variables, a moisture index, yield, and past-year yield. Annual water-balance variables were derived from monthly water balances based on precipitation and temperature (Thornthwaite and Mather 1955, 1957) and calculated by computer programs (Wisiol 1981). These variables included actual and potential evapotranspiration, ratio of actual to potential evapotranspiration, and ratio of precipitation to potential evapotranspiration. A moisture index (Thornthwaite 1948) was calculated as: annual surplus minus 0.6 times deficit, the difference taken as a percent of potential evapotranspiration. Other variables included past-year values, ratios, reciprocals, logarithms, and products of basic variables.

The study compared annual yields estimated by models with reported yields, with error expressed as a percent of yield. Variables not used in the formulas were tested for effect on accuracy, often graphically. Logarithmic transformation helped normalize distributions for parametric analysis. Regression, covariance, and time series methods (Johnston 1972) were used to test effect of variables on accuracy as well as to modify original formulas. Tests used a single year of data per site except where multiple-year data were appropriate. Details are given by Wisiol (1981). Spearman rank correlations (Snedecor and Cochran 1967) were taken as primary measures of association.

Modified formulas, built using data from some of the sites, were tested on data from the remaining sites. Versions based on identical rangeland data were derived (in logarithmic form) for all the original formulas, to focus on differences due to structure or

---

Author was a graduate student, Department of Biological Sciences, Univ. of Illinois at Chicago, and is currently plant physiologist, USDA Agricultural Research Service, Soil-Water-Plant Research, Urbana, Ill. Author's mailing address is 614 Indian Rd., Glenview, Ill. 60025.

The author is especially indebted to Drs. J. Lussenhop, D.B. Carter, W.D. O'Neill, J.A. Teeri, G. Agarwal, and the late G.M. Van Dyne for their support. Funds came from fellowships of the Univ. of Illinois at Chicago and a National Science Foundation grant, DEB-7826735.

The study includes data shared by D.A. Duncan, C. Floret, L.A. Joyce, W.K. Lauenroth, J.E. Radcliffe, D.S. Rickard, J.S. Singh, T.E. Skinner, S. Smoliak, F.A. Sneva, G.M. Van Dyne, H. Van Keulen, J.P. Van Niekirk, P.J. Vickery, D.G. Wilcox, and P.S. Yadava. Data were also made available by the Canadian Climate Center, New Zealand Meteorological Service, U.S. National Climate Center, Natural Resource Ecology Laboratory at Colorado State Univ., and Tunisian Institut des Régions Arides. Agriculture Canada, Ottawa, supplied a computer program by G.W. Robertson and D.A. Russelo for daylength calculation.

Manuscript accepted December 15, 1983.

Table 1. Published yield equations tested.

Source	Equation[1]	Climate variables[2]
Czarnowski 1973	$y = 0.176(V)(L)(1 - e^{-P/PE})$	P/PE = precipitation ÷ potential evapotranspiration V = saturated vapor pressure, mm Hg, for mean temperature T of growing season (monthly T>3°C) L = length growing season, hrs. daylight
Lieth 1973	y = minimum of (Y1, Y2) $Y1 = 15000\ (1 - e^{-0.000664P})$ $Y2 = 15000/(1 + e^{1.315 - 0.119T})$	P = precipitation, mm T = mean temperature, ° C
Lieth & Box 1972	$y = 15000\ (1 - e^{-0.0009695(AE - 20)})$	AE = actual evapotranspiration, mm
Le Houerou & Hoste 1977 (Mediterranean model)	$y = 3.89\ (P^{1.09})$	P = precipitation, mm
Le Houerou & Hoste 1977 (Sahel-Sudan model)	$y = 2.643(P^{1.001})$	P = precipitation, mm
Rosenzweig 1968	$y = 10(10^{1.66(\log}10^{AE-1)})$	AE = actual evapotranspiration, mm

[1] y = annual aboveground dry matter yield, kg/ha; e = base of natural logarithms, 2.718. Form, units, and symbols are those allowing easy comparison. Czarnowski, Lieth, and Lieth-Box estimates were intended for total vs. aboveground yield and are shown halved here. Rosenzweig model was published in logarithmic form: $\log y = 1 + 1.66 (\log AE - 1)$, where y was in g/sq m.

[2] Annual totals except for temperature (mean).

Table 2. Grazingland sites, with annual climate and yield. Mean ± standard deviation is given if data covered a series of years. AE and PE = actual and potential evapotranspiration. Yield = aboveground dry matter.

Location	Latitude (deg.)	Longitude (deg.)	Elevation (m)	Site[1]	Temp. (°C)	Precip. (mm)	AE (mm)	PE (mm)	Yield (kg/ha)
Aiken (AEC Plant), SC, USA	33.6 N	81.7 W	161	1	17	1013	668	901	5855
				2	17	1013	668	901	4851
Ale IBP Site, Richland, WA, USA	46.4 N	119.5 W	365	3	12 ± 0.4	190 ± 14	168 ± 14	748 ± 4	1500 ± 396
Armidale, N.S.W., Australia	30.6 S	151.5 E	1046	4	12 ± 0.1	868 ± 182	595	650 ± 4	6320 ± 1541
				5	12 ± 0.1	868 ± 182	595	650 ± 4	9405 ± 3444
Cañas, Costa Rica	10.3 N	85.2 W	45	6	28	2044	1352	1787	9676
				7	28	2044	1352	1787	13870
Claresholm, Alberta, Canada	50.0 N	113.7 W	1018	8	4.7 ± 1.1	458 ± 93	385 ± 46	535 ± 23	2755 ± 493
				9	4.8 ± 1.2	462 ± 102	387 ± 50	533 ± 24	3364 ± 485
Craigmyle, Alberta, Canada	51.6 N	112.2 W	841	10	2.6 ± 1.3	392 ± 81	379 ± 33	520 ± 26	6734 ± 1727
Desert Exptl. Range, Utah, USA	38.6 N	113.8 W	1600	11	9.9 ± 0.8	170 ± 53	170 ± 52	711 ± 42	249 ± 103
Dickinson, ND, USA	46.9 N	102.8 W	750	12	4.4 ± 0.3	420 ± 73	373 ± 46	560 ± 35	2237 ± 734
				13	4.4 ± 0.3	420 ± 73	373 ± 46	560 ± 35	2671 ± 489
Dissa, Tunisia	33.9 N	9.9 E	61	14	19 ± 0.6	208 ± 48	189 ± 20	994 ± 60	333 ± 71
Gabes, Tunisia	33.9 N	10.0 E	56	15	20 ± 0.8	227 ± 69	191 ± 64	1019 ± 54	269 ± 123
Gellap-Ost, SW Africa (Namibia)	26.4 S	18.1 E	1040	16	20 ± 0.6	218 ± 154	216 ± 149	1031 ± 53	433 ± 237
Kalahari, SW Africa (Namibia)	24.1 S	18.5 E	1200	17	19 ± 0.3	315 ± 149	300 ± 117	923 ± 25	766 ± 370
Kurukshetra, India	30.0 N	76.8 E	250	18	24	676	495	1421	19740
				19	24	676	495	1421	20466
				20	24	676	495	1421	26636
Lamto, Ivory Coast	6.2 N	5.0 W	120	21	27	1189	1089	1638	6940
				22	27	1189	1072	1638	7010
				23	27	1189	1072	1638	9880
				24	27	1189	1111	1638	11110
				25	27	1189	1111	1638	6960
				26	27	1189	1111	1638	8300
				27	27	1189	1111	1638	6890
				28	27	1189	1111	1638	13380
Lanzhot, Czechoslovakia	48.8 N	17.0 E	155	29	8.8	537	451	648	5270
				30	8.8	537	428	648	3940
Mandan, ND, USA	46.8 N	100.9 W	610	31	5.3 ± 1.2	394 ± 97	385 ± 74	604 ± 31	2345 ± 548
Manyberries, Alberta, Canada	49.1 N	110.5 W	934	32	3.8 ± 0.8	336 ± 89	323 ± 58	545 ± 24	1372 ± 257
Maraekakaho, N.I., New Zealand	39.6 S	176.3 E	120	33	14 ± 0.3	810 ± 45	615 ± 94	725 ± 27	7280 ± 915
Meekatharra, W.A. Australia	26.6 S	118.5 E	517	34	22 ± 0.7	306 ± 148	305 ± 152	1173 ± 72	268 ± 105
Migda, Israel	31.4 N	34.4 E	100	35	20 ± 0.2	289 ± 86	288 ± 83	992 ± 16	2644 ± 1081
Mt. Washington, NH, USA	44.3 N	71.3 W	1840	36	−2 ± 0.7	1620 ± 447	329 ± 61	367 ± 54	1938 ± 418
			1665	37	−2 ± 0.8	1609 ± 431	342 ± 70	404 ± 53	1167 ± 278
Osage (IBP), Pawhuska, OK, USA	36.9 N	96.5 W	380	38	15 ± 0.2	827 ± 157	633 ± 117	867 ± 10	5977 ± 1250
Rajkot, India	22.3 N	70.9 E	136	39	26	702	617	1562	3692
San Joaquin Exp. Range, CA, USA	37.1 N	119.7 W	370	40	15 ± 0.7	474 ± 154	300 ± 33	827 ± 54	2626 ± 830
Scotfield, Alberta, Canada	51.6 N	111.3 W	762	41	3.3 ± 0.1	317 ± 46	315 ± 37	558 ± 35	1374 ± 326
Squaw Butte Exp. Sta., OR, USA	43.5 N	119.7 W	1425	42	7.7 ± 0.5	291 ± 82	215 ± 39	577 ± 20	836 ± 313
Westport, S.I., New Zealand	41.8 S	171.6 E	60	43	12 ± 0.7	2178 ± 267	680 ± 46	684 ± 47	10922 ± 741
Winchmore, S.I., New Zealand	43.8 S	171.8 E	160	44	11 ± 0.3	724 ± 118	545 ± 49	633 ± 23	5867 ± 1111

[1] Multiple sites for same location differed in at least vegetation type, soil texture, or method of sampling yield. Full descriptions and sources are given by Wisiol (1981). Maraekakaho data was used only as series data.

variables. Rangeland versions took this form, after back-transformation:

$$\text{Rangeland Yield} = a \bullet \text{Original Formula's Estimate}^{b}$$

where log a and b were estimated by regression.

Tests of estimates or of year-ahead predictions examined ability to estimate or predict a single year's yield at a variety of sites. Two indexes of accuracy were calculated for each model: (1) average percentage error over test sites; and (2) maximum percentage error for the 9/10 of the test sites at which model was most accurate, taken as a reasonable measure of the maximum error to be expected at most new sites.

Results

Accuracy of Original and Rangeland-based Formulas

The 6 published formulas erred by an average 67% to 128% in estimating annual yields for grazingland sites (Table 3). All tended strongly to overestimate, as much as several hundred percent, except the 2 Le Houerou-Hoste equations. The Sahel-Sudan equation showed the smallest error and range of error. Each formula was able to estimate yields within 5% or better at some sites.

Basing equations on global rangeland data decreased average error by 18 to 78 percentage points at new sites and, for all except the Sahel-Sudan equation, sharply reduced maximum error (Table 3). For the Czarnowski and Lieth rangeland versions, percentage error at new sites averaged less than 40% and maximum error at 9/10 of the new sites was less than 80%.

Effect of Site and Climate on Accuracy

When mean percentage errors were tabulated by classes of vegetation, soil, and sampling method, the published formulas except the Sahel-Sudan formula (and for a few classes the Mediterranean formula) overestimated yields for all classes. They all overestimated for shrubby sites, sandy sites, and sites sampled by summing increments, and all except the Sahel-Sudan showed maximum error at such sites. Equations derived from global rangeland data overestimated yield for shrubby sites but not for sandy sites, and errors were unrelated to sampling method.

Covariance analysis of original and modified equations using time series data (N=143) explored sensitivity to qualitative site factors, while adjusting for data-base and past-year effects. Soil texture significantly affected both average accuracy and relation of estimate to yield, except for year-ahead predictive equations based on the Le Houerou-Hoste formulas. Accuracy and relation of estimate to yield for clay and loam sites differed from those for sand and silt. For equations based on the Lieth-Box and Rosenzweig formulas, the relation of estimate to yield also differed for shrubs. Method of sampling yield did not significantly affect accuracy of any equation, and neither did any interaction of vegetation with soil or sampling method.

Spearman rank correlations quantified association of percentage error with numeric site and climate variables. For all formulas except the Sahel-Sudan and Rosenzweig formulas, errors showed a highly significant negative association with actual evapotranspiration. Errors of the Czarnowski, Lieth, and Lieth-Box formulas also showed such an association with the ratio of precipitation to potential evapotranspiration, ratio of actual to potential evapotranspiration, and Thornthwaite moisture index (itself significantly associated with the ratio of precipitation to potential evapotranspiration). Errors of the Czarnowski, Lieth, and Lieth-Box formulas were also negatively associated with precipitation. Latitude, temperature, potential evapotranspiration, and growing season temperature and length were associated with error only for the Rosenzweig equation. Elevation was associated with error only for the Lieth-Box equation.

Yield itself was associated with percentage error for all formulas except the Rosenzweig formula. The association was positive for the Sahel-Sudan equation and negative for the others. Variables showing significant Spearman correlations with yield were: past-year yield; estimates of all formulas; past-year Lieth, Lieth-Box, and Czarnowski estimates; actual evapotranspiration; precipitation; and past-year moisture index.

Numeric variables that boosted precision when added to formulas were past-year yield, for all formulas, and actual evapotranspiration, for formulas based on precipitation alone. Actual evapotranspiration itself could account for almost as much variation in yield (68%) as any equation lacking past-year yield. Adding variables based on latitude, elevation, or both made coefficients highly unstable. Graphically, the relation of annual yield to distance from the equator resembled a sine function, with 3 yield maxima: between 0° and 10°, near 30°, and near 40°.

Moisture index values were related to vegetation types. For the 189 annual index values calculated, the mean for sites dominated by shrubs was -44; C3 annual herbs, -23; C4 perennial herbs, also -23; and C3 perennial herbs, +15.

Effect of Past-year Values on Accuracy

Past-year yield level affected accuracy more than any other variable, except for the Sahel-Sudan equation in which errors were linked only to current yield levels. For the other formulas, percentage error was negatively associated with past-year yield with Spearman rank correlations ranging from -0.91 to -0.62. Yield itself showed a highly significant association with past-year yield at a Spearman rank correlation of 0.95 for raw values and Pearson linear correlation of 0.90 for logarithms. When past-year yield was included as a variable, formulas could account for 87% to 94% of

Table 3. Error as percent of yield, for original, rangeland-based, and time series rangeland equations. Absolute error is mean (± standard deviation) of absolute errors for all test sites. Maximum error for 9/10 sites is maximum of yield minus estimate for the best 9/10 of the sites.

	Absolute error (%) all site			Maximum error (%) 9/10 sites	
Equation[1]	Original equation[2]	Rangeland version[3]	Time series rangeland version[4]	Original equation[2]	Rangeland version[3]
Czarnowski	117 (±187)	39 (±23)	19 (± 9)	-434	71
Lieth	97 (±147)	35 (±23)	18 (±13)	-422	78
Lieth-Box	122 (±170)	60 (±32)	28 (±20)	-432	-102
Le Houerou-Hoste Mediterranean	98 (±176)	49 (±21)	51 (±64)	-302	87
Le Houerou-Hoste Sahel-Sudan	67 (± 40)	49 (±21)	52 (±73)	91	87
Rosenzweig	128 (±120)	105 (±70)	26 (±20)	-258	-215

[1] By source; Table 1 gives form.

[2] Error is for 43 test sites, for a single year's yield per site.

[3] Linear transformation of original to general rangeland equation, by regression of logarithmic values of yields on estimates, using 2nd year of data from 21 sites having more than 2 years' data. Error is for back-transformed estimates of a single year's yield at each of 22 other sites.

[4] Incorporating past-year yield. Equations were derived in logarithmic form by regression using series data from sites having more than 3 years' data (N=131) and were tested for estimation of a year's yield at each of 9 other sites.

variation (N=25, standard error of estimate = 3.8% to 5.4% of yield). Past-year yield used alone could account for 82% of variation (standard error = 5.9% of yield).

Equations built to include past-year yield were tested for estimation of a year's yield at a new group of sites. When past-year yield was the only variable, error averaged 34% and maximum error at the 9/10 of the test sites where the equation worked best was calculated at 50%. Year-ahead predictive equations that combined lagged yield with lagged reciprocal moisture index or lagged Lieth estimate showed average errors of only 26–27% but maximum errors about the same as for past-year yield used alone. Equations combining past-year yield with Lieth or Czarnowski estimates based on current climate showed errors averaging only 18–19%, with maximum errors for the best 9/10 of the test sites calculated at 30% and 24%, respectively.

Discussion

Data-base Effect

Data base had an overriding effect on accuracy for these simple formulas. All the original formulas except the Sahel-Sudan formula tended to overestimate rangeland yields grossly; this shows that, for most regions, rangeland sites produce less aboveground biomass than one would expect from general climate-vegetation relations. Basing equations on global rangeland data reduced error by reducing overestimation.

Structure and Type of Variables

Structure and type of variable governed differences in performance of equations based on global rangeland data. Equations having equivalent structure and variables, such as the 2 Le Houerou-Hoste equations, showed equivalent errors.

The rangeland-based Lieth and Czarnowski formulas outperformed the rest. Both formulas used information about moisture and energy simultaneously, the Lieth by alternate equations and the Czarnowski by multiplicative variables. Use of such approaches may enable general grazingland models to perform more accurately at sites outside their data base. Weekly climate data might represent moisture-energy relations more precisely than the monthly data used here, especially for water-stressed sites (Webb et al. 1983).

A more mechanistic approach did not ensure greater accuracy. Accuracy of the most mechanistic of the formulas, having the most variables and most elaborate theoretical basis (Czarnowski 1964), was matched by accuracy of the Lieth formula with one equation based on precipitation and another on temperature. Apparently either structure avoided the flaws which make most multiple-variable equations based on climate unreliable (Katz 1979). Inherent nonlinearity might also afford simple formulas like these an advantage in dealing with the heterogenous character of rangeland data. Neither the Lieth nor the Czarnowski formula could be transformed to a linear equation, and the nonlinear Lieth-Box equation was twice as accurate as the Rosenzweig equation using the same variable.

Analysis of errors according to vegetation, soil, and method of sampling suggests that if algebraic formulas of this type are designed for general vegetation, they will overestimate yield seriously for nearly all rangelands. The analysis by site factor also suggests that even if equations are based on global grazingland data, those lacking a past-yield term or a term taking vegetation or soil into account will overestimate yield for sites that are both shrubby and sandy.

The effect of soil texture on accuracy of estimation and prediction, confirmed by covariance analysis, suggests that soil texture or a correlated variable holds information about yield that is not captured by information about climate or previous year's yield. Both estimative and predictive general formulas might be improved by coding for soil. Adding percent clay or a correlate as a variable might improve general yield formulas.

The fact that shrubbiness influenced relation of estimates to yield only for equations based on actual evapotranspiration may mean that the relation of effective moisture to yield differs for woody and herbaceous rangeland vegetation. The relation of the Thornthwaite moisture index to dominant vegetation type, apparently reflecting plant ability to endure longterm water stress, confirms its utility as an indicator to sites suitable for different kinds of grassland vegetation (Thornthwaite 1952), an important consideration in revegetation studies. The study extends this concept to include photosynthetic type and also indicates that the ratio of precipitation to potential evapotranspiration could be used as an alternate.

Strong correlation between many of the site and climate variables in this study emphasizes the need for caution in use of stepwise and multiple regression to build forage-weather models (Katz 1979). Accidents of sampling are also a danger. Some researchers using data largely from northern hemisphere sites have postulated a negative exponential relation between degrees of latitude and aboveground yield (Caldwell 1975, Van Dyne et al. 1979). The more complex pattern of yield peaks and troughs suggested by these data might result in part from the joint latitudinal distribution of precipitation and potential evapotranspiration. Moisture supply exceeds demand poleward of 40° and below 10°, and supply is lowest while demand is highest between 20° and 30° (Sellers 1965).

Although this study confirmed traditional wisdom that rangeland yield is closely linked to moisture-based variables, especially effective or available moisture, nutrients can be decisive on some rangelands (Bremen and De Wit 1983, Van Keulen et al. 1976).

Past Yield Effect

The best single estimator and predictor of yield, and the variable having most effect on accuracy, was past-year yield, as might be expected. Although current climate information did cut average error in half, past yield remained critical information. Its addition improved accuracy of all equations at new sites. Certain equations including past yield estimated yield within an average 18%, while the closest estimation possible without use of past yield was 39%. When past-year yield was used alone in an equation to predict yield, error averaged only 34% at new sites. Even at a single site, where one would expect most information about yield variation by year to come from climate, 6% of variation can come from past-year yield (Hanson et al. 1982). Past yield may integrate factors determining potential yield, or range condition, while current effective moisture distinguishes one year's yield from another.

Literature Cited

Breman, H., and C.T. de Wit. 1983. Rangeland productivity and exploitation in the Sahel. Science 221:1341-1347.

Caldwell, M.M. 1975. Primary production of grazing lands. p. 41-73. *In:* J.P. Cooper (ed.), Photosynthesis and productivity in different environments, International Biological Programme 3. Cambridge Univ. Press, Cambridge.

Coupland, R.T. 1979. Conclusion. p. 335-387. *In:* R.T. Coupland (ed.), Grassland ecosystems of the world; analysis of grasslands and their uses. International Biological Programme 18. Cambridge Univ. Press, Cambridge.

Czarnowski, M.S. 1964. Productive capacity of locality as a function of soil and climate with particular reference to forest land. Louisiana State Univ. Press, Baton Rouge.

Czarnowski, M.S. 1973. W sprawie mapy i modeln siedliskowej zdoldosci prodikcyincj ziemi (On the map and model of productive site-capacity on the earth). Przeglad geografieczny (Geographical Review) 45:295-308. (In Polish, with English summary.)

Downton, W.J.S. 1975. The occurrence of C4 photosynthesis among plants. Photosynthetica 9:96-105.

Hanson, C.L., J.R. Wight, J.P. Smith, and S. Smoliak. 1982. Use of historical yield data to forecast range herbage production. J. Range Manage. 35:614-616.

Johnston, J. 1972. Econometric methods. 2nd ed. McGraw-Hill, New York.

Katz, R.W. 1979. Sensitivity analysis of statistical crop-weather models. Agr. Meteorol. 20:291-300.

Kelly, J.M., G.M. Van Dyne, and W.F. Harris. 1974. Comparison of three methods of assessing grassland productivity and biomass dynamics. Amer. Midl. Natur. 92:357-369.

Lauenroth, W.K. 1979. Grassland primary production: North American grasslands in perspective. p. 3-23. *In:* N. French (ed.), Perspectives in grassland ecology. Ecological Studies 32. Springer, New York.

Le Houerou, H.N., and C.H. Hoste. 1977. Rangeland production and annual rainfall relations in the Mediterranean Basin and in the African Sahelo-Sudanian zone. J. Range Manage. 30:181-189.

Lieth, H. 1973. Primary production: terrestrial ecosystems. (Paper at 2nd Cong. of AIBS, Oct. 1971, Miami, Fla.) Human Ecol. 1:303-332.

Lieth, H. 1975. Modeling the primary productivity of the world. p. 237-263. *In:* H. Lieth and R.H. Whittaker (eds.), Primary productivity of the biosphere. Ecological Studies 14. Springer, New York.

Lieth, H., and E. Box. 1972. Evapotranspiration and primary productivity: C.W. Thornthwaite memorial model. p. 37-46. *In:* J.R. Mather (ed.), Papers on selected topics in climatology, Thornthwaite Memorial Vol. 2. C.W. Thornthwaite Associates, Elmer, N.J.

Moore, R.T. 1977. Gas exchange and photosynthetic pathways in range plants. p. 1-46. *In:* R.E. Sosebee (ed.), Rangeland plant physiology. Range Sci. Series No. 4, Soc. Range Management, Denver, Colo.

Rosenzweig, M.L. 1968. Net primary productivity of terrestrial communities: prediction from climatological data. Amer. Natur. 102:67-74.

Sellers, W.D. 1965. Physical climatology. Univ. Chicago Press, Chicago.

Singh, J.S., W.K. Lauenroth, and R.K. Steinhorst. 1975. Review and assessment of various techniques for estimating net aerial primary production in grasslands from harvest data. Bot. Rev. 41:181-232.

Singh, J.S., M.J. Trlica, P.G. Risser, R.E. Redmann, and J.K. Marshall. 1980. Autotrophic subsystem. p. 59-200. *In:* A.I. Breymeyer and G.M. Van Dyne (eds.), Grasslands, systems analysis and man. Int. Biol. Programme 19. Cambridge Univ. Press, Cambridge.

Snedecor, G.W., and W.G. Cochran. 1967. Statistical methods. 6th ed. Iowa State Univ., Ames.

Soils Survey Staff. 1951. Soil Survey Manual. Handbook No. 18, USDA, Washington. (Cited in: R.M. Basile, A geography of soils. p. 29-35, Table 2.1 and Fig. 2.16. Brown, Dubuque, Iowa, 1971.)

Thornthwaite, C.W. 1948. An approach toward a rational classification of climate. Geog. Rev. 38:55-94.

Thornthwaite, C.W. 1952. Grassland climates. (Paper given at 6th Int. Grassland Cong., Aug. 1952, State College, Penn.) Publ. in Climatology 5. Laboratory of Climatology, Centerton, N.J.

Thornthwaite, C.W., and J.R. Mather. 1955. The water balance. Pub. in Climatology 8. Laboratory of Climatology, Centerton, N.J.

Thornthwaite, C.W., and J.R. Mather. 1957. Instructions and tables for computing potential evapotranspiration and the water balance. Pub. in Climatology 10:181-311. Thornthwaite Lab. of Climatology, Centerton, N.J.

Van Dyne, G.M., L.A. Joyce, B.K. Williams, and J.E. Kautz. 1977. Data, hypotheses, simulation and optimization models, and model experiments for shortgrass prairie grazinglands: a summary of recent research. PB-273 738. U.S. Dep. Commerce, NTIS, Springfield, Va.

Van Dyne, G.M., F.M. Smith, R.L. Czaplewski, and R.G. Woodmansee. 1979. Analyses and syntheses of grassland ecosystem dynamics. p. 1-79. *In:* J.S. Singh and B. Gopal (eds.), Glimpses of ecology. Int. Sci. Pub., Jaipur, India.

Van Keulen, H., C.T. de Wit, and H. Lof. 1976. The use of simulation models for productivity studies in arid regions. p. 408-420. *In:* O.L. Lange, L. Kappen, E.-D. Schulze (eds.), Water and plant life. Ecological Studies 10. Springer, New York.

Waller, S.S., and J.K. Lewis. 1979. Occurrence of C3 and C4 photosynthetic pathways in North American grasses. J. Range Manage. 32:12-28.

Webb, W.L., W.K. Lauenroth, S.R. Szarek, and R.S. Kinerson. 1983. Primary production and abiotic controls in forests, grasslands, and desert ecosystems in the United States. Ecology 64:134-151.

Wisiol, K. 1981. Models to predict rangeland yield from common data. Ph.D. Diss. Univ. Illinois, Chicago.