MASKING DURING THE COVID-19 PANDEMIC:

A GAME THEORY ANALYSIS

By

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Abstract

Scientists have found that face masks are an effective way of preventing the transmission of the SARS-CoV-2 virus. Throughout the past two years public health officials in the United States have proposed several policies recommending Americans wear face masks when in public. However, these policies have faced severe opposition, with many people claiming that the federal government has overstepped its authority. This paper aims to determine whether federal masking policies are necessary to persuade the American public to wear masks when interacting with others. Three hypothetical scenarios will be analyzed utilizing the game theory concepts of expected utility and Bayesian Nash Equilibrium to determine if individuals will make the decision to wear a mask without government intervention. The paper will show that regardless of one's choice mechanism, individuals will only wear masks when the cost associated with wearing a mask is outweighed by the benefit associated with wearing a mask. Thus, in order to ensure that the majority of people will wear a mask in public and slow the spread of COVID-19, the United States government must enact public health policies aimed at manipulating the costs and benefits of wearing masks.

Keywords: face masks, COVID-19, public health policies, mask mandates

Contents

Introduction

On January 20, 2020, the Centers for Disease Control and Prevention (CDC) announced the first confirmed case of COVID-19 in the United States ("CDC Museum Covid-19 Timeline", 2022). The past 27 months have had unprecedented severe economic and social impacts. As of April 23, 2022, there have been 80.8 million confirmed coronavirus cases and 990 thousand deaths throughout the United States (Smith et al., 2020). In response to the global pandemic, governments across the world have begun recommending and, in some cases, mandating citizens wear face masks in public areas.

The purpose of this paper is to use game theory models to better understand how individuals behave in terms of masking decisions in the absence of government interventions and recommendations. Is it necessary for governments to implement policies to reach a Pareto optimal outcome?

1 Method

In order to determine how individuals will react in the absence of government intervention, this paper will explore several interactions between an egoist and two different types of altruists. For the purposes of this paper an egoist will be defined as an individual who is only concerned with their own well-being. A Type I altruist will refer to an individual who receives additional utility by helping others, but is not harmed when they opt not to. A Type II has the same initial characteristics of a Type I altruist but adds the additional condition that there is a cost associated harming others.

I will investigate three game models. The first model will look at the situation where both individuals in the game are egoists. The second will explore the interaction between an egoist and a Type I altruist. The third game will model the interactions between an egoist and a Type II altruist.

1.1 Code-book

The models in this paper will use the following variables to represent the different factors involved in the decision making processes.

- S = utility of being sick with COVID-19
- M = wearing a mask
- $\sim$M = not wearing a mask
- C = cost associated with wearing a mask
- V = probability of having COVID-19
- I = infection status
- B = additional altruistic benefit from helping others
- N = cost associated with harm of not helping others as an altruist

2 Alf and Betty are Both Egoists

To explore what masking decisions are made when two egoists are interacting with each other we can consider the following hypothetical scenario:

Alf and Betty, both egoists, have an upcoming exam in their philosophy course and have decided to meet at Scented Leaf to study. There are no mask mandates in effect nor has Scented Leaf posted any COVID-19 protocol signs.

Both Alf and Betty took a COVID-19 test earlier in the day. Betty received a negative test result 30 minutes prior to their meeting. Alf decided to leave for the meeting without test results. However, on his way to Scented Leaf Alf receives a notification on his phone containing his test results. Alf is very stressed about the upcoming exam and has decided that he is going to attend this study session regardless of the results.

Alf and Betty must choose independently whether or not they will wear a mask at the tea shop. Alf is made aware of Betty's health status, but Betty does not know the results of Alf's test.

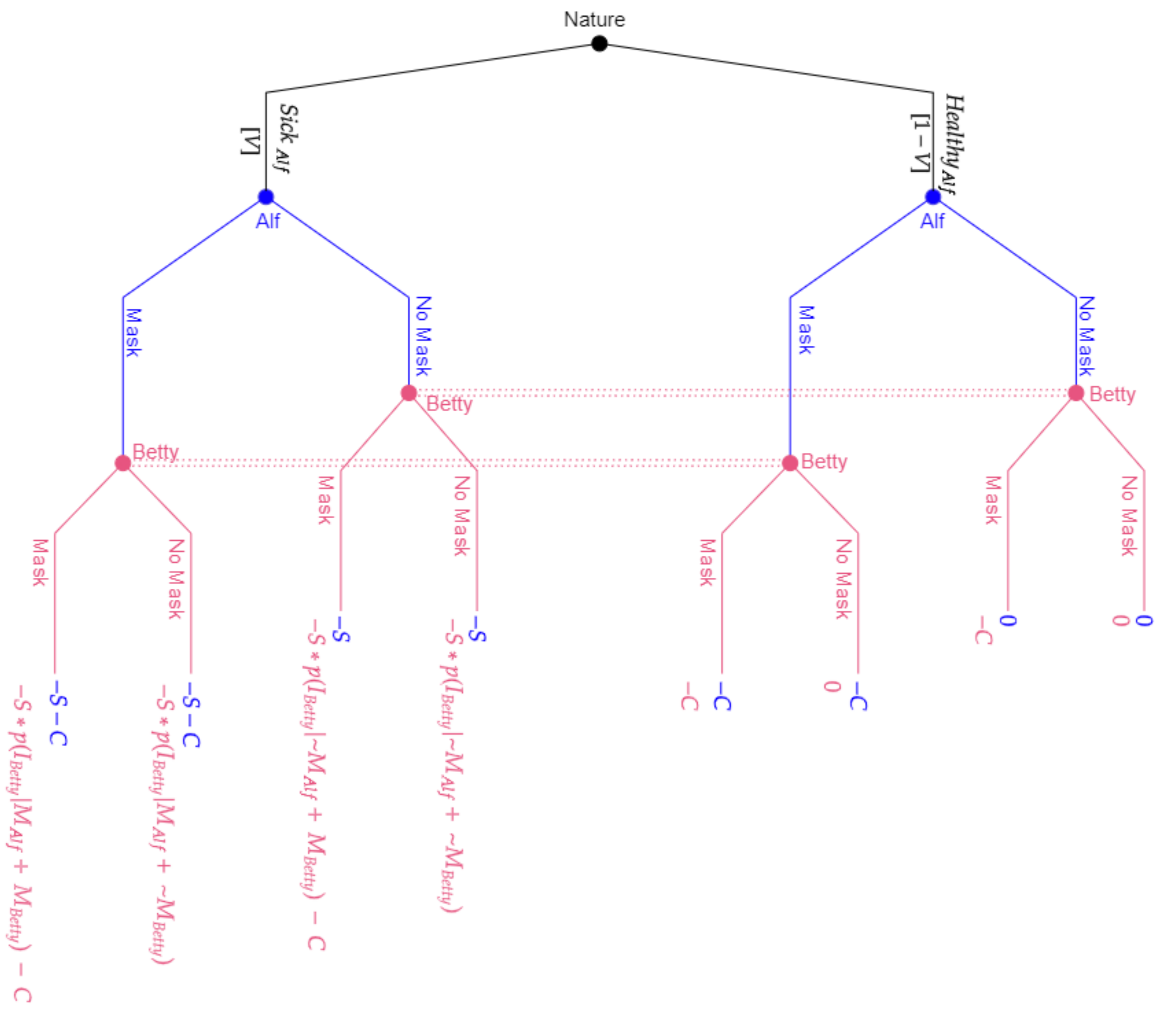

Figure 1: Egoist - Egoist Game Tree

2.1 Game Tree

The below tree maps out the eight different possible interactions between Alf and Betty. Alf's test result could have come back positive, yielding the probability V, or negative, yielding the probability $1 - V$. Then, given Alf's health status he must choose whether or not he is going to wear a mask. Betty does not know Alf's test results and thus must make her masking decision with incomplete information. The two different information sets are represented by the pink dotted lines in the game tree (Figure 1).

2.2 Game Matrices

Alf has four strategies that he can choose from when deciding whether or not to wear a mask: 1) Always Mask, 2) Only Mask if Sick, 3) Only Mask if Healthy, and 4) Never Mask. Betty also has four strategies she can choose from: 1) Always Mask, 2) Only Mask if Alf Masks, 3) Only Mask if Alf Does Not Mask, and 4) Never mask. Using these strategies, a 4x4 game matrix was created to analyze the expected payoffs for each decision (Table 1). Alf receives the highest utility by choosing the strategy Never Mask, regardless of Betty's choice:

$$-S > V * (-S - C) + (1 - V)(-C))$$

Given this assessment, Betty can rationally assume that Alf will never wear a mask.

Table 1: Egoist vs Egoist

		Betty			
		Always Mask	Only Mask If Alf Masks	Only Mask If Alf Does Not Mask	Never Mask
Alf	Always Mask	$(V)(-S-C)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S-C)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S-C)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+\ M_{Betty}))$	$(V)(-S-C)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+\ M_{Betty}))$
	Only Mask If Healthy	$(V)(-S)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|\ M_{Alf}+\ M_{Betty}))\ +(1-V)(-C)$	$(V)(-S)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|\ M_{Alf}+M_{Betty})-C)$	$(V)(-S)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|\ M_{Alf}+\ M_{Betty}))$
	Only Mask If Sick	$(V)(-S-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)$	$(V)(-S-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+\ M_{Betty}))\ +(1-V)(-C)$	$(V)(-S-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+\ M_{Betty}))$
	Never Mask	$(V)(-S)$ $(V)(-S*p(I_{Betty}\|\ M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S)$ $(V)(-S*p(I_{Betty}\|\ M_{Alf}+\ M_{Betty}))$	$(V)(-S)$ $(V)(-S*p(I_{Betty}\|\ M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S)$ $(V)(-S*p(I_{Betty}\|\ M_{Alf}+\ M_{Betty}))$

2.3 Expected Utilities

Using the payoffs listed in Table 1 we then calculated the expected utilities for both Alf and Betty in each scenario.

Given that Alf will never wear a mask, if Betty wears a mask she receives a utility of:

$$U(M) = (V * (-S * p(I_{Betty}| \sim M_{Alf} + M_{Betty}) - C)) + (-C * (1 - V))$$

$$U(M) = V(-S * p(I_{Betty}| \sim M_{Alf} + M_{Betty}) - C)$$

Given that Alf will never wear a mask, if Betty does not wear a mask she receives a utility of:

$$U(\sim M) = (V * (-S * p((I_{Betty}| \sim M_{Alf} + \sim M_{Betty})))) + ((1 - V) * 0)$$

$$U(\sim M) = V(-S * p((I_{Betty}| \sim M_{Alf} + \sim M_{Betty})$$

2.4 Bayesian Nash Equilibrium

In the Egoist versus Egoist game, Betty has incomplete information because she do not know Alf's health status. In order to calculate each player's best strategy, we must explore the game theory solution concept referred to as the Bayesian Nash Equilibrium. A Bayesian Nash Equilibrium refers to the strategy profile which maximizes the expected payoff for each player, given the strategy played by the other person and their beliefs about the state of nature (Spaniel, n.d.).

As reasoned above, Alf receives the highest utility from choosing the strategy of Never Mask regardless of Betty's strategy.

Given that Alf will Never Mask, Betty will wear a mask when:

$$V(-S * p(I_{Betty}| \sim M_{Alf} + M_{Betty}) - C) > V(-S * p(I_{Betty}| \sim M_{Alf} + \sim M_{Betty})$$

$$(-S * p(I_{Betty}| \sim M_{Alf} + M_{Betty}) - C/V) > (-S * p(I_{Betty}| \sim M_{Alf} + \sim M_{Betty})$$

Given that Alf will Never Mask, Betty will not wear a mask when:

$$(-S * p(I_{Betty}| \sim M_{Alf} + M_{Betty}) - C/V) < (-S * p(I_{Betty}| \sim M_{Alf} + \sim M_{Betty})$$

Given that Alf will Never Mask, Betty will be indifferent when:

$$(-S * p(I_{Betty}| \sim M_{Alf} + M_{Betty}) - C/V) = (-S * p(I_{Betty}| \sim M_{Alf} + \sim M_{Betty})$$

2.5 Analysis

We can rearrange the crucial inequality to get the following:

$$(-S * [p(I_{Betty}| \sim M_{Alf} + M_{Betty}) - p(I_{Betty}| \sim M_{Alf} + \sim M_{Betty})]) > C/V$$

$$(S * [p(I_{Betty}| \sim M_{Alf} + \sim M_{Betty}) - p(I_{Betty}| \sim M_{Alf} + M_{Betty})]) > C/V$$

Note that $p(I_{Betty}| \sim M_{Alf} + M_{Betty}) < p(I_{Betty}| \sim M_{Alf} + \sim M_{Betty})$, so the value on the left side of the inequality is positive.

As the disutility of being sick (S) increases, it is more likely the inequality is satisfied. As the difference between the two probabilities increases, it is more likely the inequality is satisfied. As the cost associated with masking (C) increases, it is less likely that the inequality is satisfied. As the probability of Alf having COVID-19 (V) decreases it is more likely that the inequality is satisfied.

It is important to note that "the inequality being satisfied" results in Betty wearing a mask. Therefore, Betty is more likely to wear a mask under the following conditions

1. The cost of being sick is high (S)
2. The cost of masking is low (C)
3. The likelihood that Alf is sick is high (V)
4. The amount of protection provided by the mask is high ($p(I_{Betty}| \sim M_{Alf} + M_{Betty}) << p(I_{Betty}| \sim M_{Alf} + \sim M_{Betty})$)

3 Alf is a Type I Altruist and Betty is an Egoist

While the previous model serves as a good baseline for this project, it is not very realistic. Most individuals are not fully selfish like an egoist. We must explore what happens when an egoist interacts with an altruist to have a better understanding of masking decisions in the real world. Consider the following scenario:

Alf, a Type I altruist, and Betty, an egoist, planned a date to see the new Batman movie in theaters. There are no mask mandates in effect nor has the movie theater posted any COVID-19 protocol signs.

Both Alf and Betty took a COVID-19 test earlier in the day. Betty received a negative test result a few hours prior to their date. Alf received a notification on his phone containing his test results as he was waiting in the parking lot for Betty to arrive. Alf is about to move across the country for law school and has decided that he is going to go on the date regardless of the results.

Alf and Betty must choose independently whether or not they will wear a mask on the date. Alf is made aware of Betty's health status, but Betty does not know the results of Alf's test.

3.1 Game Tree

Figure 2 shows Alf and Betty's expected utility in the eight different possible interactions. The top node, Nature, represents that Alf's test showed he is sick with COVID-19 with a probability of V and negative with probability $1 - V$. Then knowing his health status, Alf must choose whether or not he is going to wear a mask on their date. Betty does not know Alf's test results and thus must make her masking decision with incomplete information. The two different information sets are represented by the pink dotted lines in the game tree (Figure 2).

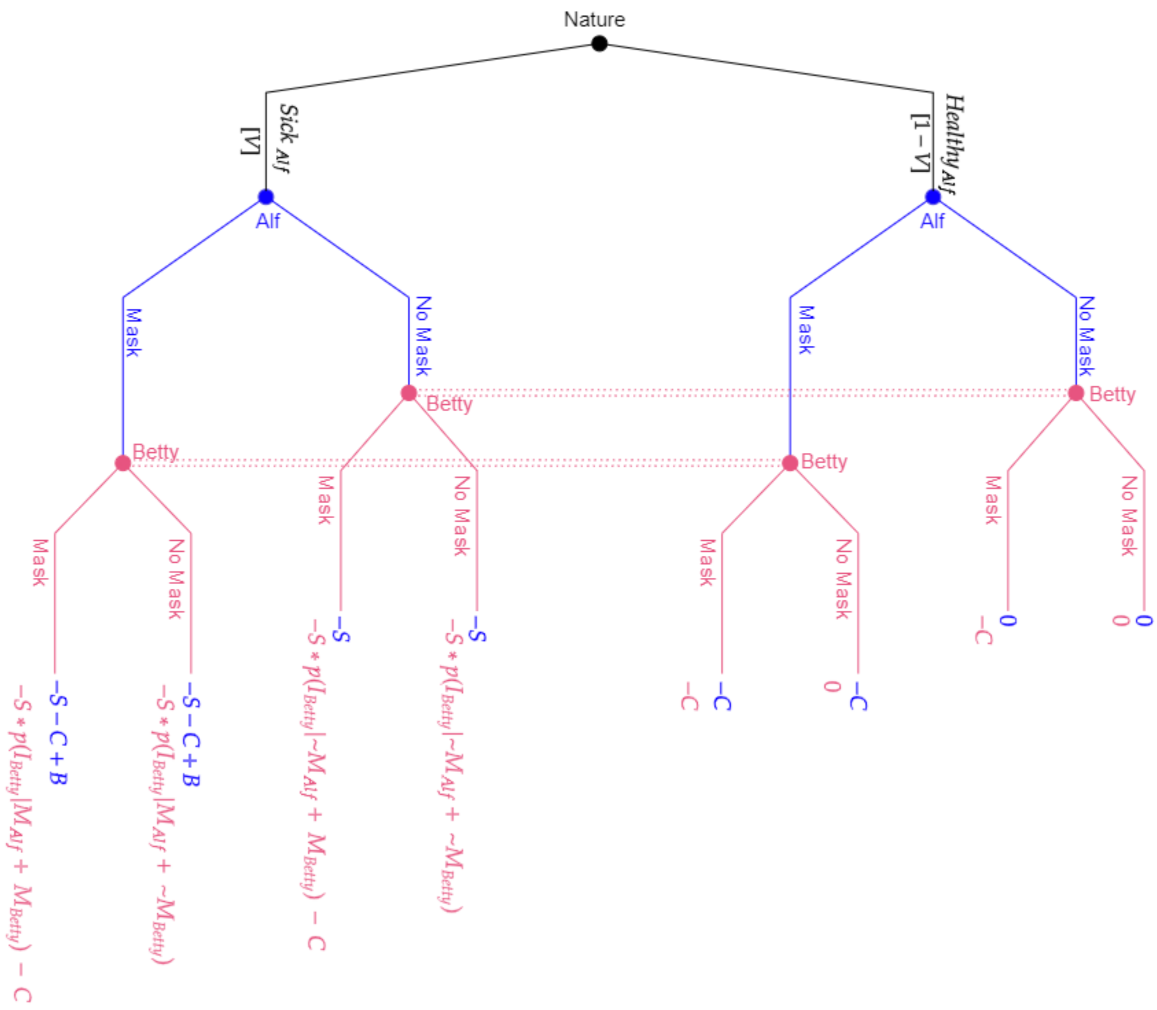

Figure 2: Type 1 Altruist - Egoist Game Tree

3.2 Game Matrices

Alf has four strategies that he can choose from when deciding if he is going to wear a mask on their date: 1) Always Mask, 2) Only Mask If Sick, 3) Only Mask If Healthy, and 4) Never Mask. Betty also has four strategies: 1) Always Mask, 2) Only Mask if Alf Masks, 3) Only Mask if Alf Does Not Mask, and 4) Never Mask.

Using this information, a 4x4 matrix was created to show the expected utilities derived at each possible outcome (Table 2). Upon analysis of this table, it is evident that Alf will never rationally choose the strategy Always Mask because the choice to Only Mask if Sick always results in a higher payoff:

$$(V)(-S-C+B) > (V)(-S-C+B)+(1-V)(-C)$$

We can also assume that Only Mask if Healthy will never be chosen as it is dominated by the strategy of Never Mask:

$$(V)(-S) > (V)(-S)+(1-V)(-C)$$

Therefore, given the design of this model, Alf will never choose to wear a mask when he is of healthy status.

Betty can eliminate two of their strategies when she knows that Alf will never wear a mask when he is healthy. Betty's strategy of Only Wear a Mask if Alf Does Not Mask is dominated by the decision to Never Mask:

$$(V)(-S*p(I_{Betty}| \sim M_{Alf}+ \sim M_{Betty})) > (V)(-S*p(I_{Betty}| \sim M_{Alf}+M_{Betty})-C)+(1-V)(-C)$$

and

$$(V)(-S*p(I_{Betty}|M_{Alf}+ \sim M_{Betty})) > (V)(-S*p(I_{Betty}|M_{Alf}+ \sim M_{Betty}))+(1-V)(-C)$$

Betty's strategy of Always Masking is dominated by Only Mask if Alf Masks:

$$(V)(-S*p(I_{Betty}|M_{Alf}+M_{+}Betty)-C) > (V)(-S*p(I_{Betty}|M_{Alf}+M_{Betty})-C)+(1-V)(-C)$$

and

$$(V)(-S*p(I_{Betty}| \sim M_{Alf}+ \sim M_{Betty})) > (V)(-S*p(I_{Betty}| \sim M_{Alf}+M_{Betty})-C)+(1-V)(-C)$$

Given this information we can simplify the 4x4 matrix into a 2x2 game matrix (Figure 3).

Table 2: Type I Altruist vs Egoist

		Betty			
		Always Mask	Only Mask If Alf Masks	Only Mask If Alf Does Not Mask	Never Mask
Alf	Always Mask	$(V)(-S-C+B)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S-C+B)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S-C+B)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+\sim M_{Betty}))$	$(V)(-S-C+B)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+\sim M_{Betty}))$
	Only Mask If Healthy	$(V)(-S)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|\sim M_{Alf}+\sim M_{Betty}))\ +(1-V)(-C)$	$(V)(-S)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|\sim M_{Alf}+M_{Betty})-C)$	$(V)(-S)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\|\sim M_{Alf}+\sim M_{Betty}))$
	Only Mask If Sick	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)$	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+\sim M_{Betty}))\ +(1-V)(-C)$	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+\sim M_{Betty}))$
	Never Mask	$(V)(-S)$ $(V)(-S*p(I_{Betty}\|\sim M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S)$ $(V)(-S*p(I_{Betty}\|\sim M_{Alf}+\sim M_{Betty}))$	$(V)(-S)$ $(V)(-S*p(I_{Betty}\|\sim M_{Alf}+M_{Betty})-C)\ +(1-V)(-C)$	$(V)(-S)$ $(V)(-S*p(I_{Betty}\|\sim M_{Alf}+\sim M_{Betty}))$

Table 3: Type I Altruist vs Egoist - Condensed

		Betty	
		Only Mask If Alf Masks	Never Mask
Alf	Only Mask If Sick	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+M_{Betty})-C)$	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\|M_{Alf}+\sim M_{Betty}))$
	Never Mask	$(V)(-S)$ $(V)(-S*p(I_{Betty}\|\sim M_{Alf}+\sim M_{Betty}))$	$(V)(-S)$ $(V)(-S*p(I_{Betty}\|\sim M_{Alf}+\sim M_{Betty}))$

3.3 Expected Utilities

Using the payoffs listed in Table 3, the expected utilities in all possible interactions were calculated for both Alf and Betty.

Alf's utility function does not factor in Betty's decisions. Therefore, their payoffs can be simplified into three possible utilities.

1. When Alf is sick and he chooses the strategy of wearing a mask when sick, he will receive a utility of:
$$(V * (-S - C + B))$$
2. When Alf is sick and he never wears a mask, he will receive a utility of:
$$(V * (-S))$$
3. When Alf is healthy and he does not wear a mask, Alf receives a utility of:
$$0$$

Betty's utility function is a bit more complicated as their payoff is directly correlated with Alf's health status and decisions.

Given that Alf only wears a mask when sick, and he is sick, Betty has the following possible utilities.

1. When Betty always wears a mask
$$(-S * (p(I_{Betty}|M_{Alf} + M_{Betty})) - C)$$
2. When Betty only wears a mask if Alf wears a mask:
$$(-S * (p(I_{Betty}|M_{Alf} + M_{Betty})) - C)$$

Given that Alf only wears a mask when sick, and he is healthy, Betty has the following possible utilities.

1. When Betty always wears a mask

$$-C$$

2. When Betty only wears a mask if Alf wears a mask:

$$0$$

Given that Alf never wears a mask and he is sick, Betty has the following possible utilities.

1. When Betty always wears a mask

$$(-S * (p(I_{Betty} | \sim M_{Alf} + M_{Betty})) - C)$$

2. When Betty only wears a mask if Alf wears a mask:

$$(-S * (p(I_{Betty} | \sim M_{Alf} + \sim M_{Betty})))$$

Given that Alf never wears a mask and he is healthy, Betty has the following possible utilities.

1. When Betty always wears a mask

$$-C$$

2. When Betty only wears a mask if Alf wears a mask:

$$0$$

3.4 Bayesian Nash Equilibrium

In the Type 1 Altruist versus Egoist game Betty has incomplete information as she does not know Alf's health status. In order determine which strategy Alf and Betty will enact the Bayesian Nash Equilibrium must be calculated.

Regardless of Betty's strategy, Alf will never choose to wear mask when he is healthy. Regardless of Betty's strategy, Alf will choose to wear a mask when he is sick when:

$$(V * (-S - C + B)) > (V * (-S))$$

$$B > C$$

Regardless of Betty's strategy, Alf will choose to never wear a mask when:

$$(V * (-S)) > (V * (-S - C + B))$$

$$C > B$$

Given that Alf Only Masks When He is Sick, Betty will play Never Mask when:

$$(-S * p(I_{B}etty|M_{Alf} + \sim M_{Betty})) > (-S * p(I_{B}etty|M_{Alf} + M_{Betty}) - \frac{C}{V})$$

$$(-S * p(I_{Betty}|M_{Alf} + \sim M_{Betty})) > (-S * p(I_{Betty}|M_{Alf} + M_{Betty}) - \frac{C}{V})$$

Given that Alf Only Masks When He is Sick, Betty will play Only Mask if Alf Masks when:

$$(-S * p(I_{Betty}|M_{Alf} + M_{Betty}) - \frac{C}{V}) > (-S * p(I_{Betty}|M_{Alf} + \sim M_{Betty})$$

Given that Alf Never Wears a Mask, Betty will be indifferent between Never Wearing a Mask and Only Wearing a Mask when Alf Wears a Mask.

3.5 Analysis

When Alf is a Type I altruist, he will:

- Never wear a mask when he is healthy.
- Wear a mask when he is sick if the utility gained by protecting others is greater than the cost associated with wearing a mask.
- Not wear a mask if the cost of wearing a mask outweighs the utility derived by protecting Betty.

When Alf decides to execute the strategy never wear a mask, Betty will be indifferent between their strategies of never wearing a mask or only wearing a mask if Alf wears a mask because in either case, she will choose not to wear a mask.

To further analyze the equilibrium where "Betty Never Masks Given that Alf Only Masks When He is Sick" we can rearrange the inequality to get the following:

$$(-S * p(I_{Betty}|M_{Alf} + \sim M_{Betty})) + S * p(I_{Betty}|M_{Alf} + M_{Betty}) > \frac{C}{V}$$

$$(S * (p(I_{Betty}|M_{Alf} + M_{Betty})) - p(I_{Betty}|M_{Alf} + \sim M_{Betty})) > \frac{C}{V})$$

This inequality states the cost of being sick times the difference between the probability that Betty gets sick when she chooses to wear a mask and the probability that Betty gets sick when she chooses not to wear a mask (assuming that Alf is Always Masking When he is sick) must outweigh the quotient of the cost of wearing a mask by the likelihood of Alf being sick. Furthermore, in this scenario the Betty can assume that Alf is sick because he is wearing a mask thus the inequality can be further simplified to reflect that the left side of the inequality only has to outweigh the cost of wearing a Mask.

$$(S * (p(I_{Betty}|M_{Alf} + M_{Betty})) - p(I_{Betty}|M_{Alf} + \sim M_{Betty})) > C)$$

In other words, when Betty sees Alf wearing a mask she can assume he is sick is COVID-19 and will subsequently choose to never wear a mask if the additional protection provided by wearing a mask is less than the cost associated with wearing a mask.

To further analyze the equilibrium where Betty only wears a mask when Alf wears a mask given that Alf Only Masks When He is Sick we can rearrange the inequality to get the following:

$$(-S * (p(I_{Betty}|M_{Alf} + M_{Betty})) + (S * p(I_{Betty}|M_{Alf} + \sim M_{Betty}) > \frac{C}{V}$$

$$S * ((p(I_{Betty}|M_{Alf} + \sim M_{Betty})) - (p(I_{Betty}|M_{Alf} + M_{Betty}) > \frac{C}{V}$$

This inequality states the cost of being sick times the difference between the probability that Betty gets sick when she chooses not mask by the probability that Betty gets sick when she chooses to mask (assuming that Alf is Always Masking When he is sick) must outweigh the quotient of the cost of wearing a mask by the likelihood of Alf being sick. Furthermore, in this scenario the Betty can assume that Alf is sick because he is wearing a mask. Thus, the inequality can be further simplified to reflect that the left side of the inequality only has to outweigh the cost of wearing a Mask.

$$S((p(I_{Betty}|M_{Alf} + \sim M_{Betty})) - (p(I_{Betty}|M_{Alf} + M_{Betty}) > C$$

In other words, when Betty sees Alf wearing a mask she can assume he is sick is COVID-19 and will subsequently choose to wear a mask if the additional protection provided by wearing a mask is greater than the cost associated with wearing a mask.

4 Alf is a Type II Altruist and Betty is an Egoist

To further expand on the previous models, in the following scenario Alf will be a Type II altruist - Alf will now have a cost associated with harming Betty by choosing to not wear a mask if they are sick.

Alf, a Type II altruist, and his sister Betty, an egoist, are planning to meet at Fashion Square Mall to go shopping for a Mother's Day present. There are no mask mandates in effect nor has the shopping mall posted any COVID-19 protocol signs.

Both Alf and Betty took a COVID-19 test earlier in the day. Betty received a negative test result a few hours prior to their meeting time. Alf received a notification on his phone containing his test results as he was waiting in the parking lot for Betty to arrive. Alf does not want to disappoint their mother and has decided that he is going to help Betty find the perfect gift regardless of the test results.

Alf and Betty must choose independently whether or not they will wear a mask on their shopping trip. Alf is made aware of Betty's health status, but Betty does not know the results of Alf's test.

4.1 Game Tree

Figure 3 maps out the eight different possible interactions between Alf and Betty. Alf's test result could have come back positive, yielding the probability V, or negative, yielding the probability $1 - V$. Then, given Alf's health status he must choose whether or not he is going to wear a mask. Betty does not know Alf's test results and thus must make her masking decision with incomplete information. The two different information sets are represented by the pink dotted lines in the game tree.

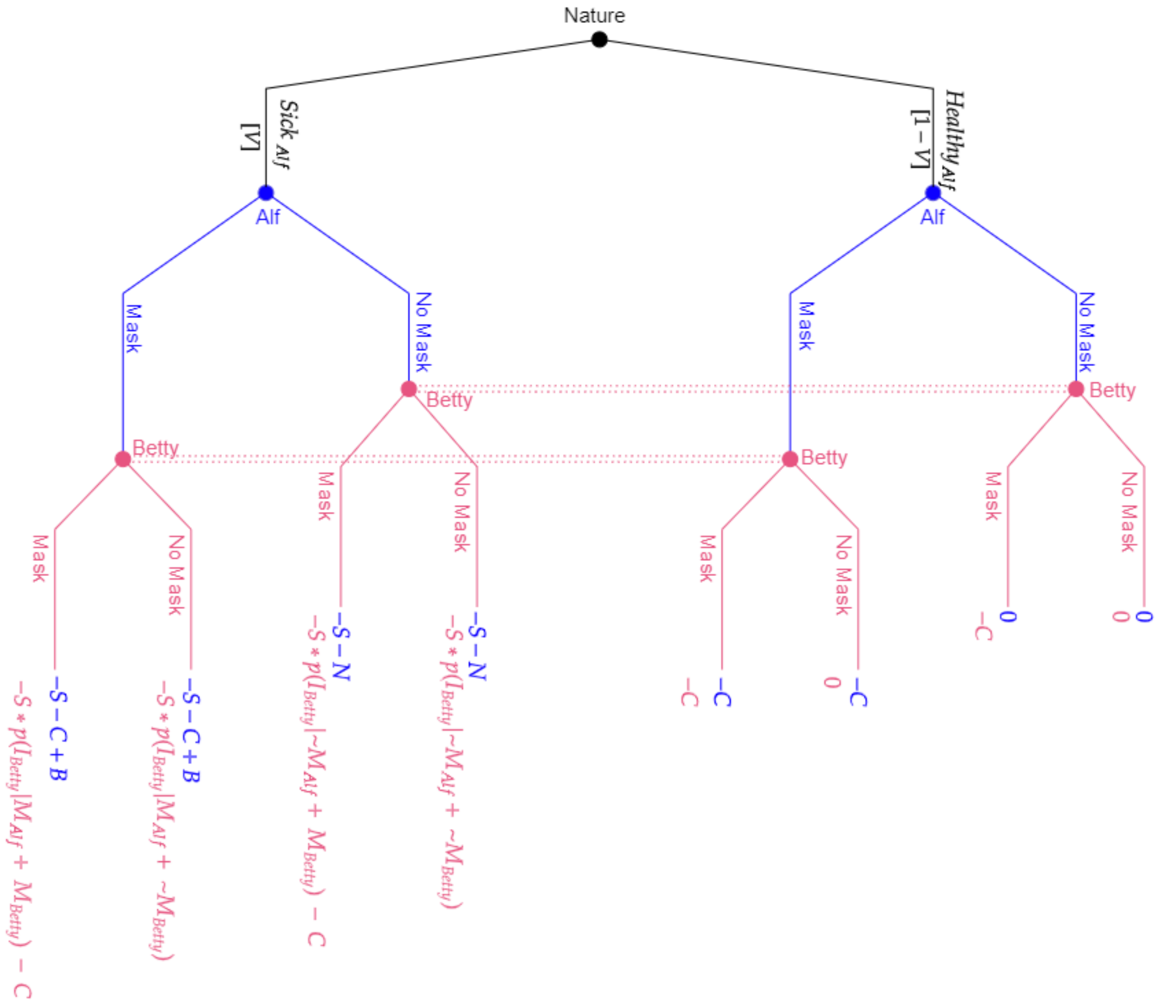

Figure 3: Type 2 Altruist - Egoist Game Tree

4.2 Game Matrices

Alf has four strategies that he can choose from when deciding if he is going to wear a mask: 1) Always Mask, 2) Only Mask If Sick, 3) Only Mask If Healthy, and 4) Never Mask. Betty also has four strategies: 1) Always Mask, 2) Only Mask if Alf Masks, 3) Only Mask if Alf Does Not Mask, and 4) Never Mask.

Using the above information, a 4x4 matrix was created to show the expected utilities derived at each possible outcome (Table 4). Upon analyzing this table, it is evident that Alf will never rationally choose the strategy Always Mask because the choice to Only Mask if Sick always results in a higher payoff:

$$(V)(-S-C+B) > (V)(-S-C+B)+(1-V)(-C)$$

We can also assume that Only Mask if Healthy will never be chosen as it is dominated by the strategy of Never Mask:

$$(V)(-S-N) > (V)(-S-N)+(1-V)(-C)$$

Therefore, Alf will never choose to wear a mask when he is of healthy status.

When Betty knows that Alf will never wear a mask if he is healthy, she can eliminate two of her own strategies. Betty's strategy of Only Wear a Mask if Alf Does Not Mask is dominated by the decision to Never Mask:

$$(V)(-S*p(I_{Betty}|\sim M_{Alf}+\sim M_{Betty})) > (V)(-S*p(I_{Betty}|\sim M_{Alf}+M_{Betty})-C)+(1-V)(-C)$$

and

$$(V)(-S*p(I_{Betty}|M_{Alf}+\sim M_{Betty})) > (V)(-S*p(I_{Betty}|M_{Alf}+\sim M_{Betty}))+(1-V)(-C)$$

Betty's strategy of Always Masking is dominated by Only Mask if Alf Masks:

$$(V)(-S*p(I_{Betty}|M_{Alf}+M_{+}Betty)-C) > (V)(-S*p(I_{Betty}|M_{Alf}+M_{Betty})-C)+(1-V)(-C)$$

and

$$(V)(-S*p(I_{Betty}|\sim M_{Alf}+\sim M_{Betty})) > (V)(-S*p(I_{Betty}|\sim M_{Alf}+M_{Betty})-C)+(1-V)(-C)$$

By eliminating these strategies, the 4x4 matrix can be simplified into a 2x2 game matrix (Figure 5).

Table 4: Type II Altruist vs Egoist

		Betty			
		Always Mask	Only Mask If Alf Masks	Only Mask If Alf Does Not Mask	Never Mask
Alf	Always Mask	$(V)(-S-C+B)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+M_{Betty})-C)+(1-V)(-C)$	$(V)(-S-C+B)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+M_{Betty})-C)+(1-V)(-C)$	$(V)(-S-C+B)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+\sim M_{Betty}))$	$(V)(-S-C+B)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+\sim M_{Betty}))$
	Only Mask If Healthy	$(V)(-S-N)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+M_{Betty})-C)+(1-V)(-C)$	$(V)(-S-N)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\mid \sim M_{Alf}+\sim M_{Betty}))+(1-V)(-C)$	$(V)(-S-N)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\mid \sim M_{Alf}+M_{Betty})-C)$	$(V)(-S-N)+(1-V)(-C)$ $(V)(-S*p(I_{Betty}\mid \sim M_{Alf}+\sim M_{Betty}))$
	Only Mask If Sick	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+M_{Betty})-C)+(1-V)(-C)$	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+M_{Betty})-C)$	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+\sim M_{Betty}))+(1-V)(-C)$	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+\sim M_{Betty}))$
	Never Mask	$(V)(-S-N)$ $(V)(-S*p(I_{Betty}\mid \sim M_{Alf}+M_{Betty})-C)+(1-V)(-C)$	$(V)(-S-N)$ $(V)(-S*p(I_{Betty}\mid \sim M_{Alf}+\sim M_{Betty}))$	$(V)(-S-N)$ $(V)(-S*p(I_{Betty}\mid \sim M_{Alf}+M_{Betty})-C)+(1-V)(-C)$	$(V)(-S-N)$ $(V)(-S*p(I_{Betty}\mid \sim M_{Alf}+\sim M_{Betty}))$

Table 5: Type II Altruist vs Egoist - Condensed

		Betty	
		Only Mask If Alf Masks	Never Mask
Alf	Only Mask If Sick	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+M_{Betty})-C)$	$(V)(-S-C+B)$ $(V)(-S*p(I_{Betty}\mid M_{Alf}+\sim M_{Betty}))$
	Never Mask	$(V)(-S-N)$ $(V)(-S*p(I_{Betty}\mid \sim M_{Alf}+\sim M_{Betty}))$	$(V)(-S-N)$ $(V)(-S*p(I_{Betty}\mid \sim M_{Alf}+\sim M_{Betty}))$

4.3 Expected Utilities

Using the payoffs listed in Table 3 we can calculate the expected utilities for both Alf and Betty for all possible interactions.

Alf's utility function does not factor in Betty's decisions. Therefore, their payoffs can be simplified into three possible utilities.

1. When Alf is sick and he wears a mask when sick:

$$(V * (-S - C + B))$$

2. When Alf is sick and he never wears a mask:

$$(V * (-S - N))$$

3. When Alf is healthy (regardless of choice of strategy):

$$0$$

Betty's utility function is a bit more complicated as their payoff is directly correlated with Alf's health status and decisions.

Given that Alf only wears a mask when sick and he is sick, Betty has the following possible utilities.

1. When Betty always wears a mask

$$(-S * (p(I_{Betty} | M_{Alf} + M_{Betty})) - C)$$

2. When Betty only wears a mask if Alf wears a mask:

$$(-S * (p(I_{Betty} | M_{Alf} + M_{Betty})) - C)$$

Given that Alf only wears a mask when sick and he is healthy, Betty has the following possible utilities.

1. When Betty always wears a mask

$$-C$$

2. When Betty only wears a mask if Alf wears a mask:

$$0$$

Given that Alf never wears a mask and he is sick, Betty has the following possible utilities.

1. When Betty always wears a mask

$$(-S * (p(I_{Betty}| \sim M_{Alf} + M_{Betty})) - C)$$

2. When Betty only wears a mask if Alf wears a mask:

$$(-S * (p(I_{Betty}| \sim M_{Alf} + \sim M_{Betty})))$$

Given that Alf never wears a mask and he is healthy, Betty has the following possible utilities.

1. When Betty always wears a mask

$$-C$$

2. When Betty only wears a mask if Alf wears a mask:

$$0$$

4.4 Bayesian Nash Equilibrium

As with the previous two models, Betty has incomplete information in the Type II Altruist versus Egoist model as she does not know Alf's health status. In order determine Alf and Betty's best responses the Bayesian Nash Equilibrium must be calculated.

Regardless of Betty's strategy, Alf will never choose to wear mask when he is healthy.

Regardless of Betty's strategy, Alf will choose to wear a mask when he is sick when:

$$(V * (-S - C + B)) > (V * (-S - N))$$

$$B - C > -N$$

Regardless of Betty's strategy, Alf will choose to never wear a mask when:

$$(V * (-S - N)) > (V * (-S - C + B))$$

$$-N > B - C$$

Given that Alf Only Masks When He is Sick, Betty will play Never Mask when:

$$(-S * p(I_Betty|M_{Alf} + \sim M_{Betty})) > (-S * p(I_Betty|M_{Alf} + M_{Betty}) - \frac{C}{V})$$

$$(-S * p(I_{Betty}|M_{Alf} + \sim M_{Betty})) > (-S * p(I_{Betty}|M_{Alf} + M_{Betty}) - \frac{C}{V})$$

Given that Alf Only Masks When He is Sick, Betty will play Only Mask if Alf Masks when:

$$(-S * p(I_{Betty}|M_{Alf} + M_{Betty}) - \frac{C}{V}) > (-S * p(I_{Betty}|M_{Alf} + \sim M_{Betty})$$

Given that Alf Never Wears a Mask, Betty will be indifferent between Never Wearing a Mask and Only Wearing a Mask when Alf Wears a Mask.

4.5 Analysis

The results of this model are very similar to those of model two. Betty will react the same way under these new conditions.

1. When Alf decides to never wear a mask, Betty will be indifferent between their strategies of never wearing a mask or only wearing a mask if Alf wears a mask because in either case, she will choose not to wear a mask.
2. When Betty sees Alf wearing a mask she can assume he is sick is COVID-19 (see analysis on Alf's decisions below for reasoning) and will subsequently choose to never wear a mask if the additional protection provided by wearing a mask is less than the cost associated with wearing a mask.
3. When Betty sees Alf wearing a mask she can assume he is sick is COVID-19 and will choose to wear a mask if the additional protection provided by wearing a mask is greater than the cost associated with wearing a mask.

Under these new conditions Alf will use slightly different logic when choosing between the strategies of never wearing a mask and only wearing a mask when he is sick.

1. When Alf is an altruist, he will never wear a mask.
2. Alf will wear a mask when he is sick if the utility gained by protecting others minus the cost of wearing a mask outweighs disutility of potentially infecting others by not wearing a mask.
3. Conversely, Alf will choose to never wear a mask when the disutility of potentially infecting others by not wearing a mask outweighs the utility gained by protecting others minus the cost of wearing a mask.

5 Limitations

There are several limitations of this paper. The Egoist Versus Egoist Model:

1. Ignores that individuals have different values and everyone is not an egoist.
2. Assumes that Betty cannot visibly observe whether or not Alf is sick and she must make her decision solely on whether or not Alf is wearing a mask.
3. Does not consider vaccination status as an unknown variable.
4. Does not consider the different types of masks that Alf and Betty might be wearing.
5. Assumes that an infected individual wearing a mask can still infect others.
6. Betty's decision whether or not to wear a mask does not impact Alf's utility.
7. Assumes that the individuals in the interaction cannot choose to leave if the other person does not wear a mask or if they find out they are sick.

In an attempt to address the first limitation, two additional choice mechanisms were introduced in the second and third models. In a future paper, I would like to explore how Betty's decision impacts Alf by changing the benefit and cost associated with Alf wearing a mask depending on Betty's choice.

6 Mask Mandates in the United States

While the above sections have proven that individuals will not wear masks unless the benefits outweigh the costs, it has still yet to be shown by this paper that wearing masks is an effective measure of slowing the spread of COVID-19 and decreasing hospitalizations and deaths. This premise is necessary in order to justify the overall conclusion that the United States government ought to either decrease the cost or increase the benefit associated with wearing face masks during a pandemic through policy policy.

In the first few months of the COVID-19 pandemic there was uncertainty surrounding the efficacy of masks. However, in early April 2020, public health officials introduced recommendations that all Americans wear face masks when they go out in public (MEGERIAN et al., 2020). This recommendation was supported by a plethora of empirical evidence. The Centers for Disease Control and Prevention published a figure summarizing the efficacy of masks. In indoor public settings, people who wore cloth masks were 56% less likely to test positive for COVID-19 than their counterparts who did not wear masks. Surgical grade masks provided an even greater protection, resulting in 66% lower odds and N95 and KN95 masks provided 83% lower odds (Andrejko et al., 2022).

The efficacy of masks is further supported by a November 2020 report by the Institute of Health Metrics and Evaluations at the University of Washington School of Medicine's report, which claims that if 95 percent of the United States population wore masks when they went into public, 130,000 lives would be saved within a few months (Collins, 2020).

Given these empirical findings, it is safe to conclude that masks are an effective means of slowing the spread of COVID-19. Therefore, justifying government intervention.

Conclusion

By using game theory to analyze three interactions between two individuals, this paper established that regardless of an individual's choice mechanism, people will choose to not wear a mask if the cost associated with masking is greater than the perceived benefit of masking. It was shown that the benefit associated with wearing a mask can be manipulated by introducing additional factors such as the gratification of protecting others (B) and through introducing a cost of harming others ($-N$). In situations where having the majority of individuals wear masks in public will save lives, the United States must find a way to either decrease the cost or increase the benefit associated with wearing face masks. To accomplish this, the federal government can enact public policies including distributing face masks for free, airing media campaigns focused on increasing the perceived benefit of wearing a mask, and reprimanding individuals for not complying with mask mandates.

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