

***B*-meson semileptonic form factors on (2+1+1)-flavor HISQ ensembles**

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We report updates to an ongoing lattice-QCD calculation of the form factors for the semileptonic decays $B \rightarrow \pi \ell \nu$, $B_s \rightarrow K \ell \nu$, $B \rightarrow \pi \ell^+ \ell^-$, and $B \rightarrow K \ell^+ \ell^-$. The tree-level decays $B_{(s)} \rightarrow \pi(K) \ell \nu$ enable precise determinations of the CKM matrix element $|V_{ub}|$, while the flavor-changing neutral-current interactions $B \rightarrow \pi(K) \ell^+ \ell^-$ are sensitive to contributions from new physics. This work uses MILC's (2+1+1)-flavor HISQ ensembles at approximate lattice spacings between 0.057 and 0.15 fm, with physical sea-quark masses on four out of the seven ensembles. The valence sector is comprised of a clover b quark (in the Fermilab interpretation) and HISQ light and s quarks. We present preliminary results for the form factors f_0 , f_+ , and f_T , including studies of systematic errors.

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1. Introduction

These proceedings update an ongoing lattice-QCD calculation [1] of form factors for the semileptonic decays of B and B_s mesons to pions and kaons. Continuing from Ref. [1], we calculate the form factors f_0 , f_+ , and f_T for the decays $B \rightarrow \pi \ell \nu$, $B_s \rightarrow K \ell \nu$, $B \rightarrow \pi \ell^+ \ell^-$, and $B \rightarrow K \ell^+ \ell^-$. A major goal of this work is to improve the Standard-Model determination of the CKM matrix element $|V_{ub}|$, as informed by the tree-level charged-current decays $B_{(s)} \rightarrow \pi(K) \ell \nu$. The LHCb experiment is expected to publish results for its ongoing study of $B_s \rightarrow K \ell \nu$ [2], while the Belle II experiment [3] is also expected to study these decays.

This work uses ensembles generated by the MILC collaboration [4], with (2+1+1) flavors of HISQ sea quarks and one-loop improved Lüscher–Weisz gluons. For the valence sector, we use HISQ light and strange quarks, whose masses match those of corresponding quarks in the sea, along with clover bottom quarks in the Fermilab interpretation. Basic properties of the ensembles are listed in Table 1.

2. Chiral-continuum fits

The form factors on each ensemble and for each recoil energy are obtained from a correlation-function fitting procedure that is explained in Ref. [1], to which we refer the reader. In comparison to these earlier results, we have:

- increased the statistics on the 0.088-fm ensemble with physical-mass quarks,
- added a new 0.088-fm ensemble with heavier-than-physical light quarks, and
- added a new 0.057-fm ensemble with physical-mass quarks.

We match the lattice currents J to those in the continuum \mathcal{J} using a mostly nonperturbative renormalization procedure $\mathcal{J} \doteq Z_J J$, where $Z_{J_{bq}} = \rho_{J_{bq}} \sqrt{Z_{V_{bb}^4} Z_{V_{qq}^4}}$ [5, 6]. The nonperturbative flavor-diagonal Z_{V^4} factors are provided in Table 2. The perturbative ρ_J factors are being calculated to one-loop order by collaborators and will be provided in a blinded form when they are available.

Table 1: Simulation details of the (2+1+1)-flavor MILC ensembles used in this work. Columns are (from left to right): approximate lattice spacing, lattice size, tuned quark masses in lattice units, tuned b -quark hopping parameter, and number of configurations.

$\approx a$ (fm)	$N_s^3 \times N_t$	am'_l	am'_s	am'_c	κ'_b	N_{cfg}
0.15	$32^3 \times 48$	0.002426	0.06730	0.8447	0.07732	3630
0.12	$24^3 \times 64$	0.0102	0.0509	0.635	0.08574	1053
	$32^3 \times 64$	0.00507	0.0507	0.628	0.08574	1000
	$48^3 \times 64$	0.001907	0.05252	0.6382	0.08574	986
0.088	$48^3 \times 96$	0.00363	0.0363	0.430	0.09569	1017
	$64^3 \times 96$	0.0012	0.0363	0.432	0.09569	1535
0.057	$96^3 \times 192$	0.0008	0.022	0.260	0.10604	1027

Table 2: Nonperturbative flavor-diagonal current-renormalization factors.

$\approx a$ (fm)	$Z_{V_{ss}^4}$	$Z_{V_{bb}^4}$
0.15	1.9785(31)	0.57617(60)
0.12	1.9915(18)	0.52492(67)
0.088	1.9851(29)	0.46560(56)
0.057	1.9951(42)	0.40136(82)

The form factors are extrapolated to the continuum and corrected for slight mistunings of the sea-quark masses in a combined chiral-continuum fit, using heavy-meson rooted-staggered chiral perturbation theory (HMrS χ PT) [7, 8, 9]. We follow the procedure of Ref. [10], including both SU(2) and SU(3) formulae with terms up to next-to-next-to-next-to-leading order (N³LO).

As an example of our central fits, we now focus on the form factors from N²LO SU(2) HMrS χ PT, whose preliminary results for $B_s \rightarrow K$ are shown in Fig. 1. The expressions are as follows:

$$W_J f_J = f_J^{(0)} \times \left(c_J^0 \left[1 + \delta f_J^{\text{logs}} \right] + \delta f_J^{\text{NLO}} + \delta f_J^{\text{N}^2\text{LO}} + \dots \right) \times \left(1 + \delta f_J^b \right), \quad (2.1)$$

$$f_J^{(0)} = \frac{g_\pi}{w_0^2 f_\pi (E_L + \Delta_{B^*})}, \quad (2.2)$$

$$\delta f_J^{\text{NLO}} = c_J^l \chi_l + c_J^s \chi_s + c_J^E \chi_E + c_J^{E^2} \chi_E^2 + c_J^{a^2} \chi_{a^2}, \quad (2.3)$$

$$\delta f_J^{\text{N}^2\text{LO}} = \sum_{m,n \in \{l,s,E,E^2,a^2\}} c_J^{mn} \chi_m \chi_n, \quad (2.4)$$

where the prefactor $W_J = \{w_0^{-1/2}, w_0^{1/2}, 1\}$ for $J = \{\perp, \parallel, T\}$ accounts for the lattice units using the gradient-flow quantity w_0 [11], whose continuum value is $w_0 = 0.1714(15)$ fm [12]. In Eq. (2.1), δf_J^b accounts for b -quark discretization effects [13], while δf_J^{logs} denotes nonanalytic functions of the lattice spacing and light-quark masses. E_L , along with its abbreviation E , is the recoil energy for $L = \{\pi, K\}$. In Eq. (2.2), the pole term Δ_{B^*} arises from low-lying excited states $B_{(s)(0)}^*$ as follows:

$$\Delta_{B^*}(B_{(s)} \rightarrow L) \equiv \frac{M_{B^*}^2 - M_{B_{(s)}}^2 - M_L^2}{2M_{B_{(s)}}}. \quad (2.5)$$

The B^* states relevant to f_\perp and f_T have $J^P = 1^-$, while those relevant to f_0 have $J^P = 0^+$. For $B_s \rightarrow K$, these states are the B^* and B_0^* mesons, respectively. The B_0^* meson has not yet been observed experimentally and is thus taken from a theoretical determination [14].

Lastly, we incorporate the perturbative current-renormalization factors ρ_J into the HMrS χ PT formulae with the following priors:

$$\tilde{\rho}_J = 1 + \tilde{\rho}_J^{(1)} \alpha_s + \tilde{\rho}_J^{(2)} \alpha_s^2, \quad (2.6)$$

where $\tilde{\rho}_J^{(i)} = 0(1)$. These priors were chosen to encompass the one-loop calculations for the corresponding asqtad currents [10, 15, 16] in the absence of the ongoing HISQ calculations for $\rho_J^{(1)}$.

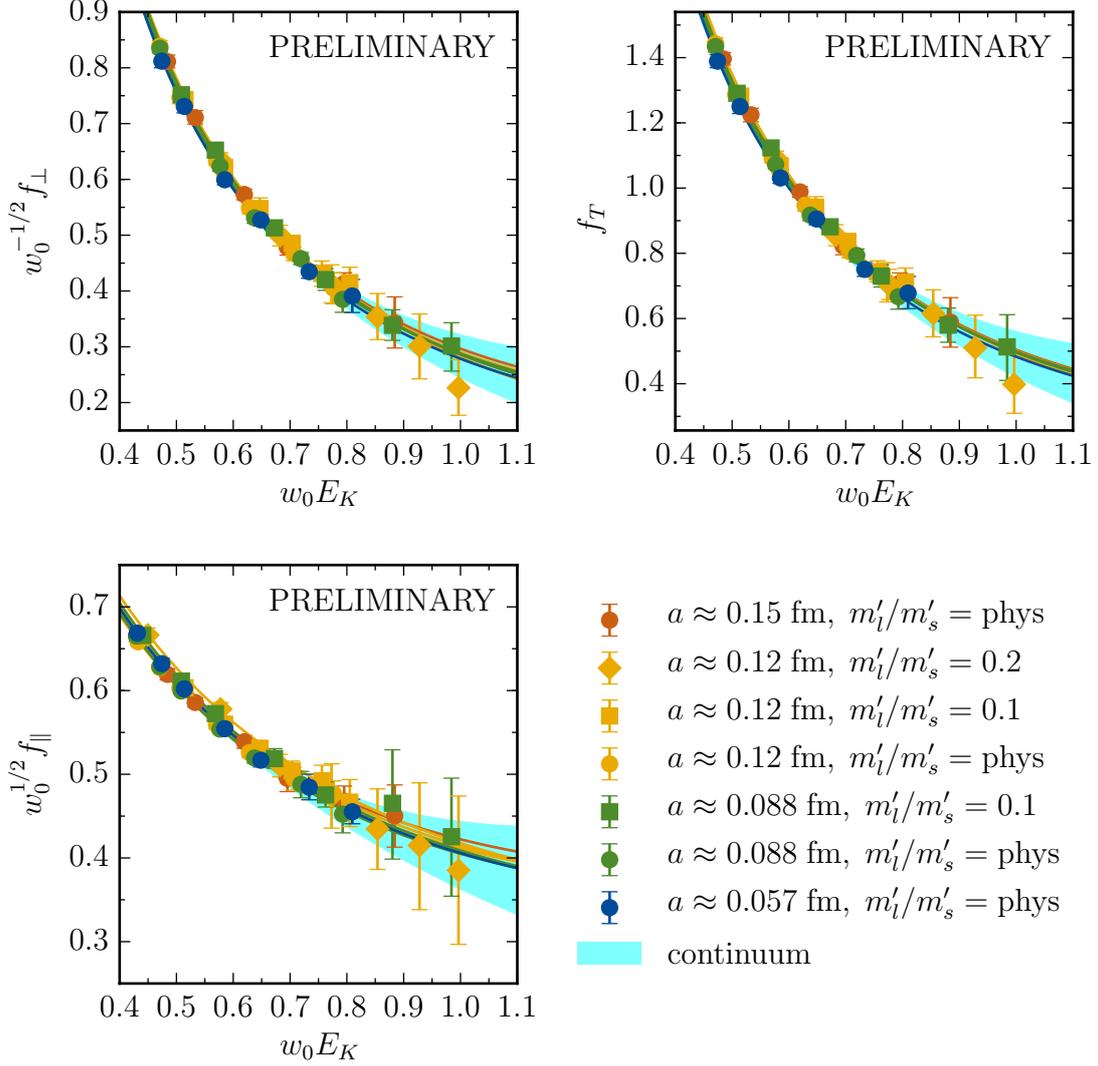


Figure 1: Preliminary chiral-continuum fits for the blinded form factors f_{\perp} , f_T , and f_{\parallel} of the decay $B_s \rightarrow K$ as a function of the recoil energy $w_0 E_K$, using N²LO SU(2) HMrS χ PT. Colors denote the lattice spacings and symbols denote the ratios of sea-quark masses. Colored lines show the fit results evaluated at the parameters of the corresponding ensembles. The cyan band shows the fit results in the chiral continuum.

3. Error budget

We are studying sources of error in order to construct a complete error budget over the range in q^2 for which we have lattice simulations, $17 \text{ (GeV)}^2 \lesssim q^2 \lesssim 24 \text{ (GeV)}^2$. The form factors from the chiral-continuum fits, as explained in Sec. 2, include errors due to statistics, χ PT, light-, strange-, and bottom-quark discretizations, current-renormalization factors Z_J , the scale-setting via w_0 , and the $B^* B \pi$ coupling constant g_{π} . The distribution of these errors is shown in Fig. 2, where the dominant source is that of statistics. The uncertainties due to current renormalization include the contributions from the fit posteriors of the perturbative factors in Eq. (2.6).

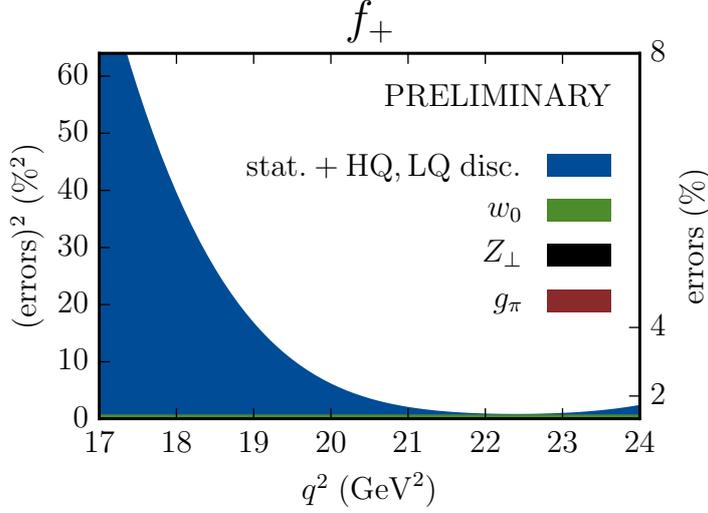


Figure 2: Preliminary distribution of errors for the blinded form factor f_+ of the decay $B_s \rightarrow K$ as a function of the momentum transfer q^2 . The squares of the errors added in quadrature are shown on the left y axis, while the errors themselves are shown on the right y axis. The blue band shows errors due to statistics, χ PT, and light-, strange-, and heavy-quark discretizations, in which the dominant contribution is that of statistics. The other bands contribute at the subpercent level.

To estimate the truncation effects of our chiral-continuum fits, we consider variations in both the fit formulae and the lattice data. Such variations are exemplified in Fig. 3, which compares the form-factor results at a recoil energy of $E_K \approx 900$ MeV. This probes the higher momenta on the lattice while remaining within the region of validity of χ PT.

Including terms up to N³LO in Eq. (2.1) provides results that agree with those of N²LO, indicating that higher-order truncation effects are negligible. Next we consider using SU(3) formulae, which include the effects of dynamical strange quarks on the nonanalytic functions in Eq. (2.1). We find SU(3) to be consistent with SU(2).

We also consider the effects of reducing the lattice data used in the fits. Excluding the coarsest ensemble (at $a \approx 0.15$ fm) typically has no significant effect on the fit results. Including only the four physical-mass ensembles has small but different effects for each form factor, which warrants further investigation.

We are actively studying other sources of error. The simulations in this work used well-tuned parameters for quark masses, including the bottom-quark hopping parameter. Preliminary estimates of the corrections due to these uncertainties are all below 0.1%. We also plan to quantify the finite-volume effects by comparing to the infinite-volume fit results in HMrS χ PT.

4. Outlook

In this work, we have shown preliminary results for the blinded form factors f_+ , f_0 , and f_T in the chiral continuum, using $B_s \rightarrow K$ form factors to illustrate our findings. Results for the other decays, $B \rightarrow \pi$ and $B \rightarrow K$, are similar both in general shape and in systematic effects. We are

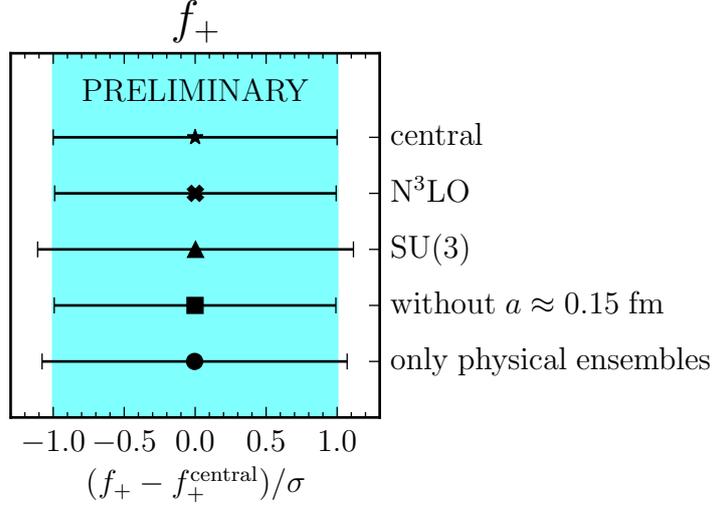


Figure 3: Preliminary chiral-continuum fit variations for the blinded form factor f_+ of the decay $B_s \rightarrow K$ compared with the central fit, whose standard error is given by σ . The comparisons are evaluated at a recoil energy of $E_K \approx 900$ MeV, which corresponds to $w_0 E_K \approx 0.78$ or $q^2 \approx 19.4$ (GeV) 2 . From top to bottom, the HMrS χ PT fit variations are as follows: the star denotes the central fit, $N^2\text{LO}$ $\text{SU}(2)$; the x denotes $N^3\text{LO}$ $\text{SU}(2)$; the upward triangle denotes $N^2\text{LO}$ $\text{SU}(3)$; the square denotes excluding the coarsest ensemble; and the circle denotes including only the physical-mass ensembles.

extrapolating in a model-independent manner to the full kinematic range accessible in experiment by using the functional implementation [17] of the BCL parametrization [18] of the z expansion [19]. Such z -expansion studies are ongoing. This work serves as a successor to the earlier asqtad analyses [10, 15, 16] and aims to reduce the errors due to scale setting and chiral-continuum fitting.

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