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ABSTRACT

Optical design of the wide field of view telescope is limited by obscuration of the secondary mirror in on-axis system and by serious linear astigmatism in conventional common off-axis system. We have developed an innovative off-axis reflective system in which the optical design is based on the confocal off-axis to completely compensate the linear astigmatism. The main objective of this paper is to compare alignment sensitivity of the confocal system to those of on-axis and common off-axis systems. All three optical designs are based on the classical Cassegrain reflecting telescope and have identical entrance pupil diameter of 1000 mm and F/8 focal ratio. Tilt and decenter of each optical component, and despace which indicates inter-mirror distance are considered as tolerancing parameters that are explored within fabrication tolerance ranges. Encircled energy diameter is evaluated as a criterion of the analysis while tilts of the secondary mirror and the focal position are set to the compensator. The statistical tolerancing method based on Monte-Carlo simulation is also performed to analyze system tolerances. From sensitivity analysis and Monte-Carlo simulation, we concluded that the confocal off-axis system is more sensitive than on-axis and common off-axis systems but it is a feasible system in terms of fabrication and alignment errors.

Keywords: Optical design, Off-axis telescope, Linear astigmatism, Tolerance analysis, Sensitivity analysis, Monte-Carlo simulation

1. INTRODUCTION

Off-axis optical systems become popular for telescopes and other optical systems since they avoid scattering, diffraction, and obscuration by the secondary mirror that exist in the on-axis system. The common off-axis system, which is an off-axis system whose optical components share the common optical axis, has limitations for wide field of view system applications because linear astigmatism degrades image quality significantly.[1,2] Linear astigmatism-free off-axis systems recently have been developed in which optical components share their focus

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instead of sharing their axes, so they are called the confocal off-axis system.[3] This system enables high quality wide field of view observations without any correcting lenses.[4,5]

Sensitivity analysis is an important step of the telescope development to improve reliability and practicality of the system. Sensitivity analyses of on-axis and common off-axis systems are widely performed to various optical systems,[6–8] and a few sensitivity analysis results have been reported for two mirror and three mirror confocal off-axis systems.[9,10] Even though sensitivity analyses are performed in many cases, each system has a different optical configuration and design parameters. Therefore, for an objective benchmark study, it is required to compare sensitivities of on-axis, common off-axis, and confocal off-axis systems with the same optical design parameters.

In this paper, we compare sensitivities of on-axis, common off-axis, and confocal off-axis Cassegrain telescopes with the same first-order-optical specifications. Optical design of three systems are introduced in Section 2. Sensitivities at the center of the field for all three systems are compared in section 3. In section 4, Monte-Carlo simulations are performed to evaluate the feasibility of the optical design by comparing the tolerance analysis result to fabrication and alignment errors. All results are summarized in section 5.

2. THREE BENCHMARK TELESCOPE DESIGNS

Optical configuration of the on-axis system follows conventional classical Cassegrain systems. Primary and secondary mirrors in both on-axis and common off-axis systems share their optical axes but only off-axis sections of mirrors are used for the common off-axis system. Optical mirrors of the confocal off-axis system share their focuses instead of sharing their axes. Figure 1 illustrates optical layout of three Cassegrain systems. Optical axis ray (OAR)[11] is indicated in a red solid line. Because all three systems are based on Cassegrain configuration, surface types of primary and secondary mirrors are paraboloid and hyperboloid, respectively.

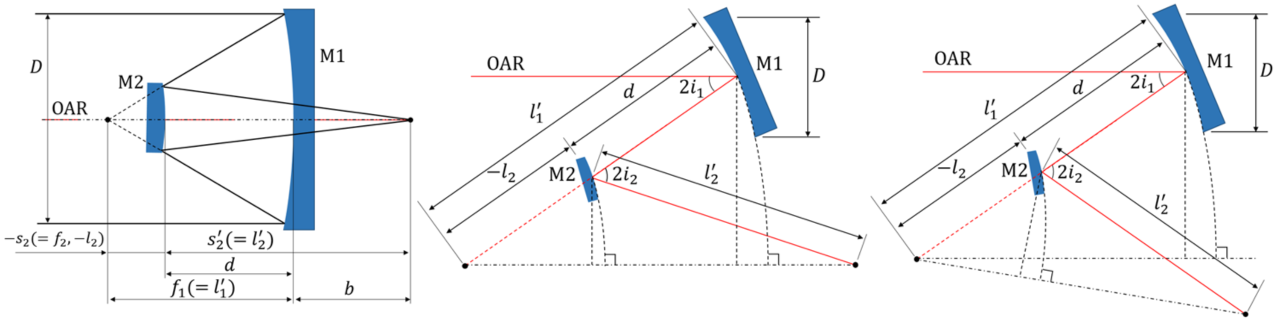

Figure 1. Optical layout of (left) classical on-axis, (center) common off-axis, (right) confocal off-axis Cassegrain systems.

Optical design methods for the on-axis, common off-axis, and confocal off-axis Cassegrain systems are methodologically summarized in Ref. 12 and 13. We set the tilt angle (i_1) of the primary mirror (M1) to 15° for two off-axis systems to design unobscured off-axis telescopes. All the Cassegrain telescopes have the same entrance pupil diameter (EPD) of 1000 mm, and effective focal length (EFL) of 8000 mm. M1 is set to the aperture stop. Calculated optical design parameters are listed in Table 1. Field of view is based on typical 36 mm × 36 mm detectors.

All systems have an ideal focus at the image center, but coma aberration exists on the other fields for the on-axis system. Optical performance of the common off-axis system dramatically degrades for the wide field angles because of linear astigmatism. The confocal off-axis system can eliminate linear astigmatism by satisfying the linear astigmatism-free condition, so optical performance of the system becomes identical to that of the on-axis system. The linear astigmatism-free condition[14] for off-axis two mirror system is

$$(1+m_1)m_2\tan i_1 + (1+m_2)\tan i_2 = 0\,, \tag{1}$$

Table 1. Specifications of on-axis, common off-axis, and confocal off-axis two mirror systems.

Type	On-axis	Common off-axis	Confocal off-axis
Primary D	1000 mm		
System f	8000 mm		
Field of view	0.26° (H) × 0.26° (V)		
Pixel scale	$0.31''$/pixel (12 μm)		
A^{a}_{eff}	0.705 m^2	0.785 m^2	0.785 m^2
f_1	2000 mm	1866.025 mm	1866.025 mm
K_1	-1	-1	-1
l'_1	2000 mm	2000 mm	2000 mm
i_1	0°	15°	15°
K_2	-2.777	-2.597	-2.577
l_2	-800 mm	-600 mm	-600 mm
l'_2	2400 mm	2400 mm	2400 mm
m_2	4	4	4
i_2	0°	-18.590°	-19.660°

[a]Effective light collecting area.

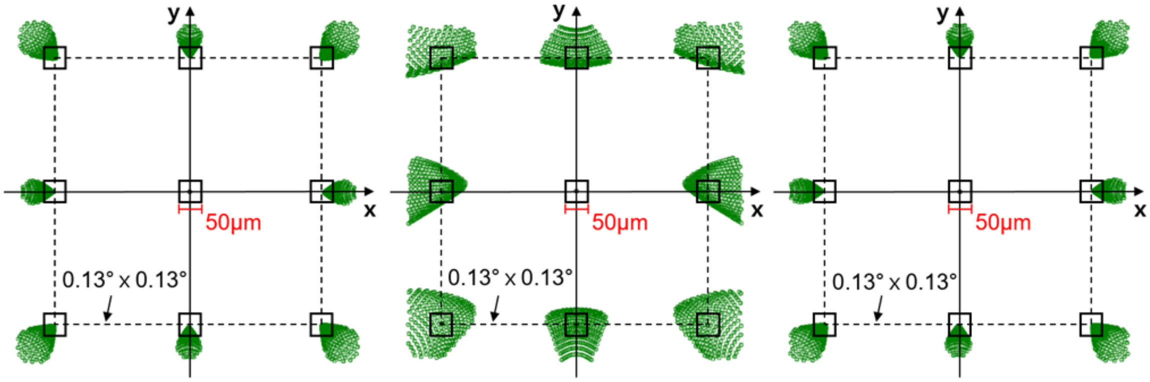

Figure 2. Spot diagrams of (left) on-axis, (center) common off-axis, and (right) confocal off-axis Cassegrain systems.

where m_k and i_k are the magnification and the incident angle of the optical axis ray for the k_{th} mirror, respectively. Spot diagrams show that coma aberration is dominant for on-axis and confocal off-axis Cassegrain systems, while linear astigmatism critically degrades the performance of the common off-axis system (Figure 2).

Figure 3 represents encircled energy diameter (EED) of three systems. EEDs of on-axis and confocal off-axis systems have the same optical performance with <0.10 mm 80 % EED for both systems. On the other hand, 80 % EED of the common off-axis system is <0.20 mm, which is the twice spot size to other systems because of linear astigmatism.

3. SENSITIVITY ANALYSIS

Sensitivities of three systems are analyzed through sensitivity analysis by individually implementing tolerance parameters.[10] Since we generally align Cassegrain systems by tilting the secondary mirror (M2), M2 α-, β-tilts, and a focal position are set to the compensator for sensitivity analysis. Note that in-plane movements of the

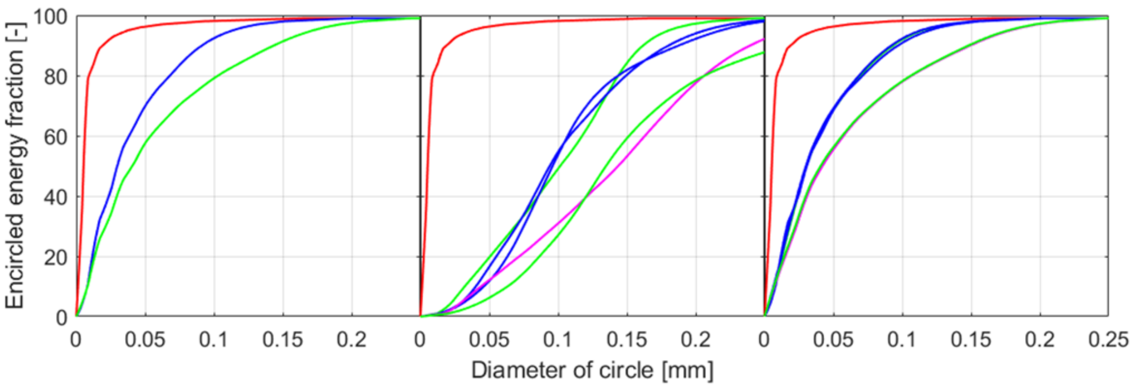

Figure 3. Encircled energy diameter of (left) on-axis, (center) common off-axis, and (right) confocal off-axis Cassegrain systems.

focal plane is fixed for all simulation cases. A right-handed coordinate system is used for sensitivity analysis. α- and β-tilts define left-handed rotations about the $+x$ and $+y$ axes, respectively. The right-handed rotation about the $+z$ axis corresponds to γ-tilt. The reference wavelength is 0.587 μm. All sensitivity parameter ranges are decided based on general fabrication or alignment tolerances that are $\pm 1'$, ± 0.2 mm, and ± 0.2 mm for tilt, decenter, and despace, respectively.

Figure 4 represents sensitivity analysis results at the center of the field. Sensitivity of tilt and decenter for all mirrors are calculated except for compensating parameters (M2 α-, β-tilts). Despace represents an inter-mirror distance of M1-M2.

The confocal off-axis system is more sensitive to α-, β-tilts of M1 (red circle and square in (g)) when comparing to those of on-axis and common off-axis systems. Decenter and despace sensitivities for two off-axis systems show the same trends. Figure 4 clearly indicates that the on-axis system is significantly insensitive for general alignment or manufacturing tolerance ranges. Despaces of off-axis systems are most sensitive parameters that degrades image quality to 23.6 μm 80 % EED. But 80 % EEDs in all the cases are smaller than the resolution limit that is calculated from the Nyquist sampling.

4. MONTE-CARLO SIMULATION

A Monte-Carlo method simulates the comprehensive performance of the system with the tolerance budgets altogether.[15,16] Tolerance parameters, their coordinates, and the reference wavelength for the Monte-Carlo simulation are the same as those of sensitivity analysis. Compensators are M2 α-, β-tilts, and the focal position. We performed 5000 Monte-Carlo trials at the image center for each system, and tolerance distributions follow a normal distribution. The performance criterion is set to 80% EED in μm. The Nyquist sampling for the analysis is based on the KAF-09000 sensor format with the 12 μm pixel size.

The initial tolerance ranges are decided by considering fabrication and alignment capabilities. Table 2 summarizes the final tolerance limits and initial conditions for Monte-Carlo simulations of each system that are optimized using the iterative method in order that each Monte-Carlo simulation result has the 0.8 cumulative probability at the Nyquist sampling (24 μm) . The histograms in Figure 5 are results of 5000 Monte-Carlo runs. Note that a few extreme cases with simulating large errors thus 80% EED is not able to calculated are excluded from each histogram but they are ingnorable comparing to the total number of Monte-Carlo tries.

As we claimed from the sensitivity analysis, the confocal off-axis system is the most sensitive system especially for tilt and despace errors (Table 2). By comparing on-axis system tolerances to those of two off-axis systems, the on-axis system is less sensitive to all tolerance parameters than other systems. Other two off-axis systems show similar tolerance ranges.

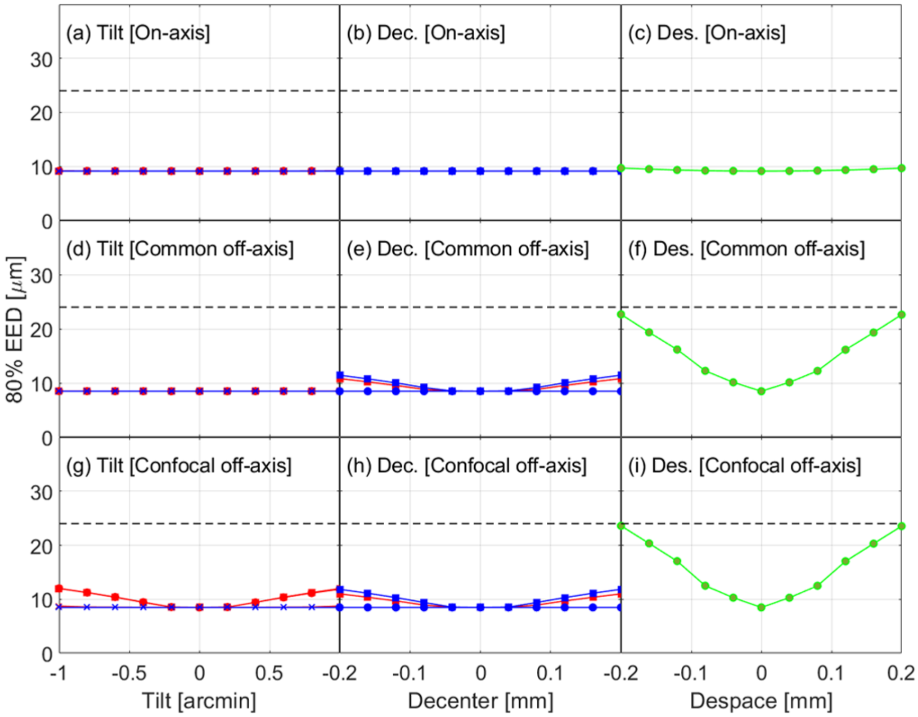

Figure 4. Sensitivity analysis results at the center of the field for (a-c) on-axis, (d-f) common off-axis, and (g-i) confocal off-axis systems. (left) Tilt, (center) decenter, and (right) despace sensitivities are calculated. M1 and M2 sensitivities are shown in red and blue, respectively. Black-dashed lines represent the resolution limit based on the Nyquist sampling.

Table 2. Tolerance limits and initial conditions for Monte-Carlo simulations.

Type	On-axis		Common off-axis		Confocal off-axis	
Mirror	M1	M2	M1	M2	M1	M2
Tilt (α, β, γ)	$\pm 9.5'$	$\pm 9.5'$ (γ only)	$\pm 3.0'$	$\pm 3.0'$ (γ only)	$\pm 2.0'$	$\pm 2.0'$ (γ only)
Decenter	±3.0 mm	±2.0 mm	±0.7 mm	±0.4 mm	±0.7 mm	±0.4 mm
Despace	±1.0 mm		±0.3 mm		±0.2 mm	
M2 tilt (comp.)	$< \pm 1.0'$ (α, β)					
Focus (comp.)	$< \pm 10.0$ mm					
Wavelength	587 nm					

5. SUMMARY AND DISCUSSION

Off-axis telescopes become more popular for wide field of view observations and observations that require low scattering and diffraction effects by the secondary mirror such as observations of low surface brightness galaxies and bright objects. The confocal off-axis system is the next generation optical configuration that has an identical optical performance to that of the on-axis system by eliminating linear astigmatism which is the biggest problem of the common off-axis system. It is required to compare sensitivity of these three optical configurations, on-axis, common off-axis, and confocal off-axis systems having the same first-order optical parameters.

Sensitivity analysis and Monte-Carlo simulation results represent that off-axis systems are more sensitive than the on-axis system to tilt, decenter, and despace errors. Stability of the systems for a temperature variation is not accounted for these simulations. We suspect that off-axis systems are more sensitive than the on-axis

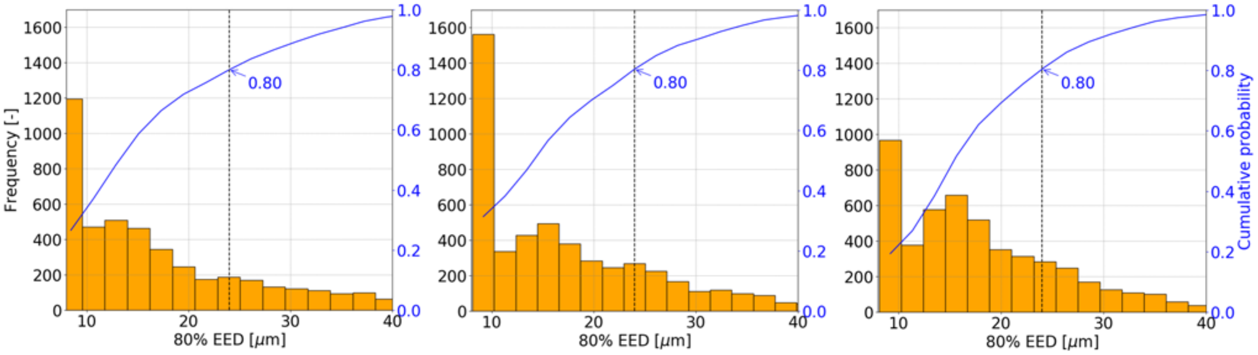

Figure 5. Monte-Carlo simulation results from 5000 Monte-Carlo trials of the (left) on-axis, (middle) common off-axis, and (right) confocal off-axis systems. The simulations for all three systems are performed with to have the same cumulative probability (blue solid lines) for the Nyquist sampling (black dashed lines).

system because the mirrors are originally tilted and decentered from the parent optical axis (dot-dashed lines in Figure 1), and additional tilt, decenter, and despace errors possibly degrade the optical performance more and shift the center of the field more.

The common and confocal off-axis systems show more sensitive tolerances for tilt, decenter, and despace than those of the on-axis system. We expect that sensitivities of off-axis systems would become less sensitive if we consider in-plane movements of the focal plane. The final tolerance ranges of all three systems are reasonable to fabricate and align the systems based on the Monte-Carlo simulation results (Table 2).

The confocal off-axis system provides the same optical performance as that of the on-axis system but without scattering, diffraction, and obscuration by the secondary mirrors, so we get a larger effective light collecting area and higher signal to noise ratio. This system is applicable to sky survey, transient detection, and other wide field of view observations. It is also useful for wide spectral range observations since it dose not use lenses for aberration corrections.

In this paper, we only consider sensitivity and tolerance at the center of the field of view for all three systems, because performance degradation by the coma aberration at the large field angles of the Cassegrain system obscures the misalignment effects on the optical performance. Follow-up researches are required in both practical and theoretical ways. For example, sensitivity and tolerance analyses over large field of view with optimized (or modified) on- and off-axis systems are necessary, and sensitivity comparisons among on-axis, common off-axis, and confocal off-axis systems are needed in different design parameters such as focal ratio or tilt angles in order to clarify relation between sensitivity and the design parameters.

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