LDPC-Coded Squeezed-Displaced States-based Quantum Communications

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Abstract: We demonstrate that LDPC-coded squeezed-displaced-states-based QPSK has better tolerance to background noise and significantly outperforms corresponding coherent states-based counterpart. For average number of thermal photons 0.3, for LDPC-code of rate 0.8, the improvement is >3 dB.

Quantum communication harnesses the quantum properties of the light to enable functionalities that cannot be realized by the classical communication techniques [1]-[4]. The coherent states are non-orthogonal states that possess the loss-tolerant property, while preserving the coherence over lossy bosonic channels. One of the key challenges is to design both the transmitter and quantum receiver to maximize the mutual information [2]. Given that the coherent states are non-orthogonal we have to use the quantum hypothesis testing to properly separate the messages being transmitted [3],[5]. The complexity of optimum quantum receivers, achieving either minimum error probability or channel capacity, is prohibitively high. To solve for this problem numerous sub-optimum receivers have been proposed [6]-[9]. In particular, the class of receivers based on beams splitters and photodetectors, implementing the displacement operators, are easy to implement and are of low-cost [2],[8].

The quantum receivers with optical squeezing can provide near optimum discrimination for QPSK coherent state signals as shown in [1],[10]. Moreover, it has been shown in [11] that squeezed-displaced states-based communications can potentially significantly outperform coherent-states based communications. However, to achieve the target bit-error rate (BER) such as 10^{-9}, in the presence of background noise, the required number of signal photons is prohibitively high. To reduce the number of signal photons required to achieve target BER performance for quantum communications, the low-density parity-check (LDPC) codes should be used. The LDPC-coded coherent states-based *M*-ary PSK has already been studied in [3].

In this paper, for the first time, we study the LDPC-coded squeezed-displaced states-based *M*-ary PSK in the presence of background noise. We demonstrate by simulations that in the presence of noise the LDPC-coded squeezed-displaced states-based QPSK significantly outperforms the corresponding LDPC-coded coherent states-based QPSK, for high-rate LDPC codes. The LDPC codes used in this paper belong to the class of quasi-cyclic (QC) codes, which are suitable for hardware implementation [4].

The squeezed-displaced coherent state $|\alpha, \underline{s}\rangle$ can be represented in terms of the ground state $|0\rangle$ as follows:

$$\left|\alpha,\underline{s}\right\rangle = D\left(\alpha\right)S\left(\underline{s}\right)\left|0\right\rangle,\; S\left(\underline{s}\right)=e^{\frac{1}{2}\left(\underline{s}a^{\dagger 2}-\underline{s}^{*}a^{2}\right)},\; D\left(\alpha\right)=e^{\alpha a^{\dagger}-\alpha^{*}a}, \tag{1}$$

where $S(\underline{s})$ is the *squeezing operator* and $\underline{s}$ is the squeezing factor, which in general is a complex number $\underline{s}=s\cdot\exp(j\theta)$. With $D(\alpha)$ we denoted the *displacement operator*, wherein $a^{\dagger}$ (a) denotes the creation (annihilation) operator. By applying the displacement operator on the vacuum state we obtain the coherent state, that is $|\alpha\rangle= D(\alpha)|0\rangle$. To generate the squeezed states we typically use the degenerate parametric down-conversion devices [12].

The binary sequence, generated by the uniform random generator, is LDPC encoded. At each channel use, $\log_2 M$ bits from the codeword are used to select a point from the signal constellation. The signal constellation points in squeezed-displaced states-based *M*-ary PSK quantum communications can be represented by the following pure states:

$$\left|\alpha_m,\underline{s}_m\right\rangle = \left|\alpha e^{j2\pi m/M}, se^{j\theta}e^{j2\pi 2m/M}\right\rangle;\; m=0,1,\cdots,M-1 \tag{2}$$

The average number of photons per symbol N_s is related to the squeezing parameter s by $N_s=|\alpha|^2+\sinh^2 s$. Therefore, the number of signal photons must be sufficient to enable both squeezing and displacement operators. To select the squeezing factor, symbol error probability plots for QPSK, based on squeezed-displaced states, in absence of noise, are shown in Fig. 1. Clearly, various squeezed-displaced states-based QPSK schemes outperform the coherent states-based QPSK. The squeezing factor needs to be properly chosen for LDPC-coded QPSK. As an illustration when large-girth, high-rate QC LDPC codes [4] are used, with threshold BER >0.1, LDPC-coded QPSK based on coherent states can outperform corresponding counterpart with $s=-0.8$, but required code rate will be too low. Here we are concerned with high code rates, similar to those used in fiber-optics communication systems.

To evaluate the performance of LDPC-coded squeezed-displaced-states-based QPSK, we assume that the pure states, defined by Eqn. (2), for M=4, are transmitted over quantum additive Gaussian channel, which maps the covariance matrix $\mathbf{\Sigma}$ to $\mathbf{\Sigma}+2N_t\boldsymbol{I}_2$, where $\boldsymbol{I}_2$ is the 2×2 identity matrix and N_t is the average number of the noise photons. This scenario is applicable in deep-space, satellite-to-satellite, near-Earth, and fiber-optics quantum communications. By setting the squeezing parameter to $\underline{s}=-0.5$ and by using the girth-8 (16935,13550) QC-LDPC code, in Fig. 2 we summarize the BER performance of LDPC-coded QPSK for two values of the average number of noise photons 0.1 and 0.3. For target BER of 10^{-7} and average number of noise photons of 0.3 the LDPC-coded squeezed-displaced-states-based QPSK outperforms the corresponding LDPC-coded coherent states-based QPSK by 3.23 dB. For the same BER and the average number of noise photons being 0.1, the improvement is lower, 0.85 dB. To reduce the gap between these two coded schemes, a lower rate LDPC code should be employed as illustrated in Fig. 2. Namely, by using the girth-10 LDPC (17076,12809) code of rate 0.75, for N_t=0.3, the gap can be reduced down to 2.72 dB. Nevertheless, the improvement of LDPC-coded squeezed-displaced-states-based QPSK over corresponding coherent states-based counterpart is still significant. Additional details of this quantum coded modulation scheme as well as the quantum detector configuration will be provided at the conference.

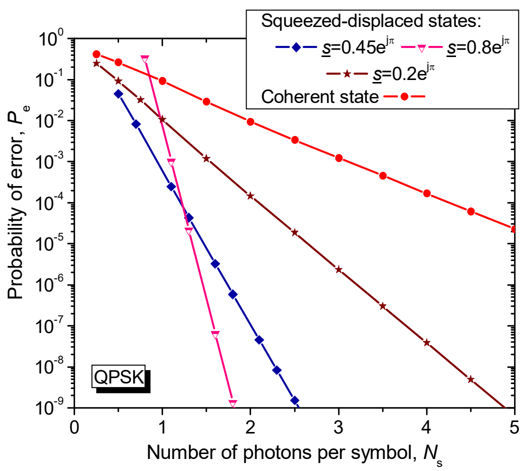

Fig. 1 Uncoded symbol error probability vs. average number of signal photons/symbol for squeezed-displaced states-based QPSK in the absence of noise.

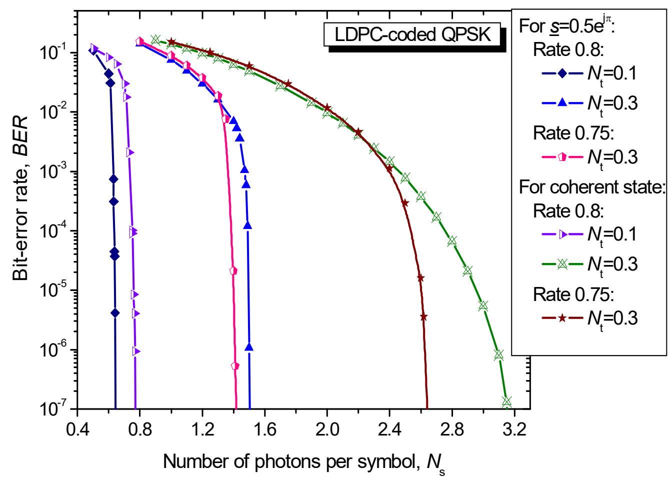

Fig. 2 BER performance of LDPC-coded squeezed-displaced states-based QPSK vs. average number of signal photons/symbol N_s for different average number of noise photons N_t.

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