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Jim Schwiegerling, Yuanxin Guan, Joseph M. Miller, Erin M. Harvey, "Remote measurement of spherocylindrical lens power and orientation through distortion analysis," Proc. SPIE 11815, Novel Optical Systems, Methods, and Applications XXIV, 1181502 (7 September 2021); doi: 10.1117/12.2594953

Event: SPIE Optical Engineering + Applications, 2021, San Diego, California, United States
Remote measurement of sphero-cylindrical lens power and orientation through distortion analysis
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ABSTRACT
A system for measuring the orientation and power of sphero-cylindrical lenses has been developed. The system attempts to minimize the need for specialized equipment and training and instead relies on the ubiquitous cell phone camera, a magnetic stripe card, and a target pattern. By capturing an image of the target through the lenses under test and analyzing the distortion in the resulting image, the orientation and powers on sphero-cylindrical lenses can be determined. In modern eye clinics, the measurement of sphero-cylindrical spectacle lenses is readily measured with a lensmeter. However, there are many examples where this measurement is not feasible. This may include remote or rural locations where access to eye care may not exist, or require impractical travel. Furthermore, the on-going global pandemic has often put restrictions on contact between the patient and the eye care provider. Telemedicine, which can connect patients to eye care providers, lacks physical access to the spectacles for measurement. The system developed in this effort overcomes this limitation by allowing remote measurement of the lenses with items found in most households. Such a system would be beneficial to often underserved populations and expand access to quality eye care.

Keywords: Spectacle lenses, power measurement, telemedicine, image processing

1. INTRODUCTION
Access to reliable eye care can dramatically improve the quality of life of individuals. For children, amblyopia can often be reduced or prevented through early intervention with spectacle lenses. In all age groups, accurate spectacle prescriptions lead to optimized vision and lets the individuals participate in all aspects of life without the hindrance of ametropia. In modern eye clinics, the measurement of spectacle lens prescription is readily measured with a lensmeter. However, there are many examples of this measurement not being feasible. This may include remote or rural locations where access to eye care may not exist, or require impractical travel. The on-going global pandemic is another example where often there are restrictions on contact between patient and the eye care provider. Telemedicine, which can serve the preceding groups by connecting the eye care provider and patient through a video link, lacks physical access to the spectacles lenses for prescription measurement and verification. A system for remotely measuring spectacle lens prescription that does not require specialized equipment would be beneficial and expand access to quality eye care. Here, a system for measuring the prescription of eye glasses is described. The system relies mainly on the ubiquitous cell phone camera and a target pattern. In the current incarnation of the system, a laptop screen is used to present the target. This requirement is not mandatory, and the target can easily be replaced with a printed card, second cell phone camera screen, or even standard sized objects such as coins. The final piece required for the system is a method for standardizing the separation between the target and the spectacle lenses. In this application, a magnetic stripe card is used. This choice again is not mandatory, but the prevalence of magnetic stripe cards and their standardized dimensions make them highly useful for the system.

2. METHODS
The Gaussian imaging equation can be rewritten from its traditional form to a form that is only dependent upon the lens power, object distance and transverse magnification such that

\[
\frac{1}{z'} - \frac{1}{z} = \phi \Rightarrow \phi = \frac{1 - m}{mz}, \tag{1}
\]

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where $\phi$ is the lens power, $z$ is the object distance, $z'$ is the image distance, $m = z'/z$ is the transverse magnification, and the lens is assumed to be in air. Eq. 1 illustrates that a prediction of the lens power can be made based solely knowing the magnification and the object distance. The proposed system exploits this relationship by standardizing the object distance, and capturing image information which allows the magnification to be directly measured. The advantage of this arrangement is that the image distance is not required, providing flexibility in the location of the image capturing system. The proposed system is shown in Figure 1. A laptop screen is used to display a checkerboard pattern. A magnetic stripe card with its standard dimensions of $85.6 \text{ mm} \times 53.98 \text{ mm}$ is used to fix the distance between laptop screen and the spectacle lens frame to $z = 85.6 \text{ mm}$. A cell phone camera is used to capture an image of the checkerboard pattern both through the lenses and in the region surrounding the lenses. The magnification $m$ is then calculated by comparing the sizes of the checkerboard squares within the lens aperture to the sizes outside the lens aperture. For typical lens powers, this configuration creates a virtual image of the checkerboard pattern near the object plane, so that both object and image planes are essentially in focus in the captured image. The advantage of this configuration is that the image distance $z'$ does not need to be known, and information regarding the cell phone optics or their location is not required. The main difficulty in obtaining the image is simultaneously holding the spectacles and magnetic stripe card with one hand while operating the cell phone camera with the other hand.

The preceding overview of the system provides the conceptual geometrical optics interpretation of the analysis. Actual implementation and extraction of the desired spectacle lens prescription requires a bit more sophistication. First, spectacle lenses are typically sphero-cylindrical, so that a cylinder component and the orientation of that cylinder need to be taken into account. Spectacle lens prescriptions are usually written as $Sph/Cyl \times Axis$, where $Sph$ and $Cyl$ are the spherical and cylindrical powers in units of diopters, and $Axis$ is the orientation of the zero power axis of the cylinder component in units of degrees. Furthermore, $Axis$ is defined as the conventional polar angle measured counterclockwise from the horizontal axis when looking at the lenses from their front side (i.e. the side of the lens facing away from the wearer). In the proposed system, the checkerboard image is captured from the viewpoint of the back of the lens, so effectively $Axis$ needs to be measured clockwise from the horizontal axis to account for the change in perspective. Consequently, a sphero-cylindrical lens can be thought of as a lens with power that oscillates as the lens is circumnavigated. The lens power
oscillates between a power $Sph$ along the axes $Axis$ and $Axis + 180^\circ$, and a power $Sph + Cyl$ along the axes $Axis + 90^\circ$ and $Axis + 270^\circ$ in a sinusoidal fashion.

To facilitate the analysis of the captured image, it is more useful to represent the prescription in terms of power vectors.[1] The representation decomposes the sphero-cylindrical lens into an average spherical equivalent lens $M$, and two crossed cylinders $J_0$ and $J_{45}$, oriented horizontally and at $45^\circ$, respectively. These power vectors are related to the original prescription components by

$$M = Sph + Cyl/2$$
$$J_0 = -(Cyl/2)\cos(2Axis)$$
$$J_{45} = -(Cyl/2)\sin(2Axis)$$

Since the lens power varies with azimuthal angle, the value of the magnification $m$ in eq. 1 will vary as well. Since sphero-cylinder lenses are not rotationally symmetric, the variable magnification introduces two additional types of distortion into the captured image: anamorphic distortion and skew distortion. Figure 2 shows examples anamorphic and skew distortion. In anamorphic distortion, the checkerboard squares become rectangular due to differences in magnification in the horizontal and vertical directions. This type of distortion occurs when the cylinder axis $Axis$ is nominally aligned with the horizontal or vertical axes. Skew distortion causes the checkerboard squares to become parallelograms and occurs when the cylinder axis $Axis$ is at an oblique angle.

![Image of checkerboard squares showing distortion](image)

Figure 2. (a) Anamorphic distortion causes the checkerboard squares to expand/compress in the horizontal and vertical directions. (b) Skew distortion causes the checkerboard squares to skew and become parallelograms

In analyzing the proposed system, the object coordinates $(x_o, y_o)$ are related to the image coordinates $(x_i, y_i)$ by

$$\begin{pmatrix} x_o \\ y_o \end{pmatrix} = \begin{pmatrix} (M + J_0 + \frac{1}{z})z & -J_{45}z \\ -J_{45}z & (M - J_0 + \frac{1}{z})z \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix}$$

Eq. 3 shows that if at least three object points can be related to their corresponding image points, then the components of the power vectors can be calculated. Three points are the minimum, but in general, more points are used to reduce noise in locating these points. For $N$ object/image points consider the matrix equation

$$C = AB,$$

where $C$ and $B$ are $2 \times N$ matrices of the object and image points, respectively. The matrix $A$ is given by
\[ A = \begin{pmatrix} \left( M + J_0 + \frac{1}{z} \right) z & -J_{45} z \\ -J_{45} z & \left( M - J_0 + \frac{1}{z} \right) z \end{pmatrix} \]  

(5)

The matrix equation can be solved by

\[ A = CB^T[BB^T]^{-1} \]  

(6)

Once the matrix \( A \) is determined from the object/image points, its diagonal components can be used to solve for \( M \) and \( J_0 \), given the object distance \( z = 85.6 \text{ mm} \). The off-diagonal components are used to determine \( J_{45} \). Any noise in the position of the object/image points will cause the off-diagonal components to differ slightly. Averaging the two off-diagonal components helps reduce this effect. Once the power vector components are determined, the components of the conventional lens prescription are calculated by

\[ Sph = M - \sqrt{J_0^2 + J_{45}^2} \]

\[ Cyl = 2 \sqrt{J_0^2 + J_{45}^2} \]

\[ Axis = -\tan^{-1}\left( \frac{Cyl/2 + J_0}{J_{45}} \right) \]  

(7)

Software has been written in MATLAB (MathWorks, Natick, MA) to analyze images captured with the system. Several preprocessing steps are used prior to calculation of the prescription. First, the checkerboard squares outside of the lens aperture are found and the image rotated to ensure the edges of these squares are horizontal and vertical. Next, if needed, the image is resampled to ensure that the external squares are indeed square. Finally, the top rim of the spectacle frame is found to quantify the tilt in the spectacle frames relative to the horizontal axis. This enables a post-hoc correction of the cylinder axis orientation if the frames are not held perfectly horizontal. After these steps, the preprocessed image is analyzed to determine the corners of the checkerboard squares located within the lens aperture and located in the region surrounding the lens. The surrounding corners are used to determine the locations of the object coordinates \((x_0, y_0)\), while the corners within the lens aperture give the image coordinates \((x_i, y_i)\). Once these points are determined, eqs. 5-7 are used to calculate the lens prescription. Figure 3 shows a couple of the intermediate steps in the image processing routines.

Figure 3. (a) Corners lying outside of the lens aperture are used to predict the location of the object points, while the image points are determined by the corners found within the lens aperture. (b) Multiple corners within the lens aperture can be found to reduce the effect of noise in determining the corner locations.
3. RESULTS

A total 24 pairs (48 lenses) of single-vision spectacle lenses were tested with the proposed system. These lenses were donations to the University of Arizona Department of Ophthalmology & Vision Sciences and had been previously catalogued and measured with a lensmeter to determine their prescriptions. The lenses had spherical powers as measured by the lensmeter ranging from $-11.25$ diopters to $+4.25$ diopters. The cylindrical powers range from $-5.25$ diopters to $-0.25$ diopters, with a wide assortment of axes. When comparing measurements from two different devices, Bland-Altman analysis is appropriate.[2] In this analysis, the mean of the measurement from the two devices is plotted against the difference between measurements from the two devices. This highlights discrepancies and biases between the two independent measurement techniques without assuming one is the “gold standard.” Figure 4 shows the resulting Bland-Altman plots for the measured lenses.

![Bland-Altman plots](https://www.spiedigitallibrary.org/conference-proceedings-of-spie)

Figure 4. (a) Power vector components of the right lens. (b) Power vector components of the left lens.

In the ophthalmic community a tolerance of $\pm 0.25$ diopters is typical accepted on power measurements of lenses. In the plots of Figure 4, the ideal case would be to have all the measurements within $\pm 0.25$ diopters on the vertical axis. The horizontal axis is representation of the range of powers of the test lenses. Many of the crossed cylinder components fall within the desired range, meaning that the cylindrical power and its orientation are well measured with the proposed
technique. The spherical equivalent component however shows a linear bias where powers of negative lenses are under-predicted and the powers of positive lenses are over-predicted. The strong linearity of these result though suggest that the errors can be calibrated based on these results. There are certainly a few outliers in the data as well and these will be examined in more detail in the Discussion.

4. DISCUSSION

A system for measuring the prescription of spectacle lenses has been demonstrated. The technique relies on measuring the magnification changes in the virtual image of a checkboard pattern. For simple spherical lenses, a simple scaling of the checkboard squares is expected. For spectacle lenses, in general, sphero-cylindrical lenses are used. Due to their lack of rotational symmetry, sphero-cylindrical lenses will also introduce anamorphic and skew distortion into the checkerboard scales, causing the squares to scale differently in the horizontal and vertical directions, as well as skew into a parallelogram shape. This analysis demonstrated that by finding the corners of checkboard squares in the object space and the image space, the prescription can be recovered. For the most part, the cylindrical component and its orientation were recovered well. For the spherical component, a linear bias was found. However, the linear nature of this bias suggests that a simple calibration curve based on these results could remove the bias and lead to results within the desired ±0.25 diopters. The data outliers were also examined. Figure 5 shows one such example. Here, the checkerboard squares exhibit not only the expected anamorphic and skew distortion, but they also have a keystone distortion component which is not taken into account in the current model. This keystone distortion could be occurring because there is marked prism in the lenses (i.e. the lenses are wedged) which is often used fix misalignment between the visual axes of the two eyes, or the lenses could be progressive addition lenses which use a freeform surface to vary the power from the top to bottom of the lens. Analysis of the outliers provides useful insights into the issues associated with this technique when applied to real-world scenarios. In all, the results are encouraging and the technique will continue to be developed to further refine the techniques outline here.

Figure 5. In the pair of spectacles, the checkerboard squares exhibit keystone distortion which would be expected from progressive addition lenses.

REFERENCES
