MODELLING ELECTRODYNAMICS IN SATURN’S UPPER ATMOSPHERE

by

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ABSTRACT

Although electrodynamics plays an important role in controlling Saturn’s thermospheric circulation and energy balance at high latitudes, less is known about its effects at lower latitudes. Recent observations from the Cassini magnetometer instrument near the equator during the Grand Finale tour revealed azimuthal magnetic field perturbations associated with ionospheric electrodynamics at low latitudes. If a significant ionospheric wind dynamo exists in Saturn’s ionosphere at low and middle latitudes, it could alter the circulation and energy balance of Saturn’s upper atmosphere and could explain the magnetic field perturbations at low latitudes. In this manuscript, we develop several models of thermospheric electrodynamics, including a new formulation for coupling the ionosphere and magnetosphere, and use them to investigate the role ion drag and resistive heating throughout Saturn’s thermosphere. Results from our models suggest that electrodynamics can generate substantial currents at low and middle latitudes that can in some cases be comparable to high-latitude currents. We find that the electrodynamics is strongly dependent on the conductivity and wind profiles. In particular, the presence of an equatorial jet is important for driving significant eastward magnetic field perturbations and electrodynamics at low latitudes. Ion drag can help lift an equatorial jet from below to higher altitudes than it could otherwise reach, and may be able to spread a narrow equatorial jet in latitude as well. Because ion drag reduces wind shears that drive electrodynamics, any evidence of significant electrodynamics implies that the winds experience continual forcing by other mechanisms. In some models, ion drag partially opposes the Coriolis force at middle latitudes, slightly weakening the Coriolis barrier and allowing some meridional transport of auroral energy. The resistive heating predicted by our models is insufficient to explain the surprisingly high temperatures at low latitudes. Finally, we discuss directions for future research.
CHAPTER 1

Introduction

Some of the most surprising discoveries from the Cassini mission were made during its Grand Finale tour. In-situ measurements indicated the presence of dust grains that are associated with so-called “ring rain” primarily at low latitudes (Hsu et al., 2018). These grains, as well as inflow of water, methane and other species, are expected to change the chemistry of the low-latitude ionosphere in ways that may be variable in latitude, longitude or time, as detected by the ion-neutral mass spectrometer (Waite et al., 2018). Dougherty et al. (2018) reported anomalous azimuthal magnetic field perturbations from the magnetometer (MAG) instrument that indicate the presence of a low-latitude current system between the D-ring and the ionosphere (Khurana et al., 2018). Analysis of plasma data from this region by (Wahlund et al., 2018) indicates that the ionosphere is dynamic and interacts with the rings. These discoveries and numerous others have driven an increased attention to Saturn’s upper atmosphere.

Although the Cassini mission has been able to address numerous problems, many other unsolved mysteries remain. For example, the reason for the effectively perfect alignment of Saturn’s magnetic and spin axes remains unknown (e.g. Cao et al., 2011; Dougherty et al., 2018). Because of this, the precise length of a Kronian day — a fundamental parameter for any model of Saturn — has proven difficult to measure (e.g. Sánchez-Lavega, 2005). Although modelling and observational efforts are constraining the role of gravity waves in Saturn’s middle and upper atmosphere, their relative importance is still unknown (e.g. Guerlet et al., 2018; Müller-Wodarg et al., 2019). The upper atmosphere of Saturn, as well as that of Jupiter, Uranus and Neptune, is several times hotter than would be expected from solar heating alone (e.g. Strobel and Smith, 1973; Yelle and Miller, 2004), yet many models have struggled to replicate the observed temperature profile. The electrical current struc-
ture throughout Saturn’s thermosphere remains unknown despite much effort made to measure and model these currents at high latitudes (e.g. Cowley et al., 2004a; Hunt et al., 2014, 2015; Dougherty et al., 2018).

Developing improved models of Saturn’s upper atmosphere will help us use the new discoveries to understand the longstanding problems. In this manuscript, we describe several models of electrodynamics in Saturn’s ionosphere and thermosphere and use them to examine how electrodynamics may affect the circulation and energy balance of Saturn’s thermosphere. This work contributes to a large and rapidly growing body of research, much of which revolves around the discoveries mentioned above. We therefore begin by placing this work in the context of a recent discovery and a longstanding question and suggest how they may be related.

1.1 Magnetic Field Constraints

Saturn’s ionosphere is of particular interest because it is the mediator between the atmosphere and the magnetosphere. Here, dynamics from the mesosphere can drive ionospheric currents which couple to the magnetosphere via magnetic field lines, and plasma flow in the magnetosphere can affect thermospheric winds. Because of this coupling, inferences about the ionosphere can be made from measurements made in the magnetosphere (e.g. Cowley et al., 2004a; Khurana et al., 2018). Because of the complex nature of these couplings, a complete model of Saturn’s thermosphere and ionosphere is still far away. However, much progress is being made to understand and constrain these interactions.

Vriesema et al. (2016) presented an exploratory model of low-latitude electrodynamics in Saturn’s thermosphere that focused on currents and resistive heating. Through discussion with Dr. Al Brauer and others, it was realized that the current system predicted by the model, peaking at $1.7 \times 10^{-7}$ A m$^{-2}$ at $1.5 \times 10^{-8}$ mbar could produce magnetic field perturbations strong enough to be detected by the Cassini MAG instrument, though no low-latitude perturbations had yet been measured in the thermosphere. Their model was updated to use a formulation similar
to that described in Vriesema et al. (2020, included in this dissertation as Chapter 3), from which azimuthal magnetic field perturbations could be easily calculated (Vriesema et al., 2017). The new model treated Saturn’s magnetic field as a dipole, used the one-dimensional conductivity profile described in Section 3.3.3, and imposed an symmetric, eastward zonal jet peaking at 400 m s$^{-1}$. It generated a strong equatorial current system with peak azimuthal magnetic field perturbations of roughly 30 nT at about $\pm 9^\circ$ latitude and at an altitude of roughly 1000 km. These perturbations, driven by gradients in the imposed zonal jet, were antisymmetric about the equator: eastward in the northern hemisphere and westward in the southern hemisphere.

Meanwhile, azimuthal magnetic field perturbations of the order 10 nT to 30 nT were reported at low latitudes by the Cassini MAG team (Dougherty et al., 2018; Khurana et al., 2018; Provan et al., 2019), confirming the presence of a low-latitude current system in Saturn’s thermosphere. Khurana et al. (2018) suggested that the magnetic field perturbations could be explained by assuming that the difference in zonal wind speeds at the conjugate footprints of a given magnetic field line could induce currents along that field line that are roughly consistent with the magnetic field measurements. They presented a simple, axisymmetric model in which a shell of southward FACs travel outward along field lines within the inner D-ring boundary and return back as a northward current sheet in the ionosphere. Using the magnetic field perturbation data, with rough estimates of the height-integrated Pedersen conductivity and zonal wind shear, they estimated the current strength to be 1.15 MA rad$^{-1}$.

Soon after, Provan et al. (2019) generalized the model of Khurana et al. (2018) to account for different height-integrated conductivities, neutral wind speeds, magnetic field strengths, and geometries at conjugate footprints in each hemisphere. Values ranging from 0.3 MA rad$^{-1}$ to 2 MA rad$^{-1}$ were derived from measurements made on subsequent orbits (identified by the spacecraft’s revolution, or “Rev”, number). These generalizations allowed them to characterize the high variability observed in the magnetic field perturbations. From this, they concluded that the variability is
not associated with pass altitude, local time, planetary period oscillation phase, or D68 ringlet phase, but may be due to variations in conductivity and/or thermospheric dynamics.

Vriesema et al. (2020) improved on the model of Khurana et al. (2018) using a 2D, axisymmetric, steady-state model of an ionospheric wind dynamo. Whereas the earlier model of Vriesema et al. (2017) had used a magnetic dipole centered in the equatorial plane for simplicity, the magnetic field of Vriesema et al. (2020) used a dipole magnetic field that was shifted northward, consistent with observations (Davis and Smith, 1990; Burton et al., 2010; Dougherty et al., 2018, e.g.). Using an assumed wind profile and conductivity profile, they found that the azimuthal magnetic field measurements could be explained by ionospheric electrodynamics, but only if an equatorial jet was present. Whereas the models of Khurana et al. (2018); Provan et al. (2019) treat the ionosphere as an infinitesimally-thin layer, the model of Vriesema et al. (2020) allowed the thermosphere to have finite thickness. This allowed them to demonstrate that ionospheric electrodynamics is sensitive to not just the meridional structure of the thermosphere, as previously explored, but also its vertical structure. This finding has two significant consequences. First, any detailed model of ionospheric electrodynamics ought to account for the vertical structure of the ionosphere rather than treating it as a thin layer. Second, this finding implies that thermospheric winds could be constrained, at least in principle, by magnetic field measurements.

The models described above are only consistent with the majority of magnetic field perturbations if the existence of an equatorial jet of order $u_\phi \sim 100 \text{ m s}^{-1}$ to $400 \text{ m s}^{-1}$ is assumed. These models predict low-latitude current systems of similar strengths which are comparable in magnitude to auroral FACs. The steady-state model of Vriesema et al. (2020) broadly predicts significant ion drag and Joule heating, both of which could be expected to alter the circulation and energy balance of Saturn’s thermosphere. Whether or not the an equatorial jet like that used in the above models actually exists in Saturn’s thermosphere is critical for interpreting other observations and ultimately for understanding the processes that govern
Saturn’s upper atmosphere.

1.2 The Energy Crisis

One of the most puzzling observations of the jovian planets to date is that the thermospheres of Jupiter, Saturn, Uranus and Neptune are all several times hotter than solar heating alone can account for (Strobel and Smith, 1973; Yelle and Miller, 2004; Müller-Wodarg et al., 2006). The observed thermospheric temperatures are 940 K for Jupiter, 420 K for Saturn, 800 K for Uranus, and 600 K for Neptune; however, the temperatures predicted by solar heating alone are 202.7 K, 177.1 K, 137.9 K and 132.3 K, respectively (Yelle and Miller, 2004). While resistive heating in the auroral regions seems adequate to explain these high temperatures at high latitudes (see Fig. 1.1) (Cowley et al., 2004a; Müller-Wodarg et al., 2006; O’Donoghue et al., 2016), the particular mechanism(s) responsible for heating the lower latitudes remains unclear. This unknown source of heating at low and middle latitude regions—dubbed the “energy crisis” by Smith et al. (2007) and “heating problem” by others—has challenged theorists and observers for decades (e.g. Hunten and Dessler, 1977; Smith et al., 2007; Koskinen et al., 2013, 2015).

Part of the reason this question has remained unsolved for so long is because the suggested mechanisms—including upward-traveling internal gravity waves (IGWs) dissipating in the thermosphere, global redistribution of auroral energy, energetic particle precipitation and resistive heating (Yelle and Miller, 2004; Müller-Wodarg et al., 2006; Smith et al., 2005)—are problematic (see below) and have not been sufficiently constrained by observations to properly test them. The wealth of data from the Cassini spacecraft has provided us with unprecedented information about Saturn’s chemical, thermal, and dynamical structure which can be used to help study these mechanisms in the context of Saturn.

Of the proposed mechanisms, the redistribution of auroral energy has been viewed as an attractive possibility because large amounts of energy are deposited near the poles from the magnetosphere (Galand et al., 2011, and others), though it
would require an efficient transport mechanism for polar energy to heat the lower latitudes. To estimate the amount of heating in the auroral regions, Cowley et al. (2004a) developed a simple, axisymmetric model of plasma flows and currents in Saturn’s polar ionosphere. They estimated the global rate of resistive heating to be roughly 10 TW, which is more than an order of magnitude larger than the globally averaged solar energy input of 0.47 TW (Smith et al., 2007) and by far the greatest heat source in their model (Cowley et al., 2004a; Müller-Wodarg et al., 2006).

Smith et al. (2005) used the Saturn Thermosphere Ionosphere General Circulation Model (STIM-GCM, or simply STIM) to study the influence of a generic polar heat source. This early version of STIM solved the 3D nonlinear Navier-Stokes equations in Saturn’s thermosphere by explicit time integration and it was capable of modeling chemistry, transport, different sources of heating, diffusive processes, and variations in wind and composition. Smith et al. (2005) found that an input of approximately 6 TW (roughly consistent with the estimates of Cowley et al., 2004a) could explain temperatures as near to the equator as 30°, but only if they greatly overestimated the polar temperature. They suggested that the electrodynamics of the equatorial region—particularly resistive heating caused by strong electrojet currents—might explain the observed temperatures at low latitudes.

Müller-Wodarg et al. (2006) also calculated how much heat would be required to explain the observed temperatures. They imposed resistive heating rates of 9.82 TW
at the poles (consistent with Cowley et al., 2004a, although the heating was not calculated self-consistently), and a uniform, generic heating source of 0.45 mW m\(^{-2}\) (14.0 TW total) at all latitudes. They found that the model could explain the temperatures of middle and high latitudes above the 0.1 nbar level only when they included both resistive heating at the poles and the generic heat source throughout the domain. Since the generic, “wave-heating” source of Müller-Wodarg et al. (2006) was imposed without a particular mechanism, it is possible that another in-situ heating mechanism—such as wind-driven resistive heating—is responsible.

Smith et al. (2007) concluded that heating at the poles tends to cool, rather than heat, low latitudes. This counter-intuitive result is indirectly caused by a “Coriolis barrier” enhanced by ion drag, which inhibits meridional energy transport. The basic mechanism for the Coriolis barrier is as follows. Magnetospheric forcing at auroral latitudes drives strong, equatorward currents associated with strong westward ion drag, which causes the neutral atmosphere to subcorotate. These currents generate significant Joule heating, which increases temperatures and pressure scale heights at high latitudes. Greater meridional temperature gradients increase the equatorward pressure gradient force, which is in turn balanced by a poleward Coriolis force associated with the subcorotating winds. This poleward Coriolis force makes it difficult for auroral energy to be transported equatorward. Although their model did not treat magnetohydrodynamics (MHD) self-consistently, Smith et al. (2007) argued that their results “rule out” the global redistribution of polar energy as an explanation for the high temperatures at low latitudes on Saturn, and that a direct (in-situ) heating mechanism, either resistive heating or the breaking of IGWs, is the more likely explanation. Later, Müller-Wodarg et al. (2012) used a more powerful version of the STIM-GCM and arrived at the same conclusions (see, e.g. Figure 1.1).

More recent studies have suggested that the Coriolis barrier could be weakened by ion drag or some other drag mechanism (Koskinen et al., 2015; Müller-Wodarg et al., 2019; Vriesema et al., 2020). Müller-Wodarg et al. (2019) presented evidence of IGWs in Saturn’s thermosphere and argued that eddy friction from wave breaking
could oppose the Coriolis barrier. They used a modified version of STIM to explore this possibility by modelling eddy friction as a generic Rayleigh drag term. They found that sufficiently strong Rayleigh drag imposed at middle and high latitudes weakened the Coriolis barrier and roughly reproduced the observed temperature profile. As with Müller-Wodarg et al. (2006), it is possible that the generic drag term they imposed could be due in part to ion drag, as suggested by Vriesema et al. (2020), although we later show that this may be unlikely, in light of new simulations.

Another mechanism that has been proposed to solve the energy crisis is heating by energetic particle precipitation. This mechanism is stronger at high latitudes where magnetic field lines converge and become nearly vertical, but transporting its heat from high latitudes equatorward may be problematic due to the Coriolis barrier (see above). There is little reason to believe that it is a significant source of heating at low-latitude regions of horizontal magnetic field lines. At any rate, energetic particle precipitation is not expected to be as strong a heat source at high latitudes as resistive heating is. Modeling upward energy transport by IGWs self-consistently requires knowledge of lower layers of the atmosphere and wave properties, which are only beginning to be constrained (Müller-Wodarg et al., 2006, 2019; Brown et al., 2020).

1.3 Outline

In summary, there is building evidence of significant electrodynamic activity at low latitudes in Saturn’s thermosphere. If this is the case, currents from an ionospheric wind dynamo would produce magnetic field perturbations that could be compared with those measured by Cassini, possibly exhibiting some of the observed variability in those measurements (Provan et al., 2019). It may therefore be possible to constrain the winds driving this system from the measured perturbations (if only roughly). These low-latitude currents would produce some amount of in-situ heating in roughly the same region where current models under-predict temperatures. If low latitude electrodynamics is able to produce sufficient resistive heating, it might
help explain the observed temperatures. Finally, the ion drag associated with these currents will affect the circulation of the thermosphere (if only slightly), and if it sufficiently opposes the Coriolis force at midlatitudes as suggested by Vriesema et al. (2020), it may be able to break the Coriolis barrier and allow equatorward transport of high-latitude heating.

The purpose of this study therefore is to develop a model of electrodynamics in Saturn’s thermosphere and ionosphere and to use it to explore the role of ion drag and resistive (Joule) heating, particularly at middle and low latitudes. In Chapter 3, we present an axisymmetric, steady-state model and demonstrate the qualitative relationship between winds and magnetic field perturbations. We also use this model to show that a wind-driven dynamo is capable of producing ion drag at midlatitudes which could alter meridional circulation in the thermosphere. In Chapter 4, we incorporate a slightly modified version of the electrodynamics formulation from Chapter 3 into STIM in order to calculate the effects of resistive heating and ion drag self-consistently. In Chapter 5, we modify STIM to better model the effects of the magnetosphere and model a self-consistent wind dynamo at low latitudes and use it to probe magnetospheric coupling and low-latitude electrodynamics. Finally, we summarize this project, highlight our major findings, and discuss directions for future study in Chapter 6.
CHAPTER 2

Model

2.1 Introduction

Accurately modelling electrodynamics in a thermosphere involves simultaneously solving a number of complex equations, and is not feasible to do analytically in any but the most highly-idealized models. Moreover, many of the quantities used in these calculations are not known precisely. A good model of electrodynamics in Saturn’s upper atmosphere requires a model of electrical conductivity, which requires a model of the chemical composition. Although Saturn’s upper atmosphere has different chemical species, the abundances for all but a few of them are negligible for our purposes, and we only consider H, H$_2$, He, CH$_4$, and H$_2$O. As light from the sun travels through the atmosphere, it is absorbed by local gas densities, driving photochemistry. Accurately modelling radiative transfer is difficult because the chemical composition of Saturn’s upper atmosphere is not well constrained globally. Even if the chemical abundances were better constrained, Saturn’s rings cast shadows which seasonally perturb the local photochemistry (e.g. Moore et al., 2004). Neutral particles falling from the rings to the equatorial region or charged particles falling down from the rings along magnetic field lines can also change the atmospheric chemistry (e.g. Yelle et al., 2018; Waite et al., 2018; O’Donoghue et al., 2017). Because the thermosphere is ionized differently at different regions, we need to use non-ideal magnetohydrodynamics. The lower-altitude regions of the thermosphere are denser, and collisions between ions, electrons and neutral particles dominate any magnetohydrodynamic effects from Saturn’s planetary magnetic field, while at higher altitudes, the more tenuous atmosphere allows charged particles to move along magnetic field lines. At higher latitudes, magnetospheric effects become important: energetic particle precipitation (e.g. Waite et al., 1983), field-aligned currents from the magne-
tosphere (e.g. Cowley et al., 2004a; Hunt et al., 2014, 2015), solar wind interactions (e.g. Jia et al., 2012a), and periodic oscillations (e.g. Provan et al., 2014) — to name only a few. The many processes listed here only begin to convey the complexity of Saturn’s upper atmosphere.

In order to make reasonable progress toward understanding Saturn’s thermosphere and ionosphere, we therefore resort to a combination of approximations, simplifications and numerical models. In Section 2.2, we describe a two-dimensional (2D) model of electrodynamics based on that of Richmond (1973a), developed originally for the terrestrial upper atmosphere. In this axisymmetric, steady-state model, we describe how to use electrical conductivities, the magnetic field and the atmospheric winds to calculate the polarization electric field, current density and azimuthal magnetic field perturbations. To our knowledge, this is the first time that a model of the terrestrial wind dynamo has been adapted to giant planets. In Section 2.4, we incorporate the 2D model into STIM in order to explore the role of ion drag and resistive heating self-consistently.

2.2 Theoretical Model

We begin by introducing notation and a derivation which will form the basis of subsequent chapters in this dissertation. Throughout this derivation, we use our own notation but will at many points compare our results to the formulae of Richmond (1973a), which are very similar but slightly less general than those in our derivation. We are intentionally verbose in the hope that all our steps will be especially clear. We use SI units throughout this derivation.

We briefly introduce the right-handed orthogonal dipole coordinate system \((\beta, \alpha, \phi)\) in terms of the spherical polar coordinates \((r, \theta_s, \phi)\), where \(r\) is radius, \(\theta_s\) is colatitude (the polar angle, measured from the north pole), and \(\phi\) is azimuth. When both coordinate systems share a common origin, the relationship between
these coordinate systems is:

\[
\beta = \frac{\cos \theta}{r^2} \quad (2.1)
\]

\[
\alpha = \frac{r}{\sin^2 \theta} \quad (2.2)
\]

\[
\phi = \phi \quad (2.3)
\]

Thus, \( \beta \) represents the distance along a dipole field line and increases from the south pole to the north pole. The coordinate \( \alpha \) is constant along a field line, increases perpendicular to dipole field lines and is related to the standard McIlwain \( L \)-value for dipole field lines by \( L = \alpha/R_S \) (McIlwain, 1966), where \( R_S \) is Saturn’s equatorial radius. Therefore, \( \alpha \) can be used to uniquely identify a dipole magnetic field line if axial symmetry is invoked (as we will later do). The azimuthal angle \( \phi \) increases eastward and is the same as in spherical polar coordinates. In Section 3.3.1, we extend this relationship for a dipole coordinate system with an origin that is shifted from the spherical polar coordinate system’s origin along the polar axis.

Finally, we include a flow chart that illustrates the relationships between the variables which we will introduce later. We encourage the reader to refer to Figure 2.1 throughout the subsequent derivation.

2.2.1 Electrodynamics Equations

We wish to calculate the current density and associated electric fields using the following continuity equation for the current density, \( \vec{j} \):

\[
\nabla \cdot \vec{j} = 0. \quad (2.4)
\]

In our notation, Ohm’s Law is:

\[
\vec{j} = \sigma_\parallel \left( \vec{E}_{CM} \cdot \hat{b} \right) \hat{b} + \sigma_P \left( \hat{b} \times \vec{E}_{CM} \times \hat{b} \right) - \sigma_H \left( \vec{E}_{CM} \times \hat{b} \right). \quad (2.5)
\]

We define \( \hat{b} \) to be in the direction of the magnetic field, which for Saturn’s flipped dipole magnetic field is given by \( \hat{b} = -\hat{\beta} \). The parallel, Pedersen, and Hall conductivities are represented by \( \sigma_\parallel \), \( \sigma_P \), and \( \sigma_H \), respectively. We define \( \vec{E}_{CM} = \vec{E} + \vec{u} \times \vec{B} \)
Figure 2.1: This directed graph illustrates the relationships between variables in our model. The color of each node indicates the approximate order that the variables are computed in, where red indicates inputs to the model and violet indicates outputs. The arrows indicate which variables depend on which others. The shape of each node indicates whether the variable is used on a dipole coordinate grid or a spherical coordinate grid: a rectangle indicates the spherical grid, an ellipse indicates the dipole grid, and a rectangle with rounded corners indicates that the variable is used on either grid. Dashed connections indicate a vector component transformation, while solid lines indicate the calculation of a new quantity.
as the electric field in the atmospheric center-of-mass (CM) frame, where \( \vec{E} \) is the electric field and \( \vec{u} \) is the atmospheric velocity, both in the Saturn III rotating reference frame. The planetary magnetic field is given by \( \vec{B} \).

We assume that \( \vec{B} \) is a dipole magnetic field, given by:

\[
\vec{B}(r, \theta) = \frac{\mu_0 m_d}{4\pi r^3} \left( 2 \cos \theta \hat{r} + \sin \theta \hat{\theta} \right)
\]

\[
= B_0 \left( \frac{2 \cos \theta}{r^3} \hat{r} + \frac{\sin \theta}{r^3} \hat{\theta} \right)
\]

\[
= \left( \frac{B_0}{r^3} \sqrt{1 + 3 \cos^2 \theta} \right) \hat{\beta},
\]

where \( B_0 \equiv \frac{\mu_0 m_d}{4\pi} \), and \( m_d = 4\pi R_S^3 g_1^0 = 4.6227 \times 10^{25} \text{ A m}^2 \) for Saturn, where \( g_1^0 = 2.11402 \times 10^4 \text{ nT} \) is Saturn’s dipole Gauss coefficient (Dougherty et al., 2018) and \( 1R_S = 6.0268 \times 10^7 \text{ m} \) is Saturn’s 1 bar equatorial radius. The spherical components of this field are

\[
B_r \equiv \vec{B} \cdot \hat{r} = B_0 \frac{2 \cos \theta}{r^3}
\]

\[
B_\theta \equiv \vec{B} \cdot \hat{\theta} = B_0 \frac{\sin \theta}{r^3},
\]

its magnitude is

\[
B(r, \theta) \equiv |\vec{B}(r, \theta)| = B_0 \frac{\sqrt{1 + 3 \cos^2 \theta}}{r^3},
\]

and its direction is

\[
\hat{b}(r, \theta) = \left( \frac{2 \cos \theta}{\sqrt{1 + 3 \cos^2 \theta}} \right) \hat{r} + \left( \frac{\sin \theta}{\sqrt{1 + 3 \cos^2 \theta}} \right) \hat{\theta}.
\]

We note that for a dipole magnetic field,

\[
\vec{u} \times \vec{B} = \begin{vmatrix}
\hat{\beta} & \hat{\alpha} & \hat{\phi} \\
\beta & \alpha & \phi \\
B_\beta & 0 & 0
\end{vmatrix}
\]

\[
= [0 - 0] \hat{\beta}
\]

\[
+ [u_\phi B_\beta - 0] \hat{\alpha}
\]

\[
+ [0 - u_\phi B_\beta] \hat{\phi}
\]

\[
= (u_\phi B_\beta) \hat{\alpha} + (-u_\alpha B_\beta) \hat{\phi}.
\]
Since for Saturn, $B_\beta \equiv \vec{B} \cdot \hat{\beta} = \vec{B} \cdot (-\hat{b}) = -B_\parallel = -B \equiv -|\vec{B}|$, Equation (2.14) could also be written for Saturn as $\vec{u} \times \vec{B} = (-u_\phi B) \hat{\alpha} + (u_\alpha B) \hat{\phi}$.

Richmond (1973a) simplifies Equation (2.5) by defining $E_\parallel \equiv E_\parallel \hat{b}$, and $E_\perp \equiv \hat{b} \times \vec{E} \times \hat{b}$. With a little algebraic manipulation, they use the following for Ohm’s Law:

$$\vec{j} = \sigma_\parallel \vec{E}_\parallel + \sigma_P \left( \vec{E}_\perp + \vec{u} \times \vec{B} \right) + \sigma_H \hat{b} \times \left( \vec{E}_\perp + \vec{u} \times \vec{B} \right),$$  \hspace{1cm} (2.15)

Richmond (1973a) also implicitly reduces Faraday’s Law to

$$\vec{\nabla} \times \vec{E} = 0,$$ \hspace{1cm} (2.16)

which is valid when $\vec{\nabla} \times \vec{E} \gg \frac{\partial \vec{B}}{\partial t}$, which will be assumed in our steady-state model.

Finally, the last equation Richmond (1973a) uses is a reduced form of Ampère’s Law ($\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$, omitting Maxwell’s addition), except in terms of $\vec{H}$ instead of $\vec{B}$:

$$\vec{j} = \vec{\nabla} \times \vec{H}.$$  \hspace{1cm} (2.17)

This is valid provided $\vec{j} \gg \epsilon_0 \frac{\partial \vec{E}}{\partial t}$, which again will be assumed in our steady-state model.

**Azimuthal Component of the Electric Field**

We claim that $E_\phi = 0$ because we assume the magnetic fields are constant in time. This holds in any reference frame because it is based on Maxwell’s equations, which are invariant under Lorentz transformations. We begin with Faraday’s Law, from which we previously derived Equation (2.16), integrate over a Gaussian surface and apply Stoke’s theorem:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0$$ \hspace{1cm} (2.18)

$$\int \int_S \left( \vec{\nabla} \times \vec{E} \right) \cdot d\hat{A} = -\int \int_S \left( \frac{\partial \vec{B}}{\partial t} \right) \cdot d\hat{A} = 0$$ \hspace{1cm} (2.19)

$$\therefore \oint_C \vec{E} \cdot d\hat{l} = 0$$ \hspace{1cm} (2.20)
This holds for any longitudinal loop (or circle of latitude):

\[ 0 = \oint_C \vec{E} \cdot d\hat{l} \]  
\[ = \int_0^{2\pi} (E_\phi \hat{\phi}) \cdot (r \sin \theta d\phi \hat{\phi}) \]  
\[ = r \sin \theta \int_0^{2\pi} E_\phi d\phi \]  
\[ = r \sin \theta E_\phi \int_0^{2\pi} d\phi \]  
\[ = 2\pi r \sin \theta E_\phi \]  
\[ \therefore E_\phi = 0 \]

Moving from Equation (2.24) to Equation (2.25), we note that our assumption of axial symmetry demands \( E_\phi \) is constant with \( \phi \).

### 2.2.2 Dipole Coordinate Representation

We now express the most important of the above equations using dipole coordinates. To do so, we use the Lamé coefficients (generalized scale factors) \( h_\beta \), \( h_\alpha \) and \( h_\phi \) associated with the dipole coordinate system.

\[ h_\beta = h_\beta = \frac{r^3}{\delta} \]  
\[ h_\alpha = h_\alpha = \frac{\sin^3 \theta}{\delta} \]  
\[ h_\phi = r \sin \theta \]

where \( \delta \equiv \sqrt{1 + 3\cos^2 \theta} \).
Ampère’s Law in Dipole Coordinates

Equation (2.17) becomes:

\[ \mathbf{j} = \nabla \times \mathbf{H} = \frac{1}{h_\beta h_\alpha h_\phi} \begin{vmatrix} h_\beta \hat{\beta} & h_\alpha \hat{\alpha} & h_\phi \hat{\phi} \\ \partial/\partial \beta & \partial/\partial \alpha & \partial/\partial \phi \\ h_\beta H_\beta & h_\alpha H_\alpha & h_\phi H_\phi \end{vmatrix} \]  

\[ = \frac{1}{h_\alpha h_\phi} \left[ \frac{\partial (h_\phi H_\phi)}{\partial \alpha} - \frac{\partial (h_\alpha H_\alpha)}{\partial \phi} \right] \hat{\beta} 
+ \frac{1}{h_\beta h_\phi} \left[ \frac{\partial (h_\beta H_\beta)}{\partial \phi} - \frac{\partial (h_\phi H_\phi)}{\partial \beta} \right] \hat{\alpha} 
+ \frac{1}{h_\beta h_\alpha} \left[ \frac{\partial (h_\alpha H_\alpha)}{\partial \beta} - \frac{\partial (h_\beta H_\beta)}{\partial \alpha} \right] \hat{\phi} \]  

(2.31)

Invoking axial symmetry \((\partial/\partial \phi \to 0)\) reduces this to:

\[ \mathbf{j} = \nabla \times \mathbf{H} = \frac{1}{h_\alpha h_\phi} \left[ \frac{\partial (h_\phi H_\phi)}{\partial \alpha} \right] \hat{\beta} 
+ \frac{1}{h_\beta h_\phi} \left[ -\frac{\partial (h_\phi H_\phi)}{\partial \beta} \right] \hat{\alpha} 
+ \frac{1}{h_\beta h_\alpha} \left[ \frac{\partial (h_\alpha H_\alpha)}{\partial \beta} - \frac{\partial (h_\beta H_\beta)}{\partial \alpha} \right] \hat{\phi} \]  

(2.32)

Richmond (1973a) point out that because \(\mathbf{j} = 0\) below the conducting region, \(\nabla \times \mathbf{H} = 0\) and therefore \(h_\phi H_\phi\) is constant in that region.

Faraday’s Law in Dipole Coordinates

Because the form of Faraday’s Law is similar to Ampère’s Law (they each have a curl), and this was expressed in Section 2.2.2, we skip to the result:

\[ 0 = \nabla \times \mathbf{E} = \frac{1}{h_\alpha h_\phi} \left[ \frac{\partial (h_\phi E_\phi)}{\partial \alpha} \right] \hat{\beta} 
+ \frac{1}{h_\beta h_\phi} \left[ -\frac{\partial (h_\phi E_\phi)}{\partial \beta} \right] \hat{\alpha} 
+ \frac{1}{h_\beta h_\alpha} \left[ \frac{\partial (h_\alpha E_\alpha)}{\partial \beta} - \frac{\partial (h_\beta E_\beta)}{\partial \alpha} \right] \hat{\phi} \]  

(2.33)
Because $\nabla \times \vec{E} = 0$ everywhere, $h_\phi E_\phi$ is also constant everywhere. While Richmond (1973a) allows $E_\phi$ to be specified, we insist that it must be zero as described in Section 2.2.1, so the constancy of $h_\phi E_\phi$ is an unimportant result for us. Thus Faraday’s Law is:

$$0 = \nabla \times \vec{E} = \frac{1}{h_\beta h_\alpha} \left[ \frac{\partial (h_\alpha E_\alpha)}{\partial \beta} - \frac{\partial (h_\beta E_\beta)}{\partial \alpha} \right] \hat{\phi}. \quad (2.35)$$

In other words,

$$\frac{\partial (h_\alpha E_\alpha)}{\partial \beta} = \frac{\partial (h_\beta E_\beta)}{\partial \alpha}. \quad (2.36)$$

Richmond (1973a) notes that when we assume magnetic field lines are equipotentials (i.e. $E_\parallel = -E_\beta = 0$), which we justify in Section 2.2.2, Equation (2.36) implies that $h_\alpha E_\alpha$ is a constant along magnetic field lines. Therefore, assuming axial symmetry, $h_\alpha E_\alpha$ is a function of $\alpha$ alone:

$$\frac{\partial (h_\alpha E_\alpha)}{\partial \beta} = \frac{\partial (h_\beta E_\beta)}{\partial \alpha}. \quad (2.37)$$

$$\therefore \frac{\partial (h_\alpha E_\alpha)}{\partial \beta} = 0. \quad (2.38)$$

This result will be used later.

**Infinite Parallel Conductivity**

The equations are greatly reduced in the limiting case wherein $\sigma_\parallel \rightarrow \infty$. We consider the parallel component of Ohm’s Law (Equation (2.5)) for this case:

$$j_\parallel = \sigma_\parallel E_\parallel \quad (2.39)$$

The only way for $\sigma_\parallel$ to be infinite but have a physically realistic (i.e., finite) current density is for $E_\parallel \rightarrow 0$. In other words, magnetic field lines must be equipotentials. Although in general, this assumption is more complicated (Vasyliunas, 2012, has an excellent discussion of this), this is appropriate for a steady-state model which has had plenty of time to equilibrate. This justifies our assumption of equipotential field lines in Section 2.2.2.
2.2.3 Integrating the Continuity Equation

The continuity equation Equation (2.4) can be expressed as:

\[ 0 = \nabla \cdot \vec{J} = \frac{1}{h_\beta h_\alpha h_\phi} \left[ \frac{\partial (h_\alpha h_\phi j_\beta)}{\partial \beta} + \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha} + \frac{\partial (h_\beta h_\alpha j_\phi)}{\partial \phi} \right] \]  

\[ (2.40) \]

We note that axisymmetry eliminates the \( \frac{\partial j_\phi}{\partial \phi} \) part of the continuity equation, so \( \frac{\partial j_\phi}{\partial \phi} = 0 \). Hence, the above equation reduces to two components:

\[ 0 = \nabla \cdot \vec{J} = \frac{1}{h_\beta h_\alpha h_\phi} \left[ \frac{\partial (h_\alpha h_\phi j_\beta)}{\partial \beta} + \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha} \right] . \]  

\[ (2.41) \]

The volume integral of the continuity equation over a meridional “slice” from \( \beta_1 \leq \beta' \leq \beta_2 \), \( \phi \leq \phi' \leq \phi + \delta \phi \), and below that field line to the altitude at which \( \vec{J} = 0 \) (i.e. “the base of the conducting region”), as depicted in Figure 1 of Richmond (1973a). Using our notation, they express this integral, for any given field line \( \alpha \), as:

\[ 0 = \delta \phi \left[ \int_{\beta_1}^{\beta_2} (h_\beta h_\phi j_\alpha) d\beta' \right] \]  

\[ (2.42) \]

This, then, is what we aim to prove rigorously.

We begin with the volume integral of the continuity equation, assuming that the lower boundary of the domain separates the conducting (\( \vec{J} \neq 0 \)) and non-conducting (\( \vec{J} = 0 \)) regions. The volume of integration is below a given surface of constant \( p \) down to where it crosses the lower boundary, and between \( \phi \) to \( \phi + \delta \phi \). We denote \( \beta_1(\alpha') \) and \( \beta_2(\alpha') \) (where \( \beta_1(\alpha') < \beta_2(\alpha') \)) as the values of \( \beta \) where a surface of constant \( \alpha' \) intersects the lower boundary. Similarly, \( \alpha_{\text{min}} \) is the value of \( \alpha' \) which crosses the lower boundary at \( \beta = 0 \).

\[ 0 = \int_V \nabla \cdot \vec{J} \, dV' = \int_{\alpha_{\text{min}}}^{\alpha} h_\alpha \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} h_\beta \int_{\phi}^{\phi + \delta \phi} h_\phi \left( \nabla \cdot \vec{J} \right) \, d\phi' \, d\beta' \, d\alpha' \]  

\[ (2.43) \]

We note that because of axial symmetry, the azimuthal integral integrates over a constant, so it reduces immediately as follows:

\[ 0 = \int_V \nabla \cdot \vec{J} \, dV' = \delta \phi \left[ \int_{\alpha_{\text{min}}}^{\alpha} h_\alpha \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} h_\beta h_\phi \left( \nabla \cdot \vec{J} \right) \right] \, d\beta' \, d\alpha'. \]  

\[ (2.44) \]
We now insert our expression for the divergence in dipole coordinates:

\[
0 = \int_V \nabla \cdot \vec{J} \, dV' = \delta \phi \left[ \int_{\alpha_{min}}^{\alpha} h_\alpha \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} h_\beta h_\phi \left( \frac{1}{h_\beta h_\alpha h_\phi} \left[ \frac{\partial (h_\alpha h_\phi j_\beta)}{\partial \beta'} + \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha'} \right] \right) \, d\beta' \, d\alpha' \right] \tag{2.45}
\]

\[
= \delta \phi \left[ \int_{\alpha_{min}}^{\alpha} \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \left( \frac{\partial (h_\alpha h_\phi j_\beta)}{\partial \beta'} + \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha'} \right) \, d\beta' \, d\alpha' \right] \tag{2.46}
\]

\[
= \delta \phi \left[ \int_{\alpha_{min}}^{\alpha} \left( \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \frac{\partial (h_\alpha h_\phi j_\beta)}{\partial \beta'} \, d\beta' \right) \, d\alpha' \right. \\
\left. + \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha'} \, d\beta' \right) \, d\alpha' \tag{2.47}
\]

\[
= \delta \phi \left[ \int_{\alpha_{min}}^{\alpha} \left( \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \frac{\partial (h_\alpha h_\phi j_\beta)}{\partial \beta'} \bigg|_{\beta_1(\alpha')}^{\beta_2(\alpha')} \right) + \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha'} \, d\beta' \bigg) \, d\alpha' \right] \tag{2.48}
\]

We note that \( \beta_1(\alpha') \) and \( \beta_2(\alpha') \) are defined for \( \alpha' \) as the points on the \( \alpha' \) field line which intersect the lower boundary of the conductive layer. At the boundary, \( \vec{j} = 0 \), so the first term in the integral evaluates to zero:

\[
(h_\alpha h_\phi j_\beta) \big|_{\beta_1(\alpha')}^{\beta_2(\alpha')} = h_\alpha(\beta_2(\alpha'), \alpha') h_\phi(\beta_2(\alpha'), \alpha') j_\beta(\beta_2(\alpha'), \alpha') - h_\alpha(\beta_1(\alpha'), \alpha') h_\phi(\beta_1(\alpha'), \alpha') j_\beta(\beta_1(\alpha'), \alpha') = 0 \tag{2.49}
\]

\[
= 0. \tag{2.50}
\]

We are left with the following for the volume integral:

\[
0 = \int_V \nabla \cdot \vec{J} \, dV' = \delta \phi \left[ \int_{\alpha_{min}}^{\alpha} \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha'} \, d\beta' \, d\alpha' \right] \tag{2.51}
\]

It is tempting to reverse the order of the integrals so that the \( \frac{\partial}{\partial \alpha'} \) term may be integrated with respect to \( \alpha' \) first — which would be trivial to evaluate — but the fact that the limits of integration for the \( \beta' \) integral depend on \( \alpha' \) prohibit us from taking this shortcut without further consideration.

Using Leibnitz’s rule for differentiation within an integral, and defining \( f(\beta', \alpha') \equiv h_\beta(\beta', \alpha') h_\phi(\beta', \alpha') j_\alpha(\beta', \alpha') \) for short, the expression inside the brackets
above becomes
\[
\int_{\alpha_{\min}}^{\alpha} \left[ \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \frac{\partial f}{\partial \alpha'} d\beta' \right] d\alpha' = \int_{\alpha_{\min}}^{\alpha} \left[ \frac{\partial}{\partial \alpha'} \left( \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} f(\beta', \alpha') d\beta' \right) \right.
- f(\beta_2(\alpha'), \alpha') \frac{\partial \beta_2(\alpha')}{\partial \alpha'}
+ f(\beta_1(\alpha'), \alpha') \frac{\partial \beta_1(\alpha')}{\partial \alpha'} \left. \right] d\alpha'.
\] (2.52)

Since the points \((\beta_1, \alpha')\) and \((\beta_2, \alpha')\) are by definition at the base of the conducting region for all \(\alpha'\), \(\vec{j} = 0\) at these points and therefore \(f(\beta_1, \alpha')\) and \(f(\beta_2, \alpha')\) are both zero as well.

\[
\int_{\alpha_{\min}}^{\alpha} \left[ \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \frac{\partial f}{\partial \alpha'} d\beta' \right] d\alpha' = \int_{\alpha_{\min}}^{\alpha} \left[ \frac{\partial}{\partial \alpha'} \left( \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} f(\beta', \alpha') d\beta' \right) \right.
- f(\beta_2(\alpha'), \alpha') \frac{\partial \beta_2(\alpha')}{\partial \alpha'}
+ f(\beta_1(\alpha'), \alpha') \frac{\partial \beta_1(\alpha')}{\partial \alpha'} \left. \right] d\alpha' (2.53)
= \int_{\alpha_{\min}}^{\alpha} \frac{\partial}{\partial \alpha'} \left( \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} f(\beta', \alpha') d\beta' \right) d\alpha' (2.54)
= \left( \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} f(\beta', \alpha') d\beta' \right) \bigg|_{\alpha_{\min}}^{\alpha} (2.55)
= \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} f(\beta', \alpha) d\beta' - \int_{\beta_1(\alpha_{\min})}^{\beta_2(\alpha_{\min})} f(\beta', \alpha_{\min}) d\beta' (2.56)

In cancelling the second term in the last step, we note that \(\beta_1(\alpha_{\min}) = \beta_2(\alpha_{\min}) = 0\), leaving the integral with no domain to integrate over. Bringing back the \(\delta\phi\) from earlier, we are left with only the volume-integrated continuity equation as stated in Richmond (1973a):

\[
0 = \int_V \vec{\nabla} \cdot \vec{j} dV' = \delta\phi \left[ \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} (h_\beta h_\phi j_\alpha) d\beta' \right].
\] (2.57)
2.2.4 Calculating the Polarization Electric Field

From Equation (2.15), the $\alpha$-component of $\vec{j}$ is

\[
\vec{j}_\alpha \equiv \vec{j} \cdot \hat{\alpha} \\
= \left[ \sigma \vec{E} \| + \sigma_P \left( \vec{E} \perp + \vec{u} \times \vec{B} \right) + \sigma_H \hat{b} \times \left( \vec{E} \perp + \vec{u} \times \vec{B} \right) \right] \cdot \hat{\alpha} \\
= \sigma \vec{E} \| \cdot \hat{\alpha} + \left[ \sigma_P \left( \vec{E} \perp + \vec{u} \times \vec{B} \right) + \sigma_H \hat{b} \times \left( \vec{E} \perp + \vec{u} \times \vec{B} \right) \right] \cdot \hat{\alpha} \\
= \sigma_P \left( \vec{E} \perp + \vec{u} \times \vec{B} \right) \cdot \hat{\alpha} + \sigma_H \left[ \left( -\vec{b} \right) \times \left( \vec{E} \perp + \vec{u} \times \vec{B} \right) \right] \cdot \hat{\alpha}.
\]

(2.58)
(2.59)
(2.60)
(2.61)

We know that $\vec{E} \perp = E_\alpha \hat{\alpha} + E_\phi \hat{\phi}$ (though, as mentioned earlier, we assume $E_\phi \to 0$; we retain it here for consistency with Richmond’s derivation), and that $\vec{u} \times \vec{B} = (u_\phi B_\beta) \hat{\alpha} + (-u_\alpha B_\beta) \hat{\phi}$. We use this to continue our calculation of $j_\alpha$:

\[
j_\alpha = \sigma_P \left( E_\alpha + \left[ \vec{u} \times \vec{B} \right]_\alpha \right) \\
+ \sigma_H \left[ \left( E_\phi + \left[ \vec{u} \times \vec{B} \right]_\phi \right) \hat{\alpha} + \left( -E_\alpha - \left[ \vec{u} \times \vec{B} \right]_\alpha \right) \hat{\phi} \right] \cdot \hat{\alpha} \\
= \sigma_P \left( E_\alpha + \left[ \vec{u} \times \vec{B} \right]_\alpha \right) + \sigma_H \left( E_\phi + \left[ \vec{u} \times \vec{B} \right]_\phi \right).
\]

(2.62)
(2.63)

(2.64)

We can insert Equation (2.64) into Equation (2.57), omitting the $\delta \phi$:

\[
0 = \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi j_\alpha \ d\beta' \\
= \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \left[ \sigma P \left( E_\alpha + u_\phi B_\beta \right) + \sigma_H \left( E_\phi - u_\alpha B_\beta \right) \right] d\beta' \\
= \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \left[ \sigma P E_\alpha + \sigma_P u_\phi B_\beta + \sigma_H E_\phi + \sigma_H u_\alpha B_\beta \right] d\beta' \\
= \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \left[ \sigma P E_\alpha + \sigma_H E_\phi + \left( \sigma_P u_\phi - \sigma_H u_\alpha \right) B_\beta \right] d\beta' \\
= \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \sigma_P E_\alpha d\beta' + \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \sigma_H E_\phi d\beta' \\
+ \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \left( \sigma_P u_\phi - \sigma_H u_\alpha \right) B_\beta d\beta' \\
= h_\alpha E_\alpha \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} \frac{h_\beta h_\phi \sigma_P}{h_\alpha} d\beta' + h_\phi E_\phi \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta \sigma_H d\beta' \\
+ \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \left( \sigma_P u_\phi - \sigma_H u_\alpha \right) B_\beta d\beta'.
\]

(2.65)
(2.66)
(2.67)
(2.68)
(2.69)
(2.70)
We note that in the final step, $h_\alpha E_\alpha$ and $h_\phi E_\phi$ can be moved outside their integrals because we have shown earlier that they do not vary with $\beta$.

Finally, this allows us to solve for $E_\alpha$:

$$E_\alpha = -h_\phi E_\phi \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta \sigma_H \, d\beta' - \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi (\sigma_P u_\phi - \sigma_H u_\alpha) B_\beta \, d\beta'. \tag{2.71}$$

This matches Equation 10 from Richmond (1973a) except for the sign of the first term in the numerator (involving $E_\phi$), and that of $\sigma_P u_\phi$. This discrepancy is because Richmond (1973a) assume that $\hat{\beta} = +\hat{b}$, which obscures the distinction between the dipole coordinate system’s unit vector and the magnetic field direction and makes the expressions of Richmond (1973a) nontransferable to Saturn. Tracing these sign discrepancies is non-trivial. For clarity and generality in the present derivation, we do not yet specify whether $\hat{b} = +\hat{\beta}$ or $\hat{b} = -\hat{\beta}$. As before, we set $E_\phi = 0$, although Richmond (1973a) allows it to be specified.

### 2.2.5 Calculating Currents

With $E_\alpha$ now calculated, $j_\alpha$ and $j_\phi$ are calculated (c.f. Equation (2.64)):

$$j_\alpha = \sigma_P (E_\alpha + u_\phi B_\beta) + \sigma_H (E_\phi - u_\alpha B_\beta) \tag{2.72}$$

$$j_\phi = \sigma_P (E_\phi - u_\alpha B_\beta) - \sigma_H (E_\alpha + u_\phi B_\beta) \tag{2.73}$$

We retain $E_\phi = 0$ here for comparison with Richmond (1973a), but assume $E_\phi = 0$ in practice. In Chapter 3, we simplify these expressions by assuming the wind is only in the $\phi$ direction and the Hall conductivity is zero. These simplifications give:

$$j_\alpha = \sigma_P (E_\alpha + u_\phi B_\beta) \tag{2.74}$$

$$j_\phi = -\sigma_H (E_\alpha + u_\phi B_\beta). \tag{2.75}$$

We would like to be able to calculate $j_\beta$ via $j_\beta = \sigma_\parallel E_\beta$, but we have assumed $\sigma_\parallel \to \infty$ (which corresponds to $E_\beta \to 0$) in Section 2.2.2. Therefore, we cannot calculate $j_\beta$ via this approach, but must instead implicitly calculate it some other way. Fortunately, we can use the magnetic field perturbations $H_\phi$ to calculate $j_\beta$.
via the symmetric, steady-state Ampère’s Law (Equation (2.33)), which has dipole components

\[ j_\beta = \left[ \nabla \times \vec{H} \right]_\beta = \frac{1}{h_\alpha h_\phi} \left[ \frac{\partial (h_\phi H_\phi)}{\partial \alpha} \right] \hat{\beta} \tag{2.76} \]

\[ j_\alpha = \left[ \nabla \times \vec{H} \right]_\alpha = \frac{1}{h_\beta h_\phi} \left[ -\frac{\partial (h_\phi H_\phi)}{\partial \beta} \right] \hat{\alpha} \tag{2.77} \]

First, we calculate \( H_\phi (\beta, \alpha) \) by multiplying Equation (2.77) by \( h_\phi (\beta, \alpha) \) and integrating the equation along a field line \( \alpha \), from \( \beta_1(\alpha) \) to \( \beta \), in order to get \( H_\phi (\beta, \alpha) \):

\[
\int_{\beta_1(\alpha)}^{\beta} j_\alpha h_\phi h_\beta d\beta' = -\int_{\beta_1(\alpha)}^{\beta} \left( \frac{1}{h_\beta} \left[ \frac{\partial (h_\phi H_\phi)}{\partial \beta'} \right] \right) h_\beta d\beta' \tag{2.78}
\]

\[
= -\int_{\beta_1(\alpha)}^{\beta} \left[ \frac{\partial (h_\phi H_\phi)}{\partial \beta'} \right] d\beta' \tag{2.79}
\]

\[
= -\left( h_\phi H_\phi \right) \bigg|_{\beta_1(\alpha)}^{\beta} \tag{2.80}
\]

\[
= -h_\phi (\beta, \alpha) H_\phi (\beta, \alpha) + h_\phi (\beta_1(\alpha), \alpha) H_\phi (\beta_1(\alpha), \alpha) \tag{2.81}
\]

\[ \therefore \quad H_\phi (\beta, \alpha) = \frac{h_\phi (\beta_1(\alpha), \alpha) H_\phi (\beta_1(\alpha), \alpha) - \int_{\beta_1(\alpha)}^{\beta} j_\alpha h_\phi h_\beta d\beta'}{h_\phi (\beta, \alpha)} \tag{2.82} \]

The terms inside the integral in Equation (2.82) are to be evaluated numerically at points along the field line (i.e. at \((\alpha, \beta')\)). We note that \( H_\phi \) needs to be specified at some point. We can choose one of the integrands: instead of integrating from \( \beta_1(\alpha) \rightarrow \beta \), we could have integrated backwards from \( \beta_2(\alpha) \rightarrow \beta \) — or from any \( \beta \) along that line, for that matter. This is one of the ways our derivation is more general than that of Richmond (1973a) (c.f. their Equation 11). We can simplify Equation (2.82) by assuming \( B_\phi \) and therefore \( H_\phi \) are zero below the conducting region, as is done by Richmond (1973a), leading to \( H_\phi (\beta_1(\alpha), \alpha) = 0 \). This is somewhat justified by our assumption that \( \vec{j} = 0 \) below the conducting region. This allows us to further reduce Equation (2.82) to

\[
H_\phi (\beta, \alpha) = -\frac{1}{h_\phi (\beta, \alpha)} \int_{\beta_1(\alpha)}^{\beta} j_\alpha (\beta', \alpha) h_\phi (\beta', \alpha) h_\beta (\beta', \alpha) d\beta'. \quad \tag{2.83}
\]
Our expression for $H_\phi$ is therefore in line with Equation 11 Richmond (1973a), except for a difference in sign. The reason for this difference is related to the sign differences discussed earlier in Section 2.2.4.

Finally, having solved for $H_\phi$, we are able to calculate $j_\beta$ via Equation (2.76). This model, based on Richmond (1973a), therefore allows us to calculate the polarization electric field, the azimuthal magnetic field, and all three components of current density at any altitude or latitude within the axisymmetric, steady-state approximation. We apply this model directly to Saturn in Chapter 3 and incorporate it into STIM in Chapter 4.

2.2.6 An Extension to Include External Constraints

One shortcoming of the above model is that it only uses currents that are generated in the thermosphere, ignoring any magnetospheric coupling. At high latitudes, dipole field lines in the thermosphere would undoubtedly couple to the magnetosphere. A detailed treatment of this coupling is beyond the scope of this dissertation; however, it would be advantageous to model this coupling using observational constraints or data from a magnetospheric model. We therefore extend the above derivation to include the possibility of specifying external currents and/or magnetic field perturbations along the upper boundary of the thermosphere. Accounting for these constraints is likely most important for (large $\alpha$) field lines which intersect the thermosphere at high latitudes, but it is relevant for all field lines which extend above the thermosphere. We note that the electrodynamics along (lower $\alpha$) field lines which do not extend above the thermosphere is calculated as described previously.

We begin by splitting the integral in Equation (2.46) into three regions: Region A in the southern magnetic hemisphere (south of some yet-to-be-specified boundary), Region C in the northern thermosphere (northward of another yet-to-be-specified boundary), and Region B in the outer region between these boundaries. This, along with notation that will be discussed in the paragraphs to follow, is sketched in Figure 2.2. Along a given field line, we envision the first region as the southern part
of the field line within the thermosphere, the second region as the northern part of
the field line that is in the thermosphere, and the third region as the part of the
field line that is completely above the thermosphere. Neglecting the factor of \( \delta \phi \),
we therefore have

\[
0 = \int_V \vec{\nabla} \cdot \vec{j} \, dV' = \int_{\alpha_{\text{min}}}^{\alpha} \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \left( \frac{\partial (h_\alpha h_\phi \phi_\beta)}{\partial \beta'} + \frac{\partial (h_\beta h_\phi \phi_\alpha)}{\partial \alpha'} \right) \, d\beta' \, d\alpha' \quad (2.84)
\]

\[
= \int (\ldots) \, dV'_A' + \int (\ldots) \, dV'_B' + \int (\ldots) \, dV'_C', \quad (2.85)
\]

where the dots (\ldots) indicate the contents of the integral in Equation (2.84), now
with three integration regions instead of one. Because \( \vec{\nabla} \cdot \vec{j} = 0 \) everywhere, we
observe that

\[
\int (\ldots) \, dV'_A' = \int (\ldots) \, dV'_B' = \int (\ldots) \, dV'_C' = 0. \quad (2.86)
\]

We neglect the magnetospheric region (Region B) as it is beyond our scope. Region B
is responsible for generating the FACs that we use as an upper boundary condition
for Regions A and C.

Previously, in Section 2.2.3, the integrated region was the entire volume above
the outermost field line \( \alpha_{\text{min}} \) which did not yet enter the thermosphere and below
the field line \( \alpha \) under consideration. The domain was bounded in \( \beta \) by the lower
boundary of the domain, which each field line intersects twice (once in each hemi-
sphere). In this extension, the lower boundary of Regions A and C is a field line \( \alpha_{\text{in}} \)
just below the field line \( \alpha \) under consideration. We assume that the electrodynamics
is known (or has been previously calculated) along this lower boundary. As before,
the field line \( \alpha \) serves as the outer boundary of the integration domain. Regions A
and C are each bounded once in \( \beta \) by the lower boundary of the conducing region
and once by the upper boundary. We define \( \beta_L (\alpha') \) as the \( \beta \) coordinate of the lower
(poleward) boundary of the conducing layer along the field line \( \alpha' \) and \( \beta_U (\alpha') \) as
the \( \beta \) coordinate of the upper (equatorward) boundary of the conducing layer along
the field line \( \alpha' \). Whereas we previously integrated along a given field line from \( \beta_1 \) in
the southern hemisphere to \( \beta_2 \) in the northern hemisphere, we now integrate along
field lines separately in each hemisphere, from the base of the conducing layer to
Figure 2.2: This sketch shows three field lines (dotted lines), two of which pierce the conducting layer (shaded gray). The innermost field line \( \alpha_{\text{min}} \) is the field line which just barely touches the base of the conducting layer and is inconsequential for Regions A, B and C. Field line \( \alpha' \) is a given field line of interest and field line \( \alpha_{\text{in}} \) is the next field line inward of \( \alpha' \) (the separation between these is greatly exaggerated for illustration purposes). The space between the other two field lines and above the lower boundary is divided into three regions. Region A to the south (shaded green) and Region C to the north (shaded purple) are within the conducting layer and between the field lines. We note that for some field lines with \( \alpha \) approaching the radius of the upper boundary from below, Region A can extend slightly north of the magnetic equator (dashed vertical line) due to geometrical effects related to the dipole shift. Region B (shaded orange) is between the field lines and above the conducting layer.

One might correctly expect that integrating in opposite directions in Region C as compared to Region A introduces a number of sign differences, but these cancel out.
using the formulation we describe below.

\[
\int (\ldots) \, dV_{A,C} = \int_{\alpha_{in}}^{\alpha} \int_{\beta_L(\alpha')}^{\beta_U(\alpha')} \left[ \frac{\partial}{\partial \beta'} \left( h_\alpha h_\phi j_\beta \right) + \frac{\partial}{\partial \alpha'} (h_\beta h_\phi j_\alpha) \right] \, d\beta' \, d\alpha' \quad (2.87)
\]

\[
= \int_{\alpha_{in}}^{\alpha} \left[ (h_\alpha h_\phi j_\beta) \bigg|_{\beta_L(\alpha')}^{\beta_U(\alpha')} + \int_{\beta_L(\alpha')}^{\beta_U(\alpha')} \frac{\partial}{\partial \alpha'} (h_\beta h_\phi j_\alpha) \, d\beta' \right] \, d\alpha' \quad (2.88)
\]

\[
= \int_{\alpha_{in}}^{\alpha} \left[ (h_\alpha h_\phi j_\beta) \bigg|_{\beta_U(\alpha')}^{\beta_L(\alpha')} - (h_\alpha h_\phi j_\beta) \bigg|_{\beta_L(\alpha')}^{0} \right. \\
&\quad + \left. \int_{\beta_L(\alpha')}^{\beta_U(\alpha')} \frac{\partial}{\partial \beta'} (h_\beta h_\phi j_\alpha) \, d\beta' \right] \, d\alpha'
\]

\[
= \int_{\alpha_{in}}^{\alpha} \left[ (h_\alpha h_\phi j_\beta) \bigg|_{\beta_U(\alpha')}^{\beta_L(\alpha')} + \int_{\beta_L(\alpha')}^{\beta_U(\alpha')} \frac{\partial}{\partial \beta'} (h_\beta h_\phi j_\alpha) \, d\beta' \right] \, d\alpha'. \quad (2.90)
\]

We note that in Equation (2.89), the term cancels because \( j_\beta \to 0 \) at the lower boundary, below the conducting region. As in Section 2.2.3, it would be nice to swap the order of integrals in the final term in Equation (2.90) to eliminate the integral over \( \alpha' \), but this is not trivial because the limits of integration are themselves functions of \( \alpha' \). As before, we define \( f(\beta', \alpha') \equiv h_\beta(\beta', \alpha') \, h_\phi(\beta', \alpha') \, j_\alpha(\beta', \alpha') \) and apply Leibnitz’s rule for differentiation within an integral. This gives:

\[
\int_{\beta_L(\alpha')}^{\beta_U(\alpha')} \frac{\partial f}{\partial \alpha'} \, d\beta' = \frac{\partial}{\partial \alpha'} \left( \int_{\beta_L(\alpha')}^{\beta_U(\alpha')} f(\beta', \alpha') \, d\beta' \right)
\]

\[
- \left[ f(\beta_U(\alpha'), \alpha') \frac{\partial \beta_U(\alpha')}{\partial \alpha'} \right]^{0}_{\beta_L(\alpha')} \quad (2.91)
\]

\[
= \frac{\partial}{\partial \alpha'} \left( \int_{\beta_L(\alpha')}^{\beta_U(\alpha')} f(\beta', \alpha') \, d\beta' \right). \quad (2.92)
\]

We note that \( f(\beta_L(\alpha'), \alpha') = f(\beta_U(\alpha'), \alpha') = 0 \) because \( j_\alpha \to 0 \) at the upper and lower boundaries of the conducting layer, as mentioned earlier. We combine this
result with Equation (2.90):

\[
0 = \int (...) dV_{A,C'} = \int_{\alpha_{in}}^{\alpha} \left[ (h_{\alpha} h_{\phi} j_{\beta}) \right]_{\beta_{U}(\alpha')} + \frac{\partial}{\partial \alpha'} \left( \int_{\beta_{L}(\alpha')}^{\beta_{U}(\alpha')} (h_{\beta} h_{\phi} j_{\alpha}) \, d\beta' \right) \, d\alpha'
\]

(2.93)

\[
= \int_{\alpha_{in}}^{\alpha} (h_{\alpha} h_{\phi} j_{\beta}) \left. \right|_{\beta_{U}(\alpha')} \, d\alpha' + \int_{\alpha_{in}}^{\alpha} \frac{\partial}{\partial \alpha'} \left( \int_{\beta_{L}(\alpha')}^{\beta_{U}(\alpha')} (h_{\beta} h_{\phi} j_{\alpha}) \, d\beta' \right) \, d\alpha'
\]

(2.94)

\[
= \int_{\alpha_{in}}^{\alpha} (h_{\alpha} h_{\phi} j_{\beta}) \left. \right|_{\beta_{U}(\alpha')} \, d\alpha' + \int_{\alpha_{in}}^{\alpha} \frac{\partial}{\partial \alpha'} \left( \int_{\beta_{L}(\alpha')}^{\beta_{U}(\alpha')} (h_{\beta} h_{\phi} j_{\alpha}) \, d\beta' \right) \, d\alpha'
\]

(2.95)

\[
= \int_{\alpha_{in}}^{\alpha} (h_{\alpha} h_{\phi} j_{\beta}) \left. \right|_{\beta_{U}(\alpha')} \, d\alpha' + \int_{\alpha_{in}}^{\alpha} \frac{\partial}{\partial \alpha'} \left( \int_{\beta_{L}(\alpha')}^{\beta_{U}(\alpha')} (h_{\beta} h_{\phi} j_{\alpha}) \, d\beta' \right) \, d\alpha'
\]

(2.96)

We recall that \( \int (...) dV_{A'} = \int (...) dV_{C'} = \int \left( \nabla \cdot \vec{j} \right) \, dV_{A'} = 0 \) because \( \nabla \cdot \vec{j} = 0 \) everywhere.

Equation (2.96) should be compared to Equation (2.65) from earlier. The first term in Equation (2.96) is a new term which accounts for known, externally-imposed FACs along the upper boundary of the conducting layer. We assume that these FACs are from the magnetosphere and are known (e.g. from observations) along the top of our domain. To be clear: these externally-imposed FACs are not to be confused with the FACs associated with the thermospheric wind dynamo, which are calculated via Equation (2.76), but are separate and help us use observations to further constrain thermospheric electrodynamics. We also note that the second term in Equation (2.96) is identical to the only term in Equation (2.57). The third term in Equation (2.96) is a new term which accounts for the perpendicular current along the inner (equatorward) boundary of Regions A and C. This third term is unknown at present; however, if \( j_{\alpha} \) is calculated along the \( \alpha_{in} \) field line first, we can treat this term as a known quantity.
Before proceeding, we define the following variables for convenience:

\[
J_\beta \equiv \int_{\alpha_{in}}^{\alpha} (h_\alpha h_\phi j_\beta) \left. \frac{d\alpha'}{\beta_U(\alpha')} \right|_{\beta_U(\alpha')} \tag{2.97}
\]

\[
J_\alpha \equiv \int_{\beta_L(\alpha_{in})}^{\beta_U(\alpha_{in})} (h_\beta h_\phi j_\alpha) \left. d\beta' \right|_{\alpha_{in}} \tag{2.98}
\]

These definitions allow us to rewrite Equation (2.96) as

\[
J_\alpha - J_\beta = \int_{\beta_L(\alpha)}^{\beta_U(\alpha)} (h_\beta h_\phi j_\alpha) \left. d\beta' \right|_{\alpha} \tag{2.99}
\]

It is worth noting that \( J_\alpha \) represents current density entering the region of integration from an inner field line, and \( J_\beta \) represents current density leaving the region of integration from the top of the conducting layer, between field lines \( \alpha_{in} \) and \( \alpha \). The term on the right side of Equation (2.99) is the current flowing out of the region towards outer field lines. It is worth remembering that the above equation is the integral form of the current continuity equation, with one term for each side of the integration region (the term for the lower boundary is zero because \( \vec{j} = 0 \)).

We note that whereas the integral over the entire field line in Equation (2.65) equals zero, the integral in the above equation is not along the entire field line and therefore equals \( J_\alpha - J_\beta \) instead. The value of \( J_\beta \) is known if \( j_\beta = -j_\parallel \) is known (imposed) along the upper boundary of the thermosphere. The value of \( J_\alpha \) is known if \( j_\alpha \) is known along the field line inward of the current field line \( \alpha \), which is true if we calculate the electrodynamics along field lines starting with the innermost field lines and working our way outward. In this picture, we impose FACs as a boundary condition along the upper boundary and thereby constrain electrodynamics in the thermosphere.

Using \( J_\beta \) and \( J_\alpha \) and following the derivation leading up to Equation (2.71), we
can finally derive an expression for $E_\alpha$:

$$J_\alpha - J_\beta = \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \tilde{j}_\alpha \, d\beta'$$

(2.100)

$$= \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \left[ \sigma_P \left( E_\alpha + u_\phi B_\beta \right) + \sigma_H \left( E_\phi - u_\alpha B_\beta \right) \right] \, d\beta'$$

(2.101)

$$= h_\alpha E_\alpha \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} \frac{h_\beta h_\phi \sigma_P}{h_\alpha} \, d\beta' + h_\phi E_\phi \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta \sigma_H \, d\beta'$$

$$+ \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \left( \sigma_P u_\phi - \sigma_H u_\alpha \right) B_\beta \, d\beta'$$

(2.102)

Therefore, we solve for $E_\alpha$ as follows:

$$E_\alpha = -h_\phi E_\phi \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta \sigma_H \, d\beta' - \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \left( \sigma_P u_\phi - \sigma_H u_\alpha \right) B_\beta \, d\beta' + J_\alpha - J_\beta$$

(2.103)

We impose $E_\phi \to 0$ as before. We note that Equation (2.103) differs slightly but substantially from Equation (2.71) and discuss the differences at the end of this section.

In this formulation, the PEF has contributions from both the magnetosphere (via the FACs) and from the thermosphere (via the winds). Because the terms are added, we can examine the effects of each separately. We refer to the component of the PEF that uses only FACs as the "zero-wind PEF" because the winds are effectively zero for the calculation. The zero-wind PEF is given as

$$E_\alpha = \frac{\int_{\beta_1(\alpha)}^{\beta_2(\alpha)} \left( h_\beta h_\phi \tilde{j}_\alpha \right) \, d\beta'}{h_\alpha \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} \frac{h_\beta h_\phi \sigma_P}{h_\alpha} \, d\beta'}. \quad (2.104)$$

The first term represents current flowing into field line $\alpha$ from below, and the second term represents current flowing out of the region through the top of the conducting layer.

We now return to the new terms, $J_\beta$ and $J_\alpha$. Although these terms can be computed from FACs and electrodynamics along previously-calculated field lines, these terms can be expressed more simply in terms of the azimuthal magnetic field. It is worthwhile to compare Equation (2.97) to Equation (2.76). If we multiply
Equation (2.76) by \(h_\alpha h_\phi\) and integrate with respect to \(\alpha'\) from \(\alpha_{in}\) to \(\alpha\) along the upper boundary of the domain, the result is \(J_\beta\):

\[
J_\beta \equiv \int_{\alpha_{in}}^{\alpha} (h_\alpha h_\phi j_\beta) \bigg|_{\beta_U(\alpha')} d\alpha' = \int_{\alpha_{in}}^{\alpha} \left( \frac{\partial (h_\phi H_\phi)}{\partial \alpha} \right) \bigg|_{\beta_U(\alpha')} d\alpha' \tag{2.105}
\]

\[
= (h_\phi H_\phi) \bigg|_{(\beta_U(\alpha),\alpha_{in})} \tag{2.106}
\]

\[
= (h_\phi H_\phi) \bigg|_{(\beta_U(\alpha),\alpha)} - (h_\phi H_\phi) \bigg|_{(\beta_U(\alpha_{in}),\alpha_{in})} \tag{2.107}
\]

\[
\therefore J_\beta = \left( \frac{h_\phi B_\phi}{\mu_0} \right) \bigg|_{(\beta_U(\alpha),\alpha)} - \left( \frac{h_\phi B_\phi}{\mu_0} \right) \bigg|_{(\beta_U(\alpha_{in}),\alpha_{in})} \tag{2.108}
\]

We can similarly compare Equation (2.98) to Equation (2.76) to express \(J_\alpha\) in terms of the azimuthal magnetic field. This process yields the following relationship:

\[
J_\alpha = \left( \frac{h_\phi B_\phi}{\mu_0} \right) \bigg|_{(\beta_U(\alpha_{in}),\alpha_{in})} - \left( \frac{h_\phi B_\phi}{\mu_0} \right) \bigg|_{(\beta_U(\alpha),\alpha_{in})} \tag{2.109}
\]

We assumed in Section 2.2.5 that the azimuthal magnetic field at the base of the conducting layer is zero. Using that assumption here reduces the above expression to

\[
J_\alpha = - \left( \frac{h_\phi B_\phi}{\mu_0} \right) \bigg|_{(\beta_U(\alpha_{in}),\alpha_{in})} \tag{2.110}
\]

Therefore, \(J_\alpha - J_\beta\) can be expressed in terms of the azimuthal magnetic field evaluated at different points:

\[
J_\alpha - J_\beta = \left[ - \left( \frac{h_\phi B_\phi}{\mu_0} \right) \bigg|_{(\beta_U(\alpha_{in}),\alpha_{in})} \right] - \left[ \left( \frac{h_\phi B_\phi}{\mu_0} \right) \bigg|_{(\beta_U(\alpha),\alpha)} - \left( \frac{h_\phi B_\phi}{\mu_0} \right) \bigg|_{(\beta_U(\alpha_{in}),\alpha_{in})} \right] \tag{2.111}
\]

\[
= - \left( \frac{h_\phi B_\phi}{\mu_0} \right) \bigg|_{(\beta_U(\alpha),\alpha)} \tag{2.112}
\]

We now have two ways to constrain thermospheric electrodynamics at the upper boundary of the thermosphere. We can calculate \(J_\alpha\) using either the current along
a previously-calculated inner field line via Equation (2.98) or the azimuthal magnetic field via Equation (2.110). We can calculate \( J_\beta \) using either FACs via Equation (2.97) or the azimuthal magnetic field via Equation (2.108). If FACs are used to calculate \( J_\beta \) and azimuthal magnetic fields to calculate \( J_\alpha \), they must be mutually consistent. We therefore suggest that \textit{either} FACs \textit{or} azimuthal magnetic fields are used to calculate the electric field, but not both. This gives us great flexibility in how we calculate \( J_\alpha \) and \( J_\beta \).

Before, the PEF (and all currents and magnetic field perturbations calculated using it) were those associated with the thermospheric wind dynamo. Now, the polarization electric field includes effects from the magnetosphere and all quantities should be reinterpreted appropriately. We demonstrated previously that \( h_\alpha E_\alpha \) was constant along a given magnetic field line, and this result implies that the PEF would be generated by differences in the winds, conductivities and magnetic field strength in each respective hemisphere (see Section 3.5 for detailed discussion on this). This extension of that formulation, however, splits the dipole field line, isolating each hemisphere from the other and allowing \( h_\alpha E_\alpha \) to have a different, constant value in the northern hemisphere than in the southern hemisphere. Consequentially, the two hemispheres within the scope of this model are decoupled from each other where FACs are specified. Realistically, one might expect the FACs along a given field line in one hemisphere to be related to those along the same field line in the other hemisphere. If observations indicate a relationship between FACs in each hemisphere along the upper boundary of the thermosphere, then this may indicate some degree of coupling between the two hemispheres. The ability to specify either FACs or the azimuthal magnetic field along the upper boundary of the conducting layer gives us flexibility in terms of how we solve for the polarization electric field, but it introduces extra terms to our calculation. The advantage of this more complicated formulation, of course, is that this extension allows us to use observational data or results from a magnetospheric model to constrain our model of electrodynamics in Saturn’s upper atmosphere.
2.3 The Saturn Thermosphere-Ionosphere Model (STIM)

While we can use the axisymmetric, steady state model described in Section 2.2 to make initial estimates of certain quantities, we need a three-dimensional, time-dependent model to understand the effects of ion drag and resistive heating on Saturn’s thermosphere. Towards this goal, we incorporate the model of Vriesema et al. (2020) self-consistently into STIM. In this section, we describe the previous (unmodified) version of STIM in greater detail to provide context for the changes we have made. These changes will be fully described, tested and discussed in Chapter 4.

2.3.1 Development of STIM

STIM is a general circulation model that numerically solves the coupled, nonlinear, three-dimensional Navier-Stokes equations of momentum, energy and continuity for the major species in Saturn’s upper atmosphere (e.g. Müller-Wodarg et al., 2006; Moore et al., 2010; Müller-Wodarg et al., 2012, 2019). STIM includes a suite of coupled 1-D, 2-D and 3-D models developed for Saturn, and it includes processes such as ion and neutral species transport, photochemistry, plasma diffusion, ring shadowing (disabled in the present study), and more. (Overall, the model is 3-D, but certain processes are modeled more simply.) Moore et al. (2010) has an excellent summary of STIM’s features and development history up to 2011.

More recently, electrodynamics calculations have been added to STIM to study the response of Saturn’s ionosphere and thermosphere to electron precipitation, magnetospheric forcing, and to estimate resistive heating rates. Galand et al. (2011) added electron precipitation, and Müller-Wodarg et al. (2012) added ion drag and Joule heating at high latitudes using a high-latitude electric field based on the model of Cowley et al. (2004a). Later, Müller-Wodarg et al. (2019) updated STIM to use an electric field based on simulations from the Block-Adaptive-Tree-Solarwind-Roe-Upwind-Scheme (BATSRUS) model (Jia et al., 2012a) for quiet solar wind conditions. We refer to this latter electric field throughout this paper as the magnetospheric electric field (MEF) and describe it in more detail in Section 4.2.1.
2.3.2 Overview of STIM

STIM utilizes a three-dimensional \((\log(\text{pressure}) \times \text{latitude} \times \text{longitude})\), spherical grid and a fixed time step. Most model parameters (\textit{e.g.} resolution, time step, physical constants, and others) are hard-coded, but STIM optionally accepts a set of inputs (\textit{e.g.} a startup file’s relative path, the number of rotations to model, Saturn’s season, switches, and others) from the command-line. The primary set of equations and boundary conditions were originally given in Appendix A of Müller-Wodarg et al. (2006), and due to subsequent updates and for reference within this manuscript, we include them below.

The initial setup for the neutral atmosphere (pressure, temperature, height above the 1 bar level; number densities for H, He, H\(_2\) and CH\(_4\)) is read from the one-dimensional model of Moses and Bass (2000). A typical low resolution run uses 36 vertical pressure levels with steps of 0.5 pressure scale heights, 91 latitudes and 36 longitudes. A typical high resolution run uses 72 vertical pressure levels with steps of 0.25 scale heights, 181 latitudes and 90 longitudes (Moore et al., 2004). The time steps used are typically 2 s to 5 s.

2.3.3 Governing Equations

STIM self-consistently solves the three-dimensional, time-dependent Navier-Stokes equations of energy, momentum and continuity by explicit time integration. We describe each below. For convenience, we label terms with an underscore.

Ideally, all three momentum equations would be solved simultaneously. This is numerically challenging, however, because the pressure gradient and gravity terms dominate other terms in the vertical momentum equation, causing numerical errors in the calculation of the vertical wind. To avoid this problem, vertical winds are calculated in hydrostatic equilibrium by using the continuity equation as follows:

\[
\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (u_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{\partial w}{\partial p} = 0, \tag{2.113}
\]

where \(w \equiv \frac{dp}{dt}\) is the vertical wind in the pressure frame.
Finally, we note here that STIM’s treatment of electrodynamics is based on a thin-layer conductivity approach (Baker and Martyn, 1953) in which the radial currents are zero. It also uses a calculation of conductivities that is appropriate at high altitudes, where plasma is tightly coupled to magnetic field lines, but is not appropriate at lower altitudes, where collisions are significant. STIM’s calculation of current density does not preserve $\nabla \cdot \vec{j} = 0$, and it only uses an electric field imposed from the magnetosphere, rather than a polarization electric field generated by an ionospheric dynamo. In order to investigate the effects of such a dynamo, we have updated how STIM treats electrodynamics. We defer the detailed description of these changes to Chapter 4, however, and instead focus here on the other elements of STIM.
STIM’s Colatitudinal Momentum Equation

The colatitudinal component of the momentum equation is

\[
\frac{\partial u_\theta}{\partial t} = -\left(\vec{u} \cdot \nabla\right)_{\text{horz}} u_\theta - \omega \frac{\partial u_\theta}{\partial p} + \left(2\Omega_S + \frac{u_\phi}{r \sin \theta}\right) u_\phi \cos \theta + \frac{1}{r \rho g} \frac{\partial^2 u_\theta}{\partial \phi^2} - \frac{u_\theta}{\sin^2 \theta} - 2 \frac{\cos \theta \partial u_\phi}{\sin^2 \theta \partial \phi} + 2 u_\theta
\]

Horizontal advection

Vertical advection

Coriolis and curvature effects

Second curvature term

Horizontal viscous drag

Vertical viscous drag

Geopotential

Ion drag

Rayleigh friction

where \( \omega \) is the vertical velocity relative to the pressure level, \( p \) is the pressure, \( \Omega_S = 1.6378 \times 10^{-4} \text{ rad s}^{-1} \) is Saturn’s planetary rotation rate (corresponding to a rotation period of 10 h, 39 min, 23.5689 s), \( \rho \) is the mass density of the atmosphere, \( g \) is the gravitational acceleration, \( \mu \) is the molecular viscosity coefficient (calculated from the individual gas species), \( H_p \) is the pressure scale height, \( z \) is the altitude above the 1 bar pressure level, \( \lambda_0 \) is the Rayleigh factor, \( p_{\lambda_0} \) is the Rayleigh peak pressure, and \( F(\theta) \) is a tunable function that modifies how Rayleigh drag is applied at different latitudes. The Rayleigh drag term is a generic drag term that is used to parameterize interactions between gravity waves and the mean flow (e.g., Bougher et al., 1988; Müller-Wodarg et al., 2019). To help understand the role of ion drag
in isolation from Rayleigh drag, we disable Rayleigh drag in all runs by setting \( F(\theta) = 0 \).

**STIM’s Azimuthal Momentum Equation**

The azimuthal component of the momentum equation is

\[
\frac{\partial u_\phi}{\partial t} = - \left( \vec{u} \cdot \vec{\nabla} \right)_{\text{horiz}} u_\phi + \frac{\partial u_\phi}{\partial p} \omega + \left( 2\Omega_S + \frac{u_\phi}{r \sin \theta} \right) u_\theta \cos \theta + \frac{1}{r} \frac{\omega u_\phi}{\rho g}
\]

\[
+ \mu \rho r^2 \left( \frac{\partial^2 u_\phi}{\partial \theta^2} + \cot \theta \frac{\partial u_\phi}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2 u_\phi}{\partial \phi^2} - \frac{u_\phi}{\sin^2 \theta} - \frac{\cos \theta \frac{\partial u_\theta}{\partial \theta}}{\sin^2 \theta} \frac{\partial \phi}{\partial \phi} + 2u_\phi \right)
\]

\[
+ \frac{g}{p} \left( \frac{\partial (\mu / H_p)}{\partial p} \frac{\partial u_\phi}{\partial p} + \frac{\mu}{H_p} \frac{\partial^2 u_\phi}{\partial p^2} + \frac{2}{r} \frac{\mu}{H_p} \frac{\partial u_\phi}{\partial p} \frac{\partial z}{\partial p} \right)
\]

\[
- \frac{g}{r \sin \theta} \frac{\partial z}{\partial \phi}
\]

\[
+ \frac{1}{\rho} \left[ \vec{j} \times \vec{B} \right]_{\phi}
\]

\[
- \frac{\lambda_0 u_\phi}{1 + \sqrt{\rho \chi_0 / \rho}} F(\theta),
\]

where we again disable Rayleigh drag by setting \( F(\theta) = 0 \).

**STIM’s Energy Equation**

STIM’s energy equation is expressed in terms of the combined internal and kinetic energies per unit mass, defined as

\[
\epsilon \equiv c_p T + \frac{1}{2} \left( u_\theta^2 + u_\phi^2 \right),
\]
where $c_p$ is the specific heat of the gas and $T$ is the temperature. As noted in Müller-Wodarg et al. (2006), $\epsilon + gh_p$ is the enthalpy of the gas, provided $gh_p$ is the potential energy of the gas at height $h_p$.

The energy equation is

$$\frac{\partial \epsilon}{\partial t} = -u_\theta \left( \frac{1}{r} \frac{\partial (\epsilon + gh_p)}{\partial \theta} \right) - u_\phi \left( \frac{1}{r \sin \theta} \frac{\partial (\epsilon + gh_p)}{\partial \phi} \right)$$

Horizontal advection

$$+ \omega \left( \frac{1}{p} \frac{\partial \epsilon}{\partial p} + \frac{z}{p \partial p} + \frac{1}{\rho} \right)$$

Vertical advection and adiabatic heating/cooling

$$+ \frac{1}{\rho r^2} (\kappa_M + \kappa_T) \frac{\partial^2 T}{\partial \theta^2} + \cot \theta \frac{\partial T}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2}$$

Horizontal conduction

$$+ \frac{g}{p} \left[ (\kappa_M + \kappa_T) \frac{\partial^2 T}{\partial p^2} + \frac{\partial T}{\partial p} \frac{\partial (\kappa_M + \kappa_T)}{\partial p} + \frac{2}{r} (\kappa_M + \kappa_T) \frac{\partial T}{\partial p} \frac{\partial z}{\partial p} \right]$$

Vertical conduction

$$+ \frac{g}{p} \left[ \left( \frac{\mu}{H_P} \right) \left( u_\theta \frac{\partial^2 u_\theta}{\partial p^2} + \left[ \frac{\partial u_\theta}{\partial p} \right]^2 + u_\phi \frac{\partial^2 u_\phi}{\partial p^2} + \left[ \frac{\partial u_\phi}{\partial p} \right]^2 \right) \right]$$

Vertical viscous heating

$$+ \frac{\partial (\mu/H_P)}{\partial p} \left( u_\theta \frac{\partial u_\theta}{\partial p} + u_\phi \frac{\partial u_\phi}{\partial p} \right)$$

Vertical viscous heating

$$+ \frac{1}{\rho} \left( \vec{j} \cdot \vec{E} \right)$$

Joule heating

$$+ Q_{\text{sol}} + Q_{IR} + Q_{PP} + Q_{\text{grav}},$$

(2.117)

where $\kappa_M$ is the coefficient of molecular conduction, $\kappa_M$ is the coefficient of turbulent conduction, $T$ is the temperature, $Q_{\text{sol}}$ is solar EUV heating, $Q_{IR}$ is $H_3^+$ infrared cooling, $Q_{PP}$ is heating due to particle precipitation, and $Q_{\text{grav}}$ is the heating due to a globally-imposed gravity wave profile. In all runs, we have neglected gravity wave heating by setting $Q_{\text{grav}} = 0$ in order to focus on the role of electrodynamics. Also, $Q_{PP}$ is set to zero. We note that the “Joule heating” term in this equation is actually the total effect of electrodynamics on the thermal and kinetic energy.
of the atmosphere. This term combines the effects of frictional heating, which is conventionally known as Joule heating and cannot be negative, and ion drag, which transports kinetic energy and can be negative.

2.4 Summary

In the first half of this chapter, we described an axisymmetric, steady-state model, based on Richmond (1973a), for electrodynamics in Saturn’s upper atmosphere. The model’s assumptions, in the limit that $\sigma_\parallel \rightarrow \infty$, allow us to collapse a much more difficult 4D (three spacial dimensions plus time) problem to a much more tractable 2D (axisymmetric, steady-state) problem. We can now explore how different wind and conductivity profiles could be associated with different current systems and azimuthal magnetic field perturbations. Whereas previous models of ionospheric electrodynamics in Saturn’s upper atmosphere rely on height-integrated conductivities and/or height-integrated currents, our formulation allows us to resolve the vertical structure of the thermosphere in order to determine how sensitive electrodynamics is to vertical gradients. It also establishes an analytic relationship between atmospheric winds and azimuthal magnetic field perturbations, which suggests that we may be able to constrain winds using magnetic field data. We discuss this possibility in detail in Chapter 3. We also extended this 2D model to use upper-boundary FACs at high latitudes to constrain electrodynamics. This allows us to use observations or data from a model instead of making broad assumptions about the region above the thermosphere.

The detail of the 2D model allows for future extensions and generalizations. Although this model assumes axial symmetry, it may be possible to generalize this approach for atmospheres which are locally axisymmetric. We describe other extensions in Section 6.3. With the shifted dipole generalization described in Chapter 3, the model can be applied to any planetary thermosphere for which the dipole term of the planetary magnetic field dominates and for which the winds and conductivities are approximately symmetric about the magnetic axis. The models of Khurana
et al. (2018) and Provan et al. (2019) make a slightly different assumption: that ion velocity is constant along magnetic field lines. It would therefore be an interesting future exercise to compare their modes with ours, since ours instead assumes that $h_\alpha E_\alpha$ is constant along field lines.

When applied to Saturn’s upper atmosphere with reasonable inputs, this 2D model predicts significant currents at low and high latitudes that are associated with resistive heating and ion drag (Vriesema et al., 2020). They also predict the presence of azimuthal magnetic field perturbations. In order to examine these predictions in more detail, this model of electrodynamics needs to be incorporated self-consistently into a GCM, the topic of Chapter 4. We use STIM, a GCM which self-consistently solves the coupled Navier-Stokes equations of momentum and energy and models a number of other phenomena as well in Saturn’s thermosphere and ionosphere. STIM globally calculates the transport and photochemical processes for a number of neutral and ion species. Because STIM’s treatment of electrodynamics is lacking, we make a number of updates to STIM as described in Chapter 4. With the axisymmetric model described earlier incorporated into STIM, with the extension described above, we will present results from the first comprehensive calculation of a wind dynamo in Saturn’s thermosphere and ionosphere.
CHAPTER 3

Electrodynamics in Saturn’s thermosphere at low and middle latitudes

The following peer-reviewed journal article, cited as Vriesema et al. (2020), was published by *Icarus* in its “Cassini Mission Science Results” special issue on July 1, 2020. The content of this article has not been modified beyond its published form, except that references in this manuscript are included at the end of this dissertation instead of at the end of this chapter.
3.1 Abstract

Electrodynamics plays an important role in controlling circulation and energy balance in Saturn’s auroral thermosphere, but less is known about its effects at lower latitudes. Recent observations by the Cassini magnetometer instrument, taken around the equator during the Grand Finale tour, revealed azimuthal magnetic field perturbations of the order of 10 nT to 30 nT associated with electrodynamics in Saturn’s upper atmosphere. In order to investigate the implications of these observations, we develop a steady-state, axisymmetric model adapted from terrestrial studies to simulate wind-driven electrodynamics at low to middle latitudes in Saturn’s thermosphere. Our results demonstrate, based on rigorous theory, that the magnetic field observations can be reproduced by a wind dynamo generating electric currents in the ionosphere of the order $10^{-9}$ A m$^{-2}$ to $10^{-7}$ A m$^{-2}$ provided that eastward zonal winds of the order of 100 m s$^{-1}$ exist in the equatorial thermosphere. The resistive (Joule) heating rate based on the equatorial current system, however, is significantly lower than that required to explain the temperatures in Saturn’s thermosphere. In spite of this, we find that resistive heating and ion drag due to a mid-latitude wind dynamo have the potential to alter the energy balance and general circulation in Saturn’s thermosphere and should be treated self-consistently in future global circulation models.

3.2 Introduction

Saturn’s intrinsic magnetic field is surprisingly axisymmetric, with observations placing an upper limit of only 0.0095° on the dipole tilt (Dougherty et al., 2018). Under these circumstances, observations of significant azimuthal magnetic field perturbations in the vicinity of the planet indicate the existence of field-aligned currents (FACs) which are driven by electrodynamics in the ionosphere and/or magnetosphere. Evidence of FACs at high latitudes in both the southern and northern hemispheres has been obtained by the Cassini magnetometer (e.g., Hunt et al., 2014, 2015; Dougherty et al., 2018). Azimuthal magnetic field perturbations on the order
of 20 nT to 30 nT are observed between 3–5 Saturn radii ($R_S$) and imply perturbations of about 100 nT to 200 nT at ionospheric heights (Hunt et al., 2014). These high-latitude FACs are associated with auroral current systems (e.g., Southwood and Cowley, 2014; Khurana et al., 2018). Global magnetohydrodynamic simulations by Jia et al. (2012b) and Jia and Kivelson (2012) demonstrate that wind shear in Saturn’s ionosphere can drive high-latitude FACs that are remarkably consistent with Cassini observations. Near the equatorial plane, observations made in the vicinity of the periapsides of the 22 Cassini Grand Finale orbits probe magnetic field perturbations associated with current systems at low and middle latitudes (Dougherty et al., 2018; Khurana et al., 2018) that have not yet been simulated by detailed models.

Recently, Dougherty et al. (2018) presented azimuthal magnetic field perturbations inward of the D-ring as evidence for an equatorial current system in Saturn’s ionosphere. The azimuthal magnetic field perturbations they report are generally positive (eastward), indicating a current system that flows north-to-south along field lines and closes via south-to-north currents in the ionosphere. Khurana et al. (2018) argue that this current system is driven by winds in the thermosphere and the northward offset of Saturn’s magnetic dipole moment of $\sim 0.0466R_S$ (Dougherty et al., 2018). Assuming a similar azimuthal wind structure as in the troposphere, and treating the ionosphere as a single layer, the northern footprint of a given magnetic field line inward of the D-ring is located in a region of weaker zonal wind, whereas the southern footprint is located in a region of stronger zonal wind. This difference in wind speeds causes currents to flow along the magnetic field lines from the northern hemisphere to the southern hemisphere, with a presumed northward return flow in the ionosphere. Under axial symmetry, the resulting current system forms a loop in a given meridional plane. Assuming that the current is uniform throughout the loop, Khurana et al. (2018) use the Biot-Savart law and the observed azimuthal magnetic perturbations to estimate a current of 1.15 MA rad$^{-1}$ flowing from south to north in the ionosphere. Assuming a zonal wind footprint difference of 300 m s$^{-1}$, they estimate a height-integrated Pedersen conductivity of 8.89 S and total resistive heating
of 0.22 TW generated between $\pm 20^\circ$ of Saturn’s equator. Provan et al. (2019) use a similar, though more detailed, approach to estimate a northward height-integrated current of 1 MA rad$^{-1}$, in agreement with Khurana et al. (2018).

The mechanism proposed by Khurana et al. (2018) explains how electrodynamics at low latitudes produces magnetic field perturbations and resistive heating. In order to explain the observations, however, Khurana et al. (2018) simply assume that a meridional current loop exists around the equator and do not present a detailed model to explain how electric currents and magnetic field perturbations are produced by winds in the thermosphere. In addition, they assume that currents are driven only by the meridional shear of the zonal wind between two points on opposite sides of the magnetic equator and neglect the vertical structure of the ionosphere. The complex coupling of the E and F layers in Earth’s ionosphere shows that vertical gradients in the ionosphere cannot be ignored as they play an important role in making the wind-driven dynamo near the equator possible (e.g., Richmond, 1973b). The purpose of the model presented here is to probe deeper into the driving mechanisms of the observed azimuthal magnetic field perturbations in order to better constrain the wind shear, resistive heating rates and ion drag at low to middle latitudes. As far as we are aware, this is the first time that a model of the equatorial wind dynamo has been applied to Saturn’s upper atmosphere.

A detailed look at the wind dynamo is required because electrodynamics could have profound consequences for energy balance and dynamics. The thermosphere of Saturn, along with the other giant planets in the solar system, is several times hotter than can be explained by solar heating alone (e.g., Yelle and Miller, 2004). On Saturn, models that include resistive heating by the polar aurora roughly agree with the observed temperatures at high latitudes but cannot explain the temperatures at low to middle latitudes (see Figure 3 in Koskinen et al., 2015). Instead, they predict that redistribution of energy is prevented by Saturn’s fast rotation i.e., the “Coriolis barrier”, which is enhanced by westward ion drag at high latitudes (Smith et al., 2007). The existing models, however, do not include resistive heating and ion drag by wind-driven dynamo currents. Resistive heating at low latitudes can
act as a direct heating mechanism and ion drag can change global circulation in ways that could help to redistribute energy from the poles to the equator. The observed azimuthal magnetic field perturbations around the equator indicate that these mechanisms can no longer be ignored in studies of Saturn’s thermosphere.

The magnetic field observations also provide rough constraints on the zonal winds in the upper atmosphere. Indirect evidence from occultation data and observations of the equatorial oscillation by the Cassini CIRS instrument (Hubbard et al., 1997; Guerlet et al., 2011) suggest that zonal winds may penetrate at least up to the lower thermosphere. The degree to which this is possible, however, is uncertain and until now, there have been no real observational constraints on dynamics in Saturn’s equatorial thermosphere. If the mechanism proposed by Khurana et al. (2018) is correct, the magnetic field observations indicate the presence of a relatively fast, eastward jet in the thermosphere. This is a significant departure from circulation models (Müller-Wodarg et al., 2006, 2012) and points to a possible coupling between the stratosphere and the thermosphere. The presence of an equatorial jet in the thermosphere is an exciting prospect, which can only be explored by using a detailed model to properly explore the relationship between zonal winds and magnetic field perturbations.

We construct an axisymmetric, steady-state model to demonstrate that an ionospheric wind dynamo, driven by zonal winds, produces significant currents and magnetic field perturbations in the low and middle latitude thermosphere, similar in magnitude to those proposed by Khurana et al. (2018). We explore the relationship between zonal winds and azimuthal magnetic field perturbations, also including middle latitude winds predicted by the Saturn Ionosphere-Thermosphere Model (hereafter, the STIM model, Müller-Wodarg et al., 2006, 2012) that are driven by auroral heating. Our model resolves the vertical structure of the ionosphere and calculates conductivities based on our best estimate of the atmospheric structure. The model that we have developed to reproduce the observed magnetic field perturbations allows us to estimate the importance of resistive heating and ion drag on the energy balance and dynamics of the thermosphere at different latitudes and
We describe the model and our methods in general in Section 3.3. The key results of the simulations are described in Section 3.4, which includes an exploration of several possible wind profiles and their effect on the currents and magnetic field perturbations. In Section 3.5, we demonstrate the wind dynamo mechanism and discuss the implications of our results on the interpretation of the magnetic field observations, energy balance in the thermosphere and dynamics in the upper atmosphere. Finally, we summarize our results and conclude in Section 3.6.

3.3 Model Equations and Assumptions

The existence of an ionospheric wind dynamo around the terrestrial equator is well known. Polarization electric fields are produced by electrons and ions that drift apart due to a combination of collisions with the neutral wind and gyro-motion around the magnetic field lines. Spatial variations in these electric fields give rise to perturbations to the planetary magnetic field that drive electric currents in the ionosphere (Vasyliunas, 2012; Leake et al., 2014). The same physics applies to Saturn’s ionosphere; therefore we adopt a method used in terrestrial studies to calculate the currents from wind-induced electrodynamics on Saturn (Richmond, 1973a; Forbes, 1981; Kelley, 1989; Richmond, 1995). Unlike the approach adopted by Khurana et al. (2018) that treats the ionosphere as a thin conducting layer, our formulation captures the vertical structure of the ionosphere and thermosphere. Figure 3.1 illustrates the geometry and basic mechanism for generating wind-driven currents in our model.

Our model calculates the distribution of currents in the ionosphere based on an assumed zonal wind profile. In order to achieve this, the ability to resolve the vertical structure of the ionosphere is critical. It is particularly important at low to middle latitudes where the magnetic field lines are not vertical and the use of the height-integrated conductivities, that should properly be integrated along (nearly horizontal) magnetic field lines, is questionable. Also, conductivities in Saturn’s...
Figure 3.1: A not-to-scale sketch of the geometry in a meridional plane. The two thick, black, solid lines represent the lower and upper boundaries of the thermosphere. The thick, black, dashed line indicates the location of the peak Pedersen conductivity. The thin, vertical, black line represents the planetocentric equator and the thin, vertical, dashed black line represents the magnetic equator, which is offset to the north of the planetocentric equator. The grey shading represents the magnitude of the zonal wind $u_\phi$, which here is symmetric about the planetary equator and where darker shading indicates stronger eastward winds. The thin, black, dotted lines represent two magnetic dipole field lines. Below the domain in both hemispheres, two pairs of black arrows labelled $\hat{\alpha}$ and $\hat{\beta}$ indicate the local unit vectors in the shifted dipole coordinate system (see Section 3.3.1). We denote the field lines $\alpha_i$ for the inner field line and $\alpha_o$ for the outer line. The darker blue arrows illustrate the $\alpha$-component of $\vec{u} \times \vec{B}$, equal to $-u_\phi B$, while the red arrows illustrate the $\alpha$-component of the polarization electric field, $E_\alpha$. Ignoring small geometric scale factors, $E_\alpha$ is constant along a given field line, while $-u_\phi B$ varies primarily with $u_\phi$. The sum of these terms is the $\alpha$-component of the electric field in the center of mass (CM) frame $E_{\alpha \CM}^\alpha$, which is represented by the thicker black arrows. Pedersen currents flow in the direction of $E_{\alpha \CM}^\alpha$. The thin, orange arrows sketch the direction and relative magnitude of the parallel currents at low latitudes, as required by Ampère’s Law and charge continuity. The thicker cyan arrows sketch the overall northward direction of current flow at low latitudes. Stronger winds at low latitudes cause stronger electric fields and currents, while weaker winds further from the equator generate weaker electric fields and currents. The points labelled $\beta_1(\alpha_i)$, $\beta_2(\alpha_i)$, $\beta_1(\alpha_o)$ and $\beta_2(\alpha_o)$ are the lower boundary points in our domain as described in Section 3.3.2.
ionosphere that depend on the plasma density, magnetic field strength and collision frequencies between ions, electrons and the neutral atmosphere change significantly with altitude. In this study, we use three-fluid expressions to calculate the conductivities that apply in both weak and strong ionization regimes (Koskinen et al., 2014) and allow us to calculate the conductivities and currents throughout the thermosphere. Below, we derive and describe the equations used by our model in detail.

3.3.1 Geometry and Coordinate Systems

Our model uses a dipole coordinate system shifted northward by \( \Delta z \equiv 0.0466R_S \) from Saturn’s equatorial plane along the rotation axis (Dougherty et al., 2018). The relationship between this shifted dipole coordinate system \((\beta, \alpha, \phi)\) and a spherical polar coordinate system \((r_s, \theta_s, \phi)\) with the same origin is (Swisdak, 2006; Richmond, 1973a):

\[
\beta = \frac{\cos \theta_s}{r_s^2} \quad (3.1)
\]

\[
\alpha = \frac{r_s}{\sin^2 \theta_s}. \quad (3.2)
\]

The coordinate \( \beta \) denotes the northward displacement along a magnetic field line from the magnetic equator (anti-parallel to Saturn’s dipole field lines). The coordinate \( \alpha \) is constant along magnetic field lines, increasing outward at low latitudes, and it uniquely identifies a magnetic field line in a given meridional plane. If the magnetic field lines are aligned with the planet’s rotation axis, as is the case for Saturn, the \( \alpha \) coordinate is related to the standard McIlwain \( L \)-value by \( L = \alpha/R_S \) (McIlwain, 1966). The shifted spherical coordinates \((r_s, \theta_s, \phi)\), expressed in terms of the planetocentric (unshifted) spherical coordinates \((r, \theta, \phi)\), are given by

\[
r_s^2 = (r \sin \theta)^2 + (r \cos \theta - \Delta z)^2 \quad (3.3)
\]

\[
\cos \theta_s = \frac{r \cos \theta - \Delta z}{\sqrt{(r \sin \theta)^2 + (r \cos \theta - \Delta z)^2}}. \quad (3.4)
\]

The azimuthal angle \( \phi \) is the same in all coordinate systems here.
Our magnetic field is given by $\vec{B} = B\hat{b} = B_\beta \hat{\beta}$, where $\hat{b} = -\hat{\beta}$ is the direction of the magnetic field and $B$ is its magnitude. We note that because $\beta$ increases antiparallel to Saturn’s magnetic field, $\hat{b} = -\hat{\beta}$ and $B_\beta = -B$. We set the strength of Saturn’s dipole magnetic moment to $4\pi R_3^2 g_1^0 \approx 4.6227 \times 10^{25}$ A m$^2$, where $g_1^0 = 2.11402 \times 10^4$ nT (Dougherty et al., 2018).

In order to perform our calculations, we transform the coordinates and vector components of our model inputs from the planetocentric coordinate system to the shifted spherical coordinate system and then to the shifted dipole system. We solve the electrodynamics equations in this system along magnetic field lines and then transform the results — the polarization electric field, current density, ion drag, and magnetic field perturbations — back to the planetocentric spherical coordinate system for plotting and further analysis. In principle, the basic theory we describe below may be applied to a planet with any physically realistic, axisymmetric geometry, using appropriate geometrical scale factors. However, because our numerical model uses inputs from STIM, which treats Saturn as a sphere, we also treat Saturn as a sphere.

3.3.2 Wind Dynamo Equations

We are interested in steady-state, axisymmetric currents and the associated perturbations to the magnetic field $\vec{B}$ driven by zonal winds, and therefore adopt the time-independent continuity equation

$$\vec{\nabla} \cdot \vec{j} = 0,$$

where the current density $\vec{j}$ is related to the electric field and magnetic field via the generalized Ohm’s law (e.g. Song et al., 2001; Koskinen et al., 2014)

$$\vec{j} = \sigma_\parallel \vec{E}_\parallel + \sigma_P \vec{E}_{CM}^\perp - \sigma_H \left[ \vec{E}_{CM}^\perp \times \hat{b} \right],$$
where $\sigma_\parallel$, $\sigma_P$ and $\sigma_H$ are the parallel, Pedersen and Hall conductivities, respectively. The parallel and perpendicular components of the electric fields are defined by

$$
\vec{E}_\parallel \equiv (\vec{E} \cdot \hat{b}) \hat{b}
$$

(3.7)

$$
\vec{E}^{CM}_\perp \equiv \hat{b} \times \vec{E}^{CM} \times \hat{b} = \vec{E}^{CM} - \vec{E}_\parallel,
$$

(3.8)

where $\vec{E}$ is the electric field in the System III rotating reference frame and $\vec{E}^{CM} \equiv \vec{E} + \vec{u} \times \vec{B}$ is the electric field in the atmospheric center of mass (CM) frame. The velocity of the atmosphere in the System III frame is given by $\vec{u}$. Because of the low ionization fraction on Saturn, $\vec{u}$ is to high accuracy equal to the neutral wind velocity and the CM frame is approximately the frame of the neutral wind.

We simplify Equation (3.6) by noting that $E_\phi = 0$ in an axisymmetric, steady-state model. We neglect vertical and meridional winds ($u_r$ and $u_\theta$) in the present study as they are expected to be much slower than the zonal wind ($u_\phi$). With these simplifications, the dipole components of Equation (3.6) are

$$
\vec{j}_\beta = \sigma_\parallel E_\beta
$$

(3.9)

$$
\vec{j}_\alpha = \sigma_P (E_\alpha + u_\phi B_\beta)
$$

(3.10)

$$
\vec{j}_\phi = - \sigma_H (E_\alpha + u_\phi B_\beta),
$$

(3.11)

where $\vec{j}_\alpha$ is the Pedersen current density and $\vec{j}_\phi$ is the Hall current density. We note that the Hall currents are directly proportional to the Pedersen currents via

$$
\vec{j}_\phi = - \frac{\sigma_H}{\sigma_P} \vec{j}_\alpha.
$$

(3.12)

We assume that $\sigma_P$ and $\sigma_H$ are effectively zero above and below the conducting region of the thermosphere, and that the perpendicular currents vanish ($j_\alpha = 0$ and $j_\phi = 0$) above and below our domain. We also assume that the parallel current is zero ($j_\beta = 0$) at the base of the conducting region. These assumptions are motivated in part by the fact that Hall and Pedersen conductivities become small near the boundaries of our domain (c.f. Figure 3.2). By choosing zero current boundary conditions, however, we limit ourselves to studying electrodynamics produced
exclusively by the wind dynamo in Saturn’s thermosphere. This means that we ignore magnetosphere-ionosphere coupling at high latitudes, which is the primary mechanism for generating currents in the ionosphere in addition to the wind dynamo. We also do not consider the possibility of electrodynamic coupling between the ionosphere and the rings (e.g., O’Donoghue et al., 2017; Provan et al., 2019). If necessary, externally driven currents could be included in our model by modifying the boundary conditions but we will not pursue this option here. Our aim is to show that the observed magnetic field perturbations around the equator can be produced by wind-driven electrodynamics and we do not believe that the possibility of externally driven current systems significantly alters this conclusion. Note that our boundary conditions prevent current from flowing below and perpendicular currents from flowing above the conducting layer but they do not inhibit FACs above it.

Finally, we assume that the parallel conductivity is infinite and that the parallel electric field ($E_\parallel = -E_\theta$) approaches zero. This approximation is acceptable in the thermosphere at altitudes above the $10^{-4}$ mbar pressure level where the parallel conductivity is much greater than the perpendicular conductivities (c.f. Figure 3.2). At altitudes below the $10^{-2}$ mbar pressure level, however, the three conductivities are roughly equal, suggesting that our assumption of infinite conductivity is not justified for this lower region in our domain. The conductivities are small in this region, however, and little current flow is expected. The assumption of effectively infinite parallel conductivity implies that the magnetic field lines are equipotentials. Although the justification for and the consequences of treating magnetic field lines as equipotentials are more complicated in general (Vasyliunas, 2012, has an excellent discussion of this), this assumption is appropriate for a steady-state model which has had plenty of time to equilibrate.

In Section 3.3.2, we present a general derivation of the currents and magnetic field perturbations based on Equation (3.5). The derivation is based on Richmond (1973a) and we reproduce it here in order to adapt it to Saturn and to highlight the assumptions involved.
Polarization Electric Field: \( E_\alpha \)

Assuming zonal symmetry, the continuity equation (3.5), can be expressed in dipole coordinates as:

\[
0 = \nabla \cdot \vec{j} = \frac{1}{h_\beta h_\alpha h_\phi} \left[ \frac{\partial (h_\alpha h_\phi j_\beta)}{\partial \beta} + \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha} \right], \quad (3.13)
\]

where \( h_\beta = r_s^2/\delta_s \), \( h_\alpha = \sin^3 \theta_s/\delta_s \) and \( h_\phi = r_s \sin \theta_s \) are the Lamé coefficients (geometric scale factors) associated with the dipole coordinate system, and \( \delta_s (\theta_s) \equiv \sqrt{1 + 3 \cos^2 \theta_s} \). We integrate equation (3.13) over volume within a meridional slice, as in Richmond (1973a). The integration volume lies between \( \phi \) to \( \phi + \delta \phi \) in azimuth, and it is bounded above by a surface of constant \( \alpha \) and below by the surface between the conducting and non-conducting regions. We denote \( \beta_1(\alpha') \) and \( \beta_2(\alpha') \) as the values of \( \beta \) where a surface of constant \( \alpha' \) intersects the lower boundary in the southern and northern hemispheres, respectively. We define \( \alpha_{\min} \) as the value of \( \alpha' \) which crosses the lower boundary at the magnetic equator. Mathematically:

\[
0 = \int_V \nabla \cdot \vec{j} \, dV' = \int_{\phi}^{\phi + \delta \phi} \int_{\alpha_{\min}}^{\alpha} \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} h_\beta h_\alpha h_\phi \left( \nabla \cdot \vec{j} \right) \, d\beta' \, d\alpha' \, d\phi' \quad (3.15)
\]

Due to axisymmetry, the integrand is constant in \( \phi \), so the integral over \( d\phi' \) reduces to a factor of \( \delta \phi \), which can be taken outside the integral and divided out. We insert Equation (3.13) into Equation (3.15) and simplify:

\[
0 = \int_{\alpha_{\min}}^{\alpha} \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} h_\beta h_\alpha h_\phi \left( \frac{1}{h_\beta h_\alpha h_\phi} \left[ \frac{\partial (h_\alpha h_\phi j_\beta)}{\partial \beta} + \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha} \right] \right) \, d\beta' \, d\alpha' \quad (3.16)
\]

\[
= \int_{\alpha_{\min}}^{\alpha} \left( h_\alpha h_\phi j_\beta \right)_{\beta_1(\alpha')}^{\beta_2(\alpha')} + \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha'} \, d\beta' \, d\alpha' \quad (3.17)
\]

By definition, \( \beta_1(\alpha') \) and \( \beta_2(\alpha') \) are both located at the lower boundary where we assume \( \vec{j} = 0 \), as shown in Figure 3.1. As a result, the first term in the integral disappears, leaving us with

\[
0 = \int_{\alpha_{\min}}^{\alpha} \int_{\beta_1(\alpha')}^{\beta_2(\alpha')} \frac{\partial (h_\beta h_\phi j_\alpha)}{\partial \alpha'} \, d\beta' \, d\alpha'. \quad (3.18)
\]
It can be shown, via Leibnitz’s rule for differentiation within an integral and assuming that $j_\alpha$ is zero at the lower boundary, that the above equation reduces to

$$0 = \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi j_\alpha d\beta'.$$

(3.19)

In order to express this continuity constraint in terms of the polarization electric field $\vec{E}$, we insert the $\alpha$-component of current density from Ohm’s Law (Equation (3.10)) into Equation (3.19), leaving us with

$$0 = \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \left[ \sigma_P \left( E_\alpha - u_\phi B \right) \right] d\beta'.$$

(3.20)

We note that the axisymmetric, steady-state Faraday’s Law, expressed in dipole coordinates, is:

$$0 = \vec{\nabla} \times \vec{E} = \frac{1}{h_\beta h_\alpha} \left[ \frac{\partial (h_\alpha E_\alpha)}{\partial \beta} - \frac{\partial (h_\beta E_\beta)}{\partial \alpha} \right] \hat{\phi}.$$

(3.21)

This equation gives

$$\frac{\partial (h_\alpha E_\alpha)}{\partial \beta} = \frac{\partial (h_\beta E_\beta)}{\partial \alpha},$$

(3.22)

which, due to our treatment of field lines as equipotentials ($E_\parallel = -E_\beta \to 0$), implies that $h_\alpha E_\alpha$ is also constant along field lines. We use this later in Section 3.5 to illustrate the driving mechanism for the equatorial current system on Saturn and to compare our model with that of Khurana et al. (2018). Here, it allows us to pull a factor of $h_\alpha E_\alpha$ out of the integral in Equation (3.20). Finally, we solve for $E_\alpha$:

$$E_\alpha = \frac{-\int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \sigma_P u_\phi B_\beta}{h_\alpha \int_{\beta_1(\alpha)}^{\beta_2(\alpha)} h_\beta h_\phi \sigma_P h_\alpha d\beta'} d\beta'.

(3.23)

This form of Equation (3.23) has a straightforward interpretation. If we ignore the geometric scale factors, the value of $E_\alpha$ is essentially a weighted average of $-\left[ \vec{u} \times \vec{B} \right]_\alpha = -u_\phi B_\beta$ along the field line where the weight is the Pedersen conductivity. Although $E_\alpha$ is more sensitive to zonal winds in regions where the Pedersen conductivity is larger (e.g. the ionosphere), its value is also affected by winds outside these regions. To calculate $E_\alpha$, we numerically integrate along field lines from the southern to the northern hemisphere using a cubic spline method detailed in Bhadauria and Singh (2011).
Azimuthal Magnetic Field: $B_\phi$

We use the axisymmetric, steady-state version of Ampère’s Law to calculate the magnetic field perturbations:

$$\mu_0 \vec{j} = \nabla \times \vec{B}$$

$$= \frac{1}{h_\alpha h_\phi} \left[ \frac{\partial (h_\phi B_\phi)}{\partial \alpha} \right] \hat{\beta}$$

$$+ \frac{1}{h_\beta h_\phi} \left[ \frac{- \partial (h_\phi B_\phi)}{\partial \beta} \right] \hat{\alpha}$$

$$+ \frac{1}{h_\beta h_\alpha} \left[ \frac{\partial (h_\alpha B_\alpha)}{\partial \beta} - \frac{\partial (h_\beta B_\beta)}{\partial \alpha} \right] \hat{\phi}, \quad (3.24)$$

where $\mu_0$ is the permeability of free space. We ignore the $\phi$ component of Equation (3.25) and integrate the $\alpha$-component, multiplied by $h_\beta h_\phi$, along a field line specified by $\alpha$ from one of the footprints ($\beta_1(\alpha)$ or $\beta_2(\alpha)$), to a point $\beta$ along the field line where we wish to calculate $B_\phi$. This gives

$$\mu_0 \int_{\beta_1(\alpha)}^{\beta} j_\alpha h_\phi h_\beta d\beta' = - \int_{\beta_1(\alpha)}^{\beta} \left[ \frac{\partial (h_\phi B_\phi)}{\partial \beta'} \right] d\beta'$$

$$= - h_\phi(\beta, \alpha) B_\phi(\beta, \alpha) + h_\phi(\beta_1(\alpha), \alpha) B_\phi(\beta_1(\alpha), \alpha), \quad (3.26)$$

where $j_\alpha$ is calculated via Equation (3.10). In the present study, we assume $B_\phi = 0$ at the southern footprints of magnetic field lines (i.e. where field lines in the southern hemisphere intersect the lower boundary between the conducting and non-conducting regions), leaving us with our final expression for $B_\phi$:

$$B_\phi(\beta, \alpha) = - \frac{\mu_0}{h_\phi(\beta, \alpha)} \int_{\beta_1(\alpha)}^{\beta} j_\alpha h_\phi h_\beta d\beta'. \quad (3.27)$$

We calculate $B_\phi$ via Equation (3.28) by numerically integrating along field lines from their southern footprints to their northern footprints using the same cubic spline method we use to calculate $E_\alpha$. Comparing Equations (3.19) and (3.27), one can see that imposing the condition of zero-divergence on the current density requires that $B_\phi|_{\beta_1} = B_\phi|_{\beta_2}$ for a given field line. Our assumption of $B_\phi|_{\beta_1} = 0$ implies that $B_\phi|_{\beta_2} = 0$. In practice, our results show that $B_\phi|_{\beta_2}$ is zero within numerical error for all field lines, suggesting that our calculations preserve $\nabla \cdot \vec{j} = 0$. 

We note that in regions where $j_\alpha = 0$, as is the case in our model above or below the conducting region, $h_\phi B_\phi$ is constant along field lines, as can be seen from the $\alpha$-component of Equation (3.25). Because of this, $B_\phi$ is necessarily symmetric about the magnetic equator at altitudes above our domain. Also, it is worth noting that the value of $B_\phi$ depends only on the values of $\sigma_P$, $u_\phi$ and $B$ along a given field line and on the boundary conditions at its endpoints: this calculation can be performed independently for each field line.

Having calculated $B_\phi$ via Equation (3.28), we use the $\beta$-component of Equation (3.25) to calculate $j_\beta$ directly:

$$j_\beta = \frac{1}{\mu_0 h_\alpha h_\phi} \left[ \partial (h_\phi B_\phi) / \partial \alpha \right]. \quad (3.29)$$

Along with Equations (3.10) and (3.11), Equation (3.29) allows us to calculate all components of the current density at every point within our domain. When calculating $j_\beta$, the anti-parallel component of current density, we use a finite difference scheme for arbitrary grid spacing (Fornberg, 1988) based on Lagrangian interpolating polynomials to estimate the derivative across field lines, interpolating values at points along neighboring lines as needed. The existence of Pedersen currents between 600 km to 1000 km (see Figure 3.4), along with Equation (3.12), implies the existence of primarily-eastward Hall currents in the same region. These Hall currents would generate additional perturbations to Saturn’s radial and meridional magnetic field, but we do not consider these perturbations in this study.

3.3.3 Conductivity Profile

Ionospheric conductivities depend on the ion and electron gyrofrequencies, electron-neutral, electron-ion and ion-neutral collision frequencies, the plasma density and the intrinsic magnetic field strength, as described by the generalized, three-fluid expressions given by Koskinen et al. (2014), which are simultaneously valid at low and high ionization. We use the ion mixing ratios of Kim et al. (2014) that are based on the neutral composition and mixing ratios of Vervack and Moses (2015). The atmospheric structure in these studies was derived from the Voyager 1 egress solar
occultation at the latitude of $27^\circ \text{S}$ and the models assume the thermal structure to
be independent of local time. We adapt the model to the Cassini era by using a full
atmosphere pressure-temperature profile based on Cassini/UVIS stellar occultations
(Koskinen et al., 2015) and Cassini/CIRS observations of the middle atmosphere
(Guerlet et al., 2009) to calculate ion and electron densities based on the mixing
ratios. We assume that the model mixing ratios as a function of pressure remain
unchanged. We calculate collision frequencies as described by Schunk and Nagy
(2009) and Koskinen et al. (2010), assuming that the electron temperature is equal to
the thermodynamic temperature. Our conductivity profiles are functions of pressure
only and neglect compositional variations in latitude and longitude.

The ion densities and conductivities are shown in Figure 3.2. Hydrocarbon ions
dominate at altitudes below the $10^{-4}$ mbar pressure level, while $\text{H}^+$ and $\text{H}_3^+$ dominate
above. At altitudes below the $10^{-2}$ mbar pressure level, the ion-neutral and electron-
nearl collision frequencies are higher than the ion and electron gyrofrequencies,
plasma effects are negligible, and $\sigma_\parallel \sim \sigma_P$. Between $10^{-2}$ mbar to $10^{-5}$ mbar, the
electron-neutral collision frequency is lower than the electron gyrofrequency, dynamo
action is possible, and $\sigma_\parallel > \sigma_H > \sigma_P$. At altitudes above the $10^{-5}$ mbar pressure
level, both the electron-neutral and ion-neutral collision frequencies are lower than
the electron and ion gyrofrequencies and $\sigma_\parallel \gg \sigma_P > \sigma_h$. The peak Pedersen conduc-
tivity is $3.58 \times 10^{-5} \text{ S m}^{-1}$ at $1.50 \times 10^{-5}$ mbar and the height-integrated Pedersen
conductivity is $10.7 \text{ S}$, which is consistent with several previous models of Saturn’s
ionosphere (e.g., Atreya et al., 1984; Cheng and Waite, 1988; Moore et al., 2004).
The peak Hall conductivity is $4.11 \times 10^{-5} \text{ S m}^{-1}$ at $2.29 \times 10^{-4}$ mbar and the height-
integrated Hall conductivity is $11.0 \text{ S}$.

3.3.4 Zonal Wind Profile

We have only weak constraints on the zonal winds in Saturn’s thermosphere at low
and middle latitudes and therefore consider several different wind profiles in order
to understand how electrodynamics is affected by the characteristics of the winds.
Our baseline wind model includes a strong, narrow equatorial jet superimposed on
Figure 3.2: Left: Number densities from Kim et al. (2014). Hydrocarbon ions dominate below $10^{-4}$ mbar, while $\mathrm{H}^+$ dominates above $10^{-6}$ mbar. Right: The parallel (dashed green), Pedersen (dashed/dotted orange), and Hall (dotted purple) components of the conductivity tensor.

a model zonal wind profile. We model the jet as a Gaussian centered at $\approx 1.0^\circ \mathrm{N}$ latitude, with a maximum speed $u_{\phi, \text{max}} \approx 343 \, \text{m s}^{-1}$ and latitudinal width (standard deviation) of $\approx 19.4^\circ$. This model jet is based on wind profiles at pressures of roughly $10^2$ mbar to $10^3$ mbar obtained by García-Melendo et al. (2010) via cloud-tracking, and using it in the thermosphere implies that the jet is constant with altitude. It leads to a wind shear of roughly $70 \, \text{m s}^{-1}$ between the latitudes of the magnetic field footprints considered by Khurana et al. (2018). Hubbard et al. (1997) and Guerlet et al. (2011) provide evidence that the equatorial jet extends to higher altitudes. In addition to the equatorial jet, we include a zonally-averaged, zonal wind profile from STIM (Müller-Wodarg et al., 2012), run to equilibrium during Saturn’s northern summer, which facilitates comparison with the in-situ observations made during the Cassini Grand Finale closest approaches. STIM is primarily forced by polar heating that arises from magnetosphere-ionosphere coupling. Figure 3.3 shows the resulting wind profile, obtained by adding the STIM winds to the assumed equatorial jet.
Figure 3.3: Zonal winds in our model. Magenta indicates eastward flow and green indicates westward flow. The dotted lines are pressure contours from the STIM model labeled in millibars. The solid black line at $2.8 \times 10^{-9}$ mbar is the upper boundary of the STIM model, above which we assume $u_\phi = 0$ and $\sigma_P = 0$. The dashed line at approximately $1.5 \times 10^{-5}$ mbar indicates the pressure level of peak Pedersen conductivity.

3.4 Results

The theory outlined in Section 3.3 establishes a relationship between the azimuthal magnetic field and wind patterns in the equatorial regions, as well as the electric field and current density. We first present the currents and azimuthal magnetic field associated with the baseline wind model described in Section 3.3.4, and then discuss results for several other assumed wind profiles. Finally, we present the resistive heating and ion drag results based on our models.

3.4.1 Current Density

The current density associated with our model is shown in Figure 3.4. Qualitatively, our model predicts an overall northward current across the equator in the ionosphere, consistent with Khurana et al. (2018), and two smaller, oppositely-directed current loops at midlatitudes in each hemisphere. In these smaller current loops, the current is directed equatorward below a low-current (light-colored) transition region at an
altitude of roughly 1100 km and poleward above it. On either side of the magnetic equator, Pedersen currents are of the order $10^{-8}$ A m$^{-2}$ while FACs are of the order $5 \times 10^{-8}$ A m$^{-2}$. The height-integrated current densities are shown in Figure 3.5. As must be the case for an overall northward current flow across the magnetic equator, the Pedersen current is zero and the anti-parallel current (i.e. $j_\beta$) is maximized at the magnetic equator. Our azimuth- and height-integrated current density is slightly smaller than but generally comparable to the $1.15 \text{ MA rad}^{-1}$ value of Khurana et al. (2018). Below the $10^{-5}$ mbar pressure level (located near 940 km), the Pedersen conductivity decreases with decreasing altitude, causing Pedersen currents to also decrease with decreasing altitude, approaching $j_\alpha \rightarrow 0$ at the base of our domain. We also note that Equation (3.12) predicts significant Hall currents that increase with latitude below roughly 1000 km and are eastward both in the southern hemisphere and north of 40°N.

3.4.2 Azimuthal Magnetic Field Perturbations

Figure 3.6 shows our calculated values of $B_\phi$. The dominant trend is that $B_\phi$ is positive (eastward) within roughly ±45° of the magnetic equator and negative (westward) poleward of that—at least until ±60° latitude, beyond which we do not consider in this study. At latitudes above roughly ±40° and at altitudes of roughly 900 km to 1500 km, the midlatitude current loops described in Section 3.4.1 drive enhanced magnetic perturbations that are eastward in the southern hemisphere and westward in the northern hemisphere. Because Pedersen currents decrease with decreasing altitude below the $10^{-5}$ mbar pressure level (located near 940 km) and because we assume $B_\phi = 0$ at the lower boundary, the azimuthal magnetic field perturbations also decrease with decreasing altitude and approach zero at the lower boundary. FACs produce the eastward magnetic field perturbations above the domain. The peak strength of these perturbations around the equator is about 15 nT at the altitudes sampled by the Cassini/MAG instrument i.e., comparable to the observations. This demonstrates in principle that ionospheric electrodynamics based on the wind profile described in Section 3.3.4 is capable of producing magnetic field
Figure 3.4: Plot of the current density in the meridional plane, looking westward. The solid and dotted black lines are the same domain boundary and pressure level indicators (respectively) as in Figure 3.3. The small, black arrows sketch magnitude and direction of the current density, while the background color indicates the magnitude $|\mathbf{j}| \equiv (j_r^2 + j_\theta^2)^{1/2}$. Above the upper boundary of our domain, $j_\alpha \to 0$ and we do not plot $j_\beta$ in this region. FACs above the domain generally are of the order $10^{-9}$ A m$^{-2}$ and are directed north to south.
Figure 3.5: Plot of the azimuth- and height-integrated current densities in the meridional plane, looking westward. We integrate each component in height from the lower boundary of our model to the upper boundary of our model (FACs above the domain are not included). The thick, dashed, red line indicates the $\alpha$-component of current density (Pedersen currents), the thick, dotted, blue line indicates the $\beta$-component of current density (negative FACs) and the dashed-dotted orange line indicates the $\phi$-component of current density (Hall currents). For ease of comparison, we include the $r$-component (solid green line) and the $\theta$-component (dotted purple line), as well as the total magnitude of the current density (thick, solid black line).
perturbations that are similar in magnitude and direction to those observed. The latitudinal extent of the perturbations in our model, on the other hand, exceeds the width of the observed magnetic field perturbations. Below, we modify the assumed wind profile based on different constraints to see if we can produce a better fit to the observations.

Figure 3.6: Azimuthal magnetic field perturbations $B_\phi$, looking westward. The solid, dashed and dotted black lines are the same domain boundary, Pedersen peak and pressure level indicators (respectively) as in Figure 3.3. Red indicates an eastward perturbation, while blue indicates a westward perturbation. We calculate $B_\phi$ above the $2.8 \times 10^{-9}$ mbar upper boundary level under the assumptions that $j_\alpha \to 0$, or equivalently, $\sigma_P \to 0$, above our domain, and $B_\phi = 0$ along the lower boundary.

3.4.3 Results With Different Wind Profiles

In order to highlight the relationship between winds and electrodynamics, we produce new models to explore the current systems produced by five different wind profiles at low and middle latitudes. Figure 3.7 shows the zonal wind speeds at three different altitudes in these models and Figure 3.8 shows the resulting magnetic field perturbations at the same altitudes. The first wind profile, which we refer
to as Model B (for “baseline”), is the wind profile described earlier in Section 3.3.4 where the equatorial jet is constant with altitude. In reality, the equatorial jet may not remain constant with altitude in the thermosphere – due to the effect of viscosity, it is more likely to decay with altitude. The second wind profile, which we call Model L (for “lower”), explores the effect of the equatorial jet decaying with altitude in the thermosphere. Both Model B and Model L share the same STIM wind profile and the latitudinal dependence of the imposed equatorial jet, but above 700 km, the equatorial jet in Model L decays with altitude as a one-sided, vertical Gaussian with a standard deviation of 300 km. We note that only Models B and L here include the STIM winds.

The third model, Model R (for “Read et al.”), is constant in altitude, but we interpolate the zonal wind in latitude from Figure 2a of Read et al. (2009), which is based on cloud-tracking data. Of the five wind profiles we present here, Model R is the most directly based on data (though extrapolated from the troposphere) and has the most latitudinal structure. Comparing the magnetic field perturbations associated with it to the magnetic field observations serves as a test of whether winds at the cloud level persist with similar magnitude to the thermosphere. The fourth model is Model K (for “Khurana et al.”), which we use to reproduce the assumptions of Khurana et al. (2018) with our formulation. The wind profile in this model is a Gaussian in latitude, constant in altitude, with a peak wind speed of 430 m s\(^{-1}\) at the equator and a half-width, half maximum of 20° (standard deviation of \(\approx 17°\)), as suggested by Khurana et al. (2018). This equatorial jet is therefore stronger and narrower than the imposed equatorial jets in Models B and L but comparable to the wind profile from Read et al. (2009).

The observed magnetic field perturbations, shown for example in Figure 1d in Khurana et al. (2018), show significant variability and structure. In particular, they often exhibit two maxima around the equator. In order to investigate if a wind system exists that could reproduce this structure, we created Model D (for “data”) to match the measured magnetic field perturbations from Rev 279 (see Figure 3.9). The wind profile in Model D is constant with altitude and composed
Figure 3.7: Plots of the zonal wind for the five models described in Section 3.4.3 at 1000 km, 1200 km and 1600 km. The solid black line indicates Model B, the dashed orange line indicates Model L, the dashed purple line indicates Model R, the dotted green line indicates Model K, and the solid blue line indicates Model D. We note that Models R, K and D do not vary with altitude, and the variation in altitude in Model B is due only to the STIM winds. The strongly westward winds at high altitudes and higher latitudes in Models B and L are due to the STIM wind profile.
Figure 3.8: Plots of the azimuthal magnetic field for the five models described in Section 3.4.3 and displayed at 1000 km, 1200 km and 1600 km. The solid black line indicates Model B, the dashed orange line indicates Model L, the dashed purple line indicates the Model R, the dotted green line indicates Model K, and the solid blue line indicates Model D. The azimuthal magnetic field perturbations in Models B and L are similar above roughly $\pm 30^\circ$ latitude due to being driven primarily by the local STIM winds, but are different at low latitudes due to the different height dependence of their equatorial jets. The large variations in Model R are due to large meridional gradients in zonal winds causing large gradients of $u_\phi$ along field lines. The azimuthal magnetic field perturbations in Models K and D approach zero at higher latitudes because the zonal winds in these models approach zero at higher latitudes, which causes the wind shear along field lines to also approach zero.
of six Gaussian jets in latitude. We varied the peak values, latitudinal widths and locations of the jets by hand in an attempt to fit the observations from Rev 279. While this approach is excessive and the fits are almost certainly degenerate, it shows in principle that complex magnetic field perturbations can be produced by the wind dynamo. This same conclusion is also illustrated by Model R, as we discuss below while also comparing the results from the other models.

Models B and L that include the STIM winds produce similar magnetic field perturbations at middle latitudes where the influence of their different equatorial jets is smallest. Both models have a current system in each hemisphere with similar orientation: equatorward below roughly 1100 km, but poleward above. In Model B, the lower, equatorward currents turn upward very gradually between roughly ±(20° − 40°), whereas in Model L, they turn sharply upward at the magnetic equator, creating the strong magnetic field perturbations of opposite sign within ±10° of the magnetic equator around 1000 km (see Figure 3.8). The intra-hemispheric current loops in Model B are much weaker, and the low-altitude azimuthal magnetic field perturbations associated with them are almost imperceptible in the figure. In both models, the mid-latitude loops represent the reaction of the ionosphere to the STIM winds. The similarity between the models at higher altitudes and their difference at lower altitudes highlights the fact that magnetic field measurements taken at high altitudes do not provide strong constraints on electrodynamics at lower altitudes.

The electrodynamics in Model L can be understood by considering a magnetic field line crossing the magnetic equator at 1500 km. The value of $E_\alpha$ is an average of $u_\phi B$ along the field line, weighted by the Pedersen conductivity and scale factors. Thus $E_\alpha$ is determined primarily by the value of $u_\phi B$ near the altitude where the Pedersen conductivity is maximized. Because the jet decays with altitude, points along the field line that are above this altitude have $E_\alpha > u_\phi B$, while the points below have $E_\alpha < u_\phi B$. Thus the CM electric field $E_\alpha - u_\phi B$ and the Pedersen current proportional to it are both positive (upward and/or poleward) above this transition level, but are negative (downward and/or equatorward) below it. The result is that,
whereas current at the magnetic equator flows primarily northward in Model B, it flows primarily upward in Model L. Because Model L has stronger Pedersen currents at low latitudes than Model B, it also produces significantly more resistive heating in this region (see Section 3.4.4). The difference between these models emphasizes the fact that the current systems depend sensitively on the altitude variation of the zonal wind.

The zonal wind profile in Model R is interesting because it has the most latitudinal structure of any of the wind profiles we considered. The latitudinal gradients of zonal wind have an even larger effect on the electrodynamics due to the dipole shift, as some field lines intersect the ionosphere in a region of westward winds in the southern hemisphere but eastward winds in the northern hemisphere. This causes the highly-variable, banded pattern in the azimuthal magnetic field perturbations in Figure 3.8. Like Model B, Model R generally predicts eastward magnetic field perturbations and a strong, northward current at low latitudes ($\lesssim \pm 30^\circ$), but unlike Model B, it contains a region of strong southward current within $5^\circ$ of the equator and below 1200 km. Thus parallel currents cross the equator flowing south below 1200 km and flow north across the equator above. This small equatorial current loop is associated with the westward magnetic perturbations at low latitudes. Unlike Models B and L, in which we noted two, distinct vertical regions of the thermosphere wherein the FACs and Pedersen currents flow differently, in this model the Pedersen currents are clustered between 900 km to 1100 km, where the Pedersen conductivity is greatest, and FACs are small and roughly constant along field lines at midlatitudes.

There are a few other magnetic features that are of particular interest for Model R. First, the magnetic field perturbations change significantly with both latitude and altitude near the magnetic equator. Second, the model predicts two large peaks in $B_\phi$ at roughly $\pm 10^\circ$ of the magnetic equator of roughly 30 nT to 35 nT. This is qualitatively similar to several of the Revs in the MAG data. In fact, the observed magnetic field perturbations have more in common with Model R than any other model based on tropospheric winds, and they also change significantly from one Rev
to the other. This means that the underlying wind system is likely to be more complex than a single jet and also variable with time (see Section 3.5 for more discussion on this point). Finally, it is interesting that in this model, more than any other, the hemispheres are tightly coupled. Parallel currents are stronger in this model than any other and are relatively constant along field lines at latitudes $\gtrsim \pm 35^\circ$. These bands connect to regions of $j_\alpha$ that are similar in magnitude but opposite in sign in each hemisphere, and they are capable of transporting angular momentum along field lines from one hemisphere to another via ion drag (see Section 3.4.4).

Models R and K at low latitudes ($\lesssim \pm 40^\circ$) have similar zonal wind profiles in terms of overall magnitude, direction and latitudinal width — the wind profile in Model K can almost be thought of as a single-Gaussian fit to the Model R wind profile — but these wind profiles produce different electrodynamics. The primary reason for this is that the meridional profile of zonal winds is much smoother and less structured in Model K than in Model R, so gradients in the zonal wind along the field line are also smaller and less structured. In Model K, the current flow is mostly northward throughout the domain, and the magnetic perturbations are eastward. This is because $E_\alpha > u_\phi B$ ($j_\alpha > 0$) north of the magnetic equator, and $E_\alpha < u_\phi B$ ($j_\alpha < 0$) south of it, as described in Section 3.5.1. Parallel currents are smaller in Model K than in the other models, except within $10^\circ$ of the magnetic equator and between 900 km to 1100 km, wherein they reach $10^{-7}$ A m$^{-2}$. Although the FACs in Model K are almost entirely anti-parallel to magnetic field lines, there are smaller currents parallel to magnetic field lines at high altitudes of the order $10^{-10}$ A m$^{-2}$.

When compared with Model B, Model K has a narrower equatorial jet; as a result, it produces narrower region of azimuthal magnetic field perturbations. This is broadly consistent with the magnetic field observations. Of all the models, the electrodynamics of Model K is most strongly concentrated at the altitudes where the Pedersen conductivity is greatest, and it is therefore perhaps most likely to agree with the single layer approximation (see Section 3.5).

The zonal wind profile of Model D is reminiscent of that of Model R at low latitudes ($\lesssim 20^\circ$) in that it has similar structure (e.g. both peak around 6°N lati-
tude), but the jet is roughly 200 m s\(^{-1}\) weaker than in Model R. Due to the similar structures of the zonal jet at low latitudes, these models produce magnetic field perturbations with similar structure at low latitudes. Because the zonal winds are weaker, both \(E_\alpha\) and \(u_\phi B\) have smaller magnitudes at low latitudes in Model D than in Model R. In spite of this, the difference between \(E_\alpha\) and \(u_\phi B\) is larger in Model D, causing the CM-electric fields and currents to be larger as well. This suggests that an equatorial jet does not necessarily need to be very strong in order to produce the observed magnetic field perturbations. Like Model R, Model D has a region of strong southward current within 5° of the equator and below 1200 km, which connects at midlatitudes with the northern current above it, and is associated with strong westward magnetic field perturbations at low latitudes. As in Model K, the magnetic field perturbations in Model D decay with latitude because the wind profile decays with latitude.

Figure 3.9 shows a comparison of the magnetic perturbations predicted by Model D and interpolated to Cassini’s Rev 279 trajectory with the measured values between ±60° latitude. The general form of the magnetic field perturbations at low latitudes (≤ 20°) is closely approximated by the model predictions. At midlatitudes, the measured \(B_\phi\) increases with latitude, likely due to the increasing proximity to auroral FAC systems (Dougherty et al., 2018). Because the zonal winds in Model D decay with latitude, the \(B_\phi\) predictions decay with latitude as well, so the predictions of our model diverge from the measurements at midlatitudes. As we state above, this exercise of trying to fit the data exactly goes beyond good sense because there is insufficient information to constrain such a complex wind profile and our results cannot be unique, as we discuss in Section 3.5.3. Moreover, we do not have precise knowledge of ionospheric conductivities and their variations. Nevertheless, the exercise does show that it is physically possible to explain the small scale structure of the observed azimuthal magnetic field perturbations with ionospheric electrodynamics and that if more data were available it might be possible to constrain thermospheric winds with magnetic field measurements.
3.4.4 Resistive Heating and Ion Drag

In addition to generating magnetic field perturbations, currents generate resistive heating $q$ in the frame of the neutral wind, given by (e.g., Vasyliunas and Song, 2005)

$$ q = \vec{j} \cdot \left( \vec{E} + \vec{u} \times \vec{B} \right) $$

$$ = \sigma_P \left( E_\alpha - u_\phi B \right)^2. $$

We note that the contribution of parallel currents to the heating rate is zero due to our assumption that field lines are equipotentials, and that the contribution from Hall currents is zero because Hall currents are perpendicular to the CM electric field. Figure 3.10 shows the resistive heating generated by the currents in our Model B as calculated via Equation (3.31).

Our model predicts height-integrated resistive heating rates that increase from a sharp and narrow low of $10^{-8}$ W m$^{-2}$ at the magnetic equator to several $10^{-6}$ W m$^{-2}$ at low latitudes, to several $10^{-4}$ W m$^{-2}$ at midlatitudes. The relative lack of heating at the equator is because the Pedersen currents are small at the magnetic equator,
Figure 3.10: Plot of the resistive heating generated by our model. The solid, dashed and dotted black lines are the same domain boundary, Pedersen peak and pressure level indicators (respectively) as in Figure 3.3. Darker (lighter) colors denote regions with less (more) resistive heating. The region above the domain is black because we do not calculate heating above our domain. The black regions within the domain are regions where the polarization electric field is nearly balanced with the $\vec{u} \times \vec{B}$ field (i.e. $E_\alpha \approx u_0 B$).
which in turn is because the polarization electric field is roughly equal to the opposing \( \vec{u} \times \vec{B} \) field. The resistive heating rate increasing with latitude away from the equator appears qualitatively consistent with the temperature profiles retrieved from Cassini/UVIS occultation data, which show that the temperature also increases with latitude from low to middle latitudes (Koskinen et al., 2015). Models of energy balance and dynamics, however, are required to calculate the actual temperature structure based on resistive heating before a more solid comparison with the occultation data is possible.

At midlatitudes, there are two distinct layers of heating, each separated by a thin boundary of zero heating that separates the roughly poleward Pedersen currents \((j_\alpha > 0 \text{ or } E_\alpha > u_\phi B)\) above or south of the boundary, from the roughly equatorward Pedersen currents \((j_\alpha > 0 \text{ or } E_\alpha < u_\phi B)\) below or north of the boundary. Although the heating is comparable above and below these boundaries between roughly ±(40° to 60°) latitude, heating in the higher altitude regions has a much stronger effect on the temperature profile. The thermosphere is a poor radiator and energy deposited there is transported by thermal conduction to the mesopause region at the base of the thermosphere. The downward flow of energy by thermal conduction implies a positive temperature gradient with altitude; thus, higher altitude heating results in higher temperatures.

The height-integrated heating rates for the five models described in Section 3.4.3 are shown in Figure 3.11. Model B predicts a total resistive heating rate of 0.052 TW between ±20° latitude i.e., four times lower than the value of 0.22 TW given by Khurana et al. (2018). This is the lowest resistive heating rate among the five models. Model L has the highest height-integrated heating rate at low latitudes because it has strong, upward Pedersen currents near the equator, and it predicts a total heating rate of 0.26 TW. The total heating rate based on Model R is 0.15 TW. Model K, designed to reproduce the Khurana et al. (2018) model, produces a heating rate of 0.09 TW. This indicates that the single layer approach may overestimate the heating rate. The data-based Model D predicts a relatively low heating rate of 0.072 TW. Calculated between ±60° latitude, on the other hand, the total heat-
ing rate in Model B is 2.6 TW and the latitudinally-averaged height-integrated heating is \(0.64 \times 10^{-4} \text{ W m}^{-2}\). This is comparable to the effective heat fluxes of \(0.54 \times 10^{-4} \text{ W m}^{-2}\) to \(1.2 \times 10^{-4} \text{ W m}^{-2}\) that Koskinen et al. (2015) derived by analyzing the temperature profiles retrieved from the occultation data. The heating rate in this case, however, is based on the mid-latitude winds from the STIM model that are driven by auroral heating. This means that, given the current observational constraints, the wind dynamo responsible for the equatorial magnetic field perturbations is not strong enough to directly heat Saturn’s thermosphere.

![Figure 3.11: Comparison of the height-integrated heating curves for the five models described in Section 3.4.3. The solid black line indicates Model B, the dashed orange line indicates Model L, the dashed purple line indicates Model R, the dotted green line indicates Model K, and the solid blue line indicates Model D. Most models have low heating rates near the equator because Pedersen currents are small. The heating rates of Models K and D decrease rapidly with latitude because the zonal winds in these models decay rapidly with latitude.](image)

In addition to generating resistive heating, ion-neutral collisions lead to momentum transfer between the ionosphere and neutral winds. This is commonly referred to as “ion drag”, although its direction is not always opposite to that of the wind
as the word “drag” might suggest. We calculate it in our model as follows:

\[ \vec{f}_d = \vec{j} \times \vec{B} \]  
\[ = [-j_\phi B] \hat{\alpha} + [j_\alpha B] \hat{\phi} \]  
\[ = [\sigma_H (E_\alpha - u_\phi B) B] \hat{\alpha} + [\sigma_P (E_\alpha - u_\phi B) B] \hat{\phi}. \]  

We present the azimuthal component of ion drag from Model B in Figure 3.12. Importantly, its direction is eastward at higher altitudes, opposite to the westward Coriolis force at middle to high latitudes (Smith et al., 2007; Müller-Wodarg et al., 2012). The ion drag timescale implied by our model is roughly 20 times shorter than the advection timescale at midlatitudes above 1000 km, and the magnitude of the ion drag in most places throughout the domain is several times larger than the zonal component of the Coriolis force in STIM. The midlatitude ion drag due to the Hall current is equatorward below 1000 km and has roughly the same magnitude and latitude dependence as the westward ion drag has around 1000 km. All of these considerations suggest that ion drag has the potential to alter the dynamics and redistribution of energy in the thermosphere.

3.5 Discussion

Our model shows that, consistent with Khurana et al. (2018), the azimuthal magnetic field perturbations measured during the Cassini proximal orbits can be produced by electrodynamics in Saturn’s equatorial ionosphere. In this section, we discuss more fully the implications of our results and their relationship to those of Khurana et al. (2018), using a single-layer simplification of our theory. Finally, we discuss the implications of the measured azimuthal magnetic field perturbations on the nature of the circulation and energy balance in Saturn’s thermosphere.

3.5.1 The Thin Layer Approximation

Khurana et al. (2018) estimated the ionospheric current strength using an approximation developed for magnetospheric coupling (Hill, 1979; Huang and Hill, 1989).
Figure 3.12: Plot of the zonal component of ion drag based on Model B (see text). The solid, dashed and dotted black lines are the same domain boundary, Pedersen peak and pressure level indicators (respectively) as in Figure 3.3. Red indicates an eastward force and blue indicates a westward force.
This approximation treats the ionosphere as a single layer, does not explicitly distinguish the polarization and convection electric fields, and utilizes only the radial component of the magnetic field. Here, we present a more rigorous single-layer approximation appropriate for equatorial electrodynamics, and show that the implied currents are within a factor of several of that implied by the expression used by Khurana et al. (2018). In order to formulate our single layer model, we assume that the ionosphere can be treated as an infinitely thin layer of non-zero Pedersen conductivity and consider the two points (one in each hemisphere) where a given magnetic field line (for example, the inner field line in Figure 3.1) intersects this layer, rather than considering all points along the field line. By reducing the ionosphere to a single, infinitely thin layer, we ignore the height-dependencies of the conductivity tensor. At the southern point, the Pedersen conductivity is $\sigma_S^P$, the zonal wind is $u_S^\phi$ and the magnetic field is $\vec{B}_S^S = B^S_r \hat{r}_s + B^S_\theta \hat{\theta}_s$. At the northern point, the Pedersen conductivity is $\sigma_N^P$, the zonal wind is $u_N^\phi$ and the magnetic field is $\vec{B}_N^N = B^N_r \hat{r}_s + B^N_\theta \hat{\theta}_s$. We assume that the shifted dipole field causes the northern footprint to be in a region of smaller zonal wind and the southern point to be in a region of larger zonal wind, so that $u_S^\phi > u_N^\phi$. The polarization electric field $E_\alpha$ will be communicated along the field line such that both points are assumed to share the same value of $h_\alpha E_\alpha$. The center-of-mass (CM) electric field

$$\vec{E}_{CM} \equiv \vec{E} + \vec{u} \times \vec{B},$$

however, differs between the two points due to the different values of $u_\phi$ and $B$.

In this two-point idealization, the polarization electric field calculated from Equation (3.23) for Saturn ($B_\beta \to -B$), is

$$h_\alpha E_\alpha = \frac{\sigma_P^S u_\phi^S B^S h_\phi^\theta h_\beta^\theta + \sigma_P^N u_\phi^N B^N h_\phi^\theta h_\beta^\theta}{\frac{\sigma_P^S h_\phi^\theta h_\beta^\theta}{h_\alpha^S} + \frac{\sigma_P^N h_\phi^\theta h_\beta^\theta}{h_\alpha^N}}. \quad (3.36)$$

Ignoring the geometric scale factors, which each vary by less than a factor of 1.5 along field lines in our domain, the value of the polarization electric field $E_\alpha$ is the average value of the $\alpha$-component of $-\vec{u} \times \vec{B} = u_\phi B$ weighted by the Pedersen
conductivity, as in Equation (3.23):

\[
E_\alpha \approx \frac{\sigma_P^S u^S_\phi B^S + \sigma_P^N u^N_\phi B^N}{\sigma_P^S + \sigma_P^N},
\]

which for eastward zonal winds is directed outward and perpendicular to field lines; roughly north in the northern hemisphere and south in the southern hemisphere. For ease of illustration, we here neglect the dipole shift for calculating the strength of the magnetic field at each point so that we can approximate \(B^S \approx B^N\) (i.e., \(B^S_\theta \approx B^N_\theta\) and \(B^S_r = -B^N_r\)) at the two points. If we further assume that the two points are at the same radius, then these assumptions justify neglecting the geometric scale factors in Equation (3.36). We also assume that the Pedersen conductivity is equal at both points. Then the polarization electric field is given by

\[
E_\alpha \approx \frac{(u^S_\phi + u^N_\phi) B}{2}. \tag{3.38}
\]

The CM electric field in the south is therefore

\[
E^{CM,S}_\alpha = \frac{(u^S_\phi + u^N_\phi) B}{2} - u^S_\phi B = \frac{-(u^S_\phi - u^N_\phi) B}{2}, \tag{3.39}
\]

and the CM electric field in the north is

\[
E^{CM,N}_\alpha = \frac{(u^S_\phi + u^N_\phi) B}{2} - u^N_\phi B = \frac{+(u^S_\phi - u^N_\phi) B}{2}. \tag{3.40}
\]

We can combine Equations (3.39) and (3.40) as

\[
E^{CM}_\alpha = \pm \left(\frac{u^S_\phi + u^N_\phi}{2}\right) B - u_\phi B = \pm \left(\frac{u^S_\phi - u^N_\phi}{2}\right) B, \tag{3.41}
\]

where the upper (lower) signs apply in the northern (southern) hemisphere.

This simple illustration demonstrates how a northward current in the ionosphere can be produced by the electrodynamics formulation in our model. Here, we see that \(E^{CM,N}_\alpha = -E^{CM,S}_\alpha\), meaning that the meridional component of the CM electric field points in the same direction in both hemispheres: either north, if \(u^S_\phi > u^N_\phi\), as we have assumed, or south if \(u^N_\phi > u^S_\phi\). Of course, the direction of the associated Pedersen currents in the ionosphere will follow the direction of the electric field in the
CM frame (c.f. Equation (3.10)). Because of this, if the ionospheric Pedersen current is primarily northward (southward), then the field line will be associated with an eastward (westward) magnetic field perturbation above the ionosphere, assuming that the currents along the field line act as return currents for the ionospheric current.

We can compare our simplified expression for the electric field above to Equation (1) given by Khurana et al. (2018), i.e.

\[ E = \pm \left( u^S_\phi - u^N_\phi \right) B_S \cos \theta \]

\[ = \pm \left( u^S_\phi - u^N_\phi \right) B_r \frac{2}{2}, \]

where \( B_r \) is the radial component of the magnetic field and \( E \) is the northward component of the CM electric field. This means that our form of the thin layer electric field is similar to that of Khurana et al. (2018) with the exceptions that our expression gives the electric field in the direction perpendicular to the magnetic field instead of the meridional direction and uses the absolute magnitude of the magnetic field instead of the radial component only. The meridional component of Equation (3.41) reduces to Equation (3.43) at high latitudes where magnetic field lines are nearly vertical. Within \( \pm 20^\circ \) of the magnetic equator, however, the radial component constitutes at most 59% of the magnetic field strength.

This demonstration, however, supports the validity of the Khurana et al. (2018) approach under the simplifying assumptions adopted in that work, with the exception of the use of \( B_r \). In addition, our Model K in Section 3.4.3, designed to reproduce the meridional wind shear in Khurana et al. (2018), produces height-integrated currents comparable to their value, in our case with conductivities based on the Kim et al. (2014) ionosphere model. Our total heating rate around the equator, on the other hand, is lower by a factor of about two, most likely because the single layer approximation overestimates the heating rate. To conclude, Model K agrees with the results of Khurana et al. (2018) in all important respects to a factor of about two. Their approach, however, lacks the ability to calculate the current system and therefore, it cannot be used to study magnetic field perturbations based on more
complex wind profiles in the presence of height-dependent conductivities that cause the dynamo equations to significantly deviate from the single layer approximation, a situation likely to hold in Saturn’s upper atmosphere. The contribution of Khurana et al. (2018) was to introduce a mechanism which could explain the MAG observations of $B_{\phi}$ at low-latitudes, rather than to present a detailed mathematical model, as we have done.

3.5.2 Variations in Conductivity

Just as our results show that the electrodynamics in Saturn’s upper atmosphere is sensitive to height and latitudinal variations of the zonal wind profile, it is also sensitive to variations in conductivity. Different conductivity profiles allow different wind profiles to reproduce the observed magnetic field profiles. Our 1D conductivity profile, based on low-latitude chemical models, ignores the variation of conductivity with latitude and local time (Moore et al., 2010; Sakai and Watanabe, 2016; Wahlund et al., 2018). We leave a detailed study of such variability for future work, but discuss some implications below.

First, we consider the consequences of multiplying the conductivity profile in our forward model by a uniform scalar. By Equation (3.23), multiplying the conductivities by a given factor does not change the polarization electric field; however, the increased (decreased) conductivities will increase (decrease) the currents, magnetic field perturbations, resistive heating and ion drag by the same factor. For example, if we increase our conductivities by a factor of 4, then our peak magnetic field perturbations will increase from 15 nT to 60 nT. Instead of multiplying the conductivity profile by a factor of four, we could also achieve the same result by multiplying the zonal wind profile by 4. In this case, the polarization and CM-frame electric fields, currents, magnetic field perturbations and ion drag, would be multiplied by the same factor. The resistive heating, on the other hand, would increase by the square of that factor because resistive heating is proportional to $(E_{CM}^C)^2$.

Second, we consider the effects of a conductivity profile that varies in latitude. The results presented in this paper are based on our 1D conductivity profile, which
was calculated from low-latitude chemical profiles and assumed to be constant in latitude for each pressure level. The zonally-averaged (2D) Pedersen and Hall conductivities in STIM, however, decrease from low to middle latitudes along constant pressure levels. If the Pedersen and Hall conductivities in Saturn’s upper atmosphere do decrease with latitude, then using a low-latitude conductivity profile at all latitudes, as we have done in this study, will over-estimate the conductivities at midlatitudes. This means the estimated currents, resistive heating and ion drag at midlatitudes will also be over-estimated.

3.5.3 Magnetic field perturbations and winds in the thermosphere

The observed magnetic field perturbations show significant structure and variability that cannot be captured by simple models. One persistent feature in the observations appears to be the double peaked structure around the magnetic equator (see Figure 3.9). Interestingly, this type of feature is produced by electrodynamics based on the tropospheric wind profile from Read et al. (2009). As we noted before, stellar occultations and CIRS limb scans of the stratosphere provide some evidence that eastward zonal winds penetrate to the upper atmosphere (Hubbard et al., 1997; Guerlet et al., 2011) and we argue that this is indeed required to explain the MAG data.

Beyond that, the observed magnetic field perturbations, even in the absence of variability, place only crude constraints on the nature of the winds and their speed. For example, our Model R that is based on the Read et al. (2009) wind profile (see Section 3.4.3) has a peak wind speed of about 400 m s$^{-1}$ but Model D that matches the data best has a peak wind speed of only about 200 m s$^{-1}$. The common feature between these two models is that the zonal wind is stronger on the northern side of the equator but it is easy to imagine that some other wind profile could also be invoked to match the data almost as well. In addition, the meridional wind shear invoked by Khurana et al. (2018) is not the only factor affecting the wind dynamo. Our Model L shows that vertical variations of the winds in the thermosphere play an important role in controlling the current systems and magnetic field perturbations.
at different altitudes. Based on these considerations, all that can be said of the equatorial zonal winds in the thermosphere is that they are eastward and stronger than predicted by typical circulation models. Therefore, they are probably driven by a coupling of the thermosphere to the upper stratosphere where wave breaking is believed to maintain the winds associated with the equatorial oscillation (e.g., Guerlet et al., 2011).

This brings us to the issue of variability in the MAG data, which is discussed in detail by Provan et al. (2019). It is clear that the observed magnetic field perturbations change significantly from one Rev to the other [see Figure 1d in Khurana et al. (2018)], but Provan et al. (2019) conclude that these variations are likely not related with pass altitude, local time, planetary period oscillation phase, or D68 ringlet phase. Results by our model show that different wind profiles can be invoked in each case to match the data. This exercise, however, does not feel reasonable, particularly because changes to the winds are not the only possible source of variability in the thermosphere. Waves in the neutral atmosphere, for example, lead to variability and can also alter the winds while being consistent with the proposed driving mechanism of the equatorial oscillation. Variability in the ionosphere is another obvious possibility. Provan et al. (2019) suggest that the observed variations may be related to variability in zonal winds and/or conductivity in the ionosphere, or perhaps dynamic interactions between the D-ring and the ionosphere.

Electron densities in Saturn’s ionosphere show substantial variability and the presence of high amplitude plasma waves is also well established (Kliore et al., 2009; Matcheva and Barrow, 2012). Combined with the possible variability in the ion and electron temperatures, significant variability in conductivities that can affect the magnetic field perturbations is possible. The electron densities, which increase with latitude from the equator, may also change with local time in ways not accounted for by our model. In addition, influx of plasma and possibly dust from the rings, as well as ring shadowing effects, lead to significant changes in the low to middle latitude ionosphere (O’Donoghue et al., 2017; Waite et al., 2018; Wahlund et al., 2018). Recent estimates of the mass flux from the rings, high enough to deplete the rings
over a relatively short period of time, indicate that the influx from the rings may not be continuous and instead changes with time in an episodic manner. This influx may also depend substantially on both latitude and longitude, leading to variability in ionospheric conductivities. Future studies should address the consequences of these possible sources of variability on the equatorial wind dynamo in order to properly address the question of variability in the MAG observations.

3.5.4 Energy balance and ion drag

The detection of azimuthal magnetic field perturbations raised the possibility that equatorial electrodynamics could help to solve the energy crisis in Saturn’s thermosphere. The results of Khurana et al. (2018), however, showed that the total resistive heating rate within ±20° of the equator based on the MAG observations is only about 0.22 TW i.e., comparable to the solar EUV heating rate and therefore far from sufficient to resolve the energy crisis. The corresponding total heating rates based on our models with different assumed wind profiles range from about 0.05 TW to 0.26 TW and therefore, our conclusion agrees with that of Khurana et al. (2018). The wind dynamo required for producing the observed magnetic field perturbations is not strong enough to explain the temperatures in Saturn’s thermosphere.

If we include the westward mid-latitude winds by extending the heating rate calculation to ±60°, our models produce a total resistive heating rate of about 2.6 TW that, together with auroral heating at high latitudes, is sufficient to resolve the energy crisis. This result, however, relies on the winds predicted by the STIM model that are themselves driven by auroral heating. The actual heating rate due to the mid-latitude wind dynamo is likely to be lower and self-consistent calculations including ion drag are required to calculate it. This is an important point with potentially significant consequences and an interesting direction for future research.

The mid-latitude winds in STIM are driven by a combination of auroral heating that creates a steep meridional temperature gradient, the Coriolis force and auroral ion drag. The whole system depends on the high-latitude auroral electric field that is communicated along field lines from the magnetosphere (Müller-Wodarg et al.,
In analogy with Lenz’s law, dynamo currents are expected to set up to oppose the action of the auroral electric field and that is indeed what we find. As shown by Figure 3.12, ion drag due to the wind dynamo opposes westward winds at mid-latitudes. In a self-consistent simulation, therefore, it is possible that ion drag could reduce zonal winds. This effect, together with resistive heating that reduces the meridional temperature gradient, could help to break the Coriolis barrier and allow the redistribution of auroral energy to lower latitudes. While this is possible in theory, the strength of this mechanism depends heavily on the conductivities and winds in Saturn’s upper atmosphere. Both resistive heating and ion drag are reduced by weaker winds and weaker conductivities, especially at midlatitudes. Also, if ion drag acts to remove the wind shear along field lines, then the ion drag predicted by our model must be balanced by a net force in order to maintain the assumed wind profile in steady state. If the ion drag is not balanced, then ion drag would, in a dynamic model, act to weaken the wind profile and cause it to have progressively weaker electrodynamics until equilibrium can be established.

3.6 Summary

We investigated the electrodynamics of Saturn’s thermosphere at low and middle latitudes using an axisymmetric, steady state model adapted from terrestrial studies. Assuming a dipole magnetic field, we used a one-dimensional conductivity profile and a two-dimensional model wind profile to calculate the polarization electric field, current density, azimuthal magnetic field perturbations, resistive heating and ion drag. Our model predicts northward currents in the ionosphere of the order $5 \times 10^{-8}$ A m$^{-2}$, which are associated with eastward azimuthal magnetic field perturbations of the order 10 nT to 20 nT within 30° latitude of the equator. These results are consistent with the magnetic field perturbations observed by the Cassini/MAG instrument around the equator during the Cassini Grand Finale tour (Dougherty et al., 2018; Khurana et al., 2018). Our results agree to within a factor of two with previous calculations by Khurana et al. (2018), who used a single layer description
to interpret the MAG observations. By simplifying the dynamo theory, we have also explicitly identified the conditions in which a single layer description is acceptable.

Our model is capable of exploring the relationship between zonal winds and the magnetic field perturbations that they create in detail. We created several different models based on wind profiles adopted from observations of the troposphere and models of the thermosphere. We found the magnetic field perturbations to be sensitive to both meridional and vertical structure in the assumed zonal wind profile and that the MAG observations do not contain enough information to constrain the details of the underlying wind structure. The observations do imply, however, that faster than expected eastward zonal winds persist at least in the lower thermosphere. Uniquely, our model can match the latitudinal structure in the observed magnetic field perturbations but the source of the variability that causes this structure to change from one spacecraft orbit to the other is not understood at present.

Our results indicate that thermospheric electrodynamics can generate resistive heating and ion drag in the thermosphere, both of which could also enable equatorward transport of auroral energy. However, resistive heating at low latitudes appears to be insufficient to directly heat Saturn’s thermosphere. The importance of resistive heating to the energy budget and ion drag to the momentum balance suggest that electrodynamics should be modeled self-consistently with the wind dynamo in future studies.

3.7 Acknowledgments

We thank Ingo Müller-Wodarg for the STIM wind profile, and Hao Cao and Michelle Dougherty for the Cassini MAG data, and all three for helpful discussion. We thank Jane Fox for sending us the ion densities from the Kim et al. (2014) model. Jess Vriesema was supported by the NASA Earth and Space Sciences Fellowship Program (Grant NNX16AP55H). Tommi Koskinen and Roger Yelle acknowledge support from the Cassini Data Analysis Program (Grant NNX15AN20G).
CHAPTER 4

Modelling Electrodynamics using STIM

4.1 Introduction

The modelling results of Vriesema et al. (2020) imply that electrodynamics at low and middle latitudes in Saturn’s thermosphere could be significant. Although their model demonstrated that the observed azimuthal magnetic field perturbations (e.g. Provan et al., 2019) could in theory be produced by low-latitude electrodynamics, their axisymmetric and steady-state model had several weaknesses. First, their assumed conductivity was a function of pressure only and was based on a low-latitude chemical model, and therefore ignored the variation of conductivity with latitude and local time (Moore et al., 2010; Sakai and Watanabe, 2016; Wahlund et al., 2018). Second, their model predicts significant ion drag that would act to reduce zonal wind shears along magnetic field lines over time. Because of this, the electrodynamics in their model would decay unless the wind profile were maintained by external processes (e.g. wave transport of angular momentum), which were not included in their model. Finally, and more broadly, they concluded that ion drag and resistive heating have the potential to alter the circulation and energy balance in Saturn’s thermosphere, and this would need to be calculated self-consistently in future general circulation models.

The recent study by Müller-Wodarg et al. (2019) found that Rayleigh drag, applied with sufficient strength at midlatitudes, was able to oppose the Coriolis force and allow equatorward transport of heat from high latitudes using a version of the Saturn Thermosphere-Ionosphere Model (STIM; see Section 4.2 for more info) which had been modified to model Rayleigh drag. In doing so, they were able to match the temperature observations much better than previous studies, especially at low latitudes. While Müller-Wodarg et al. (2019) suggest that Rayleigh drag may
be caused by gravity waves, it could also be attributed to other mechanisms, such as ion drag, small-scale turbulence, or a combination of several factors.

The purpose of this chapter is to improve how STIM calculates electrodynamics and examine how the presence of an imposed wind profile affects energetics and dynamics through self-consistent modelling. To do so, we performed a series of modifications to STIM and examined how each modification affected Saturn’s thermosphere. These modifications include improving STIM’s calculation of electrical conductivity and currents, and changing the magnetic field STIM uses. Finally, if low-latitude MAG observations (e.g. Dougherty et al., 2018; Khurana et al., 2018; Provan et al., 2019) are to be explained by a wind-driven dynamo in Saturn’s ionosphere, they imply the presence of an eastward equatorial jet. We therefore also impose such a jet at the lower boundary, assuming it propagates upward from the troposphere to the lower thermosphere.

In the sections that follow, we describe aspects of the electrodynamics in detail and use a sequence of models to study how each update affects Saturn’s thermosphere. In the four models that we will describe (the previous STIM model and Models 1–3), we run STIM to 500 rotations and check that the models have effectively reached a steady state. (Typically, STIM runs have nearly reached steady-state by 200 rotations, so running 500 rotations is mostly a safeguard.) All models presented in this chapter were run at low resolution, without ring shadowing, without Rayleigh drag, at low solar activity, and at northern summer solstice. We describe our formulation and models in Section 4.2. Next, we present our results in Section 4.3. Finally, we discuss the implications of our models in Section 4.4. We will investigate magnetosphere-ionosphere coupling at high latitudes and the effect of the polarization electric field on wind-driven electrodynamics in Chapter 5.

4.2 Updates to STIM

As discussed in Chapter 2, STIM is a general circulation model that numerically solves the coupled, nonlinear, three-dimensional Navier-Stokes equations of momen-
tum, energy and continuity for the major species in Saturn’s upper atmosphere (e.g. Müller-Wodarg et al., 2006; Moore et al., 2010; Müller-Wodarg et al., 2012, 2019).

STIM includes a suite of coupled 1-D, 2-D and 3-D models developed for Saturn, and it includes processes such as ion and neutral species transport, photochemistry, plasma diffusion, ring shadowing (disabled in the present study), and more. In Section 2.3, we summarized the coupled, nonlinear, three-dimensional Navier-Stokes equations of momentum, energy and continuity that it solves. Previous versions of STIM used an electric field at high latitudes based on models of the magnetosphere and winds to calculate currents, which were used to calculate ion drag and Joule heating in the momentum and energy equations. These additions were intended to be used at high latitudes to study magnetospheric coupling, and additional updates are required to model electrodynamics at low and middle latitudes.

In this section, we focus on describing the specific updates we made to STIM in order to better model electrodynamics globally. Previous versions of STIM used a the magnetospheric electric field (MEF) to model coupling to the magnetosphere, and we describe it in more detail in Section 4.2.1. We describe our updates to the calculation of the Pedersen and Hall conductivities in Section 4.2.2. We also use a more general formulation of Ohm’s law to calculate the current densities and describe this update in Section 4.2.3. We describe the magnetic field used by STIM in Section 4.2.4. Finally, STIM has the ability to impose a zonal jet along the lower boundary, a feature we describe in Section 4.2.5. We discuss updates to STIM’s treatment of the high-latitude coupling between the magnetosphere and ionosphere in Chapter 5.

To determine how each of these updates affect electrodynamics, we compare a sequence of four models. Each of these models is based on the model before, using all the same inputs and parameters, save for the noted update. As an experimental control, we begin with a previous version of STIM, run at northern solstice, that uses the same electrodynamics formulation as in Müller-Wodarg et al. (2019). Next, we explore the effects of the newly-implemented, generalized conductivity and current density calculations in Model 1. We use Model 2 to determine how using a shifted
dipole magnetic field affects electrodynamics. Finally, we introduce the imposed zonal jet in Model 3. These models are summarized in Table 4.1.

<table>
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<tr>
<th>Model Name</th>
<th>Season</th>
<th>Conductivity</th>
<th>Current Density</th>
<th>Magnetic Field</th>
<th>Imposed Wind</th>
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<td>BMRG</td>
<td>Layer</td>
<td>SPV</td>
<td>None</td>
</tr>
<tr>
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<td>N. Solstice</td>
<td>SN</td>
<td>General</td>
<td>SPV</td>
<td>None</td>
</tr>
<tr>
<td>Model 2</td>
<td>N. Solstice</td>
<td>SN</td>
<td>General</td>
<td>Shifted Dipole</td>
<td>None</td>
</tr>
<tr>
<td>Model 3</td>
<td>N. Solstice</td>
<td>SN</td>
<td>General</td>
<td>Shifted Dipole</td>
<td>FM</td>
</tr>
</tbody>
</table>

Table 4.1: This table provides a convenient way of comparing the models used in this study. In the Season column, N. Solstice indicates northern solstice. Under the “Conductivity” column, BMRG indicates that the Pedersen and Hall conductivities were calculated according to the outdated expressions given in Baker and Martyn (1953) and Rishbeth and Garriott (1969), while SN indicates that the more general expressions in Schunk and Nagy (2009) were used. Under the “Current Density” heading, Layer indicates the current density was calculated via the approximated layer conductivity formulation (Baker and Martyn, 1953; Fuller-Rowell, 1981), while General indicates that the complete, general formulation was used. In the “Magnetic Field” column, SPV indicates that the model’s planetary magnetic field was the Saturn Pioneer Voyager magnetic field model introduced by Davis and Smith (1990), while Shifted Dipole indicates that the shifted dipole magnetic field was used instead. In the “Imposed Wind” column, FM indicates that a wind profile based on Friedson and Moses (2012) was imposed at the lower boundary.

4.2.1 Magnetospheric Electric Field

MEFs are generated in the magnetosphere by complex interactions with the ionosphere, solar wind and magnetospheric plasma, and they are communicated along magnetic field lines between the ionosphere and magnetosphere. In the high latitude ionosphere, these fields drive strong electric currents which interact with the neutral atmosphere via ion drag and Joule heating. Recently, a model MEF was added to STIM at high latitudes in order to approximate atmosphere-magnetosphere coupling (Müller-Wodarg et al., 2019). The MEF used by STIM, of the form

\[
\vec{E}^m(\theta, \phi) = E^m_\theta(\theta, \phi) \hat{\theta} + E^m_\phi(\theta, \phi) \hat{\phi},
\]

is based on magnetosphere simulations using the BATSRUS model (Jia et al., 2012a). The BATSRUS model, focusing more on the magnetosphere, treats the ionosphere...
simply as a thin layer at an altitude of 1000 km above the 1 bar level, with uniform Pedersen conductance (height-integrated conductivity) of 0.5 S and zero Hall conductance. It also uses a centered (unshifted) dipole magnetic field. In their model, FACs are calculated at 4 $R_S$ and are mapped to the ionosphere along dipole magnetic field lines, under the assumption of ideal magnetohydrodynamics. These FACs close via horizontal currents in the ionosphere to satisfy current continuity. Their model uses a Poisson solver to calculate the electrostatic potential poleward of roughly $\pm 55^\circ$ latitude, from which the electric field is calculated. We interpolate values from the BATSRUS MEF onto the rotating STIM grid such that the MEF is constant. We apply this field only at latitudes poleward of $\pm 56^\circ$ and assume it to be zero at lower latitudes. Finally, STIM scales the field by a constant scalar value of $\epsilon = 4.0$ in order to better reproduce the observed high-latitude temperatures.

4.2.2 Calculation of Electrical Conductivity

Previously, STIM calculated the Pedersen and Hall conductivities as follows:

$$\sigma_P = \sum_{i}^{\text{ions}} n_i e \frac{k_i}{B} \frac{1}{1 + k_i^2}$$  \hspace{1cm} (4.2)

$$\sigma_H = \sum_{i}^{\text{ions}} n_i e \frac{k_i^2}{B} \frac{1}{1 + k_i^2}$$  \hspace{1cm} (4.3)

based on Baker and Martyn (1953); Rishbeth and Garriott (1969), where $e$ is the elementary charge, $B$ is the magnitude of the magnetic field, $n_i$ is the number density of ion species $i$, and

$$k_i = \frac{m_i \nu_i}{eB} = \frac{\nu_i}{\omega_i}$$  \hspace{1cm} (4.4)

where $m_i$ is the mass of ion species $i$, $\nu_i \equiv \sum_n \nu_{im}$ is the ion-neutral momentum-transfer collision frequency between ion species $i$ and all neutral species, $\nu_{im}$ is the momentum-transfer collision frequency between ion species $i$ and neutral species $n$, and $\omega_i = eB/m_i$ is the gyrofrequency of ion species $i$. In this formulation, the electron-neutral terms are absent.
The Pedersen and Hall conductivities in STIM are now calculated according to the expressions given in Schunk and Nagy (2009), which do not neglect the electron-neutral terms:

\[
\sigma_P = \frac{\nu_e^2}{\nu_e^2 + \omega_e^2} + \sum_i \frac{\sigma_i \nu_i^2}{\nu_i^2 + \omega_i^2}
\]

\[
\sigma_H = \frac{\nu_e \omega_e}{\nu_e^2 + \omega_e^2} - \sum_i \frac{\sigma_i \nu_i \omega_i}{\nu_i^2 + \omega_i^2}
\]

where \(\nu_e \equiv \sum_n \nu_{en}\) is the momentum-transfer collision frequency of electrons with all neutral species, \(\nu_{en}\) is the momentum-transfer collision frequency between electrons and neutral species \(n\), \(\omega_e = eB/m_e\) is the electron gyrofrequency, \(\omega_i = e_i B/m_i\) is the ion gyrofrequency, \(e_i\) is the charge of ion species \(i\), \(m_e\) is the electron mass, \(m_i\) is the molecular mass of ion species \(i\), \(B\) is the magnitude of the magnetic field, and

\[
\sigma_e = \frac{n_e e^2}{m_e \nu_e}
\]

\[
\sigma_i = \frac{n_i e_i^2}{m_i \nu_{im}}
\]

where \(n_e\) is the electron number density and \(n_i\) is the number density of ion species \(i\). In essence, the new formulation introduces the electron-neutral collision terms, which are significant at lower altitudes/greater pressures. We found that the new calculation of conductivities did not significantly change the Hall conductivities but greatly increased the Pedersen conductivities at pressures greater than \(10^{-4}\) mbar. A visual comparison of the previous and updated formulations is provided in Figure 4.1.

### 4.2.3 Current Density

The electrodynamics used by STIM is based on a layer conductivity formulation (Baker and Martyn, 1953) in which currents are constrained to flow only in a thin, horizontal, conducting sheet. In this picture, currents perpendicular to the sheet must vanish to ensure horizontal flow. This is facilitated by assuming the existence of a polarization electric field which adjusts itself such that the vertical current is
Figure 4.1: A comparison of the Pedersen (blue) and Hall (red) conductivities as computed in the previous version of STIM (open circles) and via the expressions in Schunk and Nagy (2009) (crosses and lines). The introduction of the electron-neutral terms does not greatly change the Hall conductivity, but it significantly alters the Pedersen conductivity at pressures greater than $10^{-4}$ mbar.
zero. These expressions were

\[
\begin{align*}
\mathbf{j}_\theta &= \left[ \frac{\sigma_\parallel \sigma_P}{\sigma_\parallel \sin^2 I + \sigma_P \cos^2 I} \right] E_{\theta}^{CM} + \left[ \frac{\sigma_\parallel \sigma_H \sin I}{\sigma_\parallel \sin^2 I + \sigma_P \cos^2 I} \right] E_{\phi}^{CM} \\
\mathbf{j}_\phi &= \left[ -\frac{\sigma_\parallel \sigma_H \sin I}{\sigma_\parallel \sin^2 I + \sigma_P \cos^2 I} \right] E_{\theta}^{CM} + \left[ \frac{\sigma_\parallel \sigma_P \sin^2 I + (\sigma_P^2 + \sigma_H^2) \cos^2 I}{\sigma_\parallel \sin^2 I + \sigma_P \cos^2 I} \right] E_{\phi}^{CM},
\end{align*}
\]

where \( \sigma_\parallel, \sigma_P \) and \( \sigma_H \) are the parallel, Pedersen and Hall conductivities, respectively, and \( I \) is the magnetic dip angle. The electric field in the center-of-mass frame of the atmosphere (approximately the frame of the neutral winds) is given by

\[
\vec{E}^{CM} = \vec{E} + \vec{u} \times \vec{B}
\]

\[
= \left[ E_r + u_\theta B_\phi - u_\phi B_\theta \right] \hat{r} + \left[ E_\theta + u_\phi B_r - u_r B_\phi \right] \hat{\theta} + \left[ E_\phi + u_r B_\theta - u_\theta B_r \right] \hat{\phi},
\]

where \( \vec{B} \) is the planetary magnetic field, \( \vec{u} \) and \( \vec{E} \) are the wind velocity and electric field, respectively, in Saturn’s System III co-rotating reference frame, and \( \hat{r}, \hat{\theta} \) and \( \hat{\phi} \) are the radial (positive upward), colatitudinal (positive southward) and azimuthal (positive eastward) unit vectors, respectively.

The actual formulation used by STIM (Rishbeth and Garriott, 1969; Fuller-Rowell, 1981) is an approximation of the earlier formulation which is valid when \( \sigma_P \cos^2 I \ll \sigma_\parallel \sin^2 I \) — i.e. for high latitudes and/or when the parallel conductivity \( \sigma_\parallel \) is much greater than the Pedersen conductivity. This formulation also assumes \( u_r B_\theta \ll u_\theta B_r \) and \( u_r B_\phi \ll u_\phi B_r \), which are reasonable assumptions because \( u_r \) is almost always smaller than the horizontal wind components and Saturn’s magnetic field is almost perfectly aligned with its spin axis, so \( B_\phi \to 0 \) (Dougherty et al., 2018). Although \( u_r B_\theta \) may be greater than \( u_\theta B_r \) at the magnetic equator, these expressions are used only at high latitudes where the above assumptions are satisfied.

The expressions previously used by STIM are:

\[
\begin{align*}
\mathbf{j}_\theta &= \frac{\sigma_P}{\sin^2 I} (E_\theta + u_\phi B_r) + \frac{\sigma_H}{\sin I} (E_\phi - u_\theta B_r) \\
\mathbf{j}_\phi &= -\frac{\sigma_H}{\sin I} (E_\theta + u_\phi B_r) + \sigma_P (E_\phi - u_\theta B_r).
\end{align*}
\]
Within 2° of the magnetic equator, STIM manually set \( I \) to \( \pm 2° \) when calculating the current density to avoid numerical instabilities in Equations (4.13) and (4.14) stemming from \( \sin I \) approaching zero in the denominator of the above equations. In previous versions of STIM, this formulation was applied independently at each pressure level, allowing the horizontal components of current density to vary with height but still assuming the vertical current density to be zero everywhere. In doing so, STIM effectively ignored FACs at high latitudes (e.g., Hunt et al., 2014, 2015) that would have required more detailed coupling with the magnetosphere. Moreover, there was no direct vertical coupling because no information about adjacent pressure levels was used to compute the electrodynamics at a given pressure level. This precluded STIM from being able to properly model the wind dynamo effect because it is driven by winds that vary with height and magnetic field lines couple the thermosphere vertically via the Lorentz force.

In the present study, we relax the assumption of zero vertical current and express the current density in full generality as follows:

\[
\vec{j} = \sigma_{\parallel} \left( \vec{E}_{\text{CM}} \cdot \hat{b} \right) \hat{b} + \sigma_{P} \left( \hat{b} \times \vec{E}_{\text{CM}} \times \hat{b} \right) - \sigma_{H} \left( \vec{E}_{\text{CM}} \times \hat{b} \right), \tag{4.15}
\]

where \( \sigma_{\parallel}, \sigma_{P}, \) and \( \sigma_{H} \) are the parallel, Pedersen and Hall conductivities, respectively, \( \vec{B} \) is the magnetic field, \( \hat{b} \equiv \vec{B} / |\vec{B}| \) is the unit vector pointing in the direction of the magnetic field. We express the vector components of Equation (4.15) as follows, where the subscripts \( r, \theta \) and \( \phi \) indicate the radial, meridional (positive southward) and azimuthal (positive eastward) vector components, respectively:

\[
j_r = \sigma_{rr} E_{r}^{\text{CM}} + \sigma_{r\theta} E_{\theta}^{\text{CM}} + \sigma_{r\phi} E_{\phi}^{\text{CM}} \tag{4.16}
\]
\[
j_\theta = \sigma_{\theta r} E_{r}^{\text{CM}} + \sigma_{\theta\theta} E_{\theta}^{\text{CM}} + \sigma_{\theta\phi} E_{\phi}^{\text{CM}} \tag{4.17}
\]
\[
j_\phi = \sigma_{\phi r} E_{r}^{\text{CM}} + \sigma_{\phi\theta} E_{\theta}^{\text{CM}} + \sigma_{\phi\phi} E_{\phi}^{\text{CM}}. \tag{4.18}
\]
The components of the conductivity tensor in the above equations are given by

\[
\sigma_{rr} \equiv \sigma_\parallel b_r^2 + \sigma_P \left( b_\theta^2 + b_\phi^2 \right)
\]
\[
\sigma_{r\theta} \equiv \sigma_\parallel b_r b_\theta - \sigma_P b_r b_\theta - \sigma_H b_\phi
\]
\[
\sigma_{r\phi} \equiv \sigma_\parallel b_r b_\phi - \sigma_P b_r b_\phi + \sigma_H b_\theta
\]
\[
\sigma_{\theta r} \equiv \sigma_\parallel b_\theta b_r - \sigma_P b_\theta b_r - \sigma_H b_\phi
\]
\[
\sigma_{\theta\theta} \equiv \sigma_\parallel b_\theta^2 + \sigma_P \left( b_r^2 + b_\theta^2 \right)
\]
\[
\sigma_{\theta\phi} \equiv \sigma_\parallel b_\theta b_\phi - \sigma_P b_\theta b_\phi - \sigma_H b_r
\]
\[
\sigma_{\phi r} \equiv \sigma_\parallel b_\phi b_r - \sigma_P b_\phi b_r - \sigma_H b_\theta
\]
\[
\sigma_{\phi\phi} \equiv \sigma_\parallel b_\phi^2 + \sigma_P \left( b_r^2 + b_\theta^2 \right),
\]

(4.19)

or, with a little rearrangement,

\[
\sigma_{rr} \equiv \left( \sigma_\parallel - \sigma_P \right) b_r^2 + \sigma_P
\]
\[
\sigma_{r\theta} \equiv \left( \sigma_\parallel - \sigma_P \right) b_r b_\theta - \sigma_H b_\phi
\]
\[
\sigma_{r\phi} \equiv \left( \sigma_\parallel - \sigma_P \right) b_r b_\phi + \sigma_H b_\theta
\]
\[
\sigma_{\theta r} \equiv \left( \sigma_\parallel - \sigma_P \right) b_\theta b_r + \sigma_H b_\phi
\]
\[
\sigma_{\theta\theta} \equiv \left( \sigma_\parallel - \sigma_P \right) b_\theta^2 + \sigma_P
\]
\[
\sigma_{\theta\phi} \equiv \left( \sigma_\parallel - \sigma_P \right) b_\theta b_\phi - \sigma_H b_r
\]
\[
\sigma_{\phi r} \equiv \left( \sigma_\parallel - \sigma_P \right) b_\phi b_r - \sigma_H b_\theta
\]
\[
\sigma_{\phi\phi} \equiv \left( \sigma_\parallel - \sigma_P \right) b_\phi^2 + \sigma_P.
\]

(4.20)

This formulation is a restatement of Equation (4.15) and is therefore valid for any system in which Equation (4.15) is valid: any conductivity profile, magnetic field geometry, wind or electric field structure. We compare these expressions to those previously used by STIM and described in Equations 7 and 8 of (Müller-Wodarg et al., 2012) in Table 4.2. We note that the previous STIM formulation has factors of \( b_r^{-1} = \csc I \) where the general formulation has factors of \( b_r = \sin I \). At high latitudes,
this difference is less noticeable. The changes between the way STIM used to handle
electrodynamics and the way it does now are summarized in Table 4.2.

We note that Hall currents will be reversed (see $\sigma_{\theta\phi}$ and $\sigma_{\phi\theta}$ in Table 4.2). The
layer conductivity formulation, originally developed for the terrestrial ionosphere,
worked for Earth-based models. However, this formulation appears to have been
applied to Saturn without accounting for Saturn’s flipped magnetic field relative to
the Earth’s. As a result, there is a difference in sign in all terms that involve odd
factors of $b_r$ or $b_\theta$. No terms involve factors of $b_\theta$, and only the Hall terms involve
odd terms of $b_r$, so ultimately only the Hall terms have the wrong sign.

Because parallel currents do not enter the equations for resistive heating or ion
drag (c.f. Equations (4.21) to (4.23)), we ignore them in the updated version of
STIM. When we compute the components of current density via Equations (4.16)
to (4.18), we remove FACs from the equations by removing all terms involving $\sigma_\parallel$
from the conductivity tensor components. Apart from ignoring parallel currents,
we have updated STIM to calculate the current density using the expressions in the
“General Terms” column of Table 4.2 rather than those from the layer conductivity
approach it had previously been using. Because the layer conductivity assumed zero
vertical current ($j_r = 0$), the expressions STIM used to calculate the ion drag and
Joule heating did not include $j_r$. We therefore also updated the ion drag expressions
in the momentum equation and Joule heating expression to include these terms.
The horizontal components of ion drag, previously given in Equations 9 and 10 of
(Müller-Wodarg et al., 2012), are now

\[
\begin{align*}
a_{ID,\theta} &= \frac{1}{\rho} \left[ j \times \vec{B} \right]_\theta = \frac{1}{\rho} (j_\phi B_r - j_r B_\phi) \\
a_{ID,\phi} &= \frac{1}{\rho} \left[ j \times \vec{B} \right]_\phi = \frac{1}{\rho} (j_r B_\theta - j_\theta B_r),
\end{align*}
\]

where $\rho$ is the mass density. Because solving the vertical momentum equation is
numerically difficult, STIM uses the divergence of horizontal winds from the contin-
uuity equation to calculate vertical winds. As a result, it does not use the vertical
Table 4.2: Comparison of the important terms in the electrodynamics equations for the electrodynamics approaches we discuss. The layer conductivity approach is what STIM used previously, the general approach follows directly from Equation (4.15) and — apart from STIM ignoring parallel currents — is the new approach we have implemented in STIM. The Effective Terms are simplified from the General Terms by assuming $B_\phi = 0$ (as is the case for the magnetic field STIM uses) and $\sigma_\parallel = 0$ (forcing STIM to compute only field-perpendicular currents), and therefore more clearly expresses the equations that STIM effectively solves. We note that in the Layer Conductivity Approach, $E_r$ is eliminated from the equations and is therefore not used, though it could be calculated as described in (Baker and Martyn, 1953).

The radial component of the ion drag is neglected in STIM due to the assumption of hydrostatic equilibrium. The Joule energy term $q$ used by STIM is $\vec{j} \cdot \vec{E}$ rather than the more conventional Joule heating term $\vec{j} \cdot \vec{E}^{CM}$, for reasons discussed in Section 2.3.3.
component of ion drag. The updated Joule energy term is

\[ q = \frac{1}{\rho} \vec{j} \cdot \vec{E} = \frac{1}{\rho} (j_r E_r + j_\theta E_\theta + j_\phi E_\phi). \]

(4.23)

We note that this is not the “conventional ionospheric Joule heating” rate \( \vec{j} \cdot \vec{E} \) typically used in upper atmosphere electrodynamics (Vasyliunas and Song, 2005). The Joule energy term used by STIM is the dot product between the current densities and the electric field \( \vec{E} \), whereas Joule heating is calculated as the dot product between the current densities and the center-of-mass electric field \( \vec{E}^{CM} \), which includes the \( \vec{u} \times \vec{B} \) terms. The \( \vec{j} \cdot \vec{E} \) Joule energy term contains a contribution from frictional heating, conventionally known as Joule heating, as well as a contribution from ion drag, which transports kinetic energy throughout the system. Although the frictional heating component is positive-definite, the ion drag component can be negative, and this can cause the overall term to be negative in some regions.

4.2.4 Magnetic Field

In Section 2.2, we described an approach to calculate the polarization electric field (PEF) assuming that the magnetic field is a dipole magnetic field shifted northward by \( \Delta z = 0.0466 R_S \) from Saturn’s equatorial plane along the rotation axis (Dougherty et al., 2018). While the previous STIM model and Model 1 use the Saturn Pioneer Voyager (SPV) model magnetic field (Davis and Smith, 1990), Models 2–3 use a shifted magnetic dipole field with a dipole magnetic moment of \( 4\pi R_S^3 g_1^0 \approx 4.6227 \times 10^{25} \text{ A m}^2 \), where \( g_1^0 = 2.11402 \times 10^4 \text{ nT} \) (Dougherty et al., 2018). These magnetic fields are shown together in Figure 4.2. The reason for this change is because we use a formulation in Chapter 5 that requires a dipole field geometry. As a result of this simplifying assumption, the radial component of the dipole magnetic field is relatively weaker than its SPV counterpart poleward of roughly \( \pm 50^\circ \) latitude, and stronger equatorward of this latitude. The colatitude component of the shifted dipole field is weaker than the SPV field at latitudes poleward of roughly \( -25^\circ \) and \( +35^\circ \), and stronger at lower latitudes. Comparing
Models 2 and 3 allow us to study how approximating Saturn’s magnetic field as a shifted dipole affects the thermosphere.

Figure 4.2: A comparison of the SPV magnetic field used in previous versions of STIM and in Model 1 with the shifted dipole magnetic field used in Chapter 3 and in Models 2–3. Altitudes indicated here represent the distance above 1 $R_S$ (assuming spherical geometry).
4.2.5 Equatorial Jet

There is building evidence that a significant eastward jet exists in Saturn’s low-latitude thermosphere (Hubbard et al., 1997; Guerlet et al., 2011; Khurana et al., 2018; Brown et al., 2020). Khurana et al. (2018) suggested that an eastward jet that is symmetric about the equator could produce eastward magnetic field perturbations that were of the same order of magnitude as those observed by the Cassini MAG instrument during Cassini’s Grand Finale tour. Vriesema et al. (2020) tested this idea in more detail and arrived at similar conclusions. We therefore impose a constant azimuthal wind profile \( u_\phi(\theta) \) along STIM’s lower boundary pressure level and modify the altitudes of this pressure level accordingly. This wind profile, of the form \( u_\phi(\theta) \), may be specified in STIM, allowing one to use improved wind profiles in later runs. The zonal wind profile we impose, shown in Figure 4.3, is based on that of the upper (i.e. highest-altitude) boundary in the model presented by Friedson and Moses (2012). This boundary condition is implemented as a perturbation to the heights of the lower-boundary pressure level assuming a balance between the Coriolis, centrifugal and pressure-gradient forces (i.e. gradient balance). The height perturbations \( z'(\theta) \) are calculated via

\[
\frac{\partial z'}{\partial \theta} = -\frac{1}{g_0} \left( \frac{u_\phi^2}{\tan \theta} + 2r \Omega_S u_\phi \cos \theta \right),
\]

(4.24)

where \( g_0 \) is the gravitational acceleration at zero altitude (i.e. the 1 bar pressure level) and \( \Omega_S \) is the angular rotation rate of Saturn, and by assuming \( z' = 0 \) at the south pole. The heights of the lower boundary level are never modified by STIM, so the imposed zonal wind profile does not decay in time. Rather, the zonal wind profile propagates upward via advection and diffusion of momentum.

4.3 Results

The modified STIM outlined in Section 4.2 allows us to better model electrodynamics and an equatorial jet in Saturn’s thermosphere. In this section, we walk through the modifications in order to understand how each affects STIM and how
Figure 4.3: The zonal wind profile we impose in Model 3 at the lower boundary, based on the zonal wind profile of the upper boundary in the model presented by Friedson and Moses (2012). Positive values, such as those at low latitudes, indicate eastward flow.

each might contribute to the final model. We note that although modifying the calculations of the Pedersen and Hall conductivities significantly changed the Pedersen conductivity at pressures greater than $10^{-4}$ mbar, the overall results were not significantly changed, so we do not discuss them here. In the present study, we present only longitudinally-averaged results; we will investigate longitudinal variations in a follow-up study. In Section 4.3.1, we discuss the results from Model 1 by comparing them to the previous STIM model. In Section 4.3.2 we describe how the imposed zonal wind profile affected STIM and compare the results of Model 3 to those of Models 1 and 2.

4.3.1 Effects of Using Updated Electrodynamics

We begin by comparing the previous STIM model to Model 1 in order to see how the updated conductivities and generalized electrodynamics affects STIM. We first examine the electric current, then ion drag, and finally the temperature. Because the zonally-averaged resistive heating profile at high latitudes is similar in both models, we do not discuss resistive heating further here.

Perhaps the most relevant comparison is that of the electric currents, shown in Figures 4.4 to 4.6. The radial component of the current density perpendicular to field lines in Figure 4.4b are the first radial currents calculated in Saturn’s thermosphere
from a general circulation model and are of the order $10^{-8}$ A m$^{-2}$ at high latitudes. (We note that because our model ignores FACs, these radial currents are the radial projection of the perpendicular component of current density, and are not the radial component of the total current.) The colatitudinal current $j_\theta$ in Figure 4.5, which is mostly perpendicular to magnetic field lines at high latitudes, is significantly changed at high latitudes. At low altitudes and high latitudes, the sign of the currents flip, indicating a significant departure from the previous model. The zonal current density $j_\phi$ in Figure 4.6b is similar to that in Figure 4.6a except for a factor of roughly $-0.9$. This discrepancy is primarily because the $\sigma_{\phi\theta}$ element of the conductivity tensor is $-\sigma_H/b_r$ in the previous model but is effectively $+\sigma_H b_r$ in Model 1. The new zonal currents are in at least qualitative agreement with the model of Cowley et al. (2004b), which describes primarily Hall currents flowing eastward at high latitudes. Our updates represent a significant qualitative improvement to modeling electrodynamics in Saturn’s thermosphere.

Figure 4.4: A comparison of the zonally-averaged radial current density for the previous STIM model (Figure 4.4a) and Model 1 with the updated calculation of current density (Figure 4.4b). Each line represents a different pressure level as shown in the legend. We note that in the previous STIM model, radial current was assumed to be zero.

These currents induce ion drag throughout the thermosphere. The zonal com-
Figure 4.5: A comparison of the zonally-averaged colatitudinal (southward) current density profile for the previous STIM model (Figure 4.5a) and Model 1 with the updated calculation of current density (Figure 4.5b). Each line represents a different pressure level as shown in the legend.

Figure 4.6: A comparison of the zonally-averaged zonal (eastward) current density profile for the previous STIM model (Figure 4.6a) and Model 1 with the updated calculation of current density (Figure 4.6b). Each line represents a different pressure level as shown in the legend.
ponent of ion drag was not significantly changed, but the meridional component, compared in Figure 4.7, is quite different. While the primarily poleward ion drag above roughly $10^{-5}$ mbar is unchanged in both hemispheres, ion drag is directed equatorward below this pressure level and is somewhat weaker in Model 1. This reversal below $10^{-5}$ mbar is due to the high-latitude Hall currents now flowing eastward instead of westward. This reversal highlights the importance of modelling the vertical structure of the ionosphere and of using an accurate electrodynamics formulation.

![Figure 4.7: A comparison of the zonally-averaged meridional ion drag profile for the previous STIM model (Figure 4.7) and Model 1 with the updated calculation of current density (Figure 4.7). Each line represents a different pressure level as shown in the legend.](image)

We now consider how the updated formulation affects temperature. Figure 4.8 compares the zonally-averaged temperature profiles of the previous STIM model and Model 1 with the updated current density calculations. The updated electrodynamics generated less heating at high latitudes, causing the polar temperatures in Model 1 to decrease by 25 K to 100 K. Part of the reason for this is because the previous STIM model’s conductivity is enhanced at low and middle latitudes due to $\sigma_{\theta \theta}$ being proportional to $b_r^{-1}$. In Model 1, the conductivity is proportional to $b_r$ and is therefore reduced relative to the previous STIM model at middle and low
latitudes. Another reason is because the westward auroral jets are broader in the previous STIM model than in Model 1, which causes the $u_\phi B_r$ term of $E_{CM}$ to be larger at midlatitudes than in Model 1. Lower latitudes are cooler in Model 1 due to reduced meridional transport of auroral energy. In the previous STIM model, equatorward winds at high altitudes (low pressures) were broader and persisted to lower latitudes. Those meridional winds were better able to transport auroral energy to middle and low latitudes than in Model 1, wherein meridional circulation is greatly reduced at middle latitudes. This helps bring STIM closer to the observed temperature profile at high latitudes. However, the conventional ionospheric Joule heating is reduced in Model 1 relative to the previous STIM model — in some places by an order of magnitude or more — because meridional currents are reduced. The increased meridional advection is insufficient to make up for the loss of Joule heating. This brings the low latitude temperatures in Model 1 further from the observations.

![Figure 4.8](image_url)

(a) Previous STIM model  
(b) Model 1

Figure 4.8: A comparison of the zonally-averaged temperature profile for the previous STIM model (Figure 4.8a) and Model 1 with the updated calculation of current density (Figure 4.8b). Each line represents a different pressure level as shown in the legend.
4.3.2 Effects of Imposing an Equatorial Jet

Based on Khurana et al. (2018) and Vriesema et al. (2020), we expect that the presence of an equatorial jet should produce more electrodynamic activity at low latitudes. We begin by presenting equatorial currents and then the temperature and zonal wind profiles.

The presence of the imposed equatorial jet in Model 3 generates significant electrodynamic currents at low latitudes. Figure 4.9 shows the presence of radial current densities of up to $\sim 10^{-8}$ A m$^{-2}$ near the $10^{-5}$ mbar pressure level. The equatorward auroral current densities reach $3 \times 10^{-7}$ A m$^{-2}$, and zonal currents in the Hall conducting region are roughly $1.2 \times 10^{-6}$ A m$^{-2}$. STIM does not calculate FACs, so all currents — horizontal at the magnetic equator and vertical at high latitudes — are a combination of Pedersen and Hall currents.

These low-latitude currents are generally an order of magnitude larger than in the previous STIM model or in Models 1–2. This suggests that any observations of significant electrodynamic activity at low latitudes may be due to the presence of significant zonal winds in Saturn’s thermosphere as suggested by Khurana et al. (2018); Vriesema et al. (2020).

The winds in Model 3, shown in Figure 4.10, are not greatly changed at middle and high latitudes by the addition of the zonal wind profile. At low latitudes, the imposed zonal wind profile can be seen to have propagated upward in Figure 4.10b. While the meridional gradient of meridional wind (i.e. $\frac{\partial u_s}{\partial \theta}$) had in Model 2 been steep at middle latitudes and shallow at low latitudes, the profile has smoothed out such that the gradient is smaller. This is most pronounced for the $10^{-8}$ mbar pressure level, showing a stronger convergence near the equator than before. Also, the region of convergence has moved from roughly $-20^\circ$ latitude to roughly $+5^\circ$ latitude.

We can also examine how the presence of the equatorial jet influences the zonal force balance, shown in Figure 4.11 at the $10^{-5}$ mbar pressure level. At this pressure level, the eastward Coriolis force counteracts the westward ion drag. At higher alti-
Figure 4.9: The radial (Figure 4.9a), southward (Figure 4.9b) and eastward (Figure 4.9c) components of current density in Model 3 with the imposed zonal wind profile. Each line represents a different pressure level as shown in the legend.
Figure 4.10: The southward (Figure 4.10a) and eastward (Figure 4.10b) components of wind velocity in Model 3 with the imposed zonal wind profile. Each line represents a different pressure level as shown in the legend. We note the large eastward jet at low altitudes (gold line).

tudes, the direction of both ion drag and the Coriolis force reverses, but they remain the dominant forces until the $10^{-7}$ mbar pressure level, at which point eastward vertical viscous drag balances the westward Coriolis force. While this is generally true for both Models 2 and 3, the presence of the equatorial jet increases the ion drag and Coriolis forces between $-40^\circ$ to $+50^\circ$ latitude by an order of magnitude and changes the structure of the thermosphere, as evidenced by the increases in the other terms at low latitudes. We note also that in the meridional direction, the Coriolis and pressure gradient forces are generally 1–3 orders of magnitude higher than the other terms at altitudes below the $10^{-8}$ mbar pressure level, at which point curvature and vertical viscous drag become important. This model is therefore in good agreement with the assumptions of Brown et al. (2020) that the Coriolis, pressure gradient and ion drag forces are generally in approximate balance in Saturn’s upper atmosphere.

Because the zonal component of ion drag is able to partially oppose the Coriolis force at midlatitudes, more meridional circulation is possible than in previous models. The greater meridional circulation at low latitudes due to the equatorial jet increases the meridional advection of energy by up to several orders of magni-
Figure 4.11: A comparison of the absolute values of the zonally-averaged components of the zonal momentum equation at the $10^{-5}$ mbar pressure level for Model 2 (Figure 4.11a) and Model 3 (Figure 4.11b). Each line represents one of the forces included in the zonal momentum equation as shown in the legend.

tude at low latitudes. Vertical viscous heating experiences a similar increase. We compare these terms and others in the energy equation at the $10^{-5}$ mbar pressure level in Figure 4.12. We note that the $\vec{j} \cdot \vec{E}$ Joule energy term is not produced in either model equatorward of $\pm 56^\circ$ because the only electric field in these models is the MEF, which is restricted to high latitudes. As before, the dominant heating mechanism at low latitudes above the $10^{-5}$ mbar pressure level is solar heating, which is balanced by vertical molecular conduction. Whereas adiabatic heating was significant at midlatitudes and higher altitudes before, the addition of a zonal jet changed low-latitude circulation such that adiabatic heating is now significant near the equator. Meridional advection dominates at midlatitudes above the $10^{-6}$ mbar pressure level.

In Figure 4.13, we compare the temperature profiles from Models 2 and 3. Despite the changes to the energy equation, adding the zonal jet to STIM does not significantly alter the temperature profile except at low altitudes and low latitudes. At high altitudes near the poles, the temperatures increase by about 10 K. At low altitudes and mid-to-low latitudes, the meridional temperature profile exhibits a wave-like pattern that loosely follows the imposed zonal jet profile due to a combination of rough thermal wind balance and adiabatic heating/cooling. The dominant meridional forces therein are the pressure gradient force and the Coriolis force in
both Models 2 and 3; however, these forces (as well as ion drag and the curvature term) are 1–2 orders of magnitude stronger in Model 3 than in Model 2 in this region. Despite the increased terms, the solution remains in a relatively steady state, with the zonal Coriolis force mainly balanced by ion drag and vertical viscous drag. At the lowest latitudes and lowest altitudes, there is no clean balance between two or three terms in the energy equation, as several terms are within an order of magnitude of each other and several terms vary over small meridional length scales.

In Model 3, the top five energy terms at $10^{-3}$ mbar within $\pm 20^\circ$ of the equator — adiabatic heating/cooling, vertical molecular conduction, meridional advection, vertical viscous heating and vertical advection — are of the order $10^{-4}$ W kg$^{-1}$ to $10^{-2}$ W kg$^{-1}$, which is roughly two orders of magnitude greater than the dominant terms in Model 2 — vertical molecular conduction, adiabatic heating/cooling and vertical advection. In Model 3, upward and downward flow plays a much larger role, cooling the atmosphere adiabatically and advecting heat away at the lowest latitudes and heating it at midlatitudes. In spite of these dramatic behaviors at low latitudes and low altitudes, the overall behavior of the thermosphere as a whole is relatively unaffected. Needless to say, the complex interactions in this region merit further study.

At midlatitudes, above $10^{-6}$ mbar but especially at $10^{-8}$ mbar, meridional advect-
tion from higher latitudes is balanced by vertical viscous heating which transports heat vertically. At middle latitudes and high altitudes, the meridional temperature gradient is slightly shallower in Model 3 than in Model 2. There is also a reversal in the sign of the meridional component of ion drag in a narrow band at roughly $\pm 45^\circ$ latitude. This may allow ion drag at middle latitudes to partially oppose the so-called “Coriolis barrier”, which allows a slightly increased meridional advection of energy equatorward. These results weakly support the prediction of Vriesema et al. (2020) that an equatorial jet in Saturn’s thermosphere could increase ion drag and weaken the Coriolis barrier described by Smith et al. (2007), although in our model, the meridional advection it permits is not substantial enough to significantly alter the temperature profile.

![Figure 4.13](image)

Figure 4.13: A comparison of the zonally-averaged temperature profile for Model 2 (Figure 4.13a) and Model 3 with the imposed zonal wind profile (Figure 4.13b). Each line represents a different pressure level as shown in the legend.

4.4 Discussion

The goal of this study was to update the electrodynamics calculations in STIM and use STIM to determine how an equatorial jet would affect the circulation and energy balance in Saturn’s thermosphere. In this section, we discuss the role of resistive
heating and ion drag at low and middle latitudes. We begin by discussing Joule heating in Section 4.4.1 and then consider the dynamics in Section 4.4.2. Finally, we discuss high-latitude coupling in Section 4.4.3.

4.4.1 Joule Heating

As discussed in Section 2.3.3, STIM calculates Joule heating as $\vec{j} \cdot \vec{E}$, which combines frictional heating and the heating effects of ion drag. Because the imposed electric field in STIM is restricted to high latitudes, none of the models we consider in this study produce this $\vec{j} \cdot \vec{E}$ Joule heating at low latitudes. The conventional expression for ionospheric Joule heating is

\[ q = \vec{j} \cdot \vec{E}^{CM}, \]  

which includes a factor of $|\vec{u} \times \vec{B}|^2$, related to ion drag heating. Even in the absence of an electric field, these terms could contribute to the energy balance. This is perhaps unfortunate, because our models do predict currents at low and middle latitudes, and their contribution is not captured in the energy equation as Joule heating. Regardless, we focus on the role of Joule heating at high latitudes for the remainder of this study.

At high latitudes, the Joule heating at the $10^{-3}$ mbar pressure level increased by an order of magnitude as a result of updating the conductivities and electrodynamics formulation. From Figure 4.1, we see that this is because the Pedersen conductivity at $10^{-3}$ mbar increased by an order of magnitude. The fact that using a dipole magnetic field instead of the SPV field did not significantly change the Joule heating profile is interesting, since the conductivities depend on the magnetic field strength and the dipole magnetic field is roughly 10% weaker at high latitudes than the SPV field. This suggests that it may be a reasonable approximation that does not significantly affect results. This will be important in Chapter 5. The presence of an imposed wind profile had little effect on Joule heating at high latitudes because the wind profile was small at high latitudes and died off relatively quickly with altitude.
4.4.2 Momentum Balance

Understanding the effects of electrodynamics on the dynamics of Saturn’s thermosphere is a key part of this study. In this section, we begin by noting the general behavior of the models, and then discuss the exceptional behavior. We then discuss what implications this may have for Saturn.

In general, the meridional dynamics are dominated by the Coriolis and pressure gradient forces, indicating near geostrophic balance. However, the zonal momentum balance is more complicated. At high pressures (low altitudes), the dominant forces are the Coriolis force and vertical viscous drag (especially at low latitudes), though ion drag is significant at high latitudes. In some models, horizontal advection is also significant at low altitudes, but it tends to play a secondary role. In all models at pressures between $10^{-5}$ mbar to $10^{-7}$ mbar, ion drag is comparable to the Coriolis force, especially at midlatitudes. At pressures between $10^{-7}$ mbar to $10^{-8}$ mbar, vertical viscous drag tends to dominate with the Coriolis force. In general, this resembles the balance between the Coriolis, pressure gradient and ion drag forces discussed by Brown et al. (2020).

The biggest surprise was that the sign of the ion drag flipped at and below the $10^{-5}$ mbar pressure level when the layer conductivity formulation was replaced with the more general electrodynamics formulation. This happens because $\sigma_{\phi\theta}$ flipped sign when the electrodynamics formulation was updated, which caused the zonal Hall currents to flip direction in the Hall conducting region. Above that pressure level, meridional currents dominate the azimuthal ion drag term, and those currents did not change sign.

4.4.3 High Latitude Coupling

A complete model of electrodynamics in Saturn’s thermosphere would include dynamic forcing from the magnetosphere. STIM approximates this forcing by imposing a MEF based on the magnetospheric model of Jia et al. (2012a) at high latitudes. Their model treated Saturn’s ionosphere as a thin layer with uniform Pedersen con-
ductivity. They did not model ionospheric winds and neglect the Hall conductivity for simplicity. The FACs predicted by the BATSRUS model are field-aligned at latitudes of roughly $\pm(70^\circ - 75^\circ)$, but are anti-aligned with the magnetic field poleward of this narrow band and peak at the pole itself. This is consistent with other studies (e.g. Cowley et al., 2004a; Hunt et al., 2015) which suggest that the region of strong FACs is variable and is removed from the poles by 10° to 25°. Because of the fundamental differences between our model and that of Jia et al. (2012a), it is unclear how the electric field calculated in their model should apply to STIM (if at all). In order for our model of Saturn’s thermosphere to be consistent with their model of Saturn’s magnetosphere, it is desirable that the electric currents predicted by STIM at the outer boundary would be consistent with the currents predicted by the BATSRUS model at its inner boundary. In order for further modelling progress to be made in Saturn’s high-latitude thermosphere using the STIM model, STIM will need a better model of magnetospheric forcing. One possible way forward could be to construct a new electric field that somehow combines influences from the magnetosphere and the thermosphere such that it roughly produces currents that match those suggested by Cowley et al. (2004a) or Hunt et al. (2014, 2015).

4.4.4 Model Shortcomings

Our model uses the expressions of Schunk and Nagy (2009) to calculate the Pedersen and Hall conductivities. Although the Schunk and Nagy (2009) expressions are treated as a standard in ionospheric contexts, the more general expressions of Koskinen et al. (2014) are simultaneously valid in regions of high and low ionization and would therefore be more appropriate.

While STIM captures some chemistry, a more detailed chemical model that includes recent observations would be preferred. This is likely especially important at low latitudes, where ring material affects the chemistry (e.g. Cravens et al., 2019; Morooka et al., 2019).

There is evidence of a two-layer ionosphere in Saturn, but our current model does not extend deep enough in Saturn’s atmosphere to fully capture the lower
layer. Because of this, current is possible near the base of our domain. This violates one of the assumptions of our electrodynamics formulation. In spite of this, current density is generally very low near the base of our model, so while this assumption may not be valid in general, it does not appear to make a significant difference to STIM. More study is required to determine the effects of this treatment.
5.1 Introduction

Having determined in Chapter 4 how the updated calculation of conductivity and current density, as well the presence of an imposed equatorial jet, affect STIM, we are now prepared to consider the effects of the polarization electric field (PEF) on Saturn’s upper atmosphere. Although the PEF is microscopically due to a separation of charged particles, it is related to the coupling of the ionosphere and magnetosphere at macroscale.

The PEF arises as a result of plasma interactions in the thermosphere, where plasma is both driven by the neutral atmosphere and dragged by magnetic field lines. At lower altitudes in the thermosphere, frequent collisions between charged particles and the dense neutral atmosphere force charged particles to flow more or less with the neutral atmosphere, effectively decoupling them from the magnetosphere. At higher altitudes, however, collisions between charged particles and the tenuous neutral atmosphere are rare, and the charged particles tend to gyrate perpendicular to magnetic field lines and are influenced much less by the neutral atmosphere. According to Vasyliunas (2012), an imbalance between collisional friction and magnetic stress at different points along a given magnetic field line is what causes the dynamo currents, from which we may deduce a steady-state PEF. To model electrodynamics in the thermosphere, it is therefore necessary to track electrodynamics along a significant portion of a magnetic field line.

Tracking electrodynamics along a field line means that it must be tracked not just in the ionosphere, but also above and below it. Electrical conductivities decrease rapidly with decreasing altitude from the ionosphere, so the contributions of lower altitudes is inconsequential. Contributions from the magnetosphere, however, are
fundamental to the interaction between the ionosphere and magnetosphere (see, e.g., Cowley and Bunce, 2003; Jia et al., 2012b; Hunt et al., 2018). Despite much effort to measure and model electrical currents in Saturn’s high-latitude thermosphere Cowley et al. (e.g. 2004a); Hunt et al. (e.g. 2014, 2015); Dougherty et al. (e.g. 2018), relatively little is known.

Previously, Cowley et al. (2004a) proposed a model of coupling the ionosphere and magnetosphere by assuming that the angular velocity of ions with respect to the planet is constant along magnetic field lines. In this model, most applicable to higher altitudes where ion gyrofrequencies are much greater than ion-neutral collision frequencies, the difference between the angular velocity of the neutral atmosphere and that of the ions along field lines introduces a height-dependent slippage factor that quantifies the degree of corotation between the neutral atmosphere and magnetosphere. From this, they were able to estimate currents and resistive (Joule) heating rates. However well their model works at high altitudes, its assumption that ions are tied to magnetic field lines is not appropriate for lower altitudes where ion-neutral collision frequencies are much greater than ion gyrofrequencies. At these lower altitudes, the ions should be more tightly coupled to the neutral atmosphere than to the field lines.

Müller-Wodarg et al. (2012) modeled coupling between the ionosphere and magnetosphere by imposing a static magnetospheric electric field (MEF) based on the calculations from Cowley et al. (2004a). This was the first global model to incorporate effects from the magnetosphere down into the ionosphere. Later, Müller-Wodarg et al. (2019) updated STIM to use a MEF taken from the magnetosphere model of Jia et al. (2012a). This MEF was calculated based on the assumption of an unshifted dipole magnetic field, uniform ionospheric Pedersen conductance (i.e. height integrated conductivity) and zero Hall conductance. While the MEF as used in STIM previously is able to reproduce some of the bulk features of the ionosphere (e.g. subcorotation at high latitudes), the model of Jia et al. (2012a) is not meant to resolve the height dependence of ionospheric electrodynamics, and is therefore not suited for modelling a wind-driven ionospheric dynamo. Finally, it
is important to note that the MEF used in all previous versions of STIM does not track electrodynamics along field lines, as is required by Vasyliunas (2012). Because of this, magnetic stresses at one point along a given field line cannot affect other regions along the same field line, except indirectly (i.e., by diffusion of momentum throughout the neutral atmosphere rather than through direct electromagnetic forcing).

To date, the axisymmetric, steady-state model of Vriesema et al. (2020) is the only electrodynamics model that calculates electrodynamics along field lines throughout Saturn’s thermosphere. From this, they are able to calculate the steady-state PEF, electric current structure, magnetic field perturbations and more. This approach of calculating electrodynamics along field lines in the thermosphere (as opposed to using height-integrated quantities and/or a statically-imposed electric field) was required in order to probe low-latitude electrodynamics and the effects of winds therein. However, their model fails at high latitudes for two reasons. First, the model neglects all interactions with the magnetosphere by assuming that winds as well as the Pedersen and Hall conductivities are negligible along the entire field line above the conducting layer. A model with no Pedersen or Hall currents above the conducting layer necessarily predicts that FACs are antisymmetric about the magnetic equator (allowing for small variations due to geometry), which prohibits the FAC current system described by (e.g.) Bunce et al. (2003). Second, their conductivity profile is intended for use at low latitudes where magnetospheric coupling is less important, yet the conductivities calculated in STIM can vary by 1–2 orders of magnitude in latitude. More generally, their steady-state model requires constant influx of energy to maintain the wind profile; if it were used in a time-dependent simulation without any forcing, it would redistribute momentum through ion drag so as to reduce magnetic stresses along field lines, and it would dissipate energy as resistive heating.

In Section 2.2.6, we described an extension to the model of Vriesema et al. (2020) that allows the PEF to be constrained using FAC information (magnitude and sign) along the upper boundary of the thermosphere. These values represent
the input of the magnetosphere to the thermosphere and may be taken from either observational data or from numerical models. This allows us to calculate a system of electrodynamics throughout the thermosphere that is consistent with FACs from the magnetosphere in a relatively easy way. This is a new approach to coupling the ionosphere and magnetosphere and is the first time that a height-resolved model of Saturn’s ionosphere has been coupled to the magnetosphere via an upper boundary condition in this way. This is also the first model in which a polarization electric field (PEF) has been explicitly calculated for Saturn’s upper atmosphere, albeit still under the assumption of perfect zonal symmetry.

In this chapter, we incorporate this new model into STIM. We begin by describing the updates we have made to STIM for modelling a wind dynamo in Section 5.2. We then describe a sequence of models we use to demonstrate the effects of the wind dynamo in Section 5.2.2. In Section 5.3 we present the results of these models, and we discuss them in Section 5.4.

5.2 Methods

5.2.1 Calculating the Polarization Electric Field

In this section, we describe how we incorporate the axisymmetric, steady-state, FAC-constrained PEF described in Section 2.2.6 into STIM. Because this PEF is axisymmetric, we first calculate several zonally-averaged quantities from STIM: the vector components of the neutral wind, the Pedersen and Hall conductivities, and the altitude profile. These, along with the shifted dipole magnetic field of Vriesema et al. (2020) and an externally-imposed FAC profile in latitude, are used to calculate the axisymmetric PEF.

In calculating the PEF, we adopt the same shifted-dipole coordinate system \((\beta, \alpha, \phi)\) described in Section 2.1 of Vriesema et al. (2020), where \(\beta\) increases northward along dipole field lines and \(\alpha\) increases perpendicular to dipole field lines and is related to the standard McIlwain \(L\)-value for dipole field lines by \(L = \alpha/R_S\) (McIlwain, 1966), where \(R_S\) is Saturn’s equatorial radius, and \(\phi\) is the azimuthal
coordinate. Following Vriesema et al. (2020), we eliminate \( E_\phi \) because the model is axisymmetric. Whereas they assume winds are purely zonal and therefore neglect \( u_\alpha \), we extend their method to account for vertical and meridional winds. We create a dipole grid as in Vriesema et al. (2020) by calculating a set of field lines \( (\alpha_{\text{min}} \leq \alpha \leq \alpha_{\text{max}}) \) and then determining where each each field line intersects the upper and lower boundaries of the domain. We use several interpolation algorithms, including linear and bilinear interpolation, and 2D triangle interpolation using barycentric coordinates.

In order to calculate the FAC-constrained PEF \( E_\alpha \) via Equation (2.103), we calculate the new quantities \( J_\alpha \) and \( J_\beta \) for each field line via Equation (2.98) and Equation (2.97) (respectively) and do so separately for each hemisphere. We note that method of Vriesema et al. (2020) requires one to perform quadrature (numerical integration) along magnetic field lines from the lower footprint along the base of the domain in one hemisphere to where that field line intersects the base of the domain in the other hemisphere. In practice, their assumptions cause the contribution from the field line above the conducting layer to vanish, so for field lines that reach above the conducting layer, they effectively compute two integrals (one along a given field line in each hemisphere) and sum them, effectively computing a single integral for both hemispheres. For field lines in the present study which do not intersect the upper boundary of the domain, we follow their approach in terms of integrating along the entire field line. For field lines which do intersect the upper boundary, we also split the field lines and compute each separately. However, our upper boundary condition effectively decouples the two hemispheres for split field lines (we leave it to the magnetosphere or magnetosphere model to do the coupling), so we compute electrodynamics along a given (split) field line independently for each hemisphere and do not sum the results as was done in the earlier study. For all integrals, we numerically integrate using the cubic spline method detailed in Bhadauria and Singh (2011).

Having calculated all required values, we use Equation (2.103) to calculate the constant value of \( h_\alpha E_\alpha \) for a given field line segment in a given hemisphere. Most
points on the STIM grid are not exactly on one of the field lines on our dipole grid, so we interpolate the value of \( h_\alpha E_\alpha \) for those points and then divide by \( h_\alpha \) at those points. Although nearly all points in the zonally-averaged STIM domain are within the dipole grid, there are a few grid points at the poles and at the lowest altitudes near the magnetic equator that are difficult to include in this scheme. For these points, we use a logarithmic extrapolation method to estimate the value of \( h_\alpha E_\alpha \) instead. At the poles, we assume \( E_\alpha \rightarrow 0 \). With \( E_\alpha \) known, we can easily calculate \( E_r \) and \( E_\theta \) at all points. We note that the PEF has contributions from both the magnetosphere (via the FACs) and from the thermosphere (via the winds). Because the terms are added, we can examine the effects of each separately.

Being able to calculate the PEF in this way is a significant advancement for thermosphere-ionosphere models of gas giant planets because it allows us to incorporate a wind dynamo under the axisymmetric approximation. Computationally, it is relatively inexpensive and straightforward, but it gives results that are consistent with the magnetospheric model of Jia et al. (2012a) within scale factors of order unity. No matrix solvers are required to solve a complex partial differential equation for the electrostatic potential, as is done for some other models (e.g. Jia et al., 2012a). This is the first time, to our knowledge, that a polarization electric field (PEF) has been calculated for a gas giant planet as part of a successful GCM to calculate a wind dynamo.

5.2.2 Models

In order to determine the effects of calculating the PEF in this way, we consider a sequence of models. Unlike the models of Chapter 4 which were set to run at northern solstice, all of the models described in this chapter are run at equinox. The reason for this is because the FACs we use to calculate the PEF are based on an equinox model of Jia et al. (2012a) for low solar wind conditions, except the uniform conductance used to produce these FACs is 4 S rather than 0.5 S. These FACs are plotted in Figure 5.1. We zonally average the FAC profile and use the antiparallel component, shown in Figure 5.2, in Equations (2.97) and (2.103). Unless otherwise
stated, we ran all models to 200 rotations to help ensure that a steady state had been achieved. This is generally plenty of time for models to reach approximate equilibrium. These models are compared in Table 5.1.

Figure 5.1: The original FAC data used. The upper plot is for the northern hemisphere and the lower plot is for the southern hemisphere. The two plots are effectively mirror images of each other.

The first equinox model we present, which we call Model E1, serves as the experimental control. We run this model from the default STIM atmosphere and allow it to evolve to equilibrium. Model E1 is the equinox counterpart to Model 2 from Chapter 4 in that it uses the updated calculations of conductivity and current density, it uses the MEF rather than the new PEF, it does not impose any zonal wind profile at the lower boundary, and it uses the dipole magnetic field rather than the SPV field. Because we use STIM’s standard MEF in this model, it serves as a reference point for later models which replace the MEF with the PEF.

Next, we consider a trio of models, Models E2a, E2b and E2c. The goal of these models is to produce a model similar to Model E1 using a zero-wind PEF instead of a MEF. The zero-wind PEF, expressed in Equation (2.104), calculates the PEF using only information from the upper boundary neglecting winds in the thermosphere.
Figure 5.2: Plot of the zonally-averaged antiparallel \((j_\beta)\) component of current density, based on the equinox model of Jia et al. (2012a). These raw values are used to calculate the FAC-constrained PEF as described in Section 2.2.6 for all models in this study except Model E1, though the values are scaled as noted for each model. This current system is directed downward at the poles and upward at ±72° latitude, where the current density reaches \(±9.7 \text{ A m}^{-2}\), respectively. Strictly speaking, “FACs” refers to \(j_\parallel\), which differs from the antiparallel component shown here by a factor of -1 \((j_\beta = -j_\parallel)\). Although we discuss the “FAC-constrained PEF”, we actually constrain the anti-parallel component in practice.
The zero-wind PEF is therefore analogous to the MEF in that it represents magnetospheric forcing imposed on the thermosphere regardless of thermospheric winds or conductivities. While the MEF was assumed constant in height, the zero-wind PEF is approximately constant along field lines (within a geometric scale factor of order unity), so at high latitudes, it is approximately constant in height. Because the FACs imposed at the upper boundary which are used to calculate the zero-wind PEF are held constant, the zero-wind PEF is also constant to the degree that the geometry of the upper boundary is constant in time. Because the zero-wind PEF does not use the thermospheric winds in its calculation of the electric field, it cannot model the ionospheric wind dynamo, just as previous versions of STIM which used the MEF were unable to do.

As we will see, the FACs along the upper boundary are scaled in Model E2a such that the zero-wind PEF replicates the MEF in Model E1 as closely as possible, but the system is significantly cooler than in Model E1. In Model E2c, the upper-boundary FACs are scaled such that the temperature and winds of Model E2c better match those of Model E1, but the zero-wind PEF is stronger than the MEF in Model E1. Model E2b is intermediate between Models E2a and E2c, and we include it as a basis of comparison for upcoming models. To do this for Models E2a–c, we calculate the zero-wind PEF via Equation (2.104) using the FACs shown in Figure 5.2. Because the calculation uses the zonally-averaged FAC profile, which is presumed constant in time, the resulting PEF is constant in time and azimuth, assuming the zonally-averaged altitude grid and conductivities are also constant in time and azimuth. Because this PEF is based on externally-imposed FACs and does not include the contribution from the wind, it represents the portion of the PEF that is due to external effects, just like the original MEF was intended to do. Whereas the original MEF had an azimuthal component and no radial component, the axisymmetric PEF has a radial component but no azimuthal component.

The goal of Model 2a is to reproduce the MEF of Model E1 as closely as possible. We therefore multiply the FACs along the upper boundary of Model 2a by a factor of 0.45397 for southern magnetic field lines (i.e. Region A) and 0.30278 for northern
magnetic field lines (i.e. Region C). Although the electric field is very sensitive to these scale factors, only 3–4 of the significant digits, obtained by iterative experimentation, are likely significant. These factors cause the PEF to have approximately the same peak value as the MEF in each hemisphere. As we will see in Section 5.3.1, the temperature profile is significantly cooler in Model E2a than in Model E1, and the corresponding winds are much weaker. The goal of Model E2c, therefore, is to scale the FACs of Model E2a by a factor such that it better reproduces Model E1 in terms of the temperature and wind profiles rather than the electric field profile alone. To do this, we double the FACs in Model E2c relative to those in Model E2a. With upper-boundary FACs that are scaled 1.5x relative to those of Model E2a, Model E2b serves as an intermediate model between Models E2a and E2c. None of these models impose a zonal wind profile along the lower boundary.

Next, we present a pair of models which calculate the zero-wind PEF as in Models 2a and 2b, but they impose a zonal jet at the lower boundary. These models serve as stepping stones between Model E2a–c, which use no jet and do not include winds in the PEF calculation, and Model E4g and E4fm, which do both. Model E3g (‘g’ for “Gaussian”) imposes the Gaussian jet described in Section 3.3.4, while Model E3fm (‘fm’ for “Friedson and Moses”) imposes the more structured jet described in Section 4.2.5 and shown in Figure 4.3. For both models, we began from STIM’s starting atmosphere and ran the model for 200 rotations using STIM’s original MEF. We then replaced the MEF with the zero-wind PEF and ran the model for another 200 rotations. The FACs in Models E3g and E3fm are scaled by 1.5 times the scale factors of Model E2a, as was the case for Model E2b: 0.680955 for southern field lines and 0.45417 for northern field lines.

Next, we present a final pair of models, Model E4g and Model E4fm. The goal of both models is to calculate the effects of a wind dynamo in Saturn’s upper atmosphere, but like Models E3g and E3fm, these models use a different wind profile at the base of the domain. Model E4g (‘g’ for “Gaussian”) imposes the Gaussian jet described in Section 3.3.4, while Model E4fm (‘fm’ for “Friedson and Moses”) imposes the more structured jet described in Section 4.2.5 and shown in Figure 4.3.
To facilitate comparison between models, we scale the upper-boundary FACs by the same scale factors used in Models E3g and E3fm: 0.680955 for southern field lines and 0.45417 for northern field lines. For each model, we began from STIM’s starting atmosphere, imposing zonal wind profiles at the lower boundary as in Models E3g and E3fm, and ran the model for 200 rotations using STIM’s original MEF. Next, we replaced the MEF with the complete PEF (including the wind terms) and iterated for another 200 rotations to reach a new equilibrium state. Whereas Model E3g Model E3fm use the magnetospheric component of the PEF only (i.e. winds are ignored in the calculation of the PEF), Model E4g and Model E4fm use both the FACs and the winds to calculate the PEF. This allows us to compare the results of Vriesema et al. (2020) with our results using a time-dependent, global, and more self-consistent model.

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Season</th>
<th>Electric Field</th>
<th>FAC Scaling</th>
<th>Imposed Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>Equinox</td>
<td>MEF</td>
<td>N/A</td>
<td>None</td>
</tr>
<tr>
<td>E2a</td>
<td>Equinox</td>
<td>ZW PEF</td>
<td>1.0</td>
<td>None</td>
</tr>
<tr>
<td>E2b</td>
<td>Equinox</td>
<td>ZW PEF</td>
<td>1.5</td>
<td>None</td>
</tr>
<tr>
<td>E2c</td>
<td>Equinox</td>
<td>ZW PEF</td>
<td>2.0</td>
<td>None</td>
</tr>
<tr>
<td>E3g</td>
<td>Equinox</td>
<td>ZW PEF</td>
<td>1.5</td>
<td>Gaussian</td>
</tr>
<tr>
<td>E3fm</td>
<td>Equinox</td>
<td>ZW PEF</td>
<td>1.5</td>
<td>FM</td>
</tr>
<tr>
<td>E4g</td>
<td>Equinox</td>
<td>PEF</td>
<td>1.5</td>
<td>Gaussian</td>
</tr>
<tr>
<td>E4fm</td>
<td>Equinox</td>
<td>PEF</td>
<td>1.5</td>
<td>FM</td>
</tr>
</tbody>
</table>

Table 5.1: This table provides a convenient way of comparing the models used in this study. In the Electric Field column, MEF indicates the magnetospheric electric field based on the model of Jia et al. (2012a), while ZW PEF indicates the zero-wind polarization electric field (i.e. the PEF emulates the MEF) and PEF indicates the polarization electric field. The value in the FAC Scaling column indicates by what factor the FACs are scaled relative to those in Model E2a, which scales FACs by a factor of 0.45397 for southern magnetic field lines (i.e. Region A) and 0.30278 for northern magnetic field lines (i.e. Region C). In the “Imposed Wind” column, FM indicates that a wind profile based on Friedson and Moses (2012) was imposed at the lower boundary.
5.3 Results

Calculating the PEF via FACs and winds allow us to model electrodynamics in Saturn’s ionosphere and thermosphere in more detail than has been done before. Until the PEF was added to STIM in this step, the only electric field directly modeled by STIM has been the magnetospheric electric field and that has been only at latitudes greater than $\pm 56^\circ$. First, we compare the results of Model E1 to those of Models E2a–c in Section 5.3.1 to determine how replacing the MEF with the FAC-constrained, zero-wind PEF affects the model. In Section 5.3.2, we compare Model E2b to Models E3g and E3fm to explore the effects of imposing two wind profiles at the lower boundary. Finally, in Section 5.3.3, we present the results of Models E4g and E4fm and compare them to the previous models to help determine how a wind dynamo might affect Saturn’s upper atmosphere.

5.3.1 Comparison of Models E1, E2a, E2b and E2c

The goal of Models E2a and E2c is to reproduce Model E1 after replacing STIM’s original MEF with the component of the PEF calculated from externally-imposed FACs. We therefore begin by comparing the electric fields used by each model and then discuss the temperatures and winds. The colatitudinal component of the electric field is shown for both models in Figure 5.3.

By design, the peak southward component of the electric field values of Models E1 and E2a are similar, roughly $\pm 0.07$ V m$^{-1}$, though somewhat less in the southern hemisphere of Model E2a. The peak electric field in Model E2c is $-0.11$ V m$^{-1}$ in the southern hemisphere and $+0.12$ V m$^{-1}$ in the northern hemisphere. From this, we can infer that multiplying the FACs by a factor of two does not directly increase the peak electric field by the same factor. The width of the electric field is wider in Model E1 than in Models E2a–c. While the electric field in Model E1 peaks at $\pm 78^\circ$ latitude, it peaks at $\pm 84^\circ$ latitude in Models E2a and E2c. This poleward shift of 6° also causes the heating, temperatures and winds to be shifted poleward as well, which significantly alters Models 2a-c and subsequent models. The reasons for this
Figure 5.3: Comparison of the meridional electric fields of Models E1, E2a, E2b and E2c at 10^{-6} mbar. The electric field is constant in the vertical direction for Model E1 because the MEF is assumed constant in altitude, but varies almost negligibly for Models E2a–c (increasing with increasing pressure).
shift, as well as its consequences, is discussed in Section 5.4.2.

The poleward-shifted electric field significantly affects the current density at all latitudes. At high latitudes, the currents are associated with both the zero-wind PEF and atmospheric winds moving past the magnetic field, which are substantial. Equatorward of the imposed FAC profile, the zero-wind PEF is zero, so currents are due only to the $\vec{u} \times \vec{B}$ terms in the expression for current density Equation (2.5). A comparison of the current density for the models at the $10^{-4}$ mbar pressure level is shown in Figure 5.4. Because STIM does not calculate FACs explicitly, these represent the radial and colatitudinal components of the combined Pedersen and Hall currents.

As expected, Model E1 has strong auroral currents between roughly $\pm(70^\circ - -75^\circ)$ latitude and between pressures of $10^{-3}$ mbar to $10^{-5}$ mbar. In Models E2a–c, all components of current density are reduced by roughly an order of magnitude be-
low the $10^{-5}$ mbar pressure level. The high-latitude current system is much stronger near the poles in Models E2a–c than in Model E1 between $10^{-5}$ mbar to $10^{-6}$ mbar. The current density in Models E2a–c is generally diminished relative to Model E1 between $\pm 50^\circ$ latitude.

Despite Models E1 and E2a being similar in everything but the location and width of the electric field peak in each hemisphere, the resulting models are dramatically different. The diminished current densities reduce the Joule heating, especially at $\pm 60^\circ$ latitude. This causes temperatures to be lower in Models E2a–c except for Model E2c poleward of $\pm 65^\circ$ latitude. The reduction of midlatitude heating likely causes higher latitudinal temperature gradients and enhances the “unexpected cooling effect” described by Smith et al. (2007). We compare the temperature profiles of the models in Figure 5.5 at $10^{-8}$ mbar.

Whereas Model E1 has temperatures of roughly 550 K to 580 K at high southern latitudes, 175 K at low latitudes and 425 K to 475 K at high northern latitudes, Model E2a has temperatures of roughly 325 K at high southern latitudes, 170 K at low latitudes and 280 K to 305 K at high northern latitudes. By design, Model E2c has temperatures that are more comparable to (though slightly warmer than) those of Model E1: roughly 560 K to 620 K at high southern latitudes, 170 K at low latitudes and 460 K to 500 K at high northern latitudes. We note here, as in the previous chapter, that the Joule heating term is not produced in Model E1 equatorward of $\pm 56^\circ$ because the MEF is restricted to high latitudes. In Model E2c, FACs are restricted to $\pm 56^\circ$ at high latitudes, so Joule heating is not produced along field lines that do not intersect the upper boundary or along those that do so equatorward of $\pm 56^\circ$ latitude. As expected, Model E2b is between Models E2a and E2c. Importantly, Model E1 has the broadest transition between polar and midlatitude temperature regimes.

The winds of the three models are also quite different. The horizontal components of the wind are compared in Figure 5.6. In Model E1, zonal winds are broadly peaked around $\pm 78^\circ$ latitude, with westward peaks of roughly 2600 m s$^{-1}$ in the south and 1400 m s$^{-1}$ in the north. This coincides with the peak electric field
Figure 5.5: Comparison of the temperatures of Models E1, E2a, E2b and E2c at $10^{-8}$ mbar.
strength, as high-latitude westward jets are caused by magnetobraking. Meridional winds peak at ±62° latitude, with equatorward peaks of roughly 175 m s\(^{-1}\) in the south and 110 m s\(^{-1}\) in the north at high altitudes. The winds of Model E2a are weaker in every way. Zonal winds peak closer to the poles at ±80° with westward peaks of roughly 1250 m s\(^{-1}\) in the south and 900 m s\(^{-1}\) in the north at high altitudes. Meridional winds peak also peak near ±80°. Equatorward of roughly ±55°, zonal winds and meridional winds are negligible. Zonal winds in Model E2c peak at ±78° with westward peaks of roughly 2300 m s\(^{-1}\) in the south and 2050 m s\(^{-1}\) in the north at high altitudes. The peaks are almost as broad as in Model E1 but are shifted northward as in Model E2a. High-altitude meridional winds in Model E2c exhibit equatorward peaks of roughly 250 m s\(^{-1}\) at −66° latitude and 200 m s\(^{-1}\) at +70° latitude.

Figure 5.6: Comparison of the southward (Figure 5.6a) and eastward (Figure 5.6b) components of the wind in Models E1, E2a, E2b and E2c at 10\(^{-8}\) mbar. In both figures, it is evident that Model E1 has more structure and/or is stronger at mid-latitudes than the other models. The winds of Model E2a are stunted compared to Models E1 and E2c because weak electrodynamics causes weak Joule heating and cooler temperatures.

In summary, replacing STIM’s original MEF with the zero-wind PEF calculated from the FAC boundary condition shifts the region of peak electric field poleward.
As could be expected, stronger FACs at the upper boundary (representing stronger magnetospheric forcing) are associated with stronger electric fields, higher temperatures and stronger winds. In spite of Model E2a having peak electric fields of the same magnitude as Model E1, Model E2a is significantly cooler and has weaker winds. Model E2a, on the other hand, has FACs which are twice as strong as Model E1, yet it has similar high-latitude temperatures and winds. This demonstrates that the ionosphere and thermosphere are very sensitive to the imposed FAC profile and to the conductivity profile. We discuss these results later in Section 5.4.2.

5.3.2 Comparison of Models E2b With E3g/E3fm

We now consider the effects of imposing a wind profile at the lower boundary while using the zero-wind PEF. To do this, we set aside Models E2a and E2c and compare Models E2b, E3g and E3fm due to their common FAC scaling factor of 1.5. These models were identical except that Models E3g and E3fm have the imposed zonal wind profile at the lower boundary whereas Model E2b does not. These wind profiles are dominated by an eastward jet at low latitudes, but the imposed wind profile in Model E3fm has some structure at midlatitudes plus a \(80\text{ m s}^{-1}\) eastward jet at \(+66^\circ\) latitude. Despite these differences, the overall electric field, winds and temperature profiles are remarkably similar. At high latitudes, the similarity is presumably because the newly-imposed zonal jets are weaker at high latitudes. At low latitudes and middle latitudes, the zonal jets give more structure to the low-altitude winds, temperature and current density profiles, while keeping their overall magnitude similar.

The temperature profiles of Models E3g and E3fm are shown in Figure 5.7. The general data trends in these plots, especially at high altitudes, are strikingly similar to each other, as well as to Model E2b. The temperatures of all three models peak at about 450 K at the south pole and 375 K at the north pole, and the minimum at middle and low latitudes is roughly 160 K to 170 K. The biggest difference between these runs in the shape of the temperature profile at middle and low latitudes as well as the structure evident at ionospheric levels. In Model E2b,
the temperature at all pressure levels is nearly constant in latitude between ±45° latitude. In Model E3g, however, temperatures near the top of the domain increase at midlatitudes by a modest 30 K or so between the equator and ±45° latitude, smoothing the meridional temperature gradient somewhat. At pressures of roughly $10^{-4}$ mbar to $10^{-5}$ mbar, temperatures increase from the equator, then fall, then rise again sharply. A similar pattern is present in Model E3fm, but this is imposed on an overall trend of temperatures decreasing from north to south. This asymmetry is because the wind profile used in Model E3fm is asymmetric about the equator. The wave-like patterns in these temperature profiles, particularly at $10^{-4}$ mbar to $10^{-5}$ mbar, are a result of adiabatic heating and cooling.

![Temperature profiles of Model E3g and E3fm](image)

(a) Model E3g temperatures.  
(b) Model E3fm temperatures.

Figure 5.7: Temperature profiles of Model E3g (Figure 5.7a) and Model E3fm (Figure 5.7b). The general shape at high latitudes is very similar, but the two imposed wind profiles give different structure and, in the case of Model E3fm, a latitudinal gradient at the higher altitudes.

The zonal winds of Models E3g and E3fm are shown in Figure 5.8. The lowest pressure levels have a clear imprint from the wind profiles imposed on each respective model, but are otherwise remarkably similar to that of Model E2b in every other way. This imprint fades with increasing altitude (decreasing pressure) due to viscous effects.
Figure 5.8: Zonal wind profiles of Model E3g (Figure 5.8a) and Model E3fm (Figure 5.8b). Positive values indicate eastward velocity. The gold line, being near the base of STIM’s domain, approximately reflects the imposed wind profile. Strong westward jets at high latitudes are present at higher altitudes (lower pressures).

Perhaps the most dramatic change in Models E3g and E3fm is in the low–latitude currents. The most dramatic of these is the radial current density, shown in Figure 5.9. Whereas both radial and equatorial currents at low latitudes had been weaker than $10^{-9} \text{ A m}^{-2}$, they are now of order $10^{-8} \text{ A m}^{-2}$ at $10^{-5} \text{ mbar}$. There is a small region in Model E3g between $\pm 30^\circ$ latitude with eastward currents of roughly $4 \times 10^{-8} \text{ A m}^{-2}$. In Model E3fm, this region is broader, zonal currents are weaker, and there is a narrow region of westward current between $+30^\circ$ to $+50^\circ$ latitude.

Because the winds have not yet been included in the calculation of the PEF, this combination of Hall and (primarily) Pedersen currents is due only to the $\vec{u} \times \vec{B}$ terms in the CM electric field, which are amplified by the conductivities. These terms are associated with ion drag which almost exclusively opposes the zonal wind at low and middle latitudes. This is as it should be: if the current density were driven by primarily zonal winds, with no MEF or PEF, then it could be expressed as

$$\vec{j} \sim \sigma \left( \vec{u} \times \vec{B} \right).$$

(5.1)
The ion drag associated with this current would be given

\[ \vec{j} \times \vec{B} = \sigma \left[ \left( \vec{u} \cdot \vec{B} \right) \vec{B} - B^2 \vec{u} \right] \]

\[ \approx - \sigma B^2 \vec{u}. \]  

Later, when the winds are used to calculate the PEF, the PEF will in general oppose the \( \vec{u} \times \vec{B} \) terms and the system will have ion drag that opposes gradients in \( \vec{u} \times \vec{B} \) along field lines. Although stronger winds are therefore associated with stronger electrodynamics in general (as is the case in Models E3g and E3fm), we can expect that the effects of these currents will be carried along the entire field line when the winds are included in the calculation of the PEF. The strong low-latitude currents in Models E3g and E3fm therefore suggest that the presence of significant zonal winds at low latitudes could greatly enhance the impact of electrodynamics at low and possibly even at middle latitudes.

(a) Model E3g radial current density.  
(b) Model E3fm radial current density.

Figure 5.9: Radial currents profiles of Model E3g (Figure 5.9a) and Model E3fm (Figure 5.9b). Positive values indicate upward current flow.

5.3.3 Comparison of Models E3g/E3fm With E4g/E4fm

In this subsection, we present results from Models E4g and E4fm, the models of a wind-driven dynamo in Saturn’s ionosphere and thermosphere. The key differences
between these models and previous models (e.g. Khurana et al., 2018; Provan et al., 2019; Vriesema et al., 2020) are that these models are incorporated into the STIM GCM and they are partially coupled to a magnetosphere model through the imposition of FACs along the upper boundary. This results in a significantly different results for each wind profile.

While the temperatures of Models E3g and E3fm were generally similar to each other (and to Model E2b), the wind dynamo significantly changes the temperature profile in Models E4g and E4fm. The high-altitude temperature profiles of these four models are compared in Figure 5.10 for convenience. More detailed temperature profiles of Models E4g and E4fm are shown in Figure 5.11.

Figure 5.10: Comparison of the zonally-averaged temperature profiles of Models E3g, E3fm, E4g and E4fm at $10^{-8}$ mbar, near the top of the domain.
(a) Model E4g temperatures. (b) Model E4fm temperatures.

Figure 5.11: Temperature profiles of Model E4g (Figure 5.11a) and Model E4fm (Figure 5.11b). The general shape at high latitudes is very similar, but the two imposed wind profiles give different structure and, in the case of Model E4fm, a latitudinal gradient at the higher altitudes.

Most striking in both models is how much hotter they are than in Models E3g and E3fm. Although high-altitude temperatures are similar at low latitudes, they are 100 K to 130 K warmer at the south pole and roughly 70 K warmer at the north pole. Besides the warmer poles, the biggest difference between these models and their earlier counterparts is at midlatitudes, where the magnitude of the “wavy” latitudinal structure (due to adiabatic heating/cooling) is reduced. Models E4g and E4fm have similar temperature profiles at high latitudes, likely because the differences in their respective wind profiles are negligible near the poles. At low and middle latitudes, the temperature in both Models decreases slightly with increasing latitude, as in Model E3fm. For Model E4fm, the temperature at high altitudes decreases by roughly 4 K per 15° of latitude moving northward, but this gradient is closer to 1 K per 15° in Model E4g. This likely is a reflection of the south pole being warmer than the north pole.

The winds are another important diagnostic of what is happening in the models. The zonal winds of Models E4g and E4fm are shown together in Figure 5.12, and
the meridional winds of each are shown in Figure 5.13.

(a) Model E4g zonal winds.  
(b) Model E4fm zonal winds.

Figure 5.12: Zonal wind profiles of Model E4g (Figure 5.12a) and Model E4fm (Figure 5.12b). Positive values indicate eastward velocity. The gold line, being near the base of STIM’s domain, approximately reflects the imposed wind profile. Strong westward jets at high latitudes are present at higher altitudes (lower pressures).

(a) Model E4g meridional winds.  
(b) Model E4fm meridional winds.

Figure 5.13: Meridional wind profiles of Model E4g (Figure 5.13a) and Model E4fm (Figure 5.13b). Positive values indicate southward velocity.

At the $10^{-3}$ mbar level in each model, near STIM’s lower boundary, the zonal
winds resemble the imposed zonal wind profiles, as expected. Consistent with the higher temperatures at high latitudes, the westward jets at high latitudes and high altitudes are enhanced in Models E4g and E4fm. Model E4g has westward winds reaching 2500 m s\(^{-1}\) in the southern hemisphere and 1800 m s\(^{-1}\) in the northern hemisphere. The westward jets of Model E4fm reach 2700 m s\(^{-1}\) in the southern hemisphere and 1250 m s\(^{-1}\) in the northern hemisphere. These, along with the temperature profiles, suggest that the southern hemisphere is significantly more active in these models than in the northern hemisphere. These jets are inconsistent with previous STIM models in that the jets are far too strong. They are also inconsistent with the results of Brown et al. (2020), who infer from occultations that the westward jets may actually peak at around ±60° rather than at higher latitudes, as has been previously expected.

The electric fields of Models E3g, E3fm, E4g and E4fm are shown in Figure 5.14. The electric field is zero at low and middle latitudes for Models E3g and E3fm because the FACs used to calculate the PEF are zero between ±56° latitude, and winds are not included in the calculation. Because of this, the PEF calculated in Models E4g and E4fm equatorward of roughly ±56° latitude is driven entirely by thermospheric winds. The radial component of the PEF is upward at low latitudes with a magnitude of order 10\(^{-3}\) V m\(^{-1}\) to 10\(^{-2}\) V m\(^{-1}\). In both Models E4g and E4fm, the PEF is also roughly symmetric about the magnetic equator at low latitudes in spite of the imposed wind profile at the lower boundary not being completely symmetric. At high latitudes, the PEF of Models E4g and E4fm closely resembles that of Models E3g and E3fm with a strong equatorward component of roughly 0.1 V m\(^{-1}\), though it is slightly stronger than the previous two models due to winds being used to calculate the PEF. The peak PEF is slightly more equatorward in the later models by a degree or two, but this is difficult to assess in our model, as our latitudinal resolution is only 2°. These fields are discussed further in Section 5.4.2.

Previously, we saw that the presence of significant zonal winds at low and middle latitudes created equatorward and downward currents of order 10\(^{-8}\) A m\(^{-2}\) at the 10\(^{-5}\) mbar pressure level. We now consider how including the winds in the PEF
Figure 5.14: Comparison of the radial (Figure 5.14a) and colatitudinal (Figure 5.14b) component of the PEF of Models E3g, E3fm, E4g and E4fm along the $10^{-4}$ mbar pressure level. This profile is very roughly constant with altitude, except at low latitudes. At higher altitudes (lower pressures), the narrow peak at the magnetic equator, present in this figure, disappears.

calculation affects the current structure. The three components of current density as calculated by STIM are shown in Figures 5.15 to 5.17. We note again that because STIM does not calculate FACs, these are the Pedersen and Hall currents only. Relative to Model E3g, low-latitude currents in Model E4g are greatly reduced while auroral currents are modestly increased. The low-latitude currents are reduced because ion drag and resistive heating, now incorporated self-consistently, have brought the system to a state where the electrodynamic stresses required to balance the mechanical stresses are minimal. In Model E4fm, the low-latitude current system is somewhat reduced relative to Model E3fm, but it is still very prominent, and the radial currents at the ionospheric peak ($10^{-5}$ mbar) is clearly consistent with the predicted current system described by Khurana et al. (2018). This suggests that the wind profile imposed on Model E4fm imposes more mechanical stress on the thermosphere than does the wind profile in Model E4g, and therefore the steady-state solution requires more electrodynamic stresses in the form of ion drag to counteract it. In terms of the simple model described by Khurana et al. (2018) and Vriesema et al. (2020), the peak of the equatorial jet is shifted further south
in Model E4fm relative to the equatorial plane of the magnetic field than it is in Model E4g, and this greater offset drives stronger currents at low latitudes. The current system in Model E4fm is also much more antisymmetric in the low-latitude ionosphere than it had been previously. This is because the closed, low-latitude field lines couple the hemispheres and transport angular momentum from one side to the other, as described by Vriesema et al. (2020). From this perspective, the reason Model E4g has weaker low-latitude currents is because its equatorial jet is more closely centered on the magnetic equator, so there is a weaker overall wind shear across the hemispheres to drive a current system than in Model E4fm. Whereas convergent, meridional currents at $10^{-5}$ mbar of order $10^{-8}$ A m$^{-2}$ persist to within $\pm 20^\circ$ of the magnetic equator in Model E3g, they are 1–2 orders of magnitude weaker in Model E4g, though still convergent at the equator. In Model E4fm, the currents at this level are northward at all latitudes south of roughly $+48^\circ$, and are relatively symmetric about the magnetic equator. The previous result that zonal winds at low latitudes can drive a low-latitude current system is supported by both models, though most strongly by Model E4fm due to the greater distance of the zonal jet from the magnetic equator.

At auroral latitudes (roughly $\pm 76^\circ$ latitude), equatorward currents are approximately $1.7 \times 10^{-7}$ A m$^{-2}$ to $2.3 \times 10^{-7}$ A m$^{-2}$, with remarkably similar currents in both Models E4g and E4fm. There, primarily Hall currents of order $10^{-7}$ A m$^{-2}$ to $10^{-6}$ A m$^{-2}$ flow eastward at the $10^{-3}$ mbar to $10^{-5}$ mbar pressure levels. Interestingly, the zonal currents in Model E4fm at low and middle latitudes are eastward in the southern hemisphere and westward in the northern hemisphere, with magnitudes of order $10^{-8}$ A m$^{-2}$.

Shown in Figure 5.18 is the antiparallel component of current density, calculated outside of STIM as described in Section 3.3.2. Model E4g has negligible FACs at low latitudes, but Model E4fm has FACs flowing south-to-north across the equator, in the opposite sense as that described by Khurana et al. (2018). These FACs are of the order $2 \times 10^{-9}$ A m$^{-2}$, except at the equator, where nearly-horizontal FACs jump to almost $4 \times 10^{-8}$ A m$^{-2}$ within $5^\circ$ of the equator. At high latitudes, both
Figure 5.15: Radial current profiles of Model E4g (Figure 5.15a) and Model E4fm (Figure 5.15b). Solid lines indicate positive (upward) current flow while dotted lines indicate negative (downward) current flow.

Figure 5.16: Meridional current profiles of Model E4g (Figure 5.16a) and Model E4fm (Figure 5.16b). Solid lines indicate positive (southward) current flow while dotted lines indicate negative (northward) current flow.
models predict FAC peaks in both hemispheres of approximately $2.5 \times 10^{-8}$ A m$^{-2}$ at the $10^{-5}$ mbar pressure level.

The Pedersen and Hall components are important for ion drag and Joule heating, but without the FACs they give an incomplete picture of the ionosphere and thermosphere. A new picture develops when the FACs, calculated in post-processing as described in Vriesema et al. (2020), are included. At low and especially at middle latitudes in both Models E4g and E4fm, the radial component of current density (including FACs) is downward in the southern hemisphere near the $10^{-5}$ mbar pressure level, but upward both below and above that level. In the northern hemisphere of each model, the current density is upward near the $10^{-5}$ mbar pressure level. However, it is downward above and below the $10^{-5}$ mbar pressure level for Model E4fm but is upward for Model E4g. The current system in these models at low and middle latitudes is in the sense predicted by Khurana et al. (2018) only near the pressure level corresponding to the peak Pedersen conductivity, but that it may not persist above or below this layer. This underscores the importance of resolving the vertical structure of the ionosphere and thermosphere.
(a) Model E4g antiparallel current density. (b) Model E4fm antiparallel current density.

Figure 5.18: The anti-parallel current density profiles of Model E4g (Figure 5.18a) and Model E4fm (Figure 5.18b). Solid values indicate positive $j_\beta$ (south-to-north flow), opposite to the direction of the magnetic field, while dotted lines indicate negative values (currents pointing in the direction of the magnetic field). Values are missing near the poles because those points are beyond the domain of the grid in dipole coordinate space on which we calculate electrodynamics.

We note that Figure 5.15 shows radial currents decreasing with decreasing pressure towards the top of the domain. This is because Pedersen (and, to a lesser extent, Hall) currents rapidly die off above the $10^{-6}$ mbar pressure level due to decreasing Pedersen (Hall) conductivity. This forces FACs to flow almost entirely along field lines, with little divergence — and therefore little change — along a given field line. Indeed, the full current density profile (including FACs) predicted by these models is nearly constant with altitude at and above the $10^{-6}$ mbar pressure level at high latitudes. There, $1 \times 10^{-8}$ A m$^{-2}$ currents flow upward at roughly $\pm 72^\circ$ latitude, while downward currents of roughly $2 \times 10^{-8}$ A m$^{-2}$ increase toward the poles. As should be expected, this corresponds to the imposed FAC profile (c.f. Figure 5.2) along the upper boundary.

We note that the FACs calculated at the top of the domain in post-processing do not match the imposed values exactly, as is shown for Model E4g in Figure 5.19. Instead, the peaks in the calculated FACs profile are offset poleward of the imposed
FAC peaks by 1°. This is likely due to the numerical scheme used in post-processing, and it indicates the degree to which our results are to be trusted. In post-processing as in STIM, the PEF is calculated starting from inner field lines and working toward outer field lines because each field line requires the calculation of the field line inward of it. It is reasonable to expect that the peaks in calculated FACs would be in the same direction (either poleward or equatorward) in both hemispheres due to the propagation of numerical error. Because STIM’s latitudinal resolution in all of the models described in this study is 2°, this 1° offset does not seem significant enough to affect the models significantly.

Figure 5.19: The imposed FAC profile (scaled from the original BATSRUS profile as described in Section 5.2.2) compared to the FACs calculated from the STIM results in post-processing.

We now consider the effect of the wind dynamo on Joule heating as defined in Equation 43 of Vasyliunas and Song (2005). We note that this Joule heating term differs from the Joule heating term calculated in STIM’s energy equation (Equation (2.117)). The term used in STIM’s energy equation is $\vec{j} \cdot \vec{E}$, which includes the effects of both “conventional ionospheric Joule heating” and ion drag (Vasyliunas and Song, 2005) and is in some places negative. The term we presently consider is $\vec{j} \cdot \vec{E}_{CM}$, which is positive-definite. The Joule heating rates for Models E3g and E4g are shown in Figure 5.20. Moving from Model E3g to E4g, there is at high latitudes slightly more Joule heating, while at midlatitudes there is up to two orders of magnitude less Joule heating. At low latitudes, there is 1–3 orders of magnitude less Joule heating in the dynamo region in Model E4g than in Model E3g. This is largely because of the lack of significant radial currents at low latitudes in Model E4g compared to Model E3g. This reduced heating at low latitudes is a major result
which has significant implications for interpreting a wind dynamo in Saturn’s upper atmosphere.

Finally, we consider the effects of ion drag by comparing Models E3g and E4g. The meridional component of ion drag is shown in Figure 5.21 and the zonal component is shown in Figure 5.21. Whereas auroral ion drag was roughly $5 \times 10^{-3} \text{ m s}^{-2}$ at $10^{-5} \text{ mbar}$ in Model E3g, it grew by 10–20% in Model E4g. This is roughly the same amount that the zonal winds grew by as well. At low and middle latitudes, ion drag at the same pressure level had been of the order $10^{-4} \text{ m s}^{-2}$ and was directed equatorward, but is now negligible. The zonal component of ion drag is similarly enhanced, though by 20–80%, at high latitudes and is reduced at low latitudes.

At almost everywhere in the domain (with the poles being the most notable exception), the zonal ion drag is negligible below the $10^{-5} \text{ mbar}$ pressure level, westward at $10^{-5} \text{ mbar}$, and eastward above that pressure level. At the poles, ion drag is westward and equatorward, but it transitions to eastward and poleward at around $\pm (75^\circ - 80^\circ)$. At low and middle latitudes, the zonal ion drag is generally balanced by the Coriolis force, consistent with the “modified geostrophic balance” assumptions of Brown et al. (2020). This indicates that prograde angular momentum is
being lost in the ionosphere and is being transported upwards along magnetic field lines to the magnetosphere, causing the strong westward jets at high latitudes. Near the poles and equator, the balance is generally more complicated.

(a) Model E3g meridional ion drag.  
(b) Model E4fm meridional ion drag.

Figure 5.21: Meridional ion drag profiles in Model E4g (Figure 5.21a) and Model E4fm (Figure 5.21b). Positive values indicate a southward acceleration.

(a) Model E3g zonal ion drag.  
(b) Model E4fm zonal ion drag.

Figure 5.22: Zonal ion drag profiles in Model E4g (Figure 5.22a) and Model E4fm (Figure 5.22b). Positive values indicate an eastward acceleration.
5.4 Discussion

The goal of this study was to model a wind-driven dynamo self-consistently in STIM in order to determine how it would affect the circulation and energy balance in Saturn’s thermosphere. To do this, we extended the steady-state, axisymmetric model of Vriesema et al. (2020) to use FACs along the upper boundary of the domain as a boundary condition to constrain the PEF. We then implemented this into STIM, using FACs from a magnetosphere model at equinox (Jia et al., 2012a). We used a series of STIM models, also at equinox, to examine how these changes affected the upper atmosphere. First, we replaced STIM’s MEF with a zero-wind PEF constrained by FACs — effectively calculating the MEF differently — and saw that this shifted the electric field poleward, leading to weaker electrodynamics, weaker winds and cooler temperatures. Second, we imposed a zonal jet at the lower boundary and again used the zero-wind PEF as the electric field. We found that low-latitude electrodynamics was enhanced, but there was little overall effect on the structure and dynamics of the thermosphere. Finally, we included the winds in the PEF calculation. We found that this boosts electrodynamics at high latitudes but reduced it at low latitudes. In this section, we discuss what these results can tell us about the role of electrodynamics in Saturn’s upper atmosphere.

5.4.1 A Simple Model of the Wind Dynamo

Our electrodynamics formulation uses a scheme originally developed for Earth’s atmosphere by Richmond (1973a), adapted for Saturn by Vriesema et al. (2020), and was extended in the present study to use FACs or azimuthal magnetic field measurements along the upper boundary to constrain the electrodynamics. Conceptually, this formulation may be explained as follows. Assuming a steady-state, axisymmetric system, the PEF is assumed to satisfy the current continuity equation. Conceptually, our formulation assumes that the PEF is constant along a field line (within a geometric scale factor) and calculates that value for each field line using the winds and the Pedersen and Hall conductivities. The steady state assumption
comes from a balance of “mechanical stress balance, mediated by electromagnetic fields” (Vasyliunas, 2012). The currents resulting from this calculation induce ion drag that acts to reduce stresses caused by gradients in \((\sigma_P u_\phi - \sigma_H u_\alpha) B_\beta\) along a given field line segment. In the absence of external forcing, a given system would eventually reduce all such gradients \textit{entirely}, transporting momentum from part of the field line with an excess of momentum to part of the field line with a deficit via short-timescale Alfvén waves, as described by Vasyliunas (2012); Vriesema et al. (2020). In the process, energy is dissipated in the form of Joule heating as the system adjusts itself via collisions, allowing the system to relax into an equilibrium state. As these gradients are smoothed, the PEF and perpendicular currents decrease, causing less ion drag and less Joule heating. In a system with forcing (often in the form of boundary conditions), the gradients in the wind can be maintained with ion drag opposing the forcing. In such a system, large-scale currents can be established in a state of dynamic equilibrium,

We gain insight by exploring some of the interesting implications of our formulation in a simplified system using Cartesian geometry. We can imagine a single, straight section of a field line \(\vec{B} = B\hat{x}\) divided into three regions (or segments) of equal length labelled 1, 2 and 3 on the \(x\) axis. Each region has its own perpendicular wind velocity \(\vec{u}_i = u_i\hat{y}\) and Pedersen conductivity \(\sigma_i\). As in our formulation, we assume effectively infinite parallel conductivity. Currents are zero at either end of the field line segment. We assume the magnetic field \(B\) is uniform on the segment. Generally, the PEF \(\vec{E} = E\hat{z}\) will be constant along the whole field line section with value

\[
E = \frac{\int \sigma(x) \left[ -\vec{u}(x) \times \vec{B} \right] dx}{\int \sigma(x) dx} \tag{5.4}
\]

\[
= \frac{\int \sigma(x) u(x) B \, dx}{\int \sigma(x) \, dx} \tag{5.5}
\]

\[
\approx \frac{\sum_{i=1}^{3} \sigma_i u_i B}{\sum_{i=1}^{3} \sigma_i} \tag{5.6}
\]

\[
= \frac{(u_1\sigma_1 + u_2\sigma_2 + u_3\sigma_3) B}{\sigma_1 + \sigma_2 + \sigma_3}, \tag{5.7}
\]
the CM electric field \( \vec{E}^{CM} = E^{CM} \hat{z} \) at the \( i \)th segment as
\[
E_i^{CM} = E - u_i B, \quad (5.8)
\]
the current density \( \vec{j} = j \hat{z} \) at the \( i \)th segment as
\[
j_i = \sigma_i E_i^{CM}, \quad (5.9)
\]
and the ion drag \( \vec{f} = f \hat{y} \) acting at the \( i \)th segment as
\[
f_i = j_i B, \quad (5.10)
\]
We consider several scenarios below.

In Scenario 1, we assume uniform conductivity and uniform winds. Here, the PEF will be
\[
E = \frac{3uB\sigma}{3\sigma} = uB, \quad (5.11)
\]
and the CM electric field at the \( i \)th segment will be
\[
E_1^{CM} = (uB) - uB = 0 \quad (5.12)
E_2^{CM} = (uB) - uB = 0 \quad (5.13)
E_3^{CM} = (uB) - uB = 0. \quad (5.14)
\]
The current will be zero, and the ion drag will be zero. This system is in equilibrium and experiences no electrodynamics.

In Scenario 2, we assume uniform conductivity but let \( u_1 = +u \), \( u_2 = 0 \) and \( u_3 = -u \) to represent a wind shear. The PEF will be
\[
E = \frac{\sigma (u + 0 - u) B}{3\sigma} = 0, \quad (5.15)
\]
and the CM electric field at the \( i \)th segment will be
\[
E_1^{CM} = (0) - (+u)B = -uB \quad (5.16)
E_2^{CM} = (0) - (0)B = 0 \quad (5.17)
E_3^{CM} = (0) - (-u)B = +uB. \quad (5.18)
\]
The currents will be

\[ j_1 = -\sigma uB \]  \hspace{1cm} (5.19)  
\[ j_2 = 0 \]  \hspace{1cm} (5.20)  
\[ j_3 = +\sigma uB, \]  \hspace{1cm} (5.21)

and the ion drag

\[ f_1 = -\sigma uB^2 \]  \hspace{1cm} (5.22)  
\[ f_2 = 0 \]  \hspace{1cm} (5.23)  
\[ f_3 = +\sigma uB^2. \]  \hspace{1cm} (5.24)

We see that the sign of the ion drag for segments 1 and 3 is opposite that of the wind. This tells us that over time, the ion drag will decelerate the wind at the first segment and will accelerate the wind at the third until the wind shear has been eliminated. Similar scenarios can be considered, but this behavior will persist. This will be used later to explain how electrodynamics at low latitudes can spread an imposed equatorial jet in latitude.

Having examined the effects of wind shear, we now consider the effect of nonuniform conductivity. In Scenario 3, we assume the winds are uniform, but set \( \sigma_1 = \sigma_3 = \sigma \) and \( \sigma_2 = 0 \). Now, the PEF will be

\[ E = \frac{uB\sigma + 0 + uB\sigma}{2\sigma} = uB, \]  \hspace{1cm} (5.25)

as in Scenario 1, and the CM electric field at the \( i \)th segment will be

\[ E_{1CM}^{CM} = (uB) - uB = 0 \]  \hspace{1cm} (5.26)  
\[ E_{2CM}^{CM} = (uB) - uB = 0 \]  \hspace{1cm} (5.27)  
\[ E_{3CM}^{CM} = (uB) - uB = 0. \]  \hspace{1cm} (5.28)

As with Scenario 1, the winds in this system are “balanced” and so they experience no electrodynamics.
Next, we consider the effect of a single conducting segment. In Scenario 4, we assume the winds are as in Scenario 2 \((u_1 = +u, u_2 = 0\) and \(u_3 = -u\)), but use the conductivities of Scenario 3 \((\sigma_1 = \sigma_3 = \sigma\) and \(\sigma_2 = 0\)). The PEF will be

\[
E = \frac{uB\sigma + 0 - uB\sigma}{2\sigma} = 0.
\] (5.29)

The CM electric field at the \(i\)th segment will be

\[
E_{1}^{CM} = (0) - (+u)B = -uB
\] (5.30)
\[
E_{2}^{CM} = (0) - (0)B = 0
\] (5.31)
\[
E_{3}^{CM} = (0) - (-u)B = uB.
\] (5.32)

The currents will be

\[
\begin{align*}
  j_1 &= -\sigma uB \\
  j_2 &= 0 \\
  j_3 &= +\sigma uB,
\end{align*}
\] (5.33)
(5.34)
(5.35)

and the ion drag

\[
\begin{align*}
  f_1 &= -\sigma uB^2 \\
  f_2 &= 0 \\
  f_3 &= +\sigma uB^2.
\end{align*}
\] (5.36)
(5.37)
(5.38)

As with Scenario 2, this system will have no current in the middle segment, but will have currents and ion drag in segments 1 and 3. As in Scenario 2, the system will transport momentum along the field line to eliminate the wind shear. In fact, Scenario 4 is analogous to the thin-layer model described in Section 3.5.1 if segment 1 is the thin ionosphere layer in the northern hemisphere, segment 3 is the thin ionosphere layer in the southern hemisphere, and segment 3 is the non-conducting region between them and above the thin ionosphere.

Now we consider a case more analogous to Saturn’s upper atmosphere. We imagine a vertical field line at a northern high latitude between the upper and
lower boundaries. In Scenario 5, where the conductivity of the first segment is $\sigma_1 = 10\sigma$, $\sigma_2 = 1000\sigma$ and $\sigma_3 = \sigma$. Here, segment 1 represents the region of limited Pedersen conductivity (the Hall region), segment 2 represents the layer of peak Pedersen conductivity, and segment 3 represents the high-altitude region with almost no Pedersen conductivity. In STIM, zonal winds in the lower region at high latitudes are negligible, so we set $u_1 = 0$. Winds in the Pedersen conductivity region are slightly westward, so we set $u_2 = -u$. In the upper region, zonal winds are strongly westward, so we set $u_3 = -12u$. The PEF in this scenario will be

$$E = \frac{0 - 1000uB\sigma - 12uB\sigma}{1011\sigma} = \frac{-1012}{1011}uB.$$ (5.39)

We note that because $\sigma_2 \gg \sigma_1, \sigma_3$, $E \approx u_2B$, demonstrating that the PEF gets its value primarily from the wind speed near the peak Pedersen conductivity, in agreement with Vriesema et al. (2020). The CM electric field at the $i$th segment will be

$$E_{1}^{CM} = \frac{-1012}{1011}uB - (0)B = \frac{-1012}{1011}uB,$$ (5.40)

$$E_{2}^{CM} = \frac{-1012}{1011}uB - (-u)B = \frac{-1}{1011}uB,$$ (5.41)

$$E_{3}^{CM} = \frac{-1012}{1011}uB - (-12u)B = \frac{11120}{1011}uB.$$ (5.42)

In this example, the CM electric field was nearly zero because at the peak Pedersen conductivity, $\vec{E} \approx -\vec{u} \times \vec{B}$. This is to be expected in many cases, and perhaps lends some credibility to models which treat the ionosphere as a single, infinitely thin layer. The currents predicted by Scenario 5 are

$$j_1 = \frac{-10120}{1011}\sigma uB,$$ (5.43)

$$j_2 = \frac{-1000}{1011}\sigma uB,$$ (5.44)

$$j_3 = \frac{11120}{1011}\sigma uB.$$ (5.45)

This is qualitatively similar to Model E4g near either auroral peak, though there
are numerous other complicating factors. Finally, the ion drag will be

\begin{align*}
f_1 &= \frac{-10120}{1011} \sigma u B^2 \\ f_2 &= \frac{-1000}{1011} \sigma u B^2 \\ f_3 &= \frac{11120}{1011} \sigma u B^2.
\end{align*}

(5.46) (5.47) (5.48)

We note that the region with the greatest conductivity in this scenario experiences the least ion drag. With small adjustments to the conductivities and the winds even in this simplified model, however, the behavior of the above models can change dramatically.

Indeed, the qualitative similarities with Model E4g end with the CM electric field. The perpendicular component of the current density is almost an order of magnitude larger at $10^{-5}$ mbar (very near the Pedersen conductivity peak) than even the $10^{-4}$ mbar or $10^{-6}$ mbar pressure levels. Because of this, the westward ion drag near the peak conductivity (segment 2 in this simplified model) is much greater in magnitude than the eastward ion drag at higher altitudes (segment 3), and the ion drag below (segment 1) is almost negligible. Part of the reason for these discrepancies is because an auroral field line in Model E4g does not easily separate into three neat segments with constant length, conductivity and wind velocity. In Model E4g, the upper segment (segment 3) is several times longer than the lower segment (segment 1), owing to a hotter temperature and greater scale height. More importantly, the Pedersen conductivity jumps by an order of magnitude near its peak, and the wind gradient is large near the peak. The Hall conductivity in STIM peaks just below the Pedersen peak, and while it plays a lesser role in determining the PEF, it is still present (see below). Finally, this simple model assumes the magnetic field strength is constant along a field line, but it will change as well. In short, the simple, three-segment model is not appropriate in this part of Saturn’s thermosphere due to sharp vertical gradients.

While the simple, three-segment model is useful for gaining physical intuition, the effects of ion drag are more complicated in general. Ion drag may be better
expressed as follows, using Equation (2.5):

\[ \vec{j} \times \vec{B} = \sigma_{\|} \left( \vec{E}_{CM} \cdot \hat{b} \right) \hat{b} \times \vec{B} + \sigma_P \left( \hat{b} \times \left( \vec{E}_{CM} \times \vec{b} \right) \right) \times \vec{B} - \sigma_H \left( \vec{E}_{CM} \times \hat{b} \right) \times \vec{B} \]

\[ \quad = \sigma_P \left( \left[ \vec{E}_{CM} - \left( \hat{b} \cdot \vec{E}_{CM} \right) \hat{b} \right] \right) \times \vec{B} - \sigma_H \left[ \left( \vec{E}_{CM} \cdot \hat{b} \right) \hat{b} - \left( \hat{b} \cdot \vec{B} \right) \vec{E}_{CM} \right] \]

\[ \quad = \sigma_P \left( \vec{E}_{\perp} \times \vec{B} \right) + \sigma_H \left( B \vec{E}_{\perp} \right) \]

\[ \quad = \sigma_P \left[ \vec{E}_{\perp} \times \vec{B} - \left( \vec{u} \cdot \vec{B} \right) \vec{B} - B^2 \vec{u} \right] + \sigma_H B \left( \vec{E}_{\perp} + \vec{u} \times \vec{B} \right). \]

This shows that although ion drag does have a term that is opposite to the winds and proportional to the Pedersen conductivity, it also has contributions from other terms in other directions and from the Hall conductivity. Both the simple model above and this more detailed expression of ion drag show that it is, in general, inappropriate to assume that ion drag universally reduces wind speed (e.g. via the $-\sigma_P B^2 \vec{u}$ term), because the contribution from the PEF will in some regions dominate the $\vec{u} \times \vec{B}$ contribution and cause winds to speed up. The relative magnitudes of these terms will vary widely throughout the thermosphere, and a detailed calculation of the PEF is required to estimate the importance of these terms. Because the calculation of the PEF is based on winds and conductivities along a field line, this comparison is not easily done at a given point with knowledge of only the Pedersen conductivity and wind velocity.

In spite of the discrepancies, insight can be gained from the simple model. Ion drag indeed acts to remove gradients in the conductivity-weighted wind shear. The PEF tends to be biased towards the value of $-\vec{u} \times \vec{B}$ near the region of strongest Pedersen conductivity, though other factors influence this. The greater the overall line-integrated conductivity, the smaller the relative influence from any one part of the thermosphere will have in determining the PEF. Attempts to treat the ionosphere as a single, thin layer are justified only when there is a single, clearly-dominant conducting layer with significant wind speeds. The wind shears that complicate estimating the PEF are not only vertical gradients in the wind such as $\frac{\partial u_\phi}{\partial r}$: hori-
horizontal shears such as $\frac{\partial u}{\partial \phi}$ are important at low latitudes where field lines are more horizontal. If a clearly-dominant conducting layer is near a region of large wind shear along the field line, treating the ionosphere as a thin layer is likely to fail. One can see that if the perpendicular currents at one or even all parts of a given field line were known, it would still not be possible to extract the wind speeds along the field line. For example, if $j_1$, $j_2$ and $j_3$ were known, it would be impossible to calculate the conductivities and winds from these currents alone, as discussed earlier in Vriesema et al. (2020). One would have to have knowledge of the conductivity along the field line, including any significant gradients, in order to estimate the winds, or vice-versa.

5.4.2 Magnetospheric Influence

The MEF previously used by STIM to model the effect of the magnetosphere on Saturn’s high-latitude thermosphere and ionosphere was calculated by Jia et al. (2012a) as the electric field necessary for ionospheric current closure. Two primary inputs used to calculate that MEF were FACs at 3 $R_S$ and the height-integrated Pedersen conductance, which was assumed to be uniform. Our model calculates an electric field from the FACs using height-dependent conductivities and winds. Essentially, our model sacrifices azimuthal resolution in order to resolve the height dependence throughout the thermosphere — including at low latitudes — and include winds in the calculation of the electric field. These changes have a significant effect on the electric field (whether the MEF or the zero-wind PEF), and because of this, they greatly effect the structure of the ionosphere and thermosphere. The differences between Models E1, E2a and E2c show that the ionosphere and thermosphere are extremely sensitive to the imposed electric field. Moreover, the location and magnitude of the auroral electric field depends critically on the conductivities and imposed FACs.
5.4.3 Poleward Shift of the PEF

The poleward shift in the electric field happens in part because our model uses nonuniform conductivities, whereas the MEF is based on the BATSRUS model which uses uniform conductances (height-integrated conductivities). The Pedersen and Hall conductivities have sharp peaks at $\pm 72^\circ$ latitude and $10^{-5} \text{ mbar}$ and $10^{-4} \text{ mbar}$, respectively. The conductivity drops rapidly poleward of these peaks by 1–2 orders of magnitude at low altitudes. For reference, the zonally-averaged, height-integrated conductivities of Model E1 are plotted in Figure 5.23. The imposed FAC profile (see Figure 5.2) has upward-flowing peaks at $\pm 72^\circ$ latitude, but the currents reverse direction poleward of about $\pm 76^\circ$ latitude.

![Figure 5.23: Plot of the Pedersen and Hall conductances (height-integrated conductivities) calculated from the zonally-averaged altitude and conductivity data of Model E1. The integration is between the $4.2 \times 10^{-3} \text{ mbar}$ and $3.4533 \times 10^{-9} \text{ mbar}$ pressure levels. We note that although the height-dependent conductivities and the Hall conductance peak at $\pm 72^\circ$ latitude, the Pedersen conductance peaks at $\pm 74^\circ$ latitude.](image)

Assuming uniform conductivity, we might expect that the latitudes at which the MEF peaks would also be where the meridional current peaks. In reality, this is complicated somewhat by the $\vec{u} \times \vec{B}$ terms in Equation (2.5), such that the meridional component of the CM electric field has a sharp equatorward peak at low altitudes near $\pm 78^\circ$ (dominated by the MEF) and a broader poleward peak at high altitudes near $\pm 68^\circ$ (dominated by the winds). The nonuniform conductivity,
however, modulates the CM electric field such that the equatorward current peaks are between the peaks of the MEF and the conductivity — at approximately ±74°.

Now, we perform a similar analysis using the zero-wind PEF, constrained by FACs at the upper boundary, instead of the MEF. In this scenario, the peak meridional currents are at ±72°. The zero-wind PEF is calculated in Equation (2.104). Neglecting contributions from inward field lines (the first term in the numerator of Equation (2.104)), the PEF is proportional to the FAC imposed at the upper boundary divided by a term involving the Pedersen conductivity. At high latitudes, field lines are nearly vertical, so the lower integral is highly sensitive to the location of the narrow auroral peaks of the Pedersen conductivity. The imposed FACs and the conductivities peak at the same latitude; however, the conductivity decreases much faster than the FACs. This means that when the PEF is calculated for each field line, both the numerator and denominator decay as we move poleward, but the denominator decays faster, causing the overall expression to grow. Moving poleward still, the numerator flips sign, causing the overall expression to quickly flip sign as well. Because, we can expect the PEF to peak at a slightly more poleward latitude than the peak in either the imposed FACs or the Pedersen conductivity. In practice, however, the inward field lines are significant and the denominator term is not so simple due to the geometric scale factors. The contribution of the inward field lines \(J_\alpha\) generally opposes that of the FACs \(J_\beta\), is greater by a factor of roughly 2, and peaks poleward of the FAC term — approximately where the imposed FACs reverse direction. (This should make sense: perpendicular currents should be strongest where parallel currents are weakest in order for \(\nabla \cdot \vec{j} = 0\) to hold.) Later, when the winds are used to calculate the PEF via Equation (2.103), they contribute an integral term that must be considered alongside \(J_\alpha\) and \(J_\beta\):

\[
- \int_{\beta_L(\alpha)}^{\beta_U(\alpha)} h_\beta h_\phi (\sigma_P u_\phi - \sigma_H u_\alpha) B_\beta d\beta' \tag{5.53}
\]

In practice, this term is weaker than but has the same sign as \(J_\alpha\), and because this integral contains the Pedersen and Hall conductivities, it peaks where the conductivities peak. The overall numerator in Equation (2.103) peaks at ±74° latitude.
Due to the geometric scaling factors and the slight curvature of dipole field lines, the denominator in Equation (2.103) peaks at approximately $\pm 73^\circ$ rather than at $\pm 72^\circ$. It then decreases poleward of its peak. However, due to the factor of $h^{-1}_\alpha = \frac{\sqrt{1+3\cos^2\theta}}{\sin^3\theta}$ in this integral, the denominator term increases toward infinity at the poles. This guards the polarization electric field against being infinite at the poles. At approximately $\pm 77^\circ$, however, the denominator term has a local minimum, and dividing by this minimum value is what causes the overall PEF to sharply increase and hit its peak value. Each of these terms for the full PEF are illustrated for Model E2b in Figure 5.24. We note that the integral involving the winds was computed and used to calculate $h_\alpha E_\alpha$ only for the purposes of this comparison, but the winds were not used to calculate the PEF in Model E2b.

In summary, the location and strength of the peak PEF — and the resulting currents, Joule heating and ion drag, which are primary drivers of the high-latitude thermosphere — is particularly sensitive to the location and shape of the auroral Pedersen conductivity peaks. It is also sensitive to the currents at lower latitudes through $J_\alpha$, and the FACs imposed at the upper boundary through $J_\beta$, and the dynamics of the thermosphere through the wind terms (for Models E4g and E4fm, which use the winds to calculate the full dynamo PEF). This suggests that understanding electrodynamics at lower latitudes is important for understanding electrodynamics at higher latitudes. This is not a novel result, however, but a reflection of our requirement that $\vec{\nabla} \cdot \vec{j} = 0$ given our boundary conditions. In our model, $J_\alpha$ indicates how much current is leaving or entering a region from inward field lines, $J_\beta$ indicates how much current is leaving or entering a region from the top of the domain, and the wind terms indicate how much of the remaining current — flowing in or out of the region from or away from outward field lines — is supplied by the winds. If significant currents are flowing from low or middle latitudes towards high latitudes, they will necessarily affect electrodynamics at higher latitudes.
Figure 5.24: A comparison of different terms in Equation (2.103) for field lines in Model E2a. The latitudes on the horizontal axis are the latitudes where each field line intersected the upper boundary. The green line is $J_\beta$ as defined in Equation (2.97), and its sign matches that of the imposed FAC profile in Figure 5.2. The red line is $J_\alpha$ as defined in Equation (2.98), and its sign is based on the fact that $j_\alpha$ is generally negative, especially at high latitudes. It is negative in the southern hemisphere because integrating proceeds in the $+\hat{\beta}$ direction, and it is positive in the southern hemisphere because integration happens in the opposite direction. The blue line, dominated by the Pederssen term, indicates the contribution from the winds, and the signs at the peaks are consistent with the integrand ($\sigma_P u_\beta B_\beta$ times scale factors) being positive. The cyan line is equal to the numerator in Equation (2.103): the wind term plus $J_\alpha - J_\beta$. We note that the numerator term peaks at about $\pm 74^\circ$ latitude. The magenta line is the integral of the Pedersen conductivity weighted by various scale factors, and ultimately reduced by a factor of $10^4$ for illustrative purposes. This term goes to infinity at the poles due to the factor of $h_\alpha^{-1} = \frac{\sqrt{1+3\cos^2\theta}}{\sin^2\theta}$, which prevents $h_\alpha E_\alpha$ from being infinite at the poles. Finally, the black line is $h_\alpha E_\alpha$, calculated as the cyan line divided by the magenta line, and then multiplied by $5 \times 10^9$ for illustrative purposes. The data in this plot do not extend to the poles because the outermost field lines intersect the upper boundary of the domain at the points shown.
5.4.4 Shifting the PEF to Match Observations

The poleward-shifted PEF moves the relevant models further from the results of (e.g.) Brown et al. (2020), which suggests that the high-latitude westward jets peak at ±60°. It is tempting to consider looking for a different FAC profile to impose along the upper boundary of STIM that would better reproduce these observations. In the models discussed in this study, the PEF is shifted poleward because the conductivity decreases more rapidly towards the poles than the imposed FAC profile. This poleward shift then causes the heating, temperatures and winds to be shifted poleward as well. In order to shift these quantities equatorward, the electric field may need to peak equatorward of the conductivity peaks. This could easily be accomplished in a numerical model by shifting the imposed FAC peak equatorward. Then, ion drag would likely peak at a somewhat lower latitude, driving westward jets at a lower latitude as well, which is at least more qualitatively consistent with Brown et al. (2020). Ion drag could also contribute toward the midlatitude-peaking Rayleigh drag profile that Müller-Wodarg et al. (2019) determined was able reproduce the observed temperatures at low latitudes (their “Simulation B”). However, their model predicted polar temperatures which are 100 K to 200 K higher than those reported by Brown et al. (2020). If Joule heating were shifted equatorward, it would likely lower the meridional temperature gradient at midlatitudes and reduce the “Coriolis barrier”, which would help facilitate meridional advection of energy. Regardless of the mechanism, having temperatures peak at a lower latitude could help drive poleward winds at high latitudes, leading to an upwelling at the poles. The convergence of these winds in the poles would result in upwelling and adiabatic cooling, which would help lower model temperatures at the poles. It is unclear how this would arise in Saturn’s auroral thermosphere, however, as the conductivity peak and electric field are both driven by the aurorae and could be expected to be coincident.

Although this mechanism could explain the surprising temperature profile from Brown et al. (2020), its relies on shifting the FACs at the upper boundary equator-
ward, and this ought to be motivated also by observational constraints. The FAC profile we use is only roughly consistent with the PPO-independent component of the auroral current system reported by Hunt et al. (2014): it has an upward peak around $\pm 72^\circ$ and downward current of $1 \times 10^{-8} \text{A m}^{-2}$ poleward of $\pm 75^\circ$. However, they also report twin secondary FAC peaks at roughly $\pm 70^\circ$. Rather than replacing the main FAC peaks equatorward, it could be that these smaller peaks at $\pm 70^\circ$ play a larger role in the thermosphere and ionosphere than expected.

As discussed in Vriesema et al. (2020), the electrodynamics is strongly dependent on the vertical and meridional structure of the conductivity and zonal wind profiles. Significant zonal wind shear along field lines (both vertical and meridional) induce large PEFs, which are associated with stronger Pedersen and Hall currents and more intense heating. For example, a wind profile which is strong at altitudes below the $10^{-5}$ mbar pressure level and is weak above would be associated with very large electric fields, Pedersen currents, ion drag and resistive heating. The actual wind profile of Saturn may be able to drive stronger electrodynamics than we have accounted for in these models. However, it seems very unlikely that the wind profile alone could be sufficiently different enough to produce $10$–$1000$ times more resistive heating.

It should be clear that knowing the location of the auroral conductivity peaks in particular is important for understanding electrodynamics in Saturn’s global thermosphere and ionosphere. These peaks help determine the location of the peak PEF, currents, Joule heating and ion drag, which are among the principal drivers of thermospheric dynamics and energetics, particularly at high latitudes. While we could try to produce the temperature and wind profiles of Brown et al. (2020) by shifting FACs equatorward, it is possible that the same effect could be realized if the sharp conductivity peaks were shifted slightly poleward instead. In this scenario, the unshifted FACs would be associated with PEF peaks slightly equatorward of the conductivity peaks.

Unfortunately, the locations of the peak conductivities, as well as their height dependence, are relatively unknown. The conductivities are known to vary in lati-
tude and local time (Moore et al., 2010; Sakai and Watanabe, 2016; Wahlund et al., 2018). The conductivities are expected to vary seasonally, both in position and in magnitude (by perhaps a factor of 4) (e.g. Southwood and Kivelson, 2009). The Pedersen conductivity used by STIM shows some of the variability described by Moore et al. (2010), but the present models of STIM do not account for the variability caused by many other sources. The recent reports of dusty plasma, heavier ions and hydrocarbons, ring rain, and more near the equator and along field lines (e.g. O’Donoghue et al., 2017; Yelle et al., 2018; Morooka et al., 2019; Persoon et al., 2019; Shebanits et al., 2020, and references therein) both complicate and constrain our understanding of Saturn’s ionosphere. There are numerous sources of variability in the conductivities besides these, and a detailed study of any such factors is expressly not the focus of this study. These effects have yet to be incorporated into STIM, but will undoubtedly prove useful in constraining thermospheric conductivities. At equinox, there is currently no reason to expect the conductivity peaks to be markedly different in either hemisphere. The gradients of the conductivities at low and middle latitudes could help drive current systems similar to the auroral system, though perhaps smaller in magnitude and/or more transient.

In spite of the many unincorporated sources of variability in the conductivities, however, we can gain some insight from considering the effects of globally higher or lower conductivities. Higher conductivities would in general cause the wind dynamo to be more efficient, coupling the neutral atmosphere and magnetic field lines more tightly and generating more Joule heating and ion drag that reduce wind shears along magnetic field lines faster. The wind dynamo requires continuous, external forcing of the winds in order to remain in steady state. A stronger wind dynamo will more quickly draw momentum from the neutral winds, requiring stronger non-electrodynamic mechanisms to feed momentum back to the winds in order for them to remain in a steady state. Lower conductivities would in general cause a weaker wind dynamo which would take energy from the neutral atmosphere more slowly.

As should be expected, the FACs imposed at the upper boundary significantly affect the current structure throughout the thermosphere, especially at high alti-
tudes. Although FACs for both models are effectively identical (though scaled by factors of order unity) at the top of the domain as a result of our boundary condition, they diverge at lower altitudes. This difference occurs in part because the MEF imposed in STIM does not vary with height, while the PEF varies such that $h_\alpha E_\alpha$ is constant along a given field line or field line segment in a given hemisphere. The primary reason for the difference is that the PEF calculation is dynamic and is based on the changing wind profile, as discussed in Section 5.4.5. Because the peak MEF coincides closely with the peak Pedersen conductivity, the radial and colatitudinal components of current density are as much as an order of magnitude higher at $10^{-4}$ mbar in Model E1 than in Model E2c, in spite of Model E2c having a stronger peak electric field. This underscores the importance of resolving the vertical structure of the thermosphere and the sensitivity of the thermosphere and ionosphere to a nonuniform conductivity profile.

Even if our imposed FAC profile, wind profile and conductivities are correct, the location of the PEF will also be affected by differences in geometry. This is because FACs associated with a given field line in our model will intersect a given pressure level at a slightly different latitude than it would in the BATSRUS model or in the real Saturn system. This happens for several reasons. First, our PEF is calculated on a shifted magnetic dipole field whereas BATSRUS uses an unshifted dipole field. Therefore, a field line intersecting the ionosphere at a given latitude in the BATSRUS model will intersect the ionosphere of our model at a slightly more northward latitude. Second, the real Saturnian magnetic field has quadrapole, octopole and higher-order terms which lead to still different mappings of magnetospheric FACs to the ionosphere. Third and finally, both STIM and BATSRUS assume spherical geometry for Saturn, ignoring the significant oblateness of the planet. This would also cause magnetic field lines to intersect the thermosphere at different latitudes if oblateness were taken into account.

We note that the differences between a spherical and oblate model of Saturn could introduce differences in latitude of a few degrees, which is similar to the differences between the auroral peaks of conductivity, FACs and electric fields. If
the structure of the thermosphere is sensitive to the conductivity peak being within 1° to 3° of a given latitude, it stands to reason that the thermosphere would be similarly sensitive to whether or not oblateness were accounted for in a given model.

The resulting energetics and dynamics at high latitudes depend critically on the electric field and current density. At $10^{-4}$ mbar, resistive heating in Model E1 is about an order of magnitude higher than in Model E2c. At higher altitudes, however, the resistive heating of Model E2c dominates that of Model E1. At all altitudes, the heat generated in Model E2c is primarily poleward compared to that of Model E1. This shift in heating moves the temperature profile of Model E2c poleward as well, causing steeper meridional temperature gradients. At the highest altitudes in our domain ($\sim 10^{-8}$ mbar), poleward ion drag is modestly enhanced in Model E2c. These factors combined cause heat to accumulate at the poles as described in Smith et al. (2007).

5.4.5 Low Latitudes

As with high-latitude field lines, low-latitude field lines can help to smooth out gradients in the winds by transporting momentum along the field line. At high latitudes, this mechanism is responsible for coupling the magnetosphere and ionosphere. Compared to field lines at higher latitudes which may extend out to several or dozens of Saturn radii in the equatorial plane and be dominated by magnetospheric processes, field lines at low latitude field lines experience less magnetospheric influence. Less magnetospheric influence between the hemispheres would allow more direct coupling of the hemispheres in the sense discussed by Vriesema et al. (2020). Here, intuition and arguments developed at high latitudes based on vertical field lines can fail, as field lines are more horizontal than vertical.

Based on the results of Vriesema et al. (2020), one might expect that the presence of an imposed zonal jet near the equator would increase electrodynamic activity at low latitudes. By comparing Model E2b to Model E3g or E3fm, we observe that this was indeed the case. Interestingly, neither imposed jet changed the temperature profile significantly at high altitudes or high latitudes. This is likely because
the imposed jets are much weaker at high latitudes and therefore do not significantly change high-latitude electrodynamics relative to models without an imposed equatorial jet.

When the winds were included in the calculation of the PEF, the low-latitude current system was significantly reduced. The reason for this can be determined by comparing the wind profiles of Models E3g and E4g (Figures 5.8a and 5.12a). In spite of these models having the same zonal wind imposed at the base of the domain, the jet in Model E3g dies off more quickly in altitude, halving its maximum speed of \(343 \text{ m s}^{-1}\) by roughly the \(8 \times 10^{-5} \text{ mbar}\) pressure level and dropping to zero by the \(10^{-6} \text{ mbar}\) pressure level. In Model E4g, the same equatorial jet drops to \(230 \text{ m s}^{-1}\) by the \(10^{-5} \text{ mbar}\) pressure level and does not diminish further. When a zonal jet is imposed on the lower boundary at the beginning of a model calculation, viscosity helps it climb to higher altitudes, and viscosity can also limit its growth.

The crucial difference between Models E3g and E4g is their treatment of electrodynamics. In Model E3g, the zero-wind PEF at low and middle latitudes is zero, so

\[
\vec{E}^{CM} = \vec{u} \times \vec{B}
\]  

(5.54)

and currents are due entirely to these \(\vec{u} \times \vec{B}\) terms. In this treatment of electrodynamics, all currents are calculated locally and there is no meaningful coupling of the atmosphere along a given field line. In Model E4g, however, we calculate the PEF using the winds, which causes the CM electric field to contain terms that are proportional to the difference between something like a “conductivity-weighted average velocity” (from the calculation of the PEF) and the local velocity (from the \(\vec{u} \times \vec{B}\) terms in the CM electric field). A worked example of this mechanism is given in Section 3.5.1. Ion drag acts to reduce this difference in wind speeds, effectively reducing the wind shear along a given field line. In Model E4g, this mechanism acts on the equatorial jet by helping the zonal jet reach higher altitudes than it would have by viscous forcing alone.

This important result implies that electrodynamics could play a significant role in helping extend the vertical and horizontal extent of a given zonal wind profile at low
latitudes in Saturn’s thermosphere and ionosphere. It could help stabilize a wind profile at low latitudes, like that described by Brown et al. (2020), by spreading small perturbations to the wind profile along the length of a field line, thereby allowing the perturbations to viscously dissipate over a larger volume. This could also allow larger perturbations deeper in the thermosphere to be communicated upward. Due to the models’ sensitivity to the conductivity profile, FACs and wind profile, however, it will likely be difficult to conclusively infer detailed information about the lower thermosphere from above using an approach based on this theory.

In principal, this coupling mechanism could also work in reverse: field lines could help imprint the structure of the higher thermosphere onto lower altitudes. This is the basis for auroral magnetobraking at high latitudes: sub-corotating plasma from the magnetosphere imprints its angular velocity on the auroral oval below. For this to happen at lower latitudes, however, would require the upper thermosphere (or inner magnetosphere) to be driven more strongly than the lower thermosphere. In our models, the reason the zonal jet is lifted up rather than pushed down is because the zonal wind profile is fixed at the lower boundary, whereas the winds at the upper boundary are not fixed.

5.4.6 Magnetic Field Perturbations at Low Latitudes

The models we have presented do not produce magnetic field perturbations that are anything like those observed by the Cassini MAG instrument team (e.g. Provan et al., 2019) despite the inclusion of an equatorial jet as suggested by Khurana et al. (2018); Vriesema et al. (2020); Brown et al. (2020). The reason for this is because ion drag and resistive heating, when the full PEF is used, act to smooth out gradients in the winds along magnetic field lines at low latitudes, weakening any current system that could generate magnetic field perturbations like those observed. Suppose the southern footpoint of a low-latitude magnetic field line samples a region with high eastward velocity and the conjugate footpoint in the northern hemisphere samples a region with a low eastward velocity. Following the simplified model given in Section 3.5.1, a perpendicular PEF will be generated such that the CM electric
field points northward at both footpoints. Perpendicular currents will also flow northward, and ion drag will be induced through the magnetic field line that is westward at the southern footpoint and eastward at the northern footpoint. In the simplified model, this ion drag will continue to be induced until the net wind shear is eliminated. Meanwhile, the electrical currents will dissipate energy via Joule heating, which will likely be subject to advection and dissipation.

Moving beyond the simple model, we allow for vertical and latitudinal structure in the winds and conductivities, but consider a similar initial wind shear. Before equilibrium is reached, most points along the field line in the northern thermosphere will experience eastward ion drag, but some points will experience westward ion drag. Similarly, most points along the southern segment of the field line will experience westward ion drag, but some may experience eastward ion drag. Overall, a northward CM electric field will be established, but it may be southward at some points along the field lines. In this scenario, we may expect a northward current to be established in the thermosphere and ion drag will generally be in the sense described in the simplified model, but there may be variations along the field line. For field lines that connect the two hemispheres (i.e. at low and middle latitudes where FACs are not imposed), angular momentum can be transported both across hemispheres and along the field line in a given hemisphere. For field lines constrained by the imposed FACs at the upper boundary (i.e. at higher latitudes), angular momentum cannot be transported between hemispheres, but it may be transported along a given segment of the field line in either hemisphere. In the absence of external driving mechanisms, ion drag would act to equalize the angular momentum along a given field line.

In most middle and high-latitude regions of Model E4g, for example, ion drag at low altitudes is westward but eastward at high altitudes. This indicates that angular momentum is being removed at low altitudes and deposited at high altitudes for a net upward transport along field lines. Without external forcing maintaining the wind gradient, ion drag would gradually make the wind speed more or less uniform along the field line. The reason that this ion drag in this model continuously transports
angular momentum upward instead of reaching an equilibrium state is because it is being driven from below by the imposed wind profile to have slightly eastward winds and driven from above by the imposed FAC profile to have strongly westward winds.

In Model E4fm, we observe a similar forcing mechanism at work at low latitudes. There, the imposed equatorial jet peak is shifted significantly southward of the magnetic equator. At low altitudes where the conductivity is highest, this causes westward ion drag at the southern end of field lines and eastward ion drag at the northern end of field lines, as expected. Interestingly, the zonal ion drag is reversed at higher altitudes, where the eastward wind speeds are stronger in the northern hemisphere and weaker in the southern hemisphere. As a result, the higher-altitude winds are more symmetric about the magnetic equator. The end result is that the winds are more symmetrical about the magnetic equator at higher altitudes.

In order for STIM to produce azimuthal magnetic field perturbations such as those reported by Dougherty et al. (2018); Khurana et al. (2018); Provan et al. (2019), there needs to be a persistent current system at low latitudes, and such a current system requires continual forcing in order to prevent the system from exhausting itself. This idea is consistent with the high variability of the observed perturbations (Provan et al., 2019). The nature of this dynamical forcing is presently unknown. The investigation of Provan et al. (2019) suggests that the high degree of variability in the magnetic field perturbations may be caused by variations in ionospheric wind or conductivities. Based on the mechanism suggested by Khurana et al. (2018) and the theory and models developed by Vriesema et al. (2020), it seems that a model capable of reproducing the observed magnetic perturbations would have significant latitudinal and vertical structure in the winds and conductivities. It is unknown what would drive the winds and/or conductivities to have structure on such small length and time scales as to reproduce the magnetic observations. Ring shadowing is expected to cause conductivity variations in the ionosphere (Moore et al., 2004; Waite et al., 2018). It is possible that this forcing comes from gravity waves or small-scale turbulence from horizontal wind shears not resolved by STIM
(e.g. Müller-Wodarg et al., 2019), but a full discussion of this is beyond the scope of the present study.

There are other possible explanations for the azimuthal magnetic field perturbations observed at low latitudes. Our model assumes that the parallel conductivity is effectively infinite relative to the Pedersen and Hall conductivities and that the parallel component of the electric field approaches zero. If we relax this assumption, which fails at lower altitudes due to the three conductivity components being within roughly 2 orders of magnitude of each other (c.f Figure 3.2), we would need to solve a 2D partial differential equation using a modification of the procedure described in Richmond (1973a) to calculate the PEF. This would alter the current system, if only slightly, and could allow for more complex current systems at low latitudes. Because $\sigma_\parallel \gg \sigma_P$ at high altitudes, the approximation is generally valid, and therefore we would expect little change at high latitudes as a result of relaxing this assumption.

The electrodynamics formulation used in this study also assumes $B_\phi \rightarrow 0$ and $\vec{j} \rightarrow 0$ at the lower boundary. If $B_\phi$ were not zero at the lower boundary, the magnetic field perturbations in our model would be shifted up or down by a similar amount. Although we expect those assumptions to be approximately satisfied below the conducting layer due to the conductivities decreasing rapidly with decreasing altitude, STIM does not completely resolve the Hall conducting region. This matters for two interconnected reasons. First, if any significant currents extend below STIM’s lower boundary, then these would violate the $\vec{j} \rightarrow 0$ assumption of our model and would perturb electrodynamics at higher altitudes. Secondly, at these altitudes the parallel conductivity would likely be even closer to the Pedersen and Hall conductivities, which would further violate the assumptions of this model and could allow for current flow below our domain cutoff. Indeed, at low altitudes where ions and electrons are fully coupled to the neutral atmosphere, the parallel and Pedersen conductivities would be indistinguishable.

If the assumption of equipotential field lines allows angular momentum to be efficiently transported along field lines in our models, then the assumption of finite parallel conductivity could reduce or slow the redistribution of angular momentum
along field lines. This could allow dynamic forcing to play a relatively larger role than in our models by restricting the electrodynamic coupling along field lines. This effect would likely be most significant at lower altitudes, where dynamo currents are stronger and where the conductivities are closest in value. However, it could also play an important role in the equatorial ionosphere at higher altitudes, if the boundary between the chemistry-dominated region and the transport-dominated region of the ionosphere is significantly higher than at other latitudes, as suggested by Persoon et al. (2019). If this is the case, a northward current across the equator (as in Model E4fm) could potentially spread out over a greater vertical region, decreasing the current density, Joule heating density and ion drag therein. If a northward, cross-equator current flowed at a significantly higher altitude than in our models, following the contour of an equatorial bulge in the ionosphere, such a current could possibly drive westward magnetic field perturbations such as those observed during Rev 292 by Provan et al. (2019) (c.f. their Figure 12c).

Moving beyond, there are still other ways to produce azimuthal magnetic field perturbations. First, it is straightforward to estimate what FAC profile at the upper boundary would generate an observed magnetic field perturbation profile. Better yet, smoothed azimuthal magnetic field observations can easily be used in place of FACs as an upper boundary condition as described in Section 2.2.6. Second, significantly enhanced conductivities at high altitudes, which may be most likely near the equator due to interaction with the rings, could invalidate our assumptions of $\sigma_p \to 0$ and $\sigma_H \to 0$ at the top of STIM’s domain, which would allow nonzero $B_\phi$. Third, we assume axial symmetry, but relaxing this assumption would allow for different current structures which could generate azimuthal magnetic fields. A test version of STIM already allows the axisymmetric PEF to be independently calculated for each longitude in STIM, and this is a topic of an upcoming study. Fourth, our formulation assumes a steady state, but transient or cyclical events could produce currents and magnetic field perturbations. Without further constraints, this is difficult to test. While all of these mechanisms doubtless play some role, the first seems like the most productive avenue for short-term efforts. In light of the high
degree of variability in the MAG data and our conclusion that electrodynamics needs to be driven by other mechanisms, the third and fourth mechanisms are more likely to explain the magnetic field perturbations at low altitudes.

5.4.7 Axial Symmetry

A major shortcoming of our model is that our formulation for calculating the PEF is an axisymmetric, steady-state model. STIM is run until it reaches a roughly steady state, so the steady-state assumption of the PEF calculation is justified upon reaching this point. However, the model is not in general axisymmetric. In spite of Saturn rotating quickly, it still has significant dawn/dusk and day/night asymmetries. Part of the reason for this is because the ionosphere is sustained by photochemistry during the day and is balanced by recombination during the Kronian night. Because of this, the conductivities in STIM vary by an order of magnitude from day to night — at least at lower pressure levels, as shown in Figure 5.25 — though at high latitudes this difference is far smaller. Also, the meridional winds, currents, and other quantities are not axisymmetric. Also, it is difficult to imagine generating cross-polar currents with an axisymmetric PEF, though not impossible. The effects of assuming axial symmetry in our calculation of the PEF will be the topic for future study.
Figure 5.25: This plot shows the base-10 logarithm of the Pedersen conductivity in Model E1 near the $10^{-5}$ mbar pressure level.
CHAPTER 6

Conclusions

In this study, we have developed a new way to model electrodynamics in Saturn’s ionosphere and thermosphere and have used it to explore the role of resistive heating and ion drag therein. We summarize each of our three studies in Section 6.1, highlight the key results in Section 6.2, and then discuss directions for future study in Section 6.3.

6.1 Summary

For our first study, we used a formulation for calculating electrodynamics that was originally developed for Earth’s thermosphere, but adapted it to the unique geometry of Saturn’s magnetic field. We used this formulation first in an axisymmetric, steady-state model in Chapter 3, assuming a 1D conductivity profile and using altitude and wind data from STIM, to calculate the electric field, currents, resistive heating, ion drag and azimuthal magnetic field perturbations. We used our 2D model to demonstrate that the electrodynamics is very sensitive to the assumed wind profile, and constructed a proof-of-concept wind profile that closely matched the $B_\phi$ measurements made during Cassini’s Rev. 279 flyby. We found that these and similar observations are likely to be explained only if there is an equatorial jet in Saturn’s thermosphere. We compared our formulation to the highly simplified expressions of Khurana et al. (2018) and found that they are consistent in the limiting case of an infinitesimally thin ionosphere with identical magnetic fields and conductivities at conjugate field line footprints. This study indicated that although resistive heating is unlikely to resolve the energy crisis, it, together with strong ion drag at midlatitudes, might be significant enough to weaken the Coriolis barrier and thereby permit equatorward transport of polar heating. Our steady-state model
does not update the wind or temperature profiles based on electrodynamics, so we suggested that electrodynamics be calculated self-consistently in future models to better understand these interactions.

For our second study, we used STIM, a 3D, general circulation model of Saturn’s thermosphere and ionosphere, to study the effects of ion drag and resistive heating self-consistently. In order to do this, we made a series of updates to STIM and presented results from each stage to show how these updates affected the circulation and energy balance. We began by describing the previous STIM model, which neglected electron-neutral interactions when calculating the Pedersen and Hall conductivities, modeled magnetospheric forcing via an imposed magnetospheric electric field at high latitudes, used a layer conductivity formulation to calculate electrical currents and did not calculate radial currents at all. In Model 1, we updated STIM’s calculation of the Pedersen and Hall conductivities and generalized the calculation of current densities. This changed the directions of the zonal and meridional currents at low altitudes, where the electron-neutral collision terms are significant. The zonal currents are now qualitatively in agreement with Cowley et al. (2004a). Whereas the meridional component of ion drag had been poleward almost universally, the updates caused ion drag to point equatorward at low altitudes. In Model 2, we forced STIM to use a dipole magnetic field rather than the SPV field of Davis and Smith (1990). Polar temperatures increased, some meridional dynamics terms change, but little changes to the overall picture. In Model 3, we imposed a zonal jet at the lower boundary. The equatorial jet generated strong currents at low latitudes. The ion drag is strong enough to partially oppose the Coriolis force at middle latitudes, which weakens the Coriolis barrier somewhat and allows the meridional temperature gradient to be somewhat shallower.

We then extended the formulation used in our first study to constrain electrodynamics using FACs at the top of the domain as a boundary condition. Whereas STIM previously imposed a static, height-independent electric field based on the BATSRUS magnetosphere model, we modified STIM to use our new formulation to calculate a PEF based on an equinox FAC profile from a BATSRUS model instead.
We scaled the FACs in each hemisphere so that the PEF peaks matched the MEF peaks. With this scaling, the temperatures were too low, so we further scaled the FACs by an additional factor of either 1.5 or 2. The scaled-then-doubled FACs matched the original temperature profile best, but we opted with the in-between scaling for subsequent models to build off of. Running STIM at equinox, we found that the new formulation caused the PEF to be shifted poleward of the original MEF peaks by a few degrees, due to the relative position of the FAC profile peaks and the sharply-peaked Pedersen conductivity profile at high latitudes. The resulting electrodynamics at low latitudes was shielded from the magnetosphere by imposing the zero-FAC condition. This means that the PEF was zero equatorward of roughly \( \pm 56^\circ \) and all currents were due to the local \( \vec{u} \times \vec{B} \) terms. The shifted electric field caused Joule heating to shift poleward, which enhanced the “unexpected cooling effect” described by Smith et al. (2007). Next, we imposed two wind profiles at the lower boundary to determine the effects of a zonal wind profile using the zero-wind PEF. Although the winds increased electrodynamics at low and middle latitudes, the overall dynamics was mostly unchanged. Finally, we used the winds to calculate the full PEF using the two wind profiles. This is the first time an ionospheric wind dynamo has been incorporated into Saturn’s upper atmosphere. We found that ion drag was able to lift the imposed wind profile to higher altitudes than it would have been able to extend on its own and broaden it slightly in latitude. We also found that because of our imposed FAC profile at the upper boundary, FACs were prevented from flowing into or out of the thermosphere between roughly \( \pm 56^\circ \), although Pedersen currents flow upward and downward in a similar sense as that predicted by Khurana et al. (2018). This meant that magnetic field perturbations were effectively a constant in that entire region at high altitudes. Finally, the Joule heating produced in these models is insufficient to heat the low-latitude thermosphere to the observed levels.
6.2 Key Results

We began these studies by asking whether electrodynamics could explain the observed temperatures at low latitudes, either by heating low latitudes directly, or by breaking the Coriolis barrier by heating midlatitudes and weakening thermal winds or by generating sufficient ion drag that could counteract the Coriolis force directly. We also sought to determine how the presence of an equatorial jet might influence electrodynamics at low latitudes, and whether it might be able to drive a low-latitude current system like that described by Khurana et al. (2018). From the models described above, we can infer much about the role of electrodynamics in Saturn’s upper atmosphere.

One of the most important conclusions from our studies is that the currents, ion drag and Joule heating are sensitive to both the vertical and meridional structure of wind and conductivity profiles. In Chapter 3, we explored this sensitivity using several different wind profiles using the same conductivity profile. Our results strongly support the proposed mechanism of Khurana et al. (2018) that a difference in wind velocities in the ionospheric footpoints of magnetic field lines can induce currents along those field lines and in the conducting region between those footpoints. The sense of these currents is also consistent with Khurana et al. (2018): southward FACs and northward currents in the ionosphere if the segment of the field line in the southern hemisphere has, on average, a more eastward velocity than the conjugate segment of the field line in the northern hemisphere. The magnitude of these currents (and the associated ion drag and Joule heating) is generally increased in regions of strong vertical or horizontal gradients in the zonal winds or Pedersen conductivities. It is likely also sensitive to meridional winds and Hall conductivities, due to the presence of these terms in Equation (2.103), though it will likely not be as sensitive to the Hall conductivity as to the Pedersen conductivity because only the Pedersen conductivity appears in the denominator of the right side of that equation.

We also find that relaxing the layer conductivity formulation previously used by STIM significantly changes the currents, ion drag and and temperatures. Whereas
the model prohibited vertical currents by construction, this update allowed vertical
currents of the order $10^{-9}$ A m$^{-2}$ to $10^{-8}$ A m$^{-2}$ to flow at low and middle latitudes.
This update also fixed the sign of the Hall currents, which was wrong in the previous
version of STIM. As a result, STIM now predicts eastward currents of the order
$10^{-6}$ A m$^{-2}$ in the auroral region between the $10^{-3}$ mbar and $10^{-5}$ mbar pressure
levels. This reverses the meridional ion drag that previously was thought to enhance
the Coriolis barrier by exerting a poleward force on the atmosphere (e.g. Smith et al.,
2007). Despite this reversal, the meridional ion drag is too weak to effectively depress
the Coriolis barrier and allow equatorward transport of auroral energy.

We approximate Saturn’s magnetic field as a magnetic dipole shifted northward
by $0.0466 \, R_S$. This simplified geometry allows us to use dipole coordinates to sim-
plify the problem when calculating the PEF in several of our later models. Although
the dipole magnetic field is weaker at high latitudes and stronger at low latitudes
than the SPV field previously used (Davis and Smith, 1990), we find this to be an
acceptable approximation in general.

We have developed a new way to model forcing from the magnetosphere using
FACs along the upper boundary of the domain rather than using a static MEF. This
formulation will likely prove useful for future models that couple the ionosphere and
magnetosphere, though it is limited by assuming the system is in a steady-state
and is axisymmetric. We have implemented this in two stages. First, we use the
zero-wind PEF approach to calculate an electric field from the FACs alone, ignoring
contributions from the winds. This approach mimics the MEF in form and function,
and it serves as a comparison for models which use the winds to calculate the PEF
fully self-consistently. Including the winds in the calculation of the PEF is critical,
however, as this allows field lines to transport momentum via Alfvén waves. Using
this approach, we find auroral dynamics and temperatures to be very sensitive to the
conductivity profiles, both in altitude and in latitude, and to the mapping of auroral
FACs from the magnetosphere to the ionosphere. Because this sensitivity occurs
where the magnetosphere and ionosphere are strongly coupled, the it is critical
that any model that couples the magnetosphere and ionosphere be as accurate as
possible. The MEF previously used by STIM was based on a model which treated the ionosphere as a single layer with uniform Pedersen conductivity and no Hall conductivity. The results of our study call for more self-consistent models before anything can be concluded and challenge the reliability of past predictions.

We find that including a zonal jet dramatically increases currents, ion drag and Joule heating at low latitudes, as suggested by Vriesema et al. (2020). Because of this, any observations of significant electrodynamic activity at low latitudes likely suggest the presence of significant zonal winds in Saturn’s low-latitude thermosphere, as suggested by Khurana et al. (2018) and Vriesema et al. (2020). However, in models which use the full wind dynamo form of the PEF, currents rapidly decay (over tens of rotations) as ion drag and Joule heating redistribute angular momentum and reduce wind shears along magnetic field lines. This important result implies that any significant wind structures in Saturn’s thermosphere, such as those inferred by Brown et al. (2020), must be sustained by continual forcing via some other mechanism. Therefore, evidence of electrical currents at low latitudes suggests the existence of not only zonal winds, but also mechanisms to undergird these winds and currents. We also find that when we impose a zonal jet at the lower boundary, ion drag and resistive heating together act to lift the imposed jet to higher altitudes than it would otherwise be capable of reaching on its own. In both Models E4g and E4fm, the jet extends to the top of STIM’s domain and is nearly constant with height and symmetric about the magnetic equator, leading to the reduced wind shear that would otherwise drive electrodynamics. This indicates that ion drag may be powerful enough to dominate other processes modelled by STIM at low latitudes. If there are electrical currents at low latitudes, then any mechanisms driving the required winds shears must be similarly strong to maintain wind shears against redistribution of momentum by ion drag. In light of our results and the azimuthal magnetic field perturbations at low latitudes (Provan et al., 2019, e.g.), it appears that an important mechanism(s) that drives low-latitude winds is missing from STIM. The nature of these mechanisms is unknown in large part because the winds and currents are themselves relatively unknown, though momentum
deposition by breaking gravity waves may play a role. These results further support the conclusion of many other studies that dynamics is crucial for understanding Saturn’s thermosphere.

In all of our models, electrodynamics is insufficient to explain the observed temperatures at low latitudes. The Joule heating in both Models E4g and E4fm are 1–3 orders of magnitude lower at low latitudes than the solar heat input. This may be in part because the Coriolis barrier has not been sufficiently opposed, leaving auroral energy trapped at high latitudes (e.g. Müller-Wodarg et al., 2019). However, if in-situ resistive heating is actually responsible for heating the low latitudes, then we have likely underestimated the conductivities, the wind shear along field lines, and/or the conductivity gradients along field lines. Within our model, the energy generated by Joule heating at low latitudes comes ultimately from the winds and the forces that maintain wind gradients. If more energy is to be delivered to low latitudes, it must be supplied either from high latitudes by advection or from below.

The presence of magnetic field perturbations at low latitudes (Dougherty et al., 2018; Khurana et al., 2018; Provan et al., 2019) indicates that there is a current system at low latitudes. In light of our results, the magnetic field perturbations strongly suggest that there are significant wind structures in Saturn’s equatorial thermosphere. The high degree of variability in the magnetic observations may also indicate that any forcing mechanisms maintaining the winds from below may be transient or unsteady.

6.3 Implications for Future Work

A more complete model of Saturn’s ionosphere and thermosphere would include coupling to the magnetosphere and mesosphere, accurate chemical, pressure and temperature profiles at all latitudes and longitudes, wave dynamics, interactions with the rings and the ring shadow, use of oblate coordinates, and much more. Although updating STIM to include all of these factors is beyond our grasp at the present, progress can be made in several areas.
An immediate next step will be to examine how the results of the models in this study vary in longitude. In spite of Saturn rotating quickly, there may be significant variations which could prove interesting and worthy of further study.

One of the most impactful next steps will be improving how STIM models coupling to the magnetosphere. This can be done either by using a better FAC profile as in our extended formulation or by using smoothed magnetic field perturbations at the upper boundary. The FAC profiles in Hunt et al. (2014, 2015) are a promising source.

Because the conductivity peaks and the peaks in the auroral FAC profiles are relatively narrow, it is possible that STIM under-resolves these peaks. One easy test to run is to increase the resolution in STIM to resolve these peaks better and thereby improve numerical accuracy.

We have not yet studied the time-dependence of the model. Although STIM is designed to be run to equilibrium, the process of reaching equilibrium can inform us how a wind dynamo works in Saturn’s upper atmosphere. In particular, it could help confirm the efficiency of ion drag in spreading out a given wind profile.

Due to its interactions with the rings, including loops of current, ring rain and relatively higher abundance of heavy ions, Saturn’s equatorial ionosphere and thermosphere is a hotbed of investigation (e.g. Moore et al., 2018; Cravens et al., 2019; Yelle et al., 2018). Ring shadowing further complicates the ionosphere’s composition and can introduce north-south asymmetries in the ionosphere. Given that field lines can couple the northern and southern hemispheres, these asymmetries may have interesting dynamic effects.

Our electrodynamic formulation currently assumes zero current density at the lower boundary, which it assumes is the bottom of the conducting layer. The lower boundary of STIM, however, does not extend low enough to where this would be a fair assumption. Extending the lower boundary of STIM would be useful to solidify our current electrodynamics formulation. It would also allow us to probe the Hall layer in more detail.

More advanced electrodynamic codes have been developed for Earth’s ionosphere
and thermosphere, which include varying degrees of magnetospheric forcing (Qian et al., 2014, and references therein). It would be useful to use one of these in conjunction with STIM.

While many other directions for future work could be discussed, we content ourselves with the above listing, save the following. It was the Pioneer 11, Voyagers 1 and 2 and the Cassini missions that made detailed study of Saturn possible. Therefore, it seems worthwhile, if precocious, to suggest that another mission to Saturn in the not-too-distant future will be of even greater scientific value.
REFERENCES


