

Article

Observation of Discrete Floquet Time Crystals in Periodically Driven Acoustic Bubbles

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Abstract: We show experimentally and theoretically that the translation dynamics of acoustic bubbles in an acoustic standing wave field exhibit all the attributes of a discrete time crystal, the dynamics of which is described by Mathieu's equation. Individual bubbles and synchronized bubbles in a self-organized chain undergo emergent slow persistent oscillations. The period of the emergent oscillations is longer than that of the driving acoustic wave by three orders of magnitude, therefore, breaking the discrete time translation symmetry of the driver.

Keywords: acoustic bubbles; time crystal; Mathieu's equation

1. Introduction

Spontaneous breaking of continuous time translational symmetry [1,2] in quantum equilibrium systems is forbidden [3,4]. However, discrete time translation symmetry can be broken in out-of-equilibrium periodically driven systems [5–7]. Periodically driven systems possess a discrete time translational symmetry related to the driving period. The signature of broken discrete time translation symmetry takes the form of emergent subharmonic dynamics with a period larger than that of the driver. This type of “crystallization” of time into a “superlattice” has been recently observed experimentally in many-body spin systems [8,9]. In addition to quantum systems, continuous time translation symmetry breaking in conservative classical systems was proposed [10]. Recently, we have experimentally demonstrated the existence of a thousand-fold frequency down-conversion of the translational oscillations of individual and chain-forming submillimeter acoustic bubbles in water subjected to a high-frequency ultrasonic standing wave [11]. Here, we present the interpretation of this experimental observation in the framework of classical discrete time and space-time crystals. The translation dynamics of driven acoustic bubbles meet the three criteria and conditions for a Floquet time crystal [5,6,12], namely broken time translation symmetry, rigidity and persistence. The spatial translational degree of freedom of bubbles in the acoustic standing wave field, measured as the position along the direction of the acoustic waves, breaks the time translation symmetry of the system's Hamiltonian. Bubble position exhibits slow emergent oscillations with the period exceeding that of the drive by orders of magnitudes and depends only on physical characteristics of water and of the acoustic field (rigidity). The slow oscillations lifetime is long (persistence). The observation of a classical time crystal opens the door to exploring dynamical phases of matter outside the more arduous quantum realm. Acoustic bubble-based classical time crystals offer a new platform for the exploration of the behavior of matter driven out of equilibrium.

Symmetry breaking in acoustic or elastic systems can generate states with non-conventional topology [13]. For example, broken time-reversal symmetry and parity symmetry can block the propagation of acoustic or elastic waves in one direction leading to back-scattering immune topologically protected states [14]. Breaking time-reversal symmetry and parity symmetry can be achieved by driving an elastic medium with an externally directed spatiotemporal modulation of its elastic properties [14]. Consequently, we pose



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the question: can the time translational symmetry be broken by driving acoustic systems out of equilibrium?

The fascinating dynamics of bubbles in acoustic fields result from the coupling between translational degrees of freedom and radial oscillations or surface instabilities, a complex non-linear phenomenon [15]. A number of numerical simulation studies brought to light the possibility of slow oscillatory translational motions in standing waves [16,17]. The frequency of these translational oscillations was predicted to be several orders of magnitudes lower than the frequency of the acoustic wave that drives the motion. Recently, we experimentally observed the existence of slow translational oscillations (170 Hz) of sub-millimeter size bubbles in an acoustic standing wave of high frequency (500 kHz) [11]. Here, we demonstrate that the oscillatory translation motion of acoustic bubbles satisfies the three criteria and conditions for Floquet time crystals of Ref. [12].

2. Model and Method

In the absence of any other forces such as drag, the translational dynamics of a gas bubble in water subjected to acoustic irradiation can be simply described by Newton's equation of motion:

$$m_A \ddot{r} = F(r, t) \quad (1)$$

where $m_A = \rho V_A$ is the apparent mass of the bubble with ρ being the density of water and $V_A = \frac{1}{2}V(t)$ being the apparent volume where $V(t)$ is the time-dependent volume of the bubble. $F(r, t)$ is the acoustic radiation force or Bjerknes force [15],

$$F(r, t) = -V(t)\nabla P(r, t) \quad (2)$$

where $\nabla P(r, t)$ is the local instantaneous pressure gradient. Under the influence of this effective force, a bubble may undergo oscillatory translations in response to an oscillatory pressure field associated with the sound wave, as well as its own volume pulsation.

In Ref. [11], we have shown that for an acoustic standing wave, with pressure amplitude P_a and driving angular frequency ω , Equations (1) and (2) can be reformulated in the form of Mathieu's equation:

$$\frac{d^2x}{dz^2} - 2q\cos 2zx = 0 \quad (3)$$

where x is the position of the bubble along the direction of the standing wave, the time-related variable z is defined by $2z = \omega t - \frac{\pi}{2}$ and $q = \frac{8P_a}{\rho v^2}$. v is the speed of sound in water. The parameter q depends only on the physical characteristics of the fluid and the pressure amplitude of the acoustic wave. Equation (3) can be reformulated as a Schrödinger-like Eigen value problem:

$$Hx = \left[\frac{d^2}{dz^2} - V(z) \right] x = -Ex \quad (4)$$

where the potential $V(z) = 2q\cos 2z$ is β periodic in z or $\tau = \frac{2\pi}{\omega}$ periodic in time. E is the Eigen value of the problem. The Hamiltonian, H , possesses discrete time translational symmetry $H(t + \tau) = H(t)$. We note that in the absence of a potential, Equation (4) reduces to the Eigen value problem of a free particle. However, it is also isomorphic to the equation of motion of a harmonic oscillator. Using a complex solution of the general form $x = x_0 e^{i\gamma_0 Z}$ the characteristic frequencies are given by the dispersion relation, $\gamma_0^2 = E$. This quadratic dispersion relation is illustrated schematically in Figure 1 as a band diagram (see dashed curves).

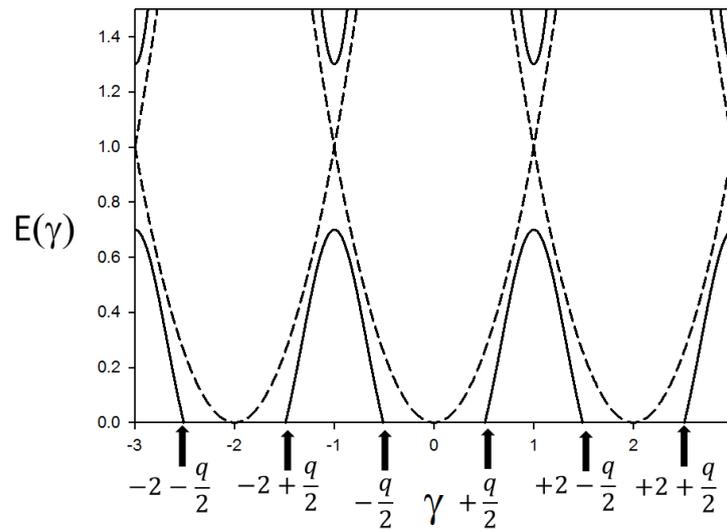


Figure 1. Schematic illustration of the band diagram for Floquet solutions of Mathieu’s equation (Equation (4)) in the first and second temporal Brillouin zones in absence of periodic potential, $E = \gamma_0^2$ (dashed curves) and with a periodic potential (solid curves). The arrows mark approximate ground state solutions of the periodically driven bubble. The interval $[-1, 1]$ constitutes the Brillouin zone in the frequency domain.

The ground state ($E = 0$) is that of a particle in uniform motion (zero frequency oscillation). The dynamics of an acoustic bubble described by Equation (3) corresponds to the Eigen ground state of Equation (4), i.e., $E = 0$, but as we will see, it may not correspond to a uniform motion of the bubble but to slow oscillatory translational motion. Floquet’s theorem can be used to seek solutions of (4) in the form:

$$x(z) = e^{i\gamma z} f(z) \tag{5}$$

where $f(z)$ is a periodic function of period β and γ is the characteristic exponent. $\gamma \in [-1, 1]$ defines a Brillouin zone in the frequency domain. Expanding the periodic function f in a Fourier series: $f(z) = \sum_{K=-\infty}^{K=+\infty} c_K e^{i2Kz}$ and inserting the sought solution (5) into Equation (4) leads to an Eigen value problem with the recurrence relation:

$$c_K \left[(2K + \gamma)^2 + E \right] + q(c_{K+1} + c_{K-1}) = 0 \tag{6}$$

This problem can be solved to various degrees of accuracy by truncating the recurrence relations to different orders in K . For instance, if we limit the recurrence to the truncated Eigen vector (c_0, c_1) , approximate solutions for γ can be obtained from the condition:

$$\begin{vmatrix} \gamma^2 + E & q \\ q & (2 + \gamma)^2 + E \end{vmatrix} = 0 \tag{7}$$

In the ground state, we find the following approximate solutions for the characteristic exponent: $\gamma_1 = -1 \pm \sqrt{1 + q}$ and $\gamma_2 = -1 \pm \sqrt{1 - q}$. Limiting the recurrence to the truncated Eigen vector (c_{-1}, c_0) gives the ground state solutions $\gamma_1 = +1 \pm \sqrt{1 + q}$ and $\gamma_2 = +1 \pm \sqrt{1 - q}$.

For water, $\rho = 1000 \text{ kg/m}^3$, $v \sim 1500 \text{ m/s}$ and $q \sim 3.5510^{-9} Pa$. In Ref. [11], we measured an experimental pressure amplitude of 2.3 atm leading to the small value $q \sim 8.16 \times 10^{-4}$. Therefore, for the second order in truncation and for small values of q , the characteristic exponent takes on the real values: $\gamma = -2 \pm \frac{q}{2}; \pm \frac{q}{2}; +2 \pm \frac{q}{2}$. These are solutions in the first and second Brillouin zone. To generate solutions in higher-order Brillouin zones, one needs to solve the recurrence Eigen value problem at degrees of truncations in $K > 1$. The four ground state exponents are marked in Figure 1, along with

a schematic representation of the complete band diagram $E(\gamma)$ in the presence of a time periodic potential. The periodic potential not only opens gaps along the E -axis but also along the γ -axis. The width of these gaps depends on the magnitude of the parameter q . The ground state solutions with $\gamma = \pm \frac{q}{2}$ correspond to sinusoidal translational oscillations with non-zero frequencies, $\frac{q}{2}\omega \sim 200$ Hz. The period, $\tau_G = \frac{2\pi}{\gamma\omega}$, of these ground-state oscillations is three orders of magnitude longer than that of the driving acoustic wave. Therefore, one can write that $x(t + \tau_G) = x(t)$ but $x(t + \tau) \neq x(t)$. The acoustic bubble satisfies the first criterion for having a Floquet crystal, namely the discrete time translation symmetry associated with the Hamiltonian H is broken. The parameter q and, therefore, the characteristics exponent γ are constants and depend only on the physical characteristics of water and of the acoustic standing wave. The frequency of the slow oscillation does not depend on fine-tuned parameters in the Hamiltonian but on the physical characteristics of the system. Therefore, the acoustic bubble translational oscillations satisfy the condition of rigidity. The low-frequency ground state solutions of Equation (4) are pure sinusoidal functions without attenuation and will persist indefinitely.

3. Results

In Figure 2a, we compare the theoretical predictions of Section 2 with the experimental dynamical behavior of the sub-millimeter acoustic bubble reported in Ref. [11]. Time translation symmetry is broken as the emergent oscillatory dynamics of the bubble's center of mass has a period longer by approximately three orders of magnitude than that of the driving acoustic wave. The slow oscillations are persistent and last for a very long time on the scale of the number of acoustic cycles. The Fourier transform of the measured oscillations can be defined unambiguously and presents a marked peak at the symmetry breaking frequency of ~ 170 Hz (in very good accord with the theoretical prediction above). Five other submillimeter-size single bubbles, with slightly different radii and locations in the water tank, were studied and showed qualitatively the same behavior and semi-quantitatively the same features in their power spectrum. Furthermore, the observed slow oscillations occur in spite of numerous parasitic effects associated with phenomena such as acoustic streaming and microstreaming or a chromatic source of the sound associated with the transducer bandwidth. The emergent oscillatory behavior is, therefore, robust against perturbations.

In Figure 2b, we show the time evolution of the position of a bubble in a chain of bubbles. The chain arrangement is due to the interaction between each bubble and the primary acoustic standing wave field and between bubbles through secondary acoustic fields (details on the experimental setup are given in Ref. [11]). Indeed, secondary Bjerknes forces [15] lead to attractive forces between bubbles oscillating radially in phase. The attractive forces organize bubbles into stable chains aligned along the direction of the standing wave. In Ref. [11], we have shown that for a bubble in a chain, the translational motion is still obeying Equation (3) but with the parameter q replaced by $Q = q \left(1 + \frac{2\omega^2}{(\omega^2 - \omega_0^2)} \frac{R_0}{\pi^2 \lambda} \sum_{p=1}^{\infty} \sin^2 p \frac{\beta \lambda}{2} \right)$, where R_0 and $\omega_0(R_0)$ are the average radius and characteristic frequency of the bubble. The quantity $\beta \in \left[-\frac{\pi}{\lambda}, \frac{\pi}{\lambda} \right]$ is a wave number associated with the spatial periodicity of bubbles in the chain. The summation accounts for the long-range nature of the interaction of bubbles, and p refers to the order of interaction with neighboring bubbles ($p = 1$ for first-nearest neighbor interactions, $p = 2$ for second-nearest neighbor interactions, etc.). As in the case of an individual bubble, bubbles forming a chain will exhibit ground state time translation symmetry breaking oscillations with a range of frequencies defined by the Floquet characteristic exponent $\Gamma(\beta) = \pm \frac{Q(\beta)}{2}$. The long wavelength slow oscillatory modes ($\beta = 0$) are those of a single bubble. For other values of the wave number, dispersion of these modes will occur due to the trigonometric terms in the expression for Q . Therefore, bubbles in a chain form a space-time crystal with spatial, $\frac{\lambda}{2}$, periodicity and time translation symmetry breaking temporal periodicity, $T_G = \frac{2\pi}{\Gamma\omega}$. The experimental measurement of the dynamics of a bubble in a chain, shown in Figure 2, shows emergent slow oscillations

that break the discrete time translation symmetry of the driving acoustic wave, rigidity of the dynamics, which is observed to be robust with respect to numerous sources of perturbations in the experiment, and persistence of the oscillations over a long time period on the scale of the period of the acoustic wave. The persistence allows us to calculate the Fourier spectrum of a bubble in a chain. This spectrum is well defined but now exhibits dispersion in the form of a spectral band characteristic of a space-time crystal.

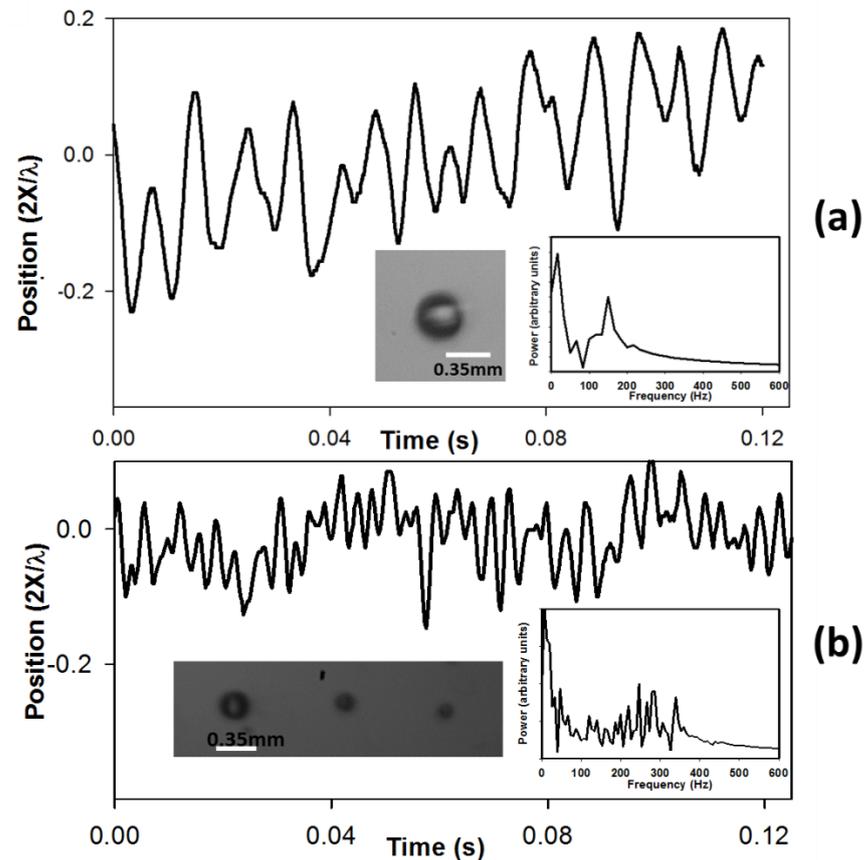


Figure 2. Measured position of (a) an isolated acoustic bubble and (b) an acoustic bubble in a self-organized chain of bubbles as a function of time. The position is that of the center of mass of the bubble along the direction of the standing wave normalized to half a wavelength, λ , of the acoustic wave. The power spectrum calculated from the Fourier transform of the normalized time-autocorrelation function of the position of the bubble (lower right inset of (a,b)). Lower left inset of (a,b): snapshot of the oscillating bubble(s).

4. Conclusions

We have shown that the translation dynamics of acoustic bubbles in an acoustic standing wave field possess all the attributes of a discrete time crystal. Individual bubbles and bubbles in a self-organized chain undergo emergent slow persistent oscillations with synchronization of the bubbles in the latter case. The period of the emergent oscillations is longer than that of the driving acoustic wave by three orders of magnitude; therefore, breaking the discrete time translation symmetry of the driver. The time (individual bubble) and space-time (chain of bubbles) crystals are rigid or robust to a number of parasitic perturbations in the systems. The acoustic bubble is but one example of a time crystal, the dynamics of which is described by Mathieu's equation. The realm of application of Mathieu's equation is quite broad and includes the prototypical inverted pendulum [18], floating vessel on a wavy sea [19], pulsating blood flow in vasculature [20] and the behavior of neutral and charged particles in electro-magnetic traps, such as the 1989 Nobel-prize winning "Paul trap" [21].

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