# OPTIMUM SENSOR DEPLOYMENT OVER TEST RANGE

Melih G. Can[1*,2], Hamza Kurt[2], Mahmut Aktaş[1]
[1] (Roketsan Missiles Industries Inc., Ankara, 06780, Turkey)
[2] (Department of Electrical and Electronics Engineering, TOBB University of Economics and Technology, Ankara, 06560, Turkey)
goktug.can@roketsan.com.tr

## ABSTRACT

In this study, link analysis calculations for ground stations have been performed. Values obtained from these calculations have been optimized to meet the given constraints for mission objective and consequently optimum location of ground stations has been obtained. Linear optimization ensures the optimum solution while satisfying the given constraints. Besides, if the given objective is constraint limited, optimization process output points to the impossibility of the desired objective. Thus, avoids user to lose time with empiric solution research.

## KEYWORDS

Radar, Link Analysis, Linear Optimization

## INTRODUCTION

In the scope of the present work it is explained how to obtain the most suitable positioning of ground station systems for military and aerospace applications depending on the target properties and various constraints. Radar systems' specifications and equations are considered as pioneering problem. As an example, you can deploy 6 different radar ground stations between 10 different locations by evaluating the target's footprint in a mission. However, if your communication network is limited to 3 satellite communication hubs, what kind of settlement should be done, what is the priorities considering the mission objectives? How to deploy 6 radars using up to 3 of the 10 possible points? Do you need all radars? Is it a more cost effective solution to reduce the number of radars and invest in a new communication station?

# METHODOLOGY

## Link Analysis

Radars would be divided into two main categories with respect to their receiver antenna position. If the receiver and transmitter antennas are deployed in same position then it is called monostatic radar else it is called bistatic radars. In the scope of the present study monostatic radars are examined (Bistatic radars and optimum positioning of transmitter-receiver couples are another phenomenon and will be discussed in further investigations). The distance which radar emission takes in the air is twice the distance, $R$, between the target and the radar. The amount of emission reflected from the target is mainly dependent on the structural parameters of the target and the angle between the direction of moment of the target and the heading of the radar. Consequently, in order to determine the parameters of the radar capable of detecting a given target from a distance, the position of the radar, the trajectory of the target, and the reflectance of the target, the radar cross-sectional area, $\sigma$, are needed. Another important parameter is observation time "$T_o$".Radar equation obtained by evaluating all these parameters is given in Equation 1 [1] while $L_{prc}$, $L_{atm}$, $L_{sys}$ are processing, atmospheric and system losses, respectively.

$$R^4 = \frac{P_t G^2 \lambda^2 T_o \sigma}{(4\pi)^3 P_r L_{atm} L_{sys} L_{prc}} \qquad (1)$$

Link analysis calculations are performed for every sample point of the trajectory. It is because of the fact that the azimuth and elevation angles between target moment vector and the ground station heading vector vary with time. These angles are determinative for the calculations as stated in the previous section. Radar cross section is mainly depended on these angles. Therefore, the angle differences $\varphi$ and $\theta$ are becomes time dependent variants. Angle differences are calculated from the reduction of the three dimensional moment vector and heading vector onto the XY plane (horizontal plane) and Z direction. Meanwhile, heading vector is the position difference between settlement area and the regarding target point. Thus, in any settlement area, at any moment of flight, the angles to which the target is observed by the ground station are found. Therefore $\varphi$ and $\theta$ becomes, $\varphi_{i,k}$ and $\theta_{i,k}$, respectively. These angle variations are implemented on the radar cross sectional area estimation. By this way, radar cross section given in the former sections becomes $\sigma(i,k)$ for radar link analysis.

## Optimization

It is always desired to utilize a minimum number of ground stations while ensuring the mission objective due to the logistics and communication requirements. During planning of a mission or feasibility search $I$ possible settlement area is considered for $J$ different radar ground stations. Number of utilized station needs to be limited with the minimum count which guaranties mission requirements.

The method to obtain the target tracking status of a radar station at different points is explained in the previous section. The resulting track matrix, given in Equation 2, is the input matrix of optimization.

$$track(i, j, k) = \{0,1\} \tag{2}$$

Track status equals to 1 if the ground station $j$ can track the target at the time $k$ of the flight in settlement area $i$. Else track status equals to 0 for the ground station at the specified time and the position.

### Linear Optimization of Track Matrix

In this section of the present study the linear optimization method for the track matrix obtained is explained. First of all a mission objective have to be specified. As an example, if the mission is safety critic then the mission objective may defined as: target must be tracked by at least two ground stations simultaneously. This statement generates an objective to be defined in the optimization. Also as mentioned above, mostly logistic sources are limited. Therefore, locating in the minimum number of settlement area is desired. This last statement generates a constraint to be defined in the optimization. Track matrix is obtained as explained in the former part of the study. Mission objective is maximizing the minimum number of simultaneous track for each time sample $k$ of the trajectory. Also a defined constraint is utilizing maximum $N$ different settlement area.

$$S(i, j) = \{1,0\} \tag{3}$$

Equation 3 is the output variable of the optimization. After linear optimization of the track matrix the optimum settlements are obtained. The $S$ variable given in Equation 3 is equal to 1 if the ground station $j$ is located in the settlement area $i$. Else it is equal to 0. In this point of the optimization, it is possible for the optimization program to locate all ground stations to all settlement areas and serving this configuration as an optimum solution. That means, $S(i, j) = 1$ for all $i$ and $j$. However, it is not possible to locate a specific ground station $j$ to all different settlement points at the same time. Therefore, a limit must be defined. This constraint is defined in Equation 4.

$$\sum_{i=1}^{I} S(i, j) \le 1 \tag{4}$$

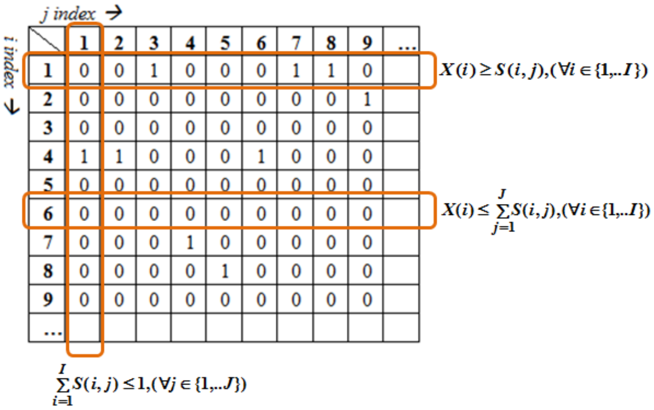
Equation 4 ensures that a specific ground station can only be deployed at only one settlement point. If a ground station $j$ is located on the settlement $i$ then $S(i,j)=1$. Equation 4 prevents S variable to be equal to 1 for same ground station $j$ in different settlements. This constraint is explained in Figure 1. To fulfill the maximum number of settlement area requirement, settlement areas which a ground station is located in the output S matrix must be considered during the optimization process. Therefore, a decision variable is defined.

$$X(i) = \{1,0\} \tag{5}$$

$$X(i) \ge S(i, j) \tag{6}$$

$$X(i) \le \sum_{j} S(i, j) \tag{7}$$

Decision variable X is only depended on position *i* variable, the settlement area index, and only can take the values 1 or 0 as stated in Equation 5. Equation 6 makes the decision variable *X (i)* = 1 if any ground station is deployed at *i* position. Equation 7 guarantees the decision variable *X (i)* = 0 if no ground station is deployed at the position *i*.



| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **...** |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | |
| **2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | |
| **3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| **4** | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | |
| **5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| **6** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| **7** | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | |
| **8** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | |
| **9** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| **...** | | | | | | | | | | |

**Fig. 1. An example S result matrix.**

Equation 8 defines the constraint so that the sum of the different points where any ground station is deployed is less than the value *N*.

$$\sum_{i} X(i) \le N \tag{8}$$

In Equation 9 obtained tracking status matrix T is given. In the optimum solution, *T (i, j, k)* = 1 if the ground station *j* in position *i* can track the target at time *k*.

$$T(i, j, k) = S(i, j) x Track(i, j, k) \tag{9}$$

$$Z(k) = \sum_{i} \sum_{j} T(i, j, k) \tag{10}$$

$$Max(\min(Z(k), \forall k) \tag{11}$$

Equation 11 is the objective function. *Z* variable gives the total number of simultaneously tracking ground station for every specific time, *k*, of the flight. The objective function is defined to maximize the minimum of the *Z*. Therefore, every sample of the flight is tracked with the maximum number of ground stations.

## NUMERICAL RESULTS

Three different scenarios have been tried to test the efficiency of the optimization algorithm. All three scenarios are implemented for the same target through a sample trajectory about 1000Km. 12 possible settlements have been selected on the footprint, approximately 100Km apart. The Universal Tracking Matrix is generated by evaluating the tracking performances of 4 continuous wave tracking radars with an antenna gain ranging from 40dB to 45dB, with an average output of 1000W.

- In the first scenario: the example trajectory is sampled with 100 Hz. Objective function is defined to maximize the minimum number of simultaneous radar track. So that the optimization algorithm forces to overlap the coverage areas of the radars while tracking the all trajectory. In addition, a constraint has been defined so that radar ground stations can be deployed on maximum of $N$=3 different locations at the same time. Station and target properties are implemented on the link analyses equation and Universal Matrix is obtained using MATLAB. Initial size of the Universal Matrix is 12 x 4 x $K$. Universal Matrix is optimized within seconds implementing the above stated approach in GAMS linear optimization program.
- In parallel, as the second scenario, same radar ground stations are randomly settled among 12 different positions. Evaluating the link analysis the tracking status of this deployment is obtained. Note that no maximum number of settlement constraints is defined.
- Radars are located heuristically. Most capable radar is located near the launch point and the others are located with respect to their estimated coverage areas. Again no constraint is defined to limit the number of settlements.

Results are given in
Table **1**. In the table deployed settlement indexes for 4 different radars are shown. It is clearly seen from the table that in the first scenario radars are deployed on 3 different settlements, number 1, 7 and 12. Radar number 1 and 3 are deployed on the settlement 1. Meanwhile, radar number 2 and 4 are deployed on settlement 7 and 12, respectively. This is the optimal case obtained from the linear optimization program with above stated constraints and objective function. In the scenario 2, the radars are randomly deployed on settlements number 9, 1, 4 and 2. In the third and final scenario, the radars are deployed on settlements number 1, 4, 8 and 12, respectively. Tracking statuses of the radars for each scenario is given in the Figure 2.

**Table 1 Deployed settlement indexes of the radars for 3 different scenarios.**

| | $j_1$ | $j_2$ | $j_3$ | $j_4$ |
|---|---|---|---|---|
| Scenario-1 | $i_1$ | $i_7$ | $i_1$ | $i_{12}$ |
| Scenario-2 | $i_9$ | $i_1$ | $i_4$ | $i_2$ |
| Scenario-3 | $i_1$ | $i_4$ | $i_8$ | $i_{12}$ |

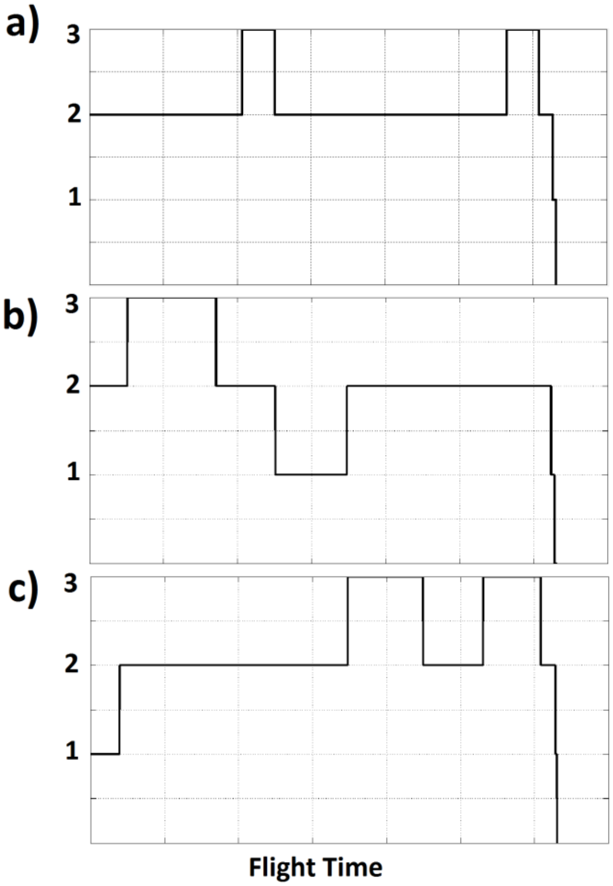


**Fig. 2. Radar Tracking Results of a) Scenario-1, b) Scenario-2 and c) Scenario-3**

In the figure vertical axis shows the number of radar which is tracking the target on related time of flight. As shown in the figure 2(a), in the optimised scenario, even though the maximum number of settlements is limited with 3, minimum number of simultaneously tracking radar is 2 through the trajectory (at the end of the trajectory no radar is tracking due to the line of sight).

As seen in figure 2(b), the randomized scenario, When the radars are placed randomly, even if the settlements where each radar can be located is not limited, only one radar can track the target around the middle of the flight.

As seen in figure 2(c), the heuristic scenario, in the early phases of the flight for a limited time, only 1 radar can track the target then the tracking is completed with at least 2 simultaneous tracks. However, in this scenario, 4 different radars are located at 4 different settlement points. In the optimized scenario, at least 2 simultaneous radar tracks are provided through all flight using only 3 settlement points.

## CONCLUSIONS

A simple optimization algorithm has been described for the best use of resources in a launch mission or military application. Optimized results are compared with random and heuristic results. Even if the random search or heuristic approach are not constraint limited, optimized results are better.

Present study is limited with only radars analysis, one objective function and two constraints. It is possible to define telemetry & telecommand problems, more parameters such as financial resources, human resource, logistics and more constraints. Objective function may be defined to minimize the operation cost. It is also possible to define a multi objective function and weight the parameters. In this study, the authors utilized a simple target with little variability of the radar cross sectional area. Obviously, the optimization algorithm will achieve even more successful results in targets that show high variability in radar cross-section depending on the angle of observation change.

In the future, a similar optimization algorithm may be used for the deployment of air defence systems warning radars. In addition, optimization of telemetry network structures and communication structures of the complex swarm applications can be studied by optimizing communication link analysis in a similar way as in radar link analysis.

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## REFERENCES

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