DEVELOPMENT OF A HIGH-CONTRAST ADAPTIVE OPTICS PHASING TESTBED FOR THE GIANT MAGELLAN TELESCOPE

by

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DEDICATION

To my wife, Kateri, and our son, Ezra, who make it all worth it.
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Our galaxy hosts $\sim 300$ billion stars. Ever since the first exoplanet was discovered in 1992, over 5,000 more exoplanet discoveries have been confirmed, and the number is still counting. As each new exoplanet is discovered, the case seems more and more likely that each star in our Milky Way galaxy must have at least one exoplanet orbiting around it. Many of these exoplanets also fall into the potentially habitable, terrestrial size category, meaning there could be billions of earth-like planets waiting to be discovered. If we hope to discover life outside of our solar system, it has been shown that directly imaging these potentially habitable exoplanets in visible light reflected from its host star is optimal. This is very possible, however difficult, since this requires Extremely Large Telescopes (ELTs; $\sim 30$ m in diameter) to achieve high angular resolutions and contrasts, extreme adaptive optics (ExAO) to suppress the effects of atmospheric seeing, and coronagraphy to block the starlight. These three technologies could coexist once the 25.4-m Giant Magellan Telescope (GMT) is completed in 2029. With the combined power of ExAO and the future GMT, the discovery of life outside of our solar system may become a reality. However, the GMT’s unique seven segmented primary mirror design raises a challenge to keep the telescope segments co-phased—a task which is critical for direct imaging of exo-earths and any other diffraction-limited science with the GMT.

This dissertation addresses the challenge of co-phasing a giant segmented telescope for exoplanet imaging and describes the development of a High-Contrast Adaptive optics phasing Testbed (HCAT) for the GMT. The testbed simulates the GMT with real optics in a lab environment with six piston, tip, and tilt actuators and tests a working concept for a “parallel deformable mirror” to optically redistribute the GMT pupil onto seven commercially available 3,000 actuator deformable mirrors. HCAT also leverages an existing ExAO system called MagAO-X to test and
demonstrate segment phase sensing and AO-control with a real ExAO system. I will first introduce the GMT and the “piston problem.” Then, I will give an introduction to adaptive optics and discuss the design and build of MagAO-X. I will then discuss the development of an early stage GMT proto-testbed which was a simple GMT simulator that provided insight into co-phasing a segmented telescope. This early stage GMT proto-testbed evolved into the official prototype version of HCAT (p-HCAT), which led to the development of a new phasing method for co-phasing the GMT using a novel optic called the “Holographic Dispersed Fringe Sensor” (HDFS). The success of the demonstrations performed with p-HCAT and the novel HDFS are discussed. These results have influenced the GMT to adapt this phasing method as their official designated phase sensor for GMT exoplanet imaging. Finally, the design and build of the full-scale HCAT testbed will be described with a demonstration of the “parallel DM” working in the lab.
There is a new generation of ground-based telescopes classified as “Extremely Large Telescopes” (ELTs). These are the Giant Magellan Telescope (GMT; Johns et al., 2012), the Thirty Meter Telescope (TMT; Johns et al., 2012), and the European Extremely Large Telescope (E-ELT; Johns et al., 2012). Figure 1.1 shows a comparison of the three telescopes, currently under construction. The GMT and the TMT are part of the U.S. ELT program, while the E-ELT is part of the European ELT program. The ELTs will be the largest telescopes ever built, providing a new era of astronomy through powerful light-collecting area and imaging resolution. Their large light-collecting area will allow us to see deeper into the universe, unlocking our ability to answer important questions about black hole formation, galaxy formation, dark matter, and more. The large diameter of the ELTs will also provide unprecedented image resolutions and contrasts that will unlock our ability to directly image potentially habitable exoplanets orbiting late type stars and could
lead to the discovery of life outside of our solar system.

1.1 The Unique Design of the GMT

The GMT design is unique from the other ELT-class telescopes in that it does not contain hundreds of small $\sim 1.0$-m hexagonal-shaped Zerodur mirrors separated by small $\sim 4$ mm gaps to make up its primary mirror, but rather seven large 8.4-m circular-shaped borosilicate glass mirrors separated by large $>30$ cm gaps. This means that the GMT will be more susceptible to temperature changes, gravity load, segment vibrations, and wind buffeting (Quirós-Pacheco et al., 2018), causing the optical path difference (OPD) between the GMT primary mirror segments to fluctuate as much as tens of microns. This is not acceptable for the GMT’s Natural Guide Star Adaptive Optics (NGSAO) science mode (e.g., direct imaging of habitable zone earth-like planets around late type stars), which requires the segments to be co-phased to well within a fraction of a wavelength. The edge sensors between the large gaps of the GMT segments are not expected to be stable enough at this level. Therefore, seven adaptive secondary mirrors (ASMs) are incorporated in the design to compensate for any OPD introduced by the primary mirror (see Figure 1.2). The ASMs will have the ability to change their shape in order to correct for any atmospheric distortions or segment piston errors introduced by the primary mirror segments.

1.1.1 The Piston Problem

Before the ASMs can correct any phase errors introduced by the primary mirror segments or the atmosphere, phase errors must first be measured with a wavefront sensor. However, phase errors in the form of piston are of particular concern, since traditional slope wavefront sensors like the Shack-Hartmann cannot sense piston across discontinuities in the telescope pupil. The atmosphere alone will introduce differential segment piston (or “petal”) errors between GMT segments that can be on the order of $\pm 5 \mu$m (Schwartz et al., 2017; Bertrou-Cantou et al., 2022). Fur-
thermore, the large gaps between the GMT segments span multiple atmospheric coherence lengths $r_0$ (for $\lambda < 1.2\,\mu m$), so atmospheric turbulence will have a differential phase error across these gaps that can be as much as $1\,\lambda$ in median seeing conditions (Schwartz et al., 2017). Visible to near-infrared adaptive optics (AO) systems (like GMagAO-X; see Males et al., 2019; Close et al., 2019) will have wavefront sensor pixels in these gaps, causing a discontinuity in the fitted wavefront data that will generate differential piston between the GMT segments (or “petals”; see Bertrou-Cantou et al., 2022). This is a problem not only for the GMT, but for the E-ELT and the TMT as well. For this reason, the U.S. Decadal Survey on Astronomy and Astrophysics 2020 (Astro2020) agrees that the highest technical risk for the GMT is the phasing and alignment of the primary mirror segments (National Academies of Sciences and Medicine, 2021).

Figure 1.3 shows an example of how differential piston errors between the GMT
Figure 1.3: A simulation showing the comparison of different levels of piston errors between the GMT segments and the resulting PSF and coronagraphic PSF (assuming a perfect coronagraph and no atmospheric turbulence) at $\lambda_c = 800$ nm, 100 nm bandwidth. (Top row): the GMT segments have zero differential piston error, so the perfect coronagraph is able to block out the light from the star perfectly. (Middle row): the GMT segments have 30 nm RMS piston error. The PSF has a Strehl ratio of 95% and the coronagraph is still functional but light is starting to leak out of the coronagraph (see Figure 1.4 for more details). (Bottom row): the GMT segments have 300 nm RMS piston error. The PSF has a Strehl ratio of 0% and most of the light is leaking out of the coronagraph, making it impossible to carry out any diffraction-limited science cases for the GMT.

segments could affect the PSF and coronagraphic PSF, assuming a perfect coronagraph (Cavarroc et al., 2006) and no atmospheric turbulence for $\lambda_c = 800$ nm, 100 nm bandwidth. These simulations were performed with High Contrast Imaging for Python (HCIPy), an open source AO and coronagraph simulator (Por et al., 2018).

Figure 1.4 shows the Strehl ratio of the PSF for a few different wavelengths
Figure 1.4: (a) The Strehl ratio of the PSF from Figure 1.3 as a function of RMS piston error for a few different wavelengths. In order to achieve high Strehl in the visible, the GMT segments must be co-phased to well within a fraction of a wavelength. (b) The radial profiles of the coronagraphic PSF from Figure 1.3 for different levels of piston errors (no other errors). We can see that in order to achieve high-contrast, the segments must be co-phased as well as possible.

and the radial profiles of the coronagraphic PSFs for the same conditions. The coronagraphic PSFs were all normalized by the reference PSF to show the relative contrast. From these simulations, we can see why it is critical for the GMT segments to be co-phased to well within a fraction of a wavelength to achieve high Strehl ratio and high contrast in the visible to near-infrared wavelengths.

1.1.2 Solving the Piston Problem

The GMT’s initial phasing strategy for NGSAO science involved a slow, off-axis, seeing-limited dispersed fringe sensor (DFS) and two different wavelength pyramid wavefront sensors (PyWFSs) to measure and control the OPD between segment pairs (Quirós-Pacheco et al., 2018; Pinna et al., 2006, 2014). The slow off-axis DFS was intended to bring the OPD to within capture range of the dual PyWFS, which in theory was capable of measuring differential piston errors from the GMT at nanometer level precision (van Dam et al., 2012). Since the phasing of the GMT segments is classified as a high risk item, it was critical that this phasing strategy be
well tested in a lab environment with real optics and AO hardware. Therefore, a risk-reduction program was contracted to the University of Arizona to build a phasing testbed that would demonstrate a working prototype of the DFS and PyWFS with a real GMT simulator.

In this dissertation, I present the development of a GMT High Contrast Adaptive optics phasing Testbed (HCAT) that simulates piston co-phasing errors between the GMT segments with real optics in a lab environment and leverages the existing AO instrument, MagAO-X (Males et al., 2020; Close et al., 2018), to test a PyWFS’s ability to control piston co-phasing errors between the GMT segments while simultaneously correcting for atmospheric turbulence. In Chapter 2, I give a brief introduction to adaptive optics. In Chapter 3, I introduce the new MagAO-X instrument. In Chapter 4, I present the first development of a GMT testbed and some of the experiments performed with it that led to the development of p-HCAT and HCAT. Chapter 5 walks through the design and build of the prototype HCAT (p-HCAT), which provided a new robust phasing method for the GMT via a novel piston sensor optic called the “Holographic Dispersed Fringe Sensor” (HDFS; see Haffert et al., 2022). The first results of closed-loop piston control with p-HCAT, MagAO-X’s PyWFS, and the novel HDFS are discussed (see Section 5.6). Finally, Chapter 6 presents the design and build of the full HCAT testbed, which incorporates all seven GMT segments with six piston, tip, and tilt actuators while demonstrating a working concept for the GMagAO-X “parallel deformable mirror” to optically distribute each GMT segment onto a commercial 3,000 actuator DM.
Adaptive Optics (AO) is a technology used for ground-based telescopes to remove the effects of atmospheric turbulence and achieve the diffraction-limit of a telescope. For direct imaging of exoplanets, AO is absolutely necessary to achieve the highest resolution and contrast possible. This chapter gives an introduction to AO.

2.1 The Point Spread Function

In astronomy, light from a star can be approximated by a point source located at infinity, emitting light in all directions as spherical waves. By the time the light reaches the top of the Earth’s atmosphere, these spherical waves become so enlarged that they appear as flat plane waves (see Figure 2.1). A telescope is designed to take these plane waves and form them into an image.

From diffraction theory, we know that for a monochromatic on-axis point source located at infinity, the image is related to the Fourier transform of the electric field in the entrance pupil. This result is known as the point spread function (PSF), which is given by

$$\text{PSF} \propto \left| \mathcal{F} \left[ T(\xi, \eta) \exp \left( j \frac{2\pi}{\lambda} W_{ab}(\xi, \eta) \right) \right] \right|^2,$$

(2.1)

where $T(\xi, \eta)$ is the entrance pupil transmission function, $W_{ab}(\xi, \eta)$ is the aberration function expressed in wavelength units, and the symbol $\mathcal{F}$ denotes a Fourier transform. Figure 2.2 shows an example of the PSF for a circular transmission function (e.g., the entrance pupil of a 6.5 m telescope) with no aberrations present. The result is the classic Airy disk, which is the diffraction-limited image of a circular aperture.
Figure 2.1: A star is approximated as a point source located at infinity (many light years away from Earth). Light emits in all directions as spherical waves, but by the time they reach Earth, the spherical waves become so enlarged that they appear to us as flat plane waves.

Figure 2.2: (Left): The entrance pupil transmission function. (Right): The Fourier Transform of the entrance pupil transmission function, also known as the point spread function, shown in logarithmic scale. For a circular aperture, the PSF is a classic Airy disk.
2.2 Atmospheric Turbulence

As light travels through the Earth’s atmosphere, the speed it travels is dependent on the refractive index of the air \( (v = c/n) \). Fluctuations in the air’s temperature causes the refractive index to change as a function of time and position in the sky. Hence, light propagating through regions of higher index will be delayed compared to light propagating through regions of lower index, changing the optical path length (OPL) of the light. The OPL is defined as

\[
\text{OPL} = \int n(z)dz, \tag{2.2}
\]

where \( n(z) \) is the refractive index as a function of position along the beam path. This informs the phase change of the light as it travels through the atmosphere, which is given by

\[
\phi = \frac{2\pi}{\lambda} \text{OPL}, \tag{2.3}
\]

where \( \lambda \) is the observation wavelength in a vacuum. If we compare the difference in phase between a given point \( x \) and a nearby point \( x + \xi \) in the sky, we can start to describe the shape of an incoming wavefront at any given position along the beam path to form a two-dimensional map of the aberrated wavefront \( W_{ab}(\xi, \eta) \) projected over the entrance pupil of a telescope and compute the resulting PSF using equation 2.1. Figure 2.4 shows an example of an aberrated wavefront (i.e., “phase screen”) and the resulting PSF. We can see that the wavefront is no longer “flat” and the resulting PSF is no longer diffraction-limited.

2.2.1 Atmospheric Seeing and the Fried Parameter

There are two parameters that characterize the strength of atmospheric turbulence: atmospheric seeing and the fried parameter.

Atmospheric Seeing

Atmospheric seeing is the full width at half maximum (FWHM) of a long exposure PSF in the presence of turbulence. Without turbulence, the apparent size of the PSF
Figure 2.3: When plane waves travel through the Earth’s atmosphere, the turbulence creates distorted wavefronts as a function of time (source: Wikipedia).

Figure 2.4: (Left): A turbulence phase screen frozen in time. (Right): The resulting PSF. We can see that in the presence of atmospheric turbulence, the PSF is no longer diffraction-limited.

would be characterized by the Airy disk, which has a FWHM of $\sim \lambda/D$. However, in the presence of atmospheric turbulence, the PSF is distorted, so a long exposure image would average out the distortions as a large “blob” (see Figure 2.5). The FWHM of this “blob” is known as the atmospheric seeing or “seeing disk.” The larger the seeing, the stronger the turbulence conditions. The best astronomical
sites in the world report seeing values as low as 0.3 arcsec at $\lambda = 500\,\text{nm}$ (Lyman et al., 2020), while average astronomical sites report seeing values of $\sim 1.0$ arcsec or more (Abt, 1980). The FWHM of the seeing disk in Figure 2.5 is 0.5 arcsec at $\lambda = 500\,\text{nm}$.

![Long Exposure PSF With Turbulence](image1)

![Long Exposure PSF With No Turbulence](image2)

Figure 2.5: (Top Left): A long exposure image of the PSF in the presence of atmospheric turbulence at an observation wavelength of 800 nm. (Top Right): A long exposure image of the PSF in the presence of no atmospheric turbulence. A cross section of each result is shown below. We can see that in the presence of atmospheric turbulence, the PSF is no longer diffraction-limited, it is considered “seeing-limited.”
Fried Parameter

The Fried parameter $r_0$, also known as the atmospheric coherence length, is the diameter of a circular region in the sky over which the root mean square (RMS) of the wavefront aberration is 1 radian. In other words, it is the size of a patch in the sky in which all light is considered coherent; a measure of how large a segment of a wavefront can be treated as a plane wave. The larger the $r_0$, the better the seeing conditions. The Fried parameter is usually reported at a wavelength of 500 nm, with typical values ranging from a few centimeters (poor seeing conditions) to 10 - 20 cm (excellent seeing conditions). The actual observed Fried parameter is dependent on the observation wavelength, which can be given by (Chromey, 2010):

$$r_0(\lambda) = r_0 \left( \frac{\lambda [\mu m]}{0.5} \right)^{6/5} (\cos \zeta)^{3/5},$$

where $r_0$ is the measured Fried parameter at 500 nm, $\lambda$ is the observation wavelength given in microns, and $\zeta$ is the zenith angle. Figure 2.6 shows a plot of equation 2.4 for a measured $r_0$ of 10 cm at 500 nm. Since $r_0 \propto \lambda^{6/5}$, $r_0$ increases almost linearly as a function of wavelength.

![Figure 2.6: The Fried parameter as a function of observation wavelength for a measured Fried parameter of 10 cm at 500 nm.](image-url)
The atmospheric seeing can be calculated from the Fried parameter by the following relation (Tokovinin, 2002):

$$\epsilon_0 = 0.98 \frac{\lambda}{r_0}. \quad (2.5)$$

### 2.3 Adaptive Optics

To compensate for the effects of atmospheric turbulence and achieve the diffraction-limit of a telescope, an AO system can be used. The design of an AO system consists of a wavefront sensor (WFS) to measure the shape of an incoming wavefront, a deformable mirror (DM) to correct for any distortions in the wavefront, and a computer to run the control loop between the WFS and DM (see Figure 2.7).

![Figure 2.7: An adaptive optics system consists of a WFS, DM, and computer to measure and control the shape of an incoming wavefront (source: pi-usa.us).](image)
2.3.1 The Shack-Hartmann Wavefront Sensor

A common example of a WFS is the Shack-Hartmann WFS (SHWFS). A SHWFS uses a lenslet array of \( N \) elements to sample an incident wavefront in the exit pupil of a telescope with entrance pupil diameter \( D \). Each lenslet creates an image of a star on the detector with a FWHM of \( \sqrt{N} \lambda / D \). When a distorted wavefront is incident onto the lenslet array, the centroid of the lenslet spots deviate, indicating the slope of the incident wavefront. Note that if the piston aberration were present, the centroids would not deviate. Hence, a SHWFS is blind to piston. This is a concern for segmented mirror telescopes like the GMT which need to measure and control the piston errors between each mirror segment. Figure 2.8 shows a schematic of the SHWFS concept. Each lenslet image is sampled by a 4x4 pixel quad cell, which allows the local \( x \) and \( y \) slopes of the wavefront to be calculated using the following equations (Roddier, 1999),

\[
\delta_x = \frac{\theta_b}{2} \frac{I_1 + I_2 - I_3 - I_4}{I_1 + I_2 + I_3 + I_4} \quad \text{and} \quad \delta_y = \frac{\theta_b}{2} \frac{I_2 + I_3 - I_1 - I_4}{I_1 + I_2 + I_3 + I_4},
\]  

(2.6)

Figure 2.8: A schematic of the Shack-Hartmann WFS. A lenslet array is placed in a pupil plane to sample an incoming wavefront. Each lenslet image is sampled by a 4x4 quad cell to calculate the local slopes of the wavefront (source: Tokovinin Tutorial Webpage).
where \( \theta_b \) is the spot size or FWHM of each lenslet image \( (\sqrt{N\lambda/D}) \), and \( I_1, I_2, I_3, I_4 \) are the intensities detected in the four pixel quadrants. The slope information is then sent to the AO system’s DM to create a shape that corrects for the wavefront distortions in real time.

### 2.3.2 Fitting Error

Depending on the number of elements used to sample an incoming wavefront (e.g., the number of lenslets in a lenslet array and/or the number of actuators on the DM), there will be some residual RMS fitting error, given by (Roddier, 1999):

\[
\sigma_{\text{fit}}^2 = 0.335 \left( \frac{D}{r_0(\lambda)} \right)^{5/3} N^{-5/6},
\]

where \( D \) is the diameter of the telescope primary mirror, \( r_0(\lambda) \) is the Fried parameter at a given wavelength, and \( N \) is the number of elements. Therefore, to reduce the RMS fitting error, an adequate number of actuators on the DM must be chosen to fit the wavefront. An acceptable number of actuators required to sample a wavefront is when the RMS fitting error is \(< 1 \) radian. Solving for \( N \), we find the following equation:

\[
N = 0.27 \left( \frac{D}{r_0(\lambda)} \right)^2,
\]

(2.8)

Since \( r_0(\lambda) \) is proportional to \( \lambda^{6/5} \), the number of actuators needed to sample a wavefront increases as \( \lambda \) decreases, making AO more challenging at shorter wavelengths (e.g., in the visible) than longer wavelengths (e.g., in the infrared).

### 2.3.3 Greenwood Time Delay

Since the atmospheric phase changes with time, the WFS of an AO system must continuously measure the incoming wavefront and apply the correction to the DM. However, if there is any time delay in between the time the phase is measured and the time the correction is applied, then there will be some error in the correction, given by (Roddier, 1999):

\[
\sigma_{\text{time}}^2 = 6.88 \left( \frac{u\tau}{r_0(\lambda)} \right)^{5/3},
\]

(2.9)
where \( v \) is the spatial average of the turbulence wind speed and \( \tau \) is the time delay. An acceptable time delay \( \tau_0 \) for the AO control loop is when the RMS phase error is \( < 1 \) radian. This is called the Greenwood time delay and is given by (Fried, 1990):

\[
\tau_0 = 0.314 \frac{r_0(\lambda)}{v},
\]  

(2.10)

Since \( r_0(\lambda) \) is proportional to \( \lambda^{6/5} \), the acceptable control loop speed increases as \( \lambda \) decreases, again making AO more challenging at shorter wavelengths (e.g., in the visible).

### 2.3.4 The Strehl Ratio

The Strehl ratio is a measure of the quality of an optical system. It is defined as the ratio of the peak intensity of the measured PSF over the peak intensity of the ideal PSF. The Strehl ratio has a value between 0.0 and 1.0, where a perfect image with no aberrations present would yield a Strehl ratio of 1.0, and an image with aberrations present would yield a Strehl ratio less than 1.0. The Strehl ratio can be approximated by

\[
S \approx e^{-\sigma^2},
\]  

(2.11)

![Figure 2.9](image-url):

Figure 2.9: (Left): Strehl ratio versus number of actuators. As \( D/r_0 \) increases, the number of actuators required to achieve a high Strehl ratio also increases. (Right): Strehl ratio versus time delay.
where $\sigma^2$ is the total RMS wavefront error of the optical system. The total RMS wavefront error is the sum of all known errors in the system,

$$\sigma^2 = \sigma^2_{\text{fit}} + \sigma^2_{\text{time}} + \sigma^2_{\text{noise}} + ...$$  \hspace{1cm} (2.12)

These equations can be used to form a wavefront error budget around a desired achievable Strehl ratio. Figure 2.9 shows the Strehl ratio plotted for two different errors: the fitting error from equation 2.7 and the time delay error from equation 2.9. As $D/r_0$ increases, the number of actuators required to achieve a high Strehl ratio also increases. As the time delay increases, the Strehl ratio decreases. By putting together an error budget that contains all known error terms and parameters, one could design an AO system around a desired final Strehl ratio.
The Magellan Extreme Adaptive Optics system (MagAO-X) is a new visible-to-near-IR ExAO instrument designed for the 6.5-m Magellan Clay telescope in Chile that was recently built (first light in fall 2019) in the University of Arizona’s Center for Astronomical Adaptive Optics (CAAO) Extreme Wavefront Control Lab (Males et al., 2020, 2018; Close et al., 2018). MagAO-X is considered “extreme” because it is designed to work at visible wavelengths (0.50 µm–1.00 µm), so it requires an “extreme” number of actuators to sample the wavefront at an “extremely” fast speed. Figure 3.1 and 3.2 show detailed views of the MagAO-X instrument. The instrument is a two-level floating optical table that sits on the Nasmyth platform.

Figure 3.1: The MagAO-X instrument is a two-level floating optical table that sits on the Nasmyth platform of the Magellan-Clay telescope. Light enters through the Nasmyth port of the telescope at F/11.
MagAO-X uses a PyWFS and three DMs (a 2,040 actuator “tweeter” DM, 97 actuator “woofer” DM, and 97 actuator “non-common path corrector” (NCPC) DM to correct for atmospheric turbulence and non-common path aberrations (NCPA) at visible wavelengths.

The three DMs in MagAO-X were optimized via a focus diversity phase retrieval method to bring the internal static wavefront error of MagAO-X down to a total of 18.7 nm RMS (Van Gorkom et al., 2018). This chapter reviews some of the main features of MagAO-X.

### 3.1 The Pyramid Wavefront Sensor

MagAO-X utilizes a special type of wavefront sensor known as a pyramid wavefront sensor (PyWFS) to measure the shape of an incoming wavefront (Esposito and Riccardi, 2001). The PyWFS combines the Shack-Hartmann WFS (SHWFS) concept with the “Foucault knife-edge test” concept to create a high precision WFS. In a
Foucault knife edge test, a sharp “knife” is placed in a focal plane. If an optical aberration is present, the knife edge will block aberrated rays coming from various regions in the pupil, causing these regions to appear dark in the exit pupil. If no aberration is present and the light comes to a perfect focus, then the knife edge will not block any rays, and the exit pupil will appear evenly illuminated. By scanning the knife through the tiny (FWHM $\sim \lambda/D$) PSF, a sensitive map of all the pupil’s aberrations can be made. Figure 3.3 shows an example of this.

Figure 3.3: A schematic of the Foucault knife-edge test. A knife edge is oriented vertically and scanned through focus. An image of the exit pupil is shown for each position of the knife edge (Goodwin and Wyant, 2006).

A PyWFS utilizes this concept. In a PyWFS, a four-sided glass pyramid is placed in the optical system such that the tip of the pyramid is located in a focal plane (see Figure 3.4). The four edges of the pyramid act as four separate “knife edges” which subdivide the focal plane into four parts. A relay lens following the pyramid then forms four separate images of the pupil onto a detector. These four images of the pupil allow the local wavefront slopes to be calculated using the same equation as the SHWFS quad cell (equation 2.6) to determine the shape of the incoming wavefront. Since the PyWFS slices a PSF of $\lambda/D$, it can sense a diffraction-limited scale $\lambda/D$ tilt easily, as opposed to the SHWFS which senses $\sqrt{N}\lambda/D$ tilts and cannot obtain the same sensitivity as the PyWFS without a massive amount of starlight.

In addition, a motorized tip/tilt mirror is placed in a conjugate pupil plane to modulate the focal plane in a circle around the pyramid tip to facilitate the scanning
Figure 3.4: A schematic of the pyramid wavefront sensor concept. A four-sided glass pyramid splits up the focal plane into four parts, while a relay lens is used to create four images of the pupil onto a detector.

of the PSF across the $x$ and $y$ edges of the pyramid, creating a 2D knife-edge test. Since we know the degree of modulation, this increases the linearity and dynamic range of the PyWFS.

3.1.1 The EMCCD

MagAO-X’s PyWFS uses an Electron Multiplying Charge-Coupled Device (EM-CCD) to achieve the highest sensitivity possible when measuring wavefront slopes within a fraction of a second (0.5 ms exposures). Contrary to the CCD, an EMCCD has the ability to detect single-photon events with almost zero readout noise and high readout speed, which is beneficial for astronomers whom constantly deal with low-light signals (Tulloch and Dhillon, 2011).

In a conventional CCD, the detector chip is made of Silicon, which is a semiconductor with a band gap of $\sim 1.1$ eV. An insulated layer of SiO$_2$ and a series of transparent electrodes are placed on top of the Silicon to generate a series of potential wells inside the Silicon, defining what we call “pixels.” When photons with more energy than the band gap ($< 1$ µm wavelength) arrive at the CCD, the photons will excite electrons from the valence band into the conduction band, where electrons are
free to move through the Silicon. These free electrons, referred to as *photoelectrons*, are then collected by the nearest potential wells, and at the end of an exposure the photoelectrons are physically transferred across the detector by applying voltage to the electrodes until they reach the charge amplifier, which converts the charge into voltage for digitization and storage.

A fast EMCCD utilizes a *frame transfer* CCD structure, where photoelectrons are quickly shifted to a separate detector area called the “storage area” to store the image prior to read out. Each row of the storage area is read out one at a time in the readout register and an output amplifier is used to convert the charge into voltage. In an EMCCD, an additional register called the multiplication register, or “EM gain register,” is placed between the readout register and the output amplifier. The gain register consists of a large number of stages ($N > 500$), each of which produce a multiplication of the photoelectrons through a process known as impact ionization. The gain probability in each register is small ($P < 2\%$), but since the number of elements in this register is large, the overall gain can be very high ($g = (1 + P)^N$), so a single photoelectron entering the gain registry can output thousands of electrons. For a single photon, the effective read noise is then considered negligible and the detection of a single photon becomes possible.

The disadvantage of the EMCCD’s EM gain mode is that for a large number of photons, the CCD outperforms the EMCCD, due to the EMCCD’s excess noise factor ($\sim \sqrt{2}$) originating from the amplification process, which reduces the SNR. However, the EMCCD can always be switched into a regular CCD by lowering the gain to 1 and reading out slowly to achieve similar performance. See Figure 3.5 for a comparison of the SNR vs number of photons for different types of detectors. For low signals ($< 10$ photons), an EMCCD outperforms the CCD. So for MagAO-X’s PyWFS EMCCD, which only ever receives a few photons/pixel, the EM gain mode is necessary to run at high speeds and achieve the highest SNR possible.
3.2 The K-Mirror

Since the 6.5-m Magellan Clay telescope is an alt-azimuth design with F/11 Gregorian foci at its Nasmyth locations, there are two forms of image rotating due to the Magellan Clay design. Pupil rotating is the result of the alt-azimuth design, while image rotating is caused by the tertiary mirror folding the light path. Pupil rotating is usually eliminated when the instrument is mounted to the Nasmyth port—which rotates with elevation (as used with MagAO)—however, MagAO-X is assembled on an optical bench that will sit in front of the Nasmyth port (see Figure 3.1), and will not be able to rotate. This will cause a clocking misalignment between the telescope pupil and the MagAO-X coronagraph mask, WFS, and DMs as the telescope tracks the sky (see Figure 3.6). Therefore, a K-mirror was introduced to the MagAO-X design to counter this rotating and keep the optical elements aligned with the image of the pupil. Initially, the K-mirror was to be placed within a 120 mm inter-OAP cavity in the MagAO-X design, but led to variable field curvature errors between the

Figure 3.5: A comparison of the SNR vs number of photons achievable with different types of detectors. For a low number of photons \((N < 10)\), the EMCCD outperforms. For high number of photons \((N > 10)\), the CCD outperforms. See andor.oxinst.com for details on plot parameters.
OAPs, so the K-mirror position was moved to a 60 mm cavity over the Gregorian focus (see Figure 3.7). This resulted in the need for a very compact K-mirror design under 60 mm in size (due to the fixed 125 mm back focal distance. Although K-mirrors are quite common in astronomical instrumentation (Morrissey et al., 2012; Buckle et al., 2009; Baudet et al., 2016), the MagAO-X K-mirror is unique due to its compactness. The MagAO-X K-mirror designing process will be discussed here, along with its final performance.

3.2.1 Optical Design

The goal for the MagAO-X K-mirror design was to create an image rotator with minimal wobble or distortion. This means that the incoming light should be coaxial

![Image](image.png)

Figure 3.6: The Magellan-Clay optical design produces a pupil that is asymmetric (top-left). Therefore, MagAO-X’s coronagraphic pupil mask must match this shape (top-right). As the telescope rotates, the coronagraphic pupil mask will be misaligned in clocking with the pupil (bottom-left). A K-mirror is therefore needed to de-rotate the pupil as the telescope tracks the sky and keep the mask aligned with the pupil (bottom-right).
Figure 3.7: (Left): The initial designed K-Mirror position between OAPs, which created variable field curvature errors. (Right): New K-mirror position with correct clocking of OAPs with minimum field curvature errors. The new K-mirror position is a tight 60 mm cavity where the Gregorian focus of the Magellan Clay is located. This results in the need for a compact K-Mirror design.

Figure 3.8: A K-Mirror consists of 3 mirrors, 2 of which are tilted by 30° with respect to the optical axis. The input light is coaxial with the output light.

with the outgoing light (along the optical axis). Three mirrors must be used, with two being 120° apart to create the correct light path (see Figure 3.8). MagAO-X was first designed in Zemax and then opto-mechanically designed in Autodesk® Fusion 360™. Fusion 360™ had errors loading the light path from the MagAO-X design, so FreeCAD and SolidWorks were used in certain cases to visualize the light footprints on the K-mirror. For example, a FreeCAD screenshot of the K-mirror section in the MagAO-X design is shown in Figure 3.9, where the K-mirror footprints are thin disks. K1 and K3 are 1.0-in in diameter while K2 is 0.5-in in diameter. These are
Figure 3.9: FreeCAD screenshot of the MagAO-X design. The three thin disks are the default K-mirror footprints.

the default K-mirror diameters set by Zemax in the MagAO-X design. In the Zemax design, a FOV of 14 arcseconds was used, creating the 1.0-in diameter necessary for K1. However, the maximum FOV that MagAO-X can observe due to its camera is approximately 6x6 arcseconds, indicating that smaller diameter mirrors may be used for the K-mirror. This information became useful when it was apparent that the initial Zemax parameters would cause a collision between K1 and the surrounding optics as the K-mirror rotated. Due to this problem, a diameter of 19 mm was chosen for K1 (and K3) in the final design.

3.2.2 The K-Mirror Housing

The main component to the K-mirror is the housing that holds the mirrors. The challenge in designing the housing was to avoid contact with the surrounding optics, the light path, and the edge of the optical bench, as seen in Figure 3.10. The main idea was to place K1 and K3 inside of a tube and mount it to a compact rotating stage. Then, mount K2 onto an arm that extends out from the tube. A Newport® SR50CC rotating stage was chosen for the K-mirror, while the housing was designed to mount onto the stage. A cylindrical wedge was designed to hold K1 and K3 at
120° and mount to the housing. The base of the housing is a disk that extends out to the K2 wedge, which bolts onto the edge of the base. Since the surrounding mirrors made it difficult to fit the K-mirror inside this cavity, it was impossible to use the usual kinematic mounts or actuators that one might use for aligning the K-mirror. For this reason, glue was the only solution to mounting these mirrors.

### 3.2.3 Gluing the Mirrors

To glue the mirrors, glue channels were machined into the wedges to allow excess glue to run free (see Figure 3.11). The K-mirror wedges were machined out of invar to reduce CTE mismatch problems between the fused silica and the wedge. Each have three equilateral hard points to which the mirrors adhere to. The glue chosen for this application was Loctite® AA 312, a clear adhesive liquid for bonding metals with

![Figure 3.11](image)

Figure 3.10: (Left): The k-mirror was input into the MagAO-X solid model to visualize the surrounding environment. There were three other mirrors surrounding the k-mirror, along with the continuing light path that extends behind the k-mirror, showing that this is an extremely tight space for the k-mirror. To allow extra space, the wedges were trimmed and the diameter of K1 was changed to 19.0 mm for the final product. (Right): The k-Mirror housing design. K1 and K3 are glued to a cylindrical wedge inside of a cylindrical housing which mounts to a compact rotating stage. K2 is mounted on an arm that extends out from the tube. This model was used for the final k-mirror product.
glass. A spray primer, Loctite® SF 736, is accompanied with the adhesive. The primer is first applied to the surface, then the adhesive is applied using a dropper. This structural adhesive dries within seconds, so the mirror must be precisely aligned on the surface fairly quickly.

### 3.2.4 Aligning the K-mirror

The alignment of a glued K-mirror is not trivial. A K-mirror system requires the output beam to be aligned precisely with the optical axis while rotating 360°. MagAO-X required its K-mirror to be aligned within 5 arcminutes. Typically K-mirror systems have actuators and/or kinematic mounts attached to the mirrors to make the alignment process easier, but for MagAO-X, glue was the only option for mounting them. This created a difficult alignment solution. The solution I developed introduces a few degrees of freedom to the K2 mirror. K2 was given the ability to shift in piston by use of a “sacrificial washer”—an aluminum spacer between the K-mirror housing and the K2 wedge (see Figure 3.12). By changing the thickness of the spacer, K2’s height could be changed. The default spacer thickness in the model was 0.25-in, but

---

Figure 3.11: A view of the K1 glue channel. The glue channels helped the mirrors sit on three hard points, while pins helped to keep the mirrors aligned while the glue dries. A primer (Loctite® SF 736) was applied to the surface of the wedge and three drops of the adhesive (Loctite® AA 312) were placed on the three hard points. The mirrors were then pressed onto the glue until dry (\(\sim\) 30 seconds).
changed to 0.36-in (a number found by trial and error) to align the K-mirror. The ability to tilt K2 was also made possible by shimming underneath the K2 wedge.

Figure 3.12: The K2 mirror may be tilted by use of shim stock. Placing shim stock underneath the K2 wedge allows the mirror to be tilted as needed. The K2 mirror may also be shifted in piston by changing the thickness of the sacrificial washer, which is just an aluminum spacer block.

**Stage Alignment**

An additional parameter for aligning the K-mirror was added by adjusting the pitch of the rotating stage itself. Since the MagAO-X optical elements are mounted 5.0-in above the optical bench, the K-mirror rotating stage was mounted on a Thorlabs adjustable optical post to adjust the K-mirror height to 5.0-in. This post caused a misalignment in the pitch of the stage, so it needed to be corrected. This was achieved by placing a long lens tube inside the K-mirror housing with 2x glass diffusers (see Figure 3.13). Using the same 1.0 mm collimated beam in the initial setup, the height and tilt of the stage was measured with the glass diffusers, and shim stock was used to correct the offset.

To avoid the unsteady Thorlabs adjustable posts, MagAO-X uses 1.0-in diameter custom stainless steel Polaris® posts for all of its optical elements (see Close et al., 2018). This method was also used for the K-mirror, but a custom adapter was needed to mount the 1-in post to the rotating stage. This custom mount was designed in Autodesk® Fusion 360™ and machined in the Steward Observatory Machine Shop out of stainless steel. An image of the custom mount and the final K-mirror product
Figure 3.13: To eliminate pitch or yaw misalignment in the stage, the mirrors were removed and a 4.0-in tube with 2x glass diffusers was inserted into the K-mirror housing (right). One glass diffuser (closest to the stage) was used to adjust the height of the K-mirror, while the other was used to measure the pitch of the stage. By seeing where the collimated beam hits the glass diffusers, the stage height and tilt was aligned (middle). Shim stock was used (left) to correct for the pitch misalignment and improve the K-mirror alignment.

are shown in Figure 3.14. The custom mount helped stabilize the rotating stage’s height, pitch, and yaw alignment due to its rigid structure.

Figure 3.14: The final MagAO-X k-mirror product. A custom stainless steel adapter was designed to mount a Polaris® 1.0-in diameter post to the Newport® SR50CC rotating stage. The custom mount was needed to properly bolt the rotating stage to the post while providing a rigid structure for holding the stage.
3.2.5 K-mirror Alignment Results

To align the K-mirror, a 1.0 mm collimated beam was set up in the lab with the K-mirror mounted on a 2.0 m optical rail (see Figure 3.15). This gave us 1.2 m of path length to measure the beam wobble. Using the SR50CC rotating stage, the K-Mirror was rotated and the beam offset was measured on a target. Then, K2 could be adjusted in piston and pitch, or the entire assembly could be adjusted in pitch and yaw to align the K-mirror. In the end, the final K-mirror product required no alignment adjustments to achieve MagAO-X’s alignment requirement. The optomechanical structure was stable enough to provide an alignment of ± 1 arcminute of wobble over 360° of rotating. The final K-mirror product installed in the MagAO-X system is shown in Figure 3.16.

Figure 3.15: The K-mirror beam alignment setup. A 1.0 mm collimated beam was set up with the K-mirror mounted on a 2.0 m rail. As the rotation stage rotated, the beam deviation was measured to determine the K-mirror’s alignment precision.

3.3 Final Build of MagAO-X

The build of MagAO-X was completed and its first on-sky observations were made at the Magellan-Clay telescope in 2019. Figure 3.17 shows the completed MagAO-X instrument installed on the telescope in Chile. MagAO-X is designed to travel back
Figure 3.16: The final K-mirror product installed in the MagAO-X system. The K-mirror fits inside the optical cavity with the ability to freely rotate 360°.

and forth between Chile (to perform science observations) and Tucson (to continue upgrades and lab experiments).

The HCAT testbed utilizes MagAO-X (when in the lab in Tucson) to demonstrate ExAO wavefront control with a “GMT-like” pupil. The next chapter discusses an early stage GMT testbed that led to the development of HCAT (discussed in Chapter 5 and 6).
Figure 3.17: (Left): The completed MagAO-X instrument in the lab. (Right): The completed MagAO-X instrument installed and aligned with the Magellan-Clay telescope in Chile during the first commissioning run in 2019.
CHAPTER 4

The GMT Proto-Testbed

The GMT proto-testbed was an early stage development of a GMT simulator that evolved into p-HCAT and HCAT. This simple testbed was an inexpensive playground for testing segment piston phasing with a real GMT-like pupil in the lab. This was an important precursor experiment (before funding for HCAT was available) to help define the final HCAT testbed design well before the HCAT final design review (FDR). Some of the methods from this preliminary testbed were adapted for p-HCAT (see Chapter 5). This chapter discusses the experiments that were performed with the GMT proto-testbed and how they provided the ground work for p-HCAT and HCAT.

4.1 The Telescope Simulator

To simulate the GMT in the lab, a four-segment GMT telescope simulator was created using a single mode fiber, an \( f = 250 \) mm collimating lens, a pupil mask, and an \( f = 250 \) mm imaging lens (see Figure 4.1). A single mode (SM) fiber was used to approximate a point source, or “star,” since its effective core diameter is very small (~4.0 µm). The output of a SM fiber is an expanding gaussian beam. Therefore, to mimic a plane wave coming from a point source, the gaussian beam was expanded until the wavefront could be considered “flat.” The further the beam is expanded and the smaller the entrance pupil (sampling a smaller portion of the gaussian beam’s core), the better the approximation of a plane wave. In this case, the beam propagates 250 mm until the light is collimated by an \( f = 250 \) mm lens. Then, a thin, 1.0-inch diameter, stainless steel etched pupil mask is placed in the beam to define the entrance pupil of the “telescope.” Finally, an \( f = 250 \) mm imaging lens creates an image of the “star,” which is given by the Fourier transform of the
entrance pupil. Figure 4.2 shows the simulated PSF from the four-segment entrance pupil transmission function to show that this telescope simulator matches theory.

Figure 4.1: The GMT telescope simulator. A single mode fiber is used to approximate a point source, while an \( f = 250 \text{ mm} \) lens collimates the light to create a “plane wave,” and a stainless steel etched pupil mask defines the entrance pupil of the “telescope.” An \( f = 250 \text{ mm} \) lens creates an image which is defined by the Fourier transform of the entrance pupil.

Figure 4.2: The simulated PSF created by the four-segment GMT pupil mask, which can be calculated by taking the Fourier transform of the entrance pupil transmission function.
4.2 The Holey Mirror

After the telescope simulator was created, a simple experiment was created to control one out of the four GMT segments in piston, tip, and tilt, while the other three segments stay fixed as a reference. To do this as simply as possible, the pupil mask was reimaged onto a flat 3.0-inch mirror with a hole in it, which allowed a 1.0-inch mirror to poke through the hole exactly around one of the reimaged GMT segments. The 1.0-inch mirror would have the ability to piston, tip, and tilt one GMT segment, while the other three segments would stay fixed (and co-phased) on the flat 3.0-inch mirror. This special mirror became known as the “holey mirror,” which was fabricated by Tucson Optical Research Corp. Figure 4.3 shows the holey mirror in the lab and Figure 4.4 shows the Zygo measurements of the holey mirror surface. The mirror was specified to be $\lambda/10$ PV, but due to the cored hole, the surface quality was measured to be $\lambda/5$ (with the edges of the hole masked out). Hence, we could expect the total wavefront error to be $\sim \lambda/5$ PV across the aperture.

Figure 4.3: The holey mirror. The four-segment GMT pupil mask is reimaged onto a 3.0-inch flat mirror with a hole that allows a 1.0-inch mirror to poke through exactly around one of the reimaged GMT segments. The 1.0-inch mirror is glued to a compact tip/tilt mount and a differential micrometer stage that allows for fine piston adjustment.
Figure 4.4: (Left): The holey mirror Zygo measurement (90% CA) (Right): The holey mirror Zygo measurement with the edges of the hole masked out. The surface was measured to be $\sim \lambda/5$ PV.

A mechanical structure was designed with commercially available parts to mount the 1.0-inch mirror with piston, tip, and tilt capability. Figure 4.5 shows the 1.0-inch mechanical mount design.

Figure 4.5: The 1.0-inch mirror mechanical mount design. The mirror was glued to a compact tip/tilt mount to allow tip and tilt control while a differential micrometer allows precise piston control. This way, one GMT segment can be controlled in piston, tip, and tilt, to create a GMT co-phasing experiment.
mechanical mount as built in the lab. The mirror is glued to a compact kinematic mount to allow tip/tilt control while a differential micrometer allows precise movements in piston.

4.3 Optical Design

To create an image of the pupil mask onto the holey mirror, a second optical relay was introduced after the telescope simulator relay with two $f = 750\text{ mm}$ commercial achromatic doublet lenses and two fold mirrors. Figure 4.6 shows the full optical design of the proto-testbed. A flat mirror is introduced in the first focal plane of the telescope simulator to fold the light over to the first $f = 750\text{ mm}$ doublet lens, which collimates the light to form an image of the pupil mask onto the holey mirror. Then, a second $f = 750\text{ mm}$ doublet lens forms an image in the final focal plane. Figure 4.7 shows a picture of the testbed as built in the lab.

Figure 4.6: The optical design of the GMT proto-testbed. The first pupil relay defines the telescope simulator, while the second pupil relay uses two $f = 750\text{ mm}$ achromatic doublet lenses to form an image of the pupil mask onto the holey mirror with $\sim 1.0$-inch segment sizes.
4.4 Co-phasing Experiments

We now have one out of four telescope segments that we can control in piston, tip, and tilt, so we can perform experiments to learn how to co-phase a segmented telescope like the GMT. Figure 4.8 shows an image of the PSF from the three segments that are fixed on the holey mirror and the PSF from the 1.0-inch mirror.

Figure 4.8: (a) An image of the three-segment PSF from the three fixed segments on the holey mirror (left) and the single segment PSF from the segment on the 1.0-inch mirror (right). (b) An image of the segments aligned and co-phased. Note that these images were taken with a laser ($\lambda = 633 \text{ nm}$) as the light source.
misaligned versus aligned and co-phased in laser light ($\lambda = 633\text{ nm}$). In this section we will explore how to co-phase the 1.0-inch mirror segment through knowledge of interference.

### 4.4.1 2 Segment Interference

One of the experiments performed with the proto-testbed was blocking out the top and bottom segments to only interfere the light between the 1.0-inch mirror segment and the central obscuration segment. By stepping the 1.0-inch mirror segment in piston, the intensity pattern of the PSF could be analyzed for finding the white light fringe (0 OPD piston). Figure 4.9 shows a simulation of the two-segment PSF at $\lambda_c = 925\text{ nm}$, 25 nm bandwidth using equation 2.1. The aperture function can be described by the following equation:

$$T(\xi, \eta) = \text{circ} \left( \frac{\rho}{D} \right) \ast \frac{1}{2} \left[ \delta(\xi - \xi_0, \eta) + \delta(\xi + \xi_0, \eta) \right], \quad (4.1)$$

where $D$ is the diameter and $\xi_0$ is the position of each segment. The PSF can be calculated by taking the Fourier transform of $T(\xi, \eta)$ and taking the modulus squared which results in

$$\text{PSF} = \left[ \frac{2J_1(r)}{r} \right]^2 \cos^2 \left( \frac{2\pi \xi_0 x}{\lambda f} + \Delta\phi / 2 \right), \quad (4.2)$$

Figure 4.9: A simulation of the two-segment PSF resulting from the interference between two of the GMT segments at $\lambda_c = 925\text{ nm}$, 25 nm bandwidth.
where $J_1$ is the first order Bessel function, $r = D\pi x/\lambda f$, and $\Delta \phi$ is the phase difference between the two segments. Figure 4.10 shows the plot of equation 4.4 for $\Delta \phi = 0$ and $\Delta \phi = \pi$. As $\Delta \phi$ increases, the fringes modulate from constructively interfering (steps of $2\pi n$) to destructively interfering (steps of $\pi n$).

![Figure 4.10: The cross section of the two-segment PSF from equation 4.4 shown for $\Delta \phi = 0$ and $\Delta \phi = \pi$. The resulting PSF is a fringe pattern defined by a cosine function inside the envelope of the first order Bessel function.](image)

Furthermore, the visibility of these fringes will decrease as $\Delta \phi$ increases for a polychromatic light source. The visibility goes to zero at the coherence length $l_c$ of the light source, which is defined as

$$l_c = \frac{\lambda_c^2}{\Delta \lambda},$$

(4.3)

where $\lambda_c$ is the central wavelength and $\Delta \lambda$ is the bandwidth. If we plot the center pixel value of the PSF as a function of OPD, we see the resulting plot shown in Figure 4.11. The modulation of the intensity follows an envelope which represents the visibility $V$ of the fringes, defined as

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}. $$

(4.4)

For a polychromatic light source with $\lambda_c = 925$ nm, 25 nm bandwidth, the fringe visibility goes to zero at a coherence length of 34 µm.

This theory was tested with the proto-testbed by moving the 1.0-inch mirror segment in piston and viewing the interference pattern in the focal plane at $\lambda_c = 925$ nm,
Figure 4.11: The center pixel intensity value of the PSF from Figure 4.9 plotted as a function of OPD. The fringes modulate as a function of OPD until the visibility goes to zero at the coherence length of the light source (34 μm in this case).

Figure 4.12: (Left): The two-segment PSF image taken with the proto-testbed at λ_c = 925 nm, 25 nm bandwidth with ∼0 μm OPD piston. (Right): The two-segment PSF image with ∼34 μm OPD piston. 25 nm bandwidth. Figure 4.12 shows two images taken with ∼0 μm OPD and ∼34 μm OPD piston introduced. At 34 μm OPD, the visibility of the fringes disappear, and the segments are no longer coherent.
4.4.2 Dispersed Fringes

A simple and powerful way of co-phasing two segments to find the white light fringe is by utilizing the concept of diffraction gratings to disperse the light into a spectrum and create dispersed fringes in the focal plane. An example of a diffraction grating is a binary mask with periodic slits etched across the aperture. The aperture function of a binary transmission grating can be described by the following equation:

\[
T(\eta) = \frac{1}{2} \text{sgn}[\cos(2\pi \alpha \eta)] + \frac{1}{2},
\]

where \(\alpha\) is the number of periods across the aperture and “sgn” denotes the sign of the cosine function. For a pupil of diameter \(D\) and grating period \(d\), \(\alpha = D/d\). If this grating were placed in the pupil plane of an optical system, an incoming plane wave would diffract into multiple orders \(m\), creating a series of PSFs in the focal plane (see Figure 4.13). Each maxima is called a diffraction order and its angular

![Figure 4.13: A diagram of a diffraction grating. Incoming plane waves with beam diameter \(D\) pass through a binary transmission grating with period \(d\), resulting in a series of PSFs in the focal plane, known as diffraction orders. The angular position of each diffraction order is defined by the grating equation. The width of a slit is \(w = d/2\) and the number of slits across the aperture is \(\alpha = D/d\).](image-url)
position is defined by the grating equation,

$$\sin \theta_m = \frac{m \lambda}{d},$$

where \( \lambda \) is the observation wavelength and \( d \) is the period of the grating. As the period \( d \) increases, the angular separation of the diffracted orders increases. Figure 4.14 shows a simulation of three different gratings with different periods. The top row shows a grating with \( \alpha = 2 \), the middle row shows a grating with \( \alpha = 10 \), and the

Figure 4.14: A simulation of a few different gratings with different periods and their resulting PSF. The top row is for a grating with \( \alpha = 2 \), the middle row is for a grating with \( \alpha = 10 \), and the bottom row is for a grating with \( \alpha = 20 \). As the number of periods across the aperture increases, the angular separation of the diffracted orders increases. The narrowband PSF is for \( \lambda = 0.80 \, \mu \text{m} \) while the broadband PSF is for \( 0.60 \, \mu \text{m} – 1.00 \, \mu \text{m} \).
bottom row shows a grating with $\alpha = 20$. The resulting narrowband ($\lambda = 0.80 \mu m$) and broadband ($\lambda = 0.60 \mu m - 1.00 \mu m$) PSFs are shown to the right. Note that most of the light is diffracted into the 0th order (maximum diffraction efficiency), while the intensity of the $> 0$ orders decrease as $m$ increases. Also, since $\theta_m \propto \lambda$, the $|m| > 0$ orders contain spectra. This makes diffraction gratings useful for spectrographs and other optical systems that utilize dispersion.

When utilizing a diffraction grating for its dispersion properties, it is desirable to achieve maximum diffraction efficiency in the $\pm 1$ diffraction orders, instead of in the 0th order. This can be made possible by controlling the phase of the diffraction grating such that the amplitude is $\pi/2$ ($\lambda/4$ at one particular wavelength). Hence, the $m = 0$ order will be perfectly cancelled at one wavelength and nearly cancelled at all other wavelengths. Figure 4.15 shows a simulation of a binary transmission grating with a phase amplitude of $\lambda/4$ at $0.80 \mu m$ and the resulting narrowband PSF ($\lambda = 0.80 \mu m$) and broadband PSF ($\lambda = 0.60 \mu m - 1.00 \mu m$). We can see that the $m = 0$ order is perfectly cancelled at $0.80 \mu m$ and the $m = \pm 1$ orders have much more light. The aberration function $W_{ab}(\xi, \eta)$ from equation 2.1 is now given by

$$W_{ab}(\eta) = \frac{\lambda}{4} \text{sgn}[\cos(2\pi \alpha \eta)],$$

and the PSF can be calculated with the same equation 2.1.

Figure 4.15: A simulation of a diffraction grating with $\alpha = 20$ and a phase amplitude of $\pi/2$ ($\lambda/4$). The narrowband PSF ($\lambda = 0.80 \mu m$) and the broadband PSF ($\lambda = 0.60 \mu m - 1.00 \mu m$) are shown to the right. Since the amplitude is chosen to be $\pi/2$ at $0.80 \mu m$, the $m = 0$ order is perfectly cancelled at $0.80 \mu m$ and most of the light is diffracted into the $m = \pm 1$ orders.
To create a binary phase grating with $\pi/2$ amplitude, an optical substrate is used with etched grooves that have depths equivalent to $\pi$ radians. The etch depth in physical units can be determined by the following equation:

$$\Delta = \frac{\lambda}{2(n(\lambda) - 1)},$$

(4.8)

where $\lambda$ is the observation wavelength and $n(\lambda)$ is the refractive index of the substrate at the observed wavelength. For a fused silica glass substrate at a wavelength of 800 nm, this equates to an etch depth of 882 nm.

Now suppose we have two circular pupil segments passing through the grating. The PSF now contains the interference of the two segments (as shown in Section 4.4.1), creating “dispersed fringes” in the $m = \pm 1$ orders. When OPD in the form of piston is applied to each pupil segment, the dispersed fringes wrap into a “barber pole” shape due to the fact that phase is dependent on wavelength—indicating that the segments are not in phase. Figure 4.16 shows the result of having $0\lambda$ OPD piston error (top row) versus $4\lambda$ OPD piston error (bottom row) between the segments. The narrowband PSFs look the same in both cases since the intensity wraps by $2\pi$ for every $1\lambda$ of OPD. The broadband PSFs, however, look strikingly different due to the dispersed fringes. The segments are in phase (zero OPD) only when the dispersed fringes show a continuous straight line spectrum, indicating that all wavelengths have the same phase (on the white light fringe). The segments are not in phase when the dispersed fringes are wrapped like a barber pole. This makes dispersed fringes a powerful tool for measuring segment piston errors.

### 4.4.3 The GRISM

To utilize the concept of dispersed fringes for the proto-testbed, a GRISM (grating + prism) was used. A GRISM is a simple optic composed of a blazed diffraction grating (to maximize the diffraction efficiency in the 1st order) and a prism (to minimize coma in the 1st order) that can be placed in the converging beam of an optical system (e.g., in front of the detector near the focal plane). Without the prism, the 1st order would have some coma aberration due to the off-axis nature of
Figure 4.16: A simulation of the diffraction grating from Figure 4.15 with two circular pupil segments passing through. (Top row): the segments have $0\lambda$ OPD piston error between the segments. (Bottom row): the segments have $4\lambda$ OPD piston error. The narrowband PSFs look the same in each case since the intensity wraps by $2\pi$ for every $1\lambda$ of OPD. The broadband PSFs look very different in each case since the phase is dependent on the wavelength and the light is being dispersed into all of its colors. With no phase error, the dispersed fringes look like a continuous spectrum, but with phase error, the intensity wraps into a “barber pole” shape, indicating that the segments are not in phase.

the diffraction. Hence, the addition of the prism aligns the 1st order with the chief ray of the optical system and completely cancels coma for one wavelength, while almost cancelling coma for all other wavelengths (see Figure 4.17). The GRISM was bought from Paton Hawksley Education Limited and is composed of a 100 grooves/mm grating and a $3.8^\circ$ prism. This provided a low cost spectrograph for the proto-testbed to test piston sensing with dispersed fringes. Figure 4.18 shows the optical setup of the GRISM in the final converging beam of the proto-testbed. A 2-segment pupil mask was placed in the final converging beam of the system to block out the top and bottom segments and allow only the left and right segments to pass through to the GRISM which disperses the light to create dispersed fringes.
Figure 4.17: A diagram of a GRISM (see astrosurf.com). A prism is placed in front of a grating to align the 1st order with the chief ray of the system and reduce the coma for each wavelength in the spectrum.

on a CMOS camera. A beamsplitter was placed just before the pupil mask to send 50% of the light to another CMOS camera which observes the 4-segment PSF at any desired bandwidth. The location of the GRISM in the converging beam determines the angular size of the spectrum on the detector. Here the GRISM was placed ~100 mm away from the detector, so the number of periods across the pupil was $\alpha \sim 500$, creating a large spectrum on the detector with high spectral resolution ($R = \alpha$).

When the 1.0-inch mirror segment is not co-phased in piston, the dispersed fringes clearly show a barber pole shape to indicate that the segments are not co-phased. The science camera, on the other hand, might show a constructively interfering PSF if the piston OPD is a multiple of the observed wavelength, convincing the observer to think that the segments are co-phased. Hence, the dispersed fringes are a powerful tool for disentangling piston errors between two pupil segments. When the two segments are co-phased (on the white light fringe and 0 OPD), the dispersed fringes show a clear straight line spectrum (all colors have the same phase, which can only happen at 0 OPD). The science PSF may also be viewed to help fine tune the piston to achieve a constructively interfering PSF and ensure that we are
Figure 4.18: The optical setup of the GRISM. A 2-segment pupil mask is placed in the converging beam to block out the top and bottom segments and only pass the left and right segments towards the GRISM, which disperses the light to create broadband dispersed fringes in the focal plane. A beamsplitter is placed just before the pupil mask to send 50% of the light to a science camera to observe the full 4-segment PSF at any desired bandwidth.

These experiments from the GMT proto-testbed provided a fundamental understanding of segment co-phasing with a dispersed fringe sensor (via a GRISM) that helped build a foundation for the next phase of the testbed—the “prototype High Contrast Adaptive optics phasing Testbed” (p-HCAT). The next chapter discusses the design, build, and experiments that were performed with p-HCAT, which utilized the holey mirror from the GMT proto-testbed to create a GMT simulator for feeding the existing AO instrument, MagAO-X (introduced in Chapter 3). A new dispersed fringe sensor optic optimized for the GMT pupil was developed for p-HCAT, which will also be discussed in the next chapter.
The prototype High Contrast Adaptive Optics phasing Testbed (p-HCAT) was the start of a risk-reduction program that was contracted to the University of Arizona to build a phasing testbed that would demonstrate a working prototype of a dispersed fringe sensor and a PyWFS with a real GMT simulator (Hedglen et al., 2022). P-HCAT was the first stage of the HCAT project which adopted the holey mirror from the GMT proto-testbed in Chapter 4 to create a simple four segment M1 GMT simulator that only has one variable piston, tip, and tilt segment. The testbed’s design is an upgraded version of the GMT proto-testbed with custom triplet lenses to create a diffraction-limited achromatic beam that feeds light into MagAO-X (introduced in Chapter 3). P-HCAT allowed us to rapidly test the MagAO-X system’s AO hardware (DM, PyWFS, etc.) to test a PyWFS’s sensitivity to segment piston and develop a new piston sensing optic called the “Holographic Dispersed Fringe Sensor” (HDFS) to inform the GMT of the HDFS as a possible choice for a second channel piston phasing sensor. P-HCAT also helped pave the way for the full HCAT build (see Chapter 6), which will include all seven M1 GMT segments with six variable piston, tip, and tilt elements (Hedglen et al., 2022; Close et al., 2022; Kautz et al., 2022). This chapter discusses the optical design of p-HCAT, introduces the HDFS, and discusses the results obtained from the experiments performed with MagAO-X and a prototype HDFS.

5.1 Optical Design

The optical design of p-HCAT is shown in Figure 5.1 while Figure 5.2 shows the testbed “as built” in the lab. The design consists of two pupil relays: a commercial doublet lens relay and a custom triplet lens relay. In the first relay, a flat plane wave
Figure 5.1: The optical design of p-HCAT. A four-segment GMT pupil mask is re-imaged onto the surface of the "holey" mirror where piston errors are introduced to one of the four segments with a PI S-325 piezo (tip/tilt and piston) actuator.

Figure 5.2: p-HCAT as built in the lab. Light exits through a hole in the wall and into MagAO-X in the adjacent lab. Optionally, a fold mirror may be placed after custom triplet #2 for internal testing when needed.
Figure 5.3: (a) The Holey Mirror front view with the 1.0-inch mirror segment co-phased. (b) The Holey Mirror rear view. A 1.0-inch mirror is glued to a PI S-325 actuator which protrudes through a hole in the 3.0-inch mirror exactly over one of the re-imaged GMT segments. The piezo actuator has 30 μm mechanical piston range and 5 mrad mechanical tip/tilt range.

was simulated using a single-mode fiber point source and a \( f = 1,000 \) mm commercial doublet. A four-segment stainless steel etched GMT pupil mask was placed in the collimated beam to define the entrance pupil of the system while a \( f = 250 \) mm commercial doublet lens creates the first focal plane image. The size of the pupil mask and the focal length of the imaging lens were chosen to create an F/# of 11.24 as required for feeding MagAO-X. In this manner we effectively fill MagAO-X’s input beam and we switch from the Magellan-Clay to the GMT. In the second pupil relay, a magnified image of the pupil mask is formed on the surface of a 3.0-inch mirror with a hole cut into it (i.e., “the Holey Mirror,” see Figure 5.3). The holey mirror was adopted from the GMT proto-testbed (see Section 4.2) and modified to add a PI S-325 piezo-electric actuator to the 1.0-inch mirror that pokes through the hole in the 3.0-inch mirror exactly around one of the re-imaged GMT segments. This gave one of the GMT segments the ability to piston (30 μm mechanical range), tip, and tilt (5 mrad mechanical tip/tilt range), while the other three segments are fixed.
in piston on the flat mirror. Light is then refocused to feed light at F/11.24 into MagAO-X.

5.1.1 Custom Triplet Lenses

The MagAO-X instrument currently operates from 0.60 \( \mu \)m – 1.10 \( \mu \)m, so p-HCAT was designed to create an achromatic beam within this wavelength range using a combination of commercial doublet lenses and custom triplet lenses. The two custom triplet lenses in the second pupil relay were designed to counter the chromatic aberration from the two commercial doublet lens relay in the first pupil relay. This technique avoided the need for four custom triplet lenses and reduced the overall project cost. Figure 5.4a shows the result of adding the chromatic focal shift plots from the first commercial lens relay with the new custom triplet lens relay. Since the custom triplets were designed to have the opposite focal shift curve of the commercial lens relay, adding the focal shift curves together results in a final effective focal shift.

\[
\text{Focal Shift (mm)} \quad 0.0 \quad 0.1 \quad 0.2 \quad 0.3
\]

\[
\text{Wavelength (nm)} \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1.0 \quad 1.1
\]

\[
\text{Commercial Relay} \quad \text{Custom Relay} \quad \text{Total}
\]

Figure 5.4: (a) Chromatic focal shift plots from the first pupil lens relay (blue) and the custom triplet lens relay (red). The total resulting focal shift is shown in purple. (b) The final focal shift plot for p-HCAT with the custom triplets optimized for minimizing wavefront error. The focal shift is still considered diffraction-limited from 0.6 – 1.1 \( \mu \)m. The diffraction-limited depth of focus is shown with the dashed lines and is given by \( \pm 2\lambda F^2 \).
shift curve that is minimized (diffraction-limited from 0.60\,\mu m – 1.10\,\mu m). Figure 5.4b shows the final optimization of the triplet lens design (to minimize the total wavefront error rather than the chromatic focal shift). The focal shift changed slightly, however the final chromatic focal shift is still considered diffraction-limited from 0.60\,\mu m – 1.10\,\mu m. Each lens consists of the same design with an effective focal length of 750\,mm and an outer diameter of 3.50-inches. All surfaces are spherical with $< \lambda/10$ PV surface irregularity at 633\,nm. Table 5.1 shows the custom triplet lens prescription while Figure 5.5 shows a cross section of the lens design and an image of the finished lens on the testbed.

Table 5.1: P-HCAT Custom Triplet Lens Prescription.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Radius [mm]</th>
<th>Center Thickness [mm]</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200.839</td>
<td>6.136</td>
<td>S-BSM81</td>
</tr>
<tr>
<td>2</td>
<td>91.344</td>
<td>28.430</td>
<td>S-BSM2</td>
</tr>
<tr>
<td>3</td>
<td>-238.128</td>
<td>6.367</td>
<td>S-NBH5</td>
</tr>
<tr>
<td>4</td>
<td>460.985</td>
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</table>

Figure 5.5: (a) A cross-section of the custom triplet lens design. The orientation follows Table 5.1 such that surfaces 1 – 4 are from left to right. (b) The fabricated custom triplet lens mounted on the testbed. The triplet lens was designed to have an effective focal length of 750\,mm and an outer diameter of 3.50-inches.
5.2 Standalone Mode

To view the PSF in the final focal plane, an optional fold mirror may be placed after the final triplet lens to fold the light back onto the testbed (see Figure 5.6). This is our “standalone mode” for initial alignment/phasing of the testbed without MagAO-X if needed. This setup was adopted from the proto-testbed (see Section 4.4.3), in which a 50/50 beamsplitter splits the light into a science channel and a phasing channel. The phasing channel consists of a mask to block out the top and bottom GMT segments, only allowing the light from the piezo segment and the central obscuration segment through. The GRISM then disperses the light to create dispersed fringes in the focal plane for coarse piston phasing of the GMT segments.

Figure 5.6: p-HCAT standalone mode. A fold mirror may be placed after the last custom triplet lens to fold light back onto the testbed for internal testing without MagAO-X. A 50/50 beamsplitter splits the light into a science channel to view the 4-segment PSF and a phasing channel to observe the dispersed fringes. In the phasing channel, a pupil mask blocks out the top and bottom segments to only allow two segments to pass through to a GRISM which disperses the light to create dispersed fringes on the phasing camera.
The PSF from the science camera is shown for zero piston error (co-phased) at 925 nm while the phasing camera shows an example of dispersed fringes with several microns of piston error. With this much piston error, the dispersed fringes appear as a “barber pole” since the phase wraps many times from 600 nm to 1000 nm. Therefore, dispersed fringe sensors have the ability to linearly handle many $2\pi$ phase wraps, making dispersed fringe sensors powerful piston sensors with large dynamic ranges.

5.3 Error Budget and Strehl Ratio Measurements

Each optical surface in p-HCAT was measured using a Zygo® interferometer to estimate the total static wavefront error, and from that, the Strehl ratio of the system in the final focal plane. Table 5.2 shows a summary of the measured RMS wavefront errors for each optical element and the total root sum squared (RSS) wavefront error. From these measurements we estimated the Strehl ratio at 925 nm to be 78%. The Holey Mirror dominates the total error due to slight warping of the mirror at the edge of the post-polished cored hole (see Figure 4.4).

The Strehl ratio was then measured on the testbed, where we confirmed it fell

<table>
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<th>Optical Element</th>
<th>RMS Wavefront Error [nm]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Custom Triplet 1</td>
<td>11</td>
<td>as measured by Optimax</td>
</tr>
<tr>
<td>Fold Mirror 3</td>
<td>22.8</td>
<td>as measured on Zygo</td>
</tr>
<tr>
<td>Holey Mirror</td>
<td>59.5</td>
<td>as measured on Zygo</td>
</tr>
<tr>
<td>Fold Mirror 5</td>
<td>18.9</td>
<td>as measured on Zygo</td>
</tr>
<tr>
<td>Custom Triplet 2</td>
<td>11</td>
<td>as measured by Optimax</td>
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<td>as measured on Zygo</td>
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<td>root sum squared</td>
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</tbody>
</table>
within our error budget. The Strehl ratio was calculated by comparing a simulated reference PSF with the PSF measured in the final focal plane. The equation for calculating the Strehl ratio is given by

\[
SR = \frac{I_m/F_m}{I_0/F_0},
\]

where \(I_m\) is the peak pixel value of the measured PSF, \(F_m\) is the sum of the pixels in the measured PSF, \(I_0\) is the peak pixel value of the simulated reference PSF, and \(F_0\) is the sum of the pixels in the reference PSF. By taking the ratio of the peak pixel values normalized by the image flux (sum of pixels), an accurate measurement of the Strehl ratio is made. A non-linear least squares fit was used to fit a 2D Gaussian to the data to find the fitted peaks while the central 80 pixels were used to calculate the flux of the images. Figure 5.7 shows the radial profiles of the fitted PSF data. We measured a Strehl ratio of 84% from this data, which matches the estimations from the error budget. From these results we concluded that the p-HCAT optical quality was sufficient enough to produce coherent images in the final focal plane as needed for performing phasing tests with MagAO-X.

Figure 5.7: (a) The simulated reference PSF at \(\lambda_c = 925\) nm, 25 nm bandwidth. (b) The science PSF (final focal plane of p-HCAT) at \(\lambda_c = 925\) nm, 25 nm bandwidth. (c) The fitted radial profiles of the measured PSFs. The reference PSF is shown in blue while the output PSF is shown in green. The Strehl ratio was calculated to be 84% from this data.
5.4 Feeding MagAO-X

To feed light into MagAO-X, we send light through a hole in the wall into the adjacent lab, where MagAO-X is located (see Figure 5.8). Both the MagAO-X instrument and p-HCAT were built on floating optical tables, but the MagAO-X instrument has a TMC\textsuperscript{TM} PEPS-II closed-loop feedback system to lock the height of the floating table. This allows us to maintain a consistent alignment of the two tables from a day-to-day basis.

Figure 5.8: A solid model rendering of p-HCAT feeding MagAO-X. Light travels through a hole in the wall (not shown here) and into MagAO-X. Both systems are built on floating optical tables but MagAO-X has a TMC\textsuperscript{TM} PEPS-II closed-loop feedback system to maintain a consistent alignment between the two tables.
5.5 The Holographic Dispersed Fringe Sensor

The HDFS is a novel phase sensor that was designed as a second piston sensing channel for HCAT to sense any $2\pi$ phase ambiguities from MagAO-X’s PyWFS (see Haffert et al., 2022). The HDFS works by utilizing the concept of diffraction gratings to interfere pairs of GMT segments and create a series of dispersed fringes in the focal plane that inform the phase of each GMT segment.

5.5.1 HDFS Prototype

The first prototype of the HDFS was fabricated by BEAM Co. for p-HCAT. This prototype utilized liquid crystal geometric phase technology (see Escuti et al., 2016) to create a multiplexed continuous grating pattern that was optimized for the p-HCAT four-segment pupil geometry (see Figure 5.9). The HDFS is a single optic that can be placed in the pupil plane of an optical system. Therefore, the HDFS prototype was designed to be placed in one of MagAO-X’s pupil plane wheels (as shown in Figure 3.2) to create dispersed fringes on the science cameras. Figure 5.9 shows the designed HDFS phase pattern with the p-HCAT GMT pupil overlayed on top. A lab image of the p-HCAT pupil and a lab image of the PSF is also shown with the HDFS optic placed in the beam.

Figure 5.10 shows closed-loop images of the HDFS PSF and the science PSF (in log stretch) taken simultaneously with the MagAO-X science cameras. These were some of the calibration images that were used as a template for reconstructing the piston error (See Section 5.6.4 and Haffert et al. (2022)). The HDFS was operated on one science camera in broadband light ($0.60 \mu m - 0.90 \mu m$) while the other science camera was used to view the PSF in one astronomical band ($\lambda_c = 875 \text{ nm}$, $50 \text{ nm}$ bandwidth). The left column of Figure 5.10 shows $7.0 \mu m$ ($16\pi$ phase) of piston wavefront error introduced by the p-HCAT piezo segment, the middle column shows $3.0 \mu m$ ($7\pi$ phase) of piston error, and the right column shows $0.0 \mu m$ ($0\pi$ phase) of piston error (on the white light fringe of the system when co-phased). The Strehl ratio of the co-phased PSF with $0.0 \mu m$ piston error was measured to be $\sim 70\%$ at
Figure 5.9: (a) The HDFS phase pattern design with the p-HCAT pupil. The piezo segment is on the right hand side, so the holographic phase is a multiplex of cosine patterns to interfere this segment with the other three GMT segments. In this fashion, the differential phase between the three pairs of segments can be simultaneously measured (from 6 dispersed fringes) and solved for the GMT’s piston. (b) A lab image of the pupil with the HDFS placed in the pupil wheel as seen by the MagAO-X pupil viewer. (c) A lab image of the PSF in broadband light (0.60 µm – 0.90 µm) with the piezo segment co-phased (zero piston) as seen by the MagAO-X science camera.

875 nm while the PSF with 7.0 µm wavefront error was measured to be $\sim 60\%$ at 875 nm. These narrowband PSFs look very similar because the intensity wraps by $2\pi$ per $1\lambda$ of piston wavefront error.

On the other hand, the HDFS PSFs have differences in each image that are qualitatively obvious. To see how this works: the middle panel shows HDFS fringes with three maxima from 0.60 µm – 0.90 µm. This can only occur if the piston error at 0.60 µm is $5\lambda$ OPD, the piston error at 0.75 µm is $4\lambda$ OPD, and the piston error at 0.90 µm is $\sim 3\lambda$ OPD. Each of these phase errors correlate to an OPD of 3.0 µm, which is in perfect agreement with the piston OPD that was applied by the piezo actuator. Hence, this confirms that the HDFS fringes are measuring the exact piston OPD error. This illustrates that piston sensing with only a PSF is nearly impossible while piston sensing with the HDFS is much more deterministic and linear.
Figure 5.10: Closed-loop images of the HDFS PSF and science PSF taken simultaneously with the MagAO-X science cameras. These images are part of a series of calibration images that were taken for reconstructing the piston error. (Top row): A log stretch of the HDFS PSF in broadband light (0.60 µm – 0.90 µm) with (a) 7.0 µm (16π phase) of piston wavefront error introduced by the p-HCAT piston segment, (b) 3.0 µm (7π phase) of piston wavefront error, and (c) 0.0 µm (0π phase) of piston wavefront error. (Bottom row): A log stretch of the science PSF viewed on a separate science camera at λc = 875 nm, 50 nm bandwidth with (d) 7.0 µm of piston wavefront error introduced by the p-HCAT piston segment, (e) 3.0 µm of piston wavefront error, and (f) 0.0 µm of piston wavefront error. The PSFs look very similar with 7.0 µm and 0.0 µm piston error because the intensity wraps by 2π for each wave of piston wavefront error. Therefore, it is difficult to tell that the Strehl ratio of the PSF in (d) (~60% at 875 nm) is lower than the co-phased PSF in (f) (~70% at 875 nm).
5.6 Results of Closed-Loop Piston Control with p-HCAT

It is well known that the sensitivity of a PyWFS can be adjusted by modulating the focal plane around the tip of the pyramid via a tip/tilt mirror in a pupil plane (Esposito et al., 2000). Zero modulation results in the highest sensitivity and smallest dynamic range, while modulation results in less sensitivity and larger dynamic range. Modulation is typically needed for an AO system to close the loop on-sky since the wavefront aberration of a seeing-limited source exceeds the dynamic range of an unmodulated PyWFS. It has also been shown that a PyWFS’s response to differential piston error from the GMT segments is stronger at zero modulation and weaker as modulation increases (van Dam et al., 2012). For these reasons it is difficult to discern whether or not a PyWFS can truly measure and correct differential piston errors from the GMT segments in the presence of atmospheric turbulence without testing it with real optics and hardware in a lab environment. Here I show the first results of closed-loop piston control with p-HCAT and MagAO-X’s PyWFS with and without turbulence for several different modulation radii. I also discuss the results of closed-loop piston control with the novel HDFS as a second piston sensing channel (also see Haffert et al., 2022).

5.6.1 Calibrating the AO system

One challenge that was faced during the calibration of the AO system with p-HCAT was the low-order bench turbulence from the p-HCAT lab. Special care was taken to calibrate MagAO-X’s PyWFS under these circumstances with the presence of a time varying disturbance. The interaction matrix was determined from a long time series of random Gaussian probes. At each step, all modes were perturbed by random Gaussian noise, and at each subsequent step the opposite random probe was applied. The difference measurement for a single step is given by

\[ s_i = 2A p_i + \Delta r_i, \]

(5.2)

where \( s_i \) is the measurement at step \( i \) and \( p_i \) is the vector containing the modal coefficients of probe \( i \). The interaction matrix \( A \) converts the modal coefficients into
a wavefront sensor measurement. The low-order background signal changes between the positive and negative probe which introduces an offset in the measurement of $\Delta r_i$. The background signal can be averaged out over time if enough probes are used and the difference in the background signal is distributed as $\Delta r_i \sim \mathcal{N}(0, \Sigma)$, where $\Sigma$ is the covariance of the background process. The optimal least squares solution is given by

$$A = \frac{1}{2} S \Sigma^{-1} P^T (P \Sigma^{-1} P^T)^{-1}.$$  \hspace{1cm} (5.3)

Here, all measurements $s_i$ and probes $p_i$ have been collected and rearranged into matrices $S$ and $P$. Instead of the optimal estimator, we used the regularized normal least-squares solution,

$$A = \frac{1}{2} S P^T (P P^T + \mu I)^{-1},$$  \hspace{1cm} (5.4)

where $\mu$ is the regularization parameter. We assumed that the low-order modes are uncorrelated and identically distributed, so $\Sigma = \sigma I$. Under this assumption, and the assumption that $\mu = 0$, the normal least-squares solution is the optimal least-squares solution. While this assumption may not be completely true, it does not lead to a bad interaction matrix in practice. However, it may be possible to increase the quality of the interaction matrix if knowledge of the disturbance is added.

The modes that were calibrated for the interaction matrix were tip, tilt, the first 500 Fourier modes of the MagAO-X tweeter DM and the piston mode of the PI stage from the p-HCAT Holey Mirror. This led to a total of 503 calibrated modes. The inclusion of the piston mode as a separate mode from the PI stage during the calibration of all Fourier modes was crucial since the 2k tweeter DM is capable of reproducing piston with enough Fourier modes. Hence, if the piston mode was not explicitly added as a separate mode, it would have been corrected by the Fourier modes of the tweeter and this would not have been a valid test of piston control with a segmented telescope.
5.6.2 PyWFS: No Turbulence

First we tested the PyWFS’s sensitivity to piston with no turbulence at different modulation radii by inserting $\lambda/5$ OPD steps of piston jumps with the piezo segment from p-HCAT, measuring and controlling the induced piston errors with the PyWFS in closed-loop, and plotting the residual piston error for multiple waves of injected piston. We repeated this test multiple times to calculate the precision of the PyWFS measurements. Figure 5.11 shows the plots of the residual piston error as a function of input piston for $0\lambda/D$, $1\lambda/D$, $2\lambda/D$, $3\lambda/D$, and $5\lambda/D$ modulation.

Ideally, the residual piston error would be constant at zero for all waves of input piston error. However, since the PyWFS signal in response to wavefront phase error is sinusoidal in nature (Esposito et al., 2000), the PyWFS signal wraps by $2\pi$ as the injected piston approaches $\lambda/2$ OPD. As a result, the PyWFS finds that the phase error is “just as good” at $1\lambda$ OPD as it is at $0\lambda$ OPD, which ultimately limits the PyWFS’s dynamic range to $\pm\lambda/2$ OPD. We also found that the “bench seeing” from the p-HCAT lab created enough residual wavefront error to make the PyWFS jump by $2\pi$ earlier than expected in some cases. This can be seen by the difference in the number of data points between consecutive $2\pi$ phase jumps in the left plots of Figure 5.11. The plots on the right show the histograms of the wrapped residuals, which indicate that the precision of the reconstructed piston error was within $12 – 33$ nm RMS for $0\lambda/D – 5\lambda/D$ modulation.

Figure 5.12 shows the measured PyWFS slope response for each modulation radius with $\pm 100$ nm of piston error (wavefront). As the modulation increases, the PyWFS becomes less sensitive to piston, however we were still able to control piston up to $5\lambda/D$ modulation. These results were better than predicted, but at $5\lambda/D$ we were more sensitive to low flux conditions. From these tests we can conclude that the PyWFS has the ability to measure and correct piston errors from $0\lambda/D – 5\lambda/D$ modulation to within $12 – 33$ nm RMS with no turbulence, but has a limited dynamic range ($\pm\lambda/2$ piston wavefront error).
Figure 5.11: The PyWFS residual piston error as a function of input piston for a variety of modulation radii. We injected consistent steps of $\lambda/5$ OPD piston with the piezo segment from p-HCAT while closed-loop with the PyWFS and plotted the residual piston error as corrected by the PyWFS. We can see that the PyWFS wraps by $2\pi$ for all modulation radii, limiting the dynamic range of the PyWFS to $\pm \lambda/2$. However, the precision of the piston corrections was quite good (less than 33 nm RMS for all modulation radii).
Figure 5.12: The PyWFS piston slope response as a function of modulation radius with $\pm 100$ nm piston (wavefront). The top row shows the $x$ slope and the bottom row shows the $y$ slope. As the modulation radius approaches $5 \lambda / D$, the PyWFS’s sensitivity to piston decreases.

5.6.3 PyWFS: With Turbulence

The next stage of our tests was to inject a Kolmogorov turbulence phase screen with MagAO-X’s 2,040 actuator tweeter DM and close the loop with the PyWFS and the piezo segment. To make sure these tests were accurate, the response matrix was updated and tested successfully without turbulence first. We applied two different turbulence phase screens which correspond to 0.6 arcsec (median seeing conditions at the GMT site; see Thomas-Osip et al., 2008) and 1.2 arcsec seeing at 500 nm, then we closed the loop starting with zero piston error introduced by the piezo segment. Figure 5.13 shows the reconstructed piston error as a function of time for $2 \lambda / D$ and $3 \lambda / D$ modulation with 0.6 arcsec seeing and 1.2 arcsec seeing. For each case, the PyWFS quickly detected piston phase errors outside of its dynamic range and continuously “jumped” by $2\pi$. We performed this test for multiple iterations and different modulation radii, but we could not find any value of modulation ($0 \lambda / D$, $1 \lambda / D$, $2 \lambda / D$, $3 \lambda / D$, or $5 \lambda / D$) or gain value of the PyWFS that allowed a stable piston control for more than one minute. The results were the same: the PyWFS continuously “ran away” in piston and failed to close the loop with the piezo segment.

One reason why we may have experienced this phenomenon of the PyWFS “running away” in piston due to atmospheric turbulence is explained in detail by Bertrou-
Figure 5.13: The PyWFS residual piston error as a function of time for $2\lambda/D$ and $3\lambda/D$ modulation with 0.6 arcsec seeing and 1.2 arcsec seeing. The piston error introduced with the p-HCAT piezo segment started at zero piston error for each iteration and the PyWFS always failed to keep the segment phased. For each case, the PyWFS continuously “ran away” in piston, likely due to non-linear cross talk between modes and poor pixel sampling of the gaps between segments on the PyWFS detector. The loop was more stable with $3\lambda/D$ modulation than $2\lambda/D$ modulation because the PyWFS’s sensitivity decreases and the linearity increases with modulation, reducing the non-linear cross-talk effect.

Cantou et al. (2022). For a segmented aperture like the GMT, TMT or the ELT, which has $\sim 50 \text{cm}$ gaps that span multiple atmospheric coherence lengths in visible wavelengths, there will be a difference in the average phase value of the turbulence across each segment, referred to as a “petal.” These large gaps between segments will cause a discontinuity in the fitted wavefront data that a PyWFS will see as a differential piston error that needs to be corrected. Bertrou-Cantou et al. show
that when these petal modes are incorporated into the interaction matrix of an AO system, the PyWFS will experience a heavy non-linear cross-talk between the petal modes and high-order modes that is dependent on turbulence conditions, making it very hard to calibrate with optical gain (OG) compensation. In theory, a high fidelity OG compensation method could be used to resolve this issue, but it is very likely that there will be some error in OG measurements, especially in harsh seeing conditions (∼1 arcsec) that will push the PyWFS out of its minimal dynamic range and cause the PyWFS to continuously phase wrap by $2\pi$ and “run away” in piston.

There are also practical reasons why a PyWFS is difficult to use in turbulence. Any light that leaks (or scatters) in the gaps between the segments on the PyWFS detector will cause the PyWFS to sense a piston error, since this is the region where the strongest piston signal appears (as shown in Figure 5.12 and van Dam et al., 2012). For this reason it is important to have sufficient pixel sampling in the segment gaps. Figure 5.14 shows an image of the MagAO-X PyWFS signal with the p-HCAT pupil. We have no more than ∼1 totally isolated EMCCD pixels between the segment gaps, so any light that leaks into this region due to CCD charge diffusion (from bright high-order wavefront slopes on either side of the gap) or split frame transfer bleed, etc. may convince the PyWFS that there is a piston signal and cause the PyWFS to “run away” in piston.

5.6.4 HDFS: No Turbulence

There is a clear need for a second piston sensing channel that can either catch $2\pi$ phase jumps from the PyWFS or take complete control over piston sensing. We have explored the use of the novel HDFS as a second piston channel for this purpose. First we tested the HDFS with no turbulence and successfully demonstrated closed-loop piston control with the piezo segment from p-HCAT down to 20 nm RMS (see Haffert et al., 2022, for more details on these results). The PyWFS was used purely as a slope sensor here (piston mode gain set to zero) to correct for the low-order bench seeing and stabilize the HDFS image in the focal plane. The stabilized HDFS allowed us to take a series of calibration images by driving the piezo actuator in
Figure 5.14: The MagAO-X PyWFS signal with $3\lambda/D$ modulation and full OCAM2k sampling (bin 1 240 x 240 pixels). There is zero piston error introduced by the p-HCAT piezo segment and the wavefront was flattened with the MagAO-X 2,040 actuator tweeter DM. We can see that even with no piston error and a flat wavefront, there is still some signal in the gaps. There is only 1 pixel in between each segment, so it is very easy to scatter or diffuse light into the gaps. At lower sampling, this effect will become even stronger. Unfortunately, it is impossible to achieve better GMT pupil sampling with today’s highest speed (2 kHz frame rate) EMCCD technology.

Consecutive steps and recording the images in the focal plane. The feedback gain of the piston mode needed to be set to zero for the PyWFS since the bench seeing from the p-HCAT lab created enough differential piston to make the PyWFS jump by $2\pi$ earlier than expected in closed-loop (as seen in Section 5.6.2), which interfered with the HDFS calibrations. By nullifying the piston mode from the PyWFS, we were able to close the loop without any $2\pi$ phase jumps and take a stable series of calibration images with the HDFS. These calibration images were used as a template library. We then applied a random amount of piston to offset the system and used the template library to measure and remove the applied piston in closed-loop, while the PyWFS was also operating in closed-loop. In this manner, the PyWFS and the
HDFS worked well together to control any amount of injected piston from the piezo segment (±10 µm total range) to within 20 nm RMS (Haffert et al., 2022).

5.6.5 HDFS: With Turbulence

We performed the same test while generating 0.6 arcsec seeing turbulence (median seeing conditions at the GMT site; see Thomas-Osip et al., 2008) with the MagAO-X tweeter DM. The PyWFS was used purely as a slope sensor again with the piston mode feedback gain set to zero, while the HDFS used its calibration images to close the loop with the piezo segment. Removing the piston mode from the PyWFS’s interaction matrix allowed us to completely bypass any unwanted $2\pi$ phase ambiguities experienced in Section 5.6.3 so that the HDFS could take complete control of piston. After generating turbulence with the tweeter DM and closing the loop with the PyWFS, we injected random amounts of piston for multiple iterations and closed the loop with the HDFS. In this way the HDFS was able to successfully close the loop with the piezo segment from p-HCAT for both seeing conditions to within 50 nm RMS (see Haffert et al., 2022).

There is one problem we faced during these tests that limited our results. Since the MagAO-X science cameras could only run as fast as 20 Hz, we could only run the HDFS at 20 Hz, which was too slow to keep up with the simulated atmosphere that was injected at 300 Hz. To resolve this issue, the atmosphere was “slowed down,” meaning the loop was updated at a slower rate than the simulated atmosphere. The slower the relative loop speed, the faster the effective speed of the HDFS. However, since the HDFS was calibrated at 20 Hz while the PyWFS was controlling tip/tilt at the full frame rate of the WFS detector (300 Hz) to control the bench seeing from the p-HCAT lab, residual bench seeing dominated the reconstruction errors when slowing down the atmosphere too much. Hence, a balance was found at a relative loop speed of 60 Hz, where the HDFS reached the best correction error of 50 nm RMS.

This problem could be eliminated by simply using a faster camera to allow the HDFS to keep up with the atmosphere and fix the bench seeing errors. In Haffert
et al. (2022) we demonstrate the HDFS working in closed-loop without p-HCAT (internally inside MagAO-X without any bench seeing) to reach a correction error of $\sim 5$ nm RMS. Therefore, we expect to reach this same level of precision when we incorporate a faster camera for the next phase of the HCAT project.

5.6.6 Discussion

It is clear from Section 5.6.2 and 5.6.3 that the PyWFS, although an ideal wavefront sensor for traditional AO, is not a good piston sensor for GMT, ELT, and TMT ExAO, since the PyWFS will run away in piston due to the sinusoidal nature of the PyWFS signal in response to piston and non-linear modal cross-talk between segment/petal modes and high-order modes that depends on atmospheric turbulence conditions. Also, any light that leaks between the gaps of M1 segments on the PyWFS detector due to anything other than petal modes (CCD charge diffusion from high-order wavefront slopes, split frame transfer bleed, etc.) will be enough to push the PyWFS outside of its $\pm \lambda/2$ dynamic range, causing the PyWFS to converge to multiples of $2\pi$ and continuously run away in piston. For this reason, GMT, ELT, and TMT ExAO instruments will need a second piston sensing channel (such as the HDFS) to control segment/petal modes alone while the PyWFS works purely as a slope sensor with segment/petal modes nullified from its interaction matrix.

The HDFS does not have the issue of running away in piston since it sees multiple wavelengths all at once by creating dispersed fringes in the focal plane. In addition, the HDFS utilizes each 8.4-m segment of the GMT, creating a high signal-to-noise ratio for piston sensing with faint stars. Numerical simulations show that the HDFS running at 1 kHz only needs 10% of the light from a 12th magnitude star in J and H band ($1.1\ \mu m$ – $1.8\ \mu m$) to control piston to within 50 nm RMS (see Haffert et al., 2022). The only caveat is that the HDFS needs a stable PSF in the focal plane created with an AO system. Hence, a PyWFS used purely as a slope sensor (with segment/petal modes nullified) is the perfect partner for the HDFS.
The second stage of the HCAT project is the full seven segment GMT phasing testbed. HCAT is an upgraded version of p-HCAT that simulates all seven GMT segments with six segments that can piston, tip, and tilt using PI S-325 actuators. One of the main goals of HCAT is to test a concept for the GMagAO-X “parallel DM” that splits the pupil onto seven commercially available DMs using a reflective hexagonal pyramid (Close et al., 2019; Kautz et al., 2022). Here we discuss the parallel DM concept and the current design of HCAT.

6.1 GMagAO-X and the Parallel DM

GMagAO-X is the ExAO system and exoplanet imager for GMT high-contrast NGS AO science (Males et al., 2019; Close et al., 2019). It has just passed its GMT Conceptual Design Review (CoDR) in September 2021. As motivated in Males et al. (2019) and Close et al. (2019), using wavelengths as blue as 0.70 µm is optimal to characterize low-mass, temperate exoplanets in reflected light. For an ExAO coronagraph to function at these short wavelengths, the fitting error of the wavefront correction must be minimized to an acceptable level (<50 nm RMS). Therefore, it is necessary to have a massive (>20,000 actuator) DM for any GMT/ELT/TMT ExAO system for exoplanet science. The issue is that no commercially available DM anywhere near this size exists today. Hence, we need to “parallelize” the problem into smaller parts that can be handled by commercial DMs. This is the GMagAO-X “parallel DM” concept (Close et al., 2019).

In order to create a 21,000 actuator ELT-scale ExAO tweeter DM, GMagAO-X uses a reflective hexagonal pyramid to “slice” up the GMT pupil and optically distribute each segment onto seven commercially available Boston Micromachines...
(BMC) 3,000 actuator DMs (Close et al., 2019). Figure 6.1 shows a cartoon of this concept. The parallel DM incorporates six PI S-325 actuators which act as an interferometric beam combiner to control piston ($\pm 42 \mu$m OPD) and tip/tilt ($\pm 7$ mrad) for each off-axis segment. After wavefront and co-phasing correction, a small global $0.14^\circ$ tilt will be applied to each DM to reflect light back towards a knife-edge mirror that will pick off the beam and send light through the rest of the system. This optical design yields a 100% unvignetted field of view (FOV) of 3 arcsec x 6 arcsec on-sky with the GMT and GMagAO-X.

HCAT is designed to test this concept using a reflective hexagonal pyramid (see Section 6.1.1), six PI S-325 actuators, and flat mirrors as placeholders for the seven 3k DMs. A complete mounting structure made of Invar is incorporated in the HCAT design to mount the parallel DM.

Figure 6.1: The GMagAO-X parallel DM concept. A reflective hexagonal pyramid splits the GMT pupil onto seven commercial BMC 3k DMs. PI S-325 actuators act as an interferometric beam combiner to control piston ($\pm 42 \mu$m OPD) and segment tilt ($\pm 7$ mrad) at speeds up to 100 Hz. After wavefront and co-phasing correction, the light is reflected back with a small global $0.14^\circ$ angle and picked off by a knife-edge mirror to send light through the rest of the system.
6.1.1 The Hexpyramid

A prototype of the reflective hexagonal pyramid (i.e., the “hexpyramid”) was designed by Maggie Kautz and fabricated by Rocky Mountain Instrument Co. We have received the prototype in the lab and measured its surface quality with a Zygo® interferometer. Each surface was specified to have a protected silver coating with $< \lambda/10$ PV surface irregularity. Figure 6.2 shows the hexpyramid prototype in the lab and a screenshot of the solid model design. Figure 6.3 shows the beam footprint of the off-axis segments on the hexpyramid, the mask applied for each Zygo® surface measurement, and one of the Zygo® surface measurements. All surfaces were measured to be $< \lambda/10$ PV over the clear aperture which is in line with our requirements. We can also see that the mask applied is larger than the beam footprint, so we expect the actual wavefront error that each segment will pick up will be even less than what we measured.

![Image of hexpyramid prototype and solid model design with labels](image)

Figure 6.2: (a) The hexpyramid prototype in the lab. (b) A screenshot of the solid model design. A hole in the center of the pyramid allows the on-axis GMT segment through (see Figure 6.4).
Figure 6.3: (a) The beam footprint of one of the GMT segments on the hexpyramid. (b) The mask applied to the hexpyramid for the Zygo® measurements. (c) The Zygo® measurements of one of the hexpyramid faces with piston, tip, and tilt removed. All faces of the hexpyramid were measured to be $<\lambda/10$ PV surface irregularity within the clear aperture, which is in line with our specifications. We also note that the mask applied for these measurements is larger than the actual beam footprint, so the wavefront error for each segment is expected to be even better than the Zygo® measurements.

6.1.2 Polarization Aberrations

One of the main concerns with the parallel DM concept is the possibility of introducing polarization aberrations (Breckinridge et al., 2015). Each segment reflects at a 45° angle of incidence in different directions, so if the hexagonal pyramid is coated with a metallic coating like protected silver, its complex refractive index will create a phase shift between s- and p-polarization components that is incoherent between each segment, resulting in an incoherent sum of PSFs that drastically reduces the Strehl ratio.

We have mitigated this issue by incorporating crossed fold mirrors in the parallel DM design (see Figure 6.4). By introducing a fold mirror that is perpendicular to the plane of incidence of the hexagonal pyramid, s-polarization on the first surface becomes p-polarized on the second surface, and vice versa, so the net phase of s- and p-polarization becomes equal and the polarization aberrations perfectly cancel for the chief ray and on-axis sources, while almost cancelling perfectly for off-axis
rays (see Lam and Chipman, 2015). For coronagraphic systems like GMagAO-X that have a small field of view, this technique is sufficient. A polarization raytrace was performed in Zemax and Python to confirm this theory.

Zemax Simulations

The parallel DM was modeled in Zemax as a non-sequential component with protected silver coatings applied to each mirror surface. Collimated light rays enter into the parallel DM, reflecting through the system in double pass, where a paraxial imaging lens was introduced to create a perfectly achromatic PSF in the focal plane. An unpolarized light source was simulated using two orthogonal linearly polarized states to keep track of the polarization state through the system and calculate the resulting PSF with polarization effects included. Figure 6.5 shows a comparison
of the Zemax results with and without crossed fold mirrors incorporated into the parallel DM. The resulting Strehl ratio as a function of wavelength is shown for each case. Without crossed fold mirrors, the Strehl ratio varies drastically with wavelength, but with crossed fold mirrors, the Strehl ratio stays consistent across all wavelengths.

Figure 6.5: Zemax polarization simulation of the parallel DM (a) without crossed fold mirrors and (b) with crossed fold mirrors. (c) Without the crossed fold mirrors, the Strehl ratio varies drastically with wavelength. (d) With crossed fold mirrors, the Strehl ratio is consistent across all wavelengths.
Python Simulations

A polarization raytrace was also performed in Python to simulate the effects of polarization aberrations from the parallel DM with and without crossed fold mirrors (see Appendix A for a more detailed analysis). A simple protected silver coating (Ag + SiO₂) was applied to each surface and the Fresnel reflection coefficients for each GMT segment were calculated to generate a Jones pupil which can be used to calculate the PSF resulting from the polarization aberrations.

A Jones pupil is a 2x2 matrix that contains complex components of amplitude and phase for each point in the pupil and is given by

\[
J = \begin{bmatrix}
J_{xx}(\xi, \eta) & J_{xy}(\xi, \eta) \\
J_{yx}(\xi, \eta) & J_{yy}(\xi, \eta)
\end{bmatrix} = \begin{bmatrix}
A_{xx}(\xi, \eta)e^{i\phi_{xx}(\xi, \eta)} & A_{xy}(\xi, \eta)e^{i\phi_{xy}(\xi, \eta)} \\
A_{yx}(\xi, \eta)e^{i\phi_{yx}(\xi, \eta)} & A_{yy}(\xi, \eta)e^{i\phi_{yy}(\xi, \eta)}
\end{bmatrix}.
\] (6.1)

The \(A_{xx}\) term is the X-polarized amplitude of \(J_{xx}\) at the exit pupil resulting from an incident X-polarized field in object space. The \(\phi_{xx}\) term is the complex argument of \(J_{xx}\); the phase shift from the incident X-polarized field to the exiting X-polarized field. This is similar for the \(yy\) components. The \(yx\) components are the components of the incident X-polarized field that are coupled into the Y-polarized field. Similarly, the \(xy\) components are the components of the incident Y-polarized field coupled into the X-polarized field.

Figure 6.6 shows the Jones pupil elements for the parallel DM without crossed fold mirrors to show the amplitude and phase variations across the pupil. Ideally, the Jones pupil would be the identity matrix, for which the phase change would be independent of the polarization state. However, since the hexpyramid reflects each GMT pupil segment away in different directions, incoming unpolarized light from a star (represented by two orthogonal linearly polarized states X and Y) will have different s- and p-polarization components upon reflection depending on the face of the hexpyramid each GMT segment picks up. This causes a change in the exiting polarization state between each GMT segment that will cause the final PSF to become distorted.

To evaluate the PSF, the Amplitude Response Matrix (ARM), which is the
Figure 6.6: The Jones pupil elements at $\lambda = 650 \text{ nm}$ with no crossed fold mirrors incorporated into the parallel DM.

Figure 6.7: The resulting PSF at $\lambda = 650 \text{ nm}$ for the parallel DM without crossed fold mirrors. The Strehl ratio is 63%, resulting from the polarization aberrations. The reference PSF with no polarization effects included is shown on the right for comparison.

Fourier transform of the Jones pupil elements, can be calculated. The ARM is given by

$$\text{ARM} = \begin{bmatrix} \mathcal{F}[J_{xx}(\xi, \eta)] & \mathcal{F}[J_{xy}(\xi, \eta)] \\ \mathcal{F}[J_{yx}(\xi, \eta)] & \mathcal{F}[J_{yy}(\xi, \eta)] \end{bmatrix},$$

(6.2)
and the intensity of the PSF is given by the sum of the ARM components,

\[ I = I_x + I_y = |ARM_{xx}|^2 + |ARM_{yx}|^2 + |ARM_{xy}|^2 + |ARM_{yy}|^2. \] (6.3)

The resulting PSF at \( \lambda = 650 \text{ nm} \) for the parallel DM is shown in Figure 6.7. The Strehl ratio is 63\%, purely due to the polarization aberrations. The reference PSF (no polarization) is shown to the right for comparison.

When crossed fold mirrors are included in the parallel DM, the polarization aberrations cancel. Figure 6.8 shows the Jones pupil elements for the parallel DM with crossed fold mirrors. Now, the off-diagonal elements have zero amplitude and the result is the identity matrix. Hence, the resulting PSF, shown in Figure 6.9, is polarization aberration free. The Strehl ratio is plotted as a function of wavelength in Figure 6.10 to show the full effect of polarization aberrations in the parallel DM with and without crossed fold mirrors. Without crossed fold mirrors, the Strehl ratio varies drastically with wavelength, but with crossed fold mirrors, the Strehl ratio stays consistent across all wavelengths, matching the results from the Zemax simulations. The difference between the Zemax and Python simulations is due to the protected silver coating that was used. In the Zemax simulations, Zemax’s

![Image](image_url)

Figure 6.8: The Jones pupil elements at \( \lambda = 650 \text{ nm} \) with crossed fold mirrors included in the parallel DM. The amplitude of the off-diagonal components is now zero and the result is the identity matrix.
default two-layer protected silver coating was used, while in the Python simulations, a simple single-layer protected silver coating was used.

Figure 6.9: The resulting PSF at $\lambda = 650\,\text{nm}$ for the parallel DM with crossed fold mirrors included. The polarization aberrations cancel and the Strehl ratio is 100%. The reference PSF with no polarization effects included is shown on the right for comparison.

Figure 6.10: The Strehl ratio versus wavelength for the Python polarization raytrace simulations of the parallel DM without (left) and with (right) crossed fold mirrors included in the parallel DM.

We have found that the crossed fold mirror technique is sufficient for mitigating the polarization aberrations of the parallel DM. However, each mirror surface must have the same exact coating in order for the polarization aberrations to cancel. For
this reason, all the fold mirrors for the HCAT parallel DM concept have been coated by the same vendor as the hexpyramid to ensure that they have the same coating.

6.1.3 The Optomechanical Design of the Parallel DM

Since the parallel DM works as an interferometric beam combiner, the optics must be mounted in a mechanically and thermally stable fashion to prevent unwanted changes in OPD between the GMT segments due to temperature changes or mechanical vibrations. Therefore, a robust mechanical structure made of Invar 36 was designed for GMagAO-X to hold the seven BMC 3k DMs, the hexpyramid, and the six PI S-325 actuators. Invar 36 is an alloy that consists of 64% iron and 36% nickel in order to create a material that has a low coefficient of thermal expansion.

Figure 6.11: The solid model design of the parallel DM mount. A rigid structure made of invar 36 was designed to provide the necessary mechanical and thermal stability required for keeping the GMT segments phased at the nanometer level.
Figure 6.12: An inside look at the parallel DM mount. Six plates mount in between the walls to hold the 3k DMs and actuators in place.

Figure 6.13: A closer look at the custom hexpyramid mount and the on-axis 3k DM. The hexpyramid mount was designed to hold the hexpyramid on the front with two crossed fold mirrors on the inside which fold the light over to the on-axis 3k DM. The hexpyramid and the two crossed fold mirrors are held by gluement with alignment pins that help to glue them into place.
(Guillaume, 1904). The design of the invar structure consists of two 1.0” thick walls and six “blades” that mount in between the walls to mount the DMs and PI S-325 actuators in a crossed fold mirror configuration. Figure 6.11 shows the solid model design of the structure, while Figure 6.12 and Figure 6.13 show detailed views of the inside.

A custom mount was designed to hold the hexpyramid and two crossed fold mirrors in place via glue (see Figure 6.13). Two right angle brackets mount the custom mount to the back wall to hold these optics rigidly in place. The shape of the custom mount was also designed to accommodate the space needed for the on-axis 3k DM, which mounts to the back wall with a right angle bracket.

Once the GMagAO-X parallel DM mounting structure design was completed, the structure was fabricated for HCAT to test the parallel DM concept. All six PI S-

Mock DMs

Figure 6.14: An inside look at the HCAT parallel DM. The only difference between this version and the GMagAO-X version is that the BMC 3k DMs are replaced with oversized flat mirrors with a volume that mimics the BMC 3k DMs. The on-axis mock DM is an oversized 2.0” mirror mounted to a Thorlabs vertical drive tip/tilt mount. This allows us to align the central segment through the system with the tip/tilt mount, while aligning the off-axis segments with the PI S-325 actuators.
325 actuators were acquired for the HCAT parallel DM, but the BMC 3k DMs were replaced with “mock DMs” that are flat mirrors mounted to a box with a similar volume as the BMC 3k DMs (see Figure 6.14). If this concept proved successful with the testbed, then the plan is to use this structure for GMagAO-X with the real BMC 3k DMs in the future. Figure 6.15 shows the completed parallel DM as built in the lab.

### 6.2 The Optical Design of HCAT

The optical design of HCAT is shown in Figure 6.16. The design uses the same optics from p-HCAT, but with a third pupil relay added in the middle for the parallel DM. A knife-edge mirror placed near the first focal plane folds the light towards a new custom triplet lens that is designed to create an image of the pupil mask with 24 mm pupil sizes (the same size as the BMC 3k DMs). Light reflects through the parallel DM and travels backwards in a double-pass configuration, where each “DM”
Figure 6.16: The optical design of HCAT. The design consists of the same optics from p-HCAT with a third pupil relay added in the middle for the parallel DM. A knife-edge mirror is placed in the first focal plane to fold the light towards the parallel DM relay. Each DM (represented by fold flats) will be tilted slightly to allow the beam to travel back to the other side of the knife-edge mirror, which folds the light towards the rest of the optics and into MagAO-X.

(represented by fold flats) is tilted slightly to offset the beam on the other side of the knife-edge mirror and allow the beam to travel through the rest of the system and into MagAO-X. Figure 6.17 shows a solid model rendering of HCAT while Figure 6.18 shows the completed testbed as built in the lab.

### 6.2.1 Custom Triplet Lens

A new custom triplet lens was designed for the parallel DM pupil relay to create an image of the telescope pupil with 24 mm segment sizes (73 mm total pupil diameter) to match the size of the BMC 3k DMs. This required a large 4.0” diameter lens with a focal length of 820.66 mm. The custom triplet also needed to maintain an achromatic beam from 0.60 µm – 0.92 µm. Therefore, Zemax was used to design this lens. The final glasses that were used for this lens are S-NBH58 and S-LAM2.
from Ohara Corporation and H-LaF53 from CDGM. Since this lens was placed in between two already existing pupil relays, the optimization of the custom triplet lens
required a tight tolerance on the lens parameters for fabrication. Hence, this lens
was fabricated by Optimax with “high precision” tolerances. Figure 6.19 shows the
final chromatic focal shift plot and Strehl vs wavelength plot from the HCAT Zemax
model with the new custom triplet design. The plots are also shown with the real, “as
fabricated” lens data from Optimax inserted into the model to compare the results.

The average Strehl ratio is well above 90% across the entire bandwidth and the
chromatic focal shift is within the diffraction-limited range, showing a successful new
triplet design. Table 6.1 shows the custom triplet lens prescription while Figure 5.5
shows a cross section of the lens design and an image of the finished lens on the
testbed.

**Custom Triplet Design Problem**

Just before receiving the custom triplet lens assembly from Optimax, a problem
arose with the Zemax design due to an update with Zemax’s CDGM glass cata-
log that provided more accurate data for the H-LaF53 glass than previously used.
Table 6.1: HCAT Custom Triplet Lens Prescription.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Radius [mm]</th>
<th>Center Thickness [mm]</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>817.196</td>
<td>6.000</td>
<td>S-NBH58</td>
</tr>
<tr>
<td>2</td>
<td>184.153</td>
<td>34.898</td>
<td>S-LAM2</td>
</tr>
<tr>
<td>3</td>
<td>-93.246</td>
<td>6.000</td>
<td>H-LAF53</td>
</tr>
<tr>
<td>4</td>
<td>-1490.771</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.20: (a) A cross-section of the custom triplet lens design. The orientation follows Table 5.1 such that surfaces 1 – 4 are from left to right. (b) The fabricated custom triplet lens mounted on the testbed. The triplet lens was designed to have an effective focal length of 820.66 mm and an outer diameter of 4.0-inches.

Therefore, the custom triplet design no longer provided optimum results for the HCAT model since it was designed with glass data that was inaccurate. Figure 6.21 shows the dispersion diagrams (index of refraction vs. wavelength) for each glass data to show the difference. Figure 6.21 shows the resulting focal shift plot and Strehl vs. wavelength plot. The newly updated glass data was slightly different on the order of $10^{-5}$, which was enough to cause the HCAT design performance to decrease by a significant amount. Although this was an unfortunate coincidence, it
was a lesson learned that care should be taken when designing optics with glasses from CDGM, since the glass data may be inaccurate. Glasses from Ohara Corp. or Schott are known to be more reliable and are up-to-date with Zemax.

Figure 6.21: The index of refraction vs. wavelength plotted for each H-LaF53 glass data version in Zemax. The new, more accurate glass data from the Zemax update was slightly different (on the order of $10^{-5}$) than the old data, causing the HCAT Zemax design performance to change.

Figure 6.22: (a) The chromatic focal shift plot from the HCAT Zemax model with the new H-LaF53 glass data. (b) The Strehl ratio as a function of wavelength with the new glass data. The performance of the HCAT design is no longer within the diffraction-limited depth of focus and the average Strehl ratio is much lower than before.
To compensate for this issue, the second commercial doublet \( f = 250 \text{ mm} \) in the first pupil relay (see Figure 6.16) was substituted for a new lens. A custom doublet was explored for this solution, however it was discovered that another commercial Thorlabs doublet \( f = 300 \text{ mm} \) would fix most of the chromatic errors and the performance was competitive with the custom design. Therefore, the new Thorlabs doublet was acquired to replace the \( f = 250 \text{ mm} \) doublet. Since the focal length of this lens changed, a new pupil mask was also required to maintain an F/# of 11.24. Figure 6.23 shows the final chromatic focal shift plot and the Strehl vs. wavelength plot from the Zemax model with the new \( f = 300 \text{ mm} \) Thorlabs doublet. The chromatic errors from the H-LaF53 glass data problem are now fixed and the performance of the HCAT design is recovered. This is the final HCAT design as built in the lab as shown in Figure 6.18.

![Focal Shift Plot](a)

![Strehl Ratio](b)

Figure 6.23: (a) The chromatic focal shift plot from the HCAT Zemax model with the new \( f = 300 \text{ mm} \) commercial doublet. (b) The Strehl ratio as a function of wavelength with the new commercial doublet. The chromatic errors from the H-LaF53 Zemax glass data problem is now fixed and the performance of the HCAT design is recovered.
6.3 HCAT Standalone Mode

To observe the PSF in the final focal plane without MagAO-X, a fold mirror may be placed after the last $f = 750$ mm triplet to fold the light back onto the HCAT optical table. This method was also used for p-HCAT, as previously shown in Figure 5.6. A fourth pupil relay is created with two Thorlabs $f = 100$ mm doublets and the HDFS is placed in the pupil plane to view the dispersed fringes on a CMOS camera. Figure 6.24 shows a picture of the fourth pupil relay in standalone mode.

![Fold Mirror Binary HDFS Dispersed Fringes Science PSF (λ = 900 nm)](image)

Figure 6.24: HCAT standalone mode. A fourth pupil relay is added to utilize the HDFS for co-phasing the parallel DM. A CMOS camera is used to view the dispersed fringes in the focal plane. The HDFS may also be removed to view the science PSF.

An additional feature that may be used is to place a fold mirror in front of the parallel DM to bypass the parallel DM (see Figure 6.25). This feature was used to align the system before all of the PI S-325 actuators were obtained for the parallel DM. This also allowed us to test the binary HDFS for the first time with a flat wavefront.
Figure 6.25: A fold mirror may be placed in front of the parallel DM to bypass the parallel DM. A second fold mirror is then placed in the pupil plane to send light back through the system in double pass.

6.3.1 Error Budget and Strehl Ratio Measurements

Each optical surface in HCAT was measured using a Zygo® interferometer to estimate the total static wavefront error, and from that, the Strehl ratio of the system in the final focal plane. Table 6.2 shows a summary of the measured RMS wavefront errors for each optical element, which yielded a total root sum squared (RSS) wavefront error of 71.2 nm in standalone mode. From this number the Strehl ratio at any wavelength can be estimated using equation 2.11. For example, the Strehl ratio at 900 nm is estimated to be 78%.

The Strehl ratio was then measured on the testbed to compare with the error budget. The Strehl ratio was measured by comparing a simulated PSF with the PSF measured in the final focal plane by taking the ratio of the image peaks normalized by the image flux. A non-linear least squares fit was used to fit a 2D Gaussian to the data to find the fitted peaks while the central 80 pixel values were summed to calculate the flux of the images. Figure 6.26 shows the two images and the radial profiles of the fitted PSF data. A Strehl ratio of 85% was measured from this data,
Table 6.2: HCAT Standalone Wavefront Error Budget.

<table>
<thead>
<tr>
<th>Optical Element</th>
<th>RMS Wavefront Error [nm]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commercial Doublet 1</td>
<td>34</td>
<td>Estimated from 90% Strehl measurement at first focal plane</td>
</tr>
<tr>
<td>Commercial Doublet 2</td>
<td>34</td>
<td>Estimated from 90% Strehl measurement at first focal plane</td>
</tr>
<tr>
<td>Fold Mirror 1 (knife-edge)</td>
<td>6.5</td>
<td>Assumed to be λ/30 PV since it is located in focal plane</td>
</tr>
<tr>
<td>Fold Mirror 2</td>
<td>9.0</td>
<td>0.010λ RMS as measured on Zygo</td>
</tr>
<tr>
<td>HCAT Custom Triplet</td>
<td>10</td>
<td>As measured by Optimax</td>
</tr>
<tr>
<td>Fold Mirror 3</td>
<td>21.4</td>
<td>0.018λ RMS as measured on Zygo</td>
</tr>
<tr>
<td>Fold Mirror 4</td>
<td>19.0</td>
<td>0.015λ RMS as measured on Zygo</td>
</tr>
<tr>
<td>Fold Mirror 5</td>
<td>16.0</td>
<td>0.018λ RMS as measured on Zygo</td>
</tr>
<tr>
<td>HCAT Custom Triplet</td>
<td>10</td>
<td>As measured by Optimax</td>
</tr>
<tr>
<td>Fold Mirror 2</td>
<td>9.0</td>
<td>0.010λ RMS as measured on Zygo</td>
</tr>
<tr>
<td>Fold Mirror 5</td>
<td>16.0</td>
<td>0.018λ RMS as measured on Zygo</td>
</tr>
<tr>
<td>P-HCAT Triplet 1</td>
<td>11</td>
<td>As measured by Optimax</td>
</tr>
<tr>
<td>Fold Mirror 6</td>
<td>9.0</td>
<td>0.011λ RMS as measured on Zygo</td>
</tr>
<tr>
<td>Fold Mirror 7</td>
<td>7.5</td>
<td>0.006λ RMS as measured on Zygo</td>
</tr>
<tr>
<td>Fold Mirror 8</td>
<td>10.7</td>
<td>0.013λ RMS as measured on Zygo</td>
</tr>
<tr>
<td>P-HCAT Triplet 2</td>
<td>11</td>
<td>As measured by Optimax</td>
</tr>
<tr>
<td>Fold Mirror 9</td>
<td>17.8</td>
<td>0.015λ RMS as measured on Zygo</td>
</tr>
<tr>
<td>Total</td>
<td>71.2</td>
<td>RSS</td>
</tr>
</tbody>
</table>

which matched the estimations from the error budget.

6.3.2 The Binary HDFS

After the p-HCAT HDFS prototype (liquid crystal geometric phase) was tested with p-HCAT (see Section 5.5.1), it was found that a binary phase pattern would be more efficient and easier to fabricate than the continuous phase version (see Haffert et al.,
Figure 6.26: (a) The reference PSF (simulated) at \( \lambda_c = 900 \text{ nm} \), 10 nm bandwidth. (b) The output PSF (final focal plane image of HCAT) at \( \lambda_c = 900 \text{ nm} \), 10 nm bandwidth. (c) The fitted radial profiles of the measured PSFs. The reference PSF is shown in blue while the output PSF is shown in green. The Strehl ratio was calculated to be 85% from this data.

2022). The operational principle of the geometric phase and the binary phase types are the same. Therefore, a binary HDFS was designed by Dr. Sebastiaan Haffert and fabricated for the final HCAT build. The binary HDFS utilizes the concepts previously described to create a multiplexed pattern of binary gratings that are optimized for the GMT pupil geometry. This allows pairs of GMT segments to be interfered in the focal plane to create a series of dispersed fringes that inform the phase of each GMT segment. Figure 6.27 shows a simulation of the binary HDFS design with and without segment piston errors introduced. The broadband PSFs \((\lambda = 0.60 \mu\text{m} – 1.00 \mu\text{m})\) are shown to the right. The piston phase error of each segment can be directly measured from these images. Since the HDFS utilizes the GMT’s full aperture, the HDFS also takes advantage of the GMT’s full light-collecting power, increasing the signal-to-noise and hence the sensitivity of its measurements.

6.3.3 Fabricating The Binary HDFS For HCAT

The first binary HDFS was fabricated by OSC Master’s student Avalon McLeod by etching into a fused silica substrate to create the HDFS design pattern with depths that equate to \( \pi \) radians in phase (862 nm at a design wavelength of 782 nm). Figure
Figure 6.27: A simulation of the binary HDFS from Haffert et al. (2022). The design is comprised of a multiplexed binary phase grating that is optimized for the GMT pupil geometry, creating a series of dispersed fringes in the focal plane that inform the phase of each GMT segment. (Top row): the HDFS design with no piston errors. (Bottom row): the HDFS design with $\pm 4\lambda$ piston errors. The broadband PSFs ($\lambda = 0.60 \mu m–1.00 \mu m$) are shown to the right.

6.28 shows the design pattern and an image of the finished HDFS in the lab. The etch depths were verified by measuring the HDFS on a white light interferometer. Figure 6.29 shows the measurement results from the white light interferometer, which measured an etch depth of 863 nm, well within spec of the HDFS design requirements.

The binary HDFS was then mounted in the fourth pupil relay of HCAT (as shown in Figure 6.24) to view the dispersed fringes on the CMOS camera. A comparison of the simulated HDFS PSF and the lab image are shown in Figure 6.30. The
Figure 6.28: (Left): The binary HDFS design with depths of 862 nm. (Right): An image of the fabricated HDFS in the lab.

Figure 6.29: The white light interferometer measurement of the binary HDFS etch depth. The designed etch depth was 862 nm and the white light interferometer measured 863 nm, well within spec of the design requirements, thanks to the efforts of Avalon McLeod.
lab image looks strikingly similar to the simulated image, except for some minor wavefront errors that are present in the HCAT testbed (see Figure 6.23 and Table 6.2).

Figure 6.30: A comparison of the simulated binary HDFS image versus the real lab image taken with HCAT ($\lambda = 0.60 \mu m – 1.00 \mu m$). The images almost match perfectly, indicating that the binary HDFS was fabricated to within spec. There are some minor differences that are due to the HCAT testbed’s internal wavefront errors, such as the slight chromaticism in the testbed causing the short baseline fringes to squiggle.

6.4 Phasing the Parallel DM

The parallel DM utilizes six PI S-325 actuators to align the off-axis GMT segments with the central on-axis “reference” segment. However, the piezo-electric actuators only have so much piston, tip, and tilt range ($\pm 42 \mu m$ OPD piston, $\pm 7$ mrad OPD tip/tilt). Therefore, each segment needed to be aligned manually to the middle of the piezo-electronic range. This required an intensive phasing procedure which involved stepping each actuator in piston manually until we found the white light fringe. By interfering each segment’s PSF with the central on-axis GMT segment’s PSF in the focal plane, we could step through piston until fringes became visible via observation through a spectral filter that has a defined coherence length.
6.4.1 The Phasing Procedure

Three different spectral filters were acquired for the phasing procedure: a 633 nm, 1 nm BW filter which has a coherence length of ±400 µm, a 900 nm, 10 nm BW filter which has a coherence length of ±81 µm, and a 925 nm, 25 nm BW filter which has a coherence length of ±34 µm. The 1 nm BW filter had an optical density (OD) of 4, so we combined this filter with a slightly larger bandwidth filter (same central wavelength) that had a higher optical density (OD 8) to create an efficient but rather inexpensive 1 nm BW filter with high OD.

Once the central on-axis GMT segment was aligned through the system, each individual off-axis GMT segment was aligned in tip/tilt manually by shimming the mock DM and piezo actuator as needed. Some of the piezo-electronic tip/tilt range could be used in the end to precisely stack the PSFs on top of the reference PSF. At this point, if the OPD between the two segments were within the coherence length of the spectral filter, then interference fringes would be visible. However, this was never the case from the start, so the piezo actuator needed to be stepped in piston until fringes became visible.

To create a repeatable, precise method of stepping the piezo actuator in piston, an aluminum sleeve was designed to mount on the back of the actuator, acting as a “stop” to prevent the actuator from falling when the actuator is loosened from its clamp (see Figure 6.31). By placing precisely thick pieces of shim stock in between the sleeve clamp and the piezo clamp, the piezo could be moved in small precise steps in order to “hunt” for fringes. The phasing procedure is then as follows:

1. Use the 1 nm BW filter and move the PI S-325 actuator by hand in steps of 0.14 mm (equivalent to the coherence length of the filter in physical space) until you see fringes.

2. Switch to the 10 nm BW filter and check if you can still see fringes. If no fringes, go back to step 1 and try to move the piezo by hand another step until you see higher visibility fringes, then switch back to the 10 nm filter until you see fringes.
3. Switch to the 25 nm BW filter and see if you can still see fringes. If not, go back to step 2 and move the piezo electronically to figure out which direction has higher visibility fringes. Then, move the piezo by hand in that direction by \( \sim 0.030 \text{ mm} \). Repeat these steps until you can see fringes with the 25 nm filter and the white light fringe is in the middle of the piezo-electronic range.

Figure 6.32 shows an example of stacking two of the PSFs from the parallel DM GMT segments. If the OPD between the two segments is within the coherence length of the spectral filter (in this case the 925 nm, 25 nm BW filter which has a coherence length of \( \pm 81 \mu\text{m} \)), interference fringes become visible, indicating that the segments are co-phased. Then, by moving the piezo actuator in piston electronically, the zero fringe visibility point can be found on both ends of the fringe envelope to estimate where the white light fringe is, verifying that the white light fringe (zero OPD) is in the middle of the piezo actuator’s electronic range. This procedure was followed for each piezo until each one was co-phased with the reference segment and the white
light fringe was in the middle of the piezo-electronic range.

Figure 6.32: An example of two GMT segments from the HCAT testbed misaligned versus aligned. When the OPD between the segments is within the coherence length of the spectral filter (in this case $\lambda_c = 925$ nm, 25 nm BW), interference fringes become visible, indicating that the segments are co-phased.

6.4.2 The Fully Phased Parallel DM

The HDFS was used to co-phase the piston of each segment on the white light fringe and create the first co-phased seven-segment GMT PSF from the HCAT parallel DM. The final co-phased images of the HDFS and science PSF are shown in Figure 6.33, while Figure 6.34 shows the Strehl ratio measurement of the science PSF. The Strehl ratio was estimated to be $\sim 56\%$ at 925 nm. Most of the wavefront error is likely due to the astigmatism in the individual PSFs, which is unfortunately caused by the clamping of the piezo actuators, which causes stress on the flexure. Figure 6.35 shows a comparison of the unstacked and stacked PSFs for all seven GMT segments from the parallel DM at 20 ms exposure. Astigmatism can be seen in almost every PSF, which changes as a function of the clamping force. Therefore, the astigmatism was minimized as much as possible by loosening the piezo clamps, but unfortunately the astigmatism could not be removed fully in these images. It is estimated that $\sim \lambda/5$ PV of astigmatism is introduced by the piezo clamp, which can be shown by the example in Figure 6.36. An astigmatic surface with $\lambda/5$ PV of astigmatism ($\sim \lambda/2$ in double pass at an angle of incidence of $45^\circ$, as is the case in
the parallel DM) creates a PSF that looks similar to the real PSFs shown in Figure 6.35. One can imagine how combining several of these PSFs together to form the full GMT PSF will create a low quality image. Other wavefront errors present in the final stacked image may include tip, tilt, and defocus since no wavefront sensor was used to obtain these images. The tip, tilt, and defocus were all aligned by human eye in the presence of bench seeing, which has been known to slightly misalign the PSFs over time. Nonetheless, this image is a success because it is the first image
ever taken of a GMT pupil that has been split up into seven different directions and recombined to form a coherent image in the focal plane, proving the success of the parallel DM and opening the door to exoplanet imaging with GMagAO-X and the GMT.

Figure 6.35: (Left): an image of the seven GMT PSFs from the parallel DM unstacked. Astigmatism can be seen in each PSF due to the clamping of the piezo actuators. (Right): an image of the PSFs stacked and co-phased. Both images are shown in log stretch with 20 ms exposure.

Figure 6.36: (Left): a surface with $0.2\lambda$ PV of astigmatism at 633 nm. (Right): the resulting PSF at $\lambda = 925$ nm from the astigmatic surface in double pass at 45° angle of incidence ($2\sqrt{2}\cdot$ surface). The Strehl ratio of this image is $\sim 78\%$. This is roughly the amount of astigmatism that is suspected to be present in each PSF due to the clamping of the piezo actuators.
The future generation of ELTs (the GMT, TMT, and the E-ELT), being the largest telescopes ever built, have the potential to provide unmatched image resolutions and contrasts that could lead to the discovery of life outside of our solar system. However, due to the design nature of these extremely large telescope structures, there are issues that may prevent each telescope from achieving its full diffraction-limited power. This dissertation addressed the issue for the GMT and how its seven 8.4-m primary mirror segments act as seven individual telescopes that need to be aligned and co-phased to act as one single 25.4-m mirror. Temperature changes, gravity load, segment vibrations, wind buffeting, and atmospheric seeing will inevitably cause each segment to have optical path differences on the order of tens of microns, if not more. These optical path differences will distort the telescope’s image quality and prevent the telescope from reaching its full diffraction-limited capability, which is necessary for directly imaging habitable-zone exoplanets that could harbor life. Hence, directly imaging these potentially habitable exoplanets requires the GMT’s primary mirror segments to be co-phased to well within a fraction of a wavelength.

We discussed how piston wavefront errors alone will be enough to throw the GMT segments out of phase, preventing an adaptive optics system’s coronagraph from properly blocking out the light from a star and imaging an exoplanet. More importantly, these piston wavefront errors are immeasurable by most wavefront sensors, creating the need for a new piston wavefront sensor that can capture the piston error between each segment and update the adaptive optics system in real time while also correcting for atmospheric turbulence. The GMT’s initial phasing strategy involved a slow, seeing-limited dispersed fringe sensor and two different wavelength pyramid wavefront sensors to measure the piston between each GMT segment, however the uncertainty in the success of this strategy made it clear that a
phasing testbed was needed to test these strategies with real optics and AO hardware in a lab environment. This led to the development of a High Contrast Adaptive optics phasing Testbed (HCAT) for the GMT.

In Chapter 2, I introduced the concept of AO and showed why AO is more difficult at shorter wavelengths. In Chapter 3, I introduced a new extreme adaptive optics instrument, MagAO-X, which was designed for exoplanet imaging in the visible spectrum. The development of MagAO-X and its use of a pyramid wavefront sensor led to its involvement in the HCAT project to test its AO hardware for sensing piston errors from the GMT segments.

I reviewed the first development of a GMT proto-testbed in Chapter 4, which was created in anticipation of HCAT to gain quick insight into piston sensing with a GMT simulator. This testbed gave birth to the “holey mirror,” which was adapted for p-HCAT: the official prototype testbed for the HCAT project. The design and build of p-HCAT was revealed in Chapter 5, where I reported the first results of closed-loop piston control with MagAO-X and a new piston sensing optic called the “Holographic Dispersed Fringe Sensor” (HDFS). The success of p-HCAT and the novel HDFS led to the development of a new piston wavefront sensing architecture that was adopted by GMTO Corp. as their official phasing strategy for NGSAO science. Finally, the design and build of the full-scale testbed, HCAT, was reported in Chapter 6 along with the results of the first co-phased images from the parallel DM and the new design of the binary HDFS.

7.1 The Future of HCAT

In the coming months, HCAT will feed light into MagAO-X to test the PyWFS and HDFS wavefront sensing architecture for correcting segment piston errors from the GMT in the presence of simulated atmospheric turbulence via a turbulence plate. These tests will be the official proof of the parallel DM concept working in the lab, along with the development of the PyWFS and HDFS WFS algorithms to solve the GMT’s piston problem.
The MagAO-X AO response function can be well trained on the 85% Strehl GMT PSF from Figure 6.26 using the HCAT optics to bypass the parallel DM. Then, once the bypass mirror is removed to utilize the parallel DM, MagAO-X’s 2,040 actuator DM will automatically fix the segment tilt, focus, and astigmatism that was present in Figure 6.34. Such a correction should yield a GMT PSF with at least 85% Strehl and should only require $\sim 400$ nm PV (see Figure 6.36) of the DM’s 3,500 nm stroke range.

The final stage of the HCAT project (stage 3) will be to feed light from HCAT into the prototype Natural Guide Star Wavefront Sensor (NGWS-P), which will be the official phasing testbed for the NGWS system (Pinna et al., 2014). The current plans for feeding light from HCAT into NGWS-P are shown in Figure 7.1. A beamsplitter will be introduced in the MagAO-X converging F/57 beam to send light through the eyepiece port and into NGWS-P. This is an important part of the HCAT project as it will allow us to test the real GMT NGWS internal HDFS and

![Figure 7.1: The current plans to feed light into NGWS-P in 2023. The beam will be intercepted by a beamsplitter inside the MagAO-X system to send light to NGWS-P.](image-url)
PyWFS hardware and control algorithms for NGSAO wavefront sensing with the GMT. The stage 1 \text{p-HCAT} tests and stage 2 \text{HCAT} tests will help pave the way for stage 3, where a final robust NGWS piston sensor design will be well-tested and established with the goal of officially retiring the GMT high-risk item of phasing performance.

### 7.2 The Future of GMagAO-X

GMagAO-X is the designated ExAO coronagraphic instrument for the GMT (Males et al., 2019; Close et al., 2019). It is designed for a slot on the folded port of the GMT. In order for ExAO exoplanet imaging to be possible with the GMT, GMagAO-X must have a 21,000 actuator DM capable of $\geq 2$kHz correction speeds to meet the strict ExAO fitting and servo error requirement of $< 90$ nm RMS. Hence, the parallel DM was designed for GMagAO-X (with a prototype currently undergoing tests with HCAT) to create an effective 21,000 actuator DM that also acts as an interferometric beam combiner to minimize wavefront and segment piston errors simultaneously (see Section 6.1). The first prototype of the parallel DM has already been fabricated in the lab for HCAT (as shown in Chapter 6) and is beginning to show promising results. In the near future, successful demonstrations of the HCAT parallel DM working with MagAO-X to create high Strehl ratio images in the presence of segment piston errors and atmospheric turbulence will address the 21,000 actuator DM requirement for GMagAO-X and make ExAO exoplanet imaging a reality for the GMT.

The optomechanical design of GMagAO-X from the CoDR is shown in Figure 7.2. The instrument is designed to have its own “M3” mirror that extends out to intercept the light from the telescope. Light then enters the instrument to a 3,228 actuator “woofer” DM (to control low order wavefront errors) and a 21,000 actuator “tweeter” DM (represented by the parallel DM) on the lower optical table and folds up to an upper level optical table where the coronagraphic science cameras exist. A test fit of the parallel DM was performed in the solid model to verify that
the structure would fit in between the two optical tables. Therefore, it is possible that the current parallel DM that was built for HCAT could be used for the final GMagAO-X instrument.

The most exciting fact about the GMagAO-X instrument is that it will be the first ELT instrument able to perform reflected light characterization of potentially habitable exoplanets such as Proxima Centauri b, satisfying the number one science goal for the “Worlds and Suns in Context” section of the Astro 2020 decadal review (National Academies of Sciences and Medicine, 2021). No other ELT instrument will have this capability in the near future. With the successful completion of the CoDR, and the preliminary design review (PDR) happening in the near future, the GMagAO-X instrument is well on its way to being fully operational at or near first light with the GMT in the early 2030s and we could see the discovery of life outside of our solar system finally become a reality.
Figure 7.2: The GMagAO-X instrument as of the 2021 CoDR. The instrument is a two-level floating optical table with a custom “M3” mirror that extends out to intercept the light from the telescope. The same “woofer-tweeter” DM system was adopted from MagAO-X to create high Strehl ratio images for the coronagraphic science cameras on the upper table. The parallel DM represents the tweeter DM here, making the required 21,000 actuator DM a reality. The parallel DM that was built for HCAT was designed to fit in between the two optical tables of GMagAO-X in order to make it a possibility for the HCAT parallel DM to be adopted by GMagAO-X.
Astronomical telescopes have mirrors that are coated with metallic materials, such as aluminum, to create high surface reflection over a broad range of wavelengths. These metallic coatings have a complex refractive index which introduces diattenuation and retardance as a function of incident angle. Orthogonal polarization states, such as s and p, incident onto a telescope mirror surface will reflect with different amplitudes and phases, causing unpolarized light from a star to become partially polarized, and the resulting image to be an incoherent sum of separate point-spread functions (PSFs). This result is known as polarization aberration. Polarization aberrations in telescopes are typically very small compared to the diffraction-limited PSF and are only considered a nuisance for high-contrast imaging and precision astrometry. For the next generation of extremely large telescopes, polarization aberrations may become the limiting factor for high-contrast imaging and precision astrometry, making this a very important problem that needs to be addressed.

A.1 Unpolarized Light

To understand polarization aberrations in telescopes, we must first understand that unpolarized light from a star can be decomposed into any two orthogonal polarization states (Collett, 2005). This can be shown in terms of Stokes vectors, where the Stokes vector for unpolarized light is given by

\[
S = \begin{bmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3 \\
\end{bmatrix} = \begin{bmatrix}
I_H + I_V \\
I_H - I_V \\
I_{45} - I_{135} \\
I_R - I_L \\
\end{bmatrix} = \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
\end{bmatrix}.
\]

(A.1)
Here, $I_H$ and $I_V$ are the irradiance measurements for horizontal and vertical linearly polarized light, $I_{45}$ and $I_{135}$ are the irradiance measurements for $45^\circ$ and $135^\circ$ linearly polarized light, and $I_R$ and $I_L$ are the irradiances for right and left circular polarized light. When the Stokes vectors of two orthogonal polarization states are added, the result is the unpolarized Stokes vector. For example, adding horizontal and vertical linearly polarized light,

$$\begin{bmatrix}
\frac{1}{2} & 1 & 1 \\
0 & 1 & -1 \\
0 & 0 & 0 \\
\end{bmatrix} = \begin{bmatrix}
1 \\
0 \\
0 \\
\end{bmatrix}.$$  \hspace{1cm} (A.2)

In general, any two orthogonal elliptical polarization states with semi-major axis orientation $\psi$ and ellipticity $\chi$ can be added to represent unpolarized light,

$$\begin{bmatrix}
\frac{1}{2} \\
\frac{1}{2} \\
\end{bmatrix} \begin{bmatrix}
1 \\
\cos(2\psi) \cos(2\chi) \\
\sin(2\psi) \cos(2\chi) \\
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{1}{2} \\
\frac{1}{2} \\
\end{bmatrix} \begin{bmatrix}
1 \\
\cos(2\psi) \cos(2\chi - \pi) \\
\sin(2\psi) \cos(2\chi - \pi) \\
\end{bmatrix} = \begin{bmatrix}
1 \\
0 \\
0 \\
\end{bmatrix}.$$  \hspace{1cm} (A.3)

Two examples of orthogonal polarization ellipses are shown in Figure A.1. Figure A.1a shows two linear orthogonal polarization states with $\psi = 0^\circ$ and $\chi = 0^\circ$, while Figure A.1b shows two elliptical orthogonal polarization states with $\psi = 45^\circ$ and $\chi = 15^\circ$. The propagation direction of the electromagnetic wave in these plots is out of the page.

When describing light incident onto a mirror surface, most textbooks decompose unpolarized light into the polarized components parallel ($p$) and perpendicular ($s$) to the plane of incidence. These are the *eigenpolarizations* of reflection, because the $s$ and $p$ polarization states are unaltered after reflection, except for a possible change in amplitude and/or phase. These are commonly known as the “Fresnel coefficients,”
Figure A.1: (a) Polarization ellipses of two orthogonal linearly polarized states, horizontal (red) and vertical (green). The arrows represent the electric field vector, which oscillates back and forth in the transverse plane along the green and red lines over time as the light propagates out of the page. (b) Polarization ellipses of two orthogonal elliptically polarized states. The electric field rotates about the axis of propagation (pointing out of the page) in an elliptical pattern. These are two examples of unpolarized light decomposed into two orthogonal polarization states.

which are described by the Fresnel equations (Born and Wolf, 1999). The next section shows how the Fresnel coefficients give rise to polarization aberrations in astronomical telescopes.

A.2 The Fresnel Equations

The origin of polarization aberrations in astronomical telescopes arises from the behavior of light interacting with a mirror surface coated in metal. The Fresnel equations can be used to describe this.

Imagine unpolarized light from a star is incident onto an isotropic mirror surface at an angle of incidence $\theta_0$. The metallic coating on the mirror surface has a wavelength-dependent complex index of refraction $N_1(\lambda) = n_1(\lambda) + ik_1(\lambda)$, while the incident medium is $N_0 = 1$ for air. The unpolarized light can be decomposed into two linear orthogonal polarization states, $s$ and $p$, where the $s$-polarized component oscillates perpendicular to the plane of incidence (in and out of the page), and the
$p$-polarized component oscillates parallel to the plane of incidence. Part of the light is absorbed, or “refracted,” through the metal coating at an angle of $\theta_1$, while the rest of the light is reflected at an angle of $\theta_0$. Figure A.2 shows a diagram of this setup.

![Figure A.2: Unpolarized light, represented as two orthogonal linear polarization components $s$ and $p$, is incident onto a mirror surface with a complex refractive index of $N_1$. The $s$-polarized component oscillates in and out of the page, perpendicular to the plane of incidence. The $p$-polarized component oscillates parallel to the plane of incidence. Part of the beam is absorbed into the metal coating at a complex refraction angle $\theta_1$, while most of the light is reflected from the surface at an angle of $\theta_0$. Note how the $p$-polarized component experiences a $\pi$ phase shift after reflection.](image)

The portion of the beam that penetrates into the metal coating at a complex angle of $\theta_1$ can be calculated from Snell’s law as

$$\theta_1 = \sin^{-1} \left[ \frac{N_0 \sin(\theta_0)}{N_1} \right]. \quad \text{(A.4)}$$

The rest of the beam is reflected off of the mirror surface at an angle of $\theta_0$ from the normal. The reflectivity of the $p$-polarized component is given by the Fresnel
coefficient for \( p \)-polarized light (Azzam and Bashara, 1977),

\[
    r_p = \frac{\tan(\theta_0 - \theta_1)}{\tan(\theta_0 + \theta_1)} = |r_p|e^{i\phi_p}. \tag{A.5}
\]

Similarly, the reflectivity of the \( s \)-polarized component is given by the Fresnel coefficient for \( s \)-polarized light,

\[
    r_s = -\frac{\sin(\theta_0 - \theta_1)}{\sin(\theta_0 + \theta_1)} = |r_s|e^{i\phi_s}. \tag{A.6}
\]

The right-hand sides of equations (A.5) and (A.6) are the complex forms of \( r_p \) and \( r_s \), where \(|r_p|\) and \( \phi_p \) are the amplitude and phase of the \( p \)-polarized reflected component and \(|r_s|\) and \( \phi_s \) are the amplitude and phase of the \( s \)-polarized reflected component. Therefore, the amplitudes can be calculated by taking the complex magnitude of \( r_p \) and \( r_s \),

\[
    |r_p| = \text{Abs}[r_p] \quad \text{and} \quad |r_s| = \text{Abs}[r_s], \tag{A.7}
\]

and the phases can be calculated by taking the complex argument of \( r_p \) and \( r_s \),

\[
    \phi_p = \pi + \text{Arg}[r_p] \quad \text{and} \quad \phi_s = \text{Arg}[r_s]. \tag{A.8}
\]

The \( p \)-polarized component carries an additional \( \pi \) phase shift upon reflection (Lam and Chipman, 2015; Hecht, 2017). The amplitudes and phases from equations (A.7) and (A.8) are plotted in Figure A.3 as a function of incident angle for aluminum at 800 nm, which has a complex refractive index of \( N_1 = 2.7673 + 8.3543i \). As the angle of incidence increases, the difference in amplitude and phase between the \( s \) and \( p \)-polarized components increases. These two effects are known as \textit{diattenuation} and \textit{retardance} (Chipman et al., 2018).

Diattenuation is the property of having polarization dependent amplitude change. An example of a diattenuator is the linear polarizer, which fully transmits one linear polarization state (e.g., horizontal), but fully attenuates the orthogonal linear polarization state (e.g., vertical).
Figure A.3: (a) The amplitudes and (b) the phases of the reflection coefficients as a function of incident angle. The y-axis in (a) is in normalized percentage and the y-axis in (b) is in radians. As the angle of incidence increases, the amplitude and phase difference between the $s$ and $p$-polarized beams increases.

Retardance is the property of having an optical path length (OPL; i.e., phase) that is dependent on the incident polarization state. An example of a retarder is the half-wave plate, which introduces a $\pi$ phase shift between orthogonal polarization states.

Reflection from metals introduces both diattenuation and retardance as a function of the incident polarization state and the angle of incidence, acting as a weak polarizer. For reflections from metals, the diattenuation $D$ can be expressed in terms of the amplitudes of the Fresnel coefficients,

$$D = \frac{|r_s|^2 - |r_p|^2}{|r_s|^2 + |r_p|^2},$$

and the retardance $\delta$ can be expressed in terms of the phases of the Fresnel coefficients,

$$\delta = |\phi_p - \phi_s|.$$

Figure A.4 shows the diattenuation and retardance as a function of angle of incidence for a flat aluminum mirror at 800 nm and refractive index of $N_1 = 2.7673 + 8.3543i$. 
Figure A.4: (a) The diattenuation and (b) the retardance of the reflection coefficients as a function of incident angle. As the angle of incidence increases, the diattenuation and retardance increases.

The diattenuation and retardance resulting from the reflection of metals gives rise to polarization aberrations. Astronomical telescopes and instruments often introduce mirrors with metallic coatings, some of which have angles of incidence of $45^\circ$ or greater. The next section discusses the results of a simulated polarization raytrace for the parallel DM to show how polarization aberrations affect optical systems.

A.3 Polarization Raytrace Analysis of the Parallel DM

A polarization raytrace analysis was performed in Python for the parallel DM in Section 6.1 to examine the effects of polarization aberrations on the PSF. The methods explained in this section follow Chapter 9 from Chipman et al. (2018). Figure A.5 shows a ray diagram of the parallel DM. An incoming collimated beam from the GMT reflects off of a reflective hexagonal pyramid, which reflects each GMT pupil segment at $45^\circ$ AOI in six different directions, while the central segment propagates through a hole in the center. A fold mirror (representing a PI S-325 actuator) at $45^\circ$ AOI folds the light towards a flat mirror (representing a BMC 3k DM) which sends the light backwards through the system in double pass.
Figure A.5: A ray diagram of the parallel DM. An incoming collimated beam from the GMT intercepts a reflective hexagonal pyramid which reflects each GMT segment in six different directions at 45° AOI, while the central segment propagates through a hole in the center. A fold mirror (PI S-325 actuator) is introduced for each segment at 45° AOI which fold the light onto a flat mirror (BMC 3k DM). The light then propagates backwards in double pass.

A.3.1 Simulating a Protected Silver Coating

Each mirror surface was coated with a protected silver coating, which was simulated as a silver mirror surface coated with a single layer of SiO₂. The Fresnel coefficients for a single layer thin film coating can be represented by the following equations (Chipman et al., 2018):

\[
\begin{align*}
    r_s &= \frac{r_{01s} + r_{12s}e^{i2\beta}}{1 + r_{01s}r_{12s}e^{i2\beta}} = |r_s|e^{i\phi_s}, \\
    r_p &= \frac{r_{01p} + r_{12p}e^{i2\beta}}{1 + r_{01p}r_{12p}e^{i2\beta}} = |r_p|e^{i\phi_p},
\end{align*}
\]

where \( r_{01} \) is the Fresnel coefficient for the air/film interface and \( r_{10} \) is the Fresnel coefficient for the film/substrate layer interface, which can be calculated using equations A.4, A.5 and A.6. The phase thickness \( \beta \) of the single layer thin film is given
by
\[ \beta = \frac{2\pi}{\lambda} n_1(\lambda) \, d \cos \theta_1, \quad (A.13) \]

where \( n_1(\lambda) \) is the wavelength-dependent refractive index of the thin film, \( d \) is the thickness of the layer, and \( \theta_1 \) is the angle of refraction into the thin film layer given by equation A.4. Figure A.7 shows the resulting plots of the reflection vs. angle and phase vs. angle at \( \lambda = 800 \) nm for a protected silver coating with a single 150 nm thick SiO\(_2\) layer.

Figure A.6: (Left): The reflection vs. angle plot for the single layer protected silver coating. (Right): The phase vs. angle plot for the single layer protected silver coating. Both plots are for \( \lambda = 800 \) nm.

### A.3.2 Polarization Raytrace of the Parallel DM

Polarization raytracing calculates the evolution of a polarization state of a ray propagating through an optical system. The polarization effects at each optical surface are characterized by the polarization raytracing matrix \( P \), which is used to relate an incident electric field vector to the exiting electric field vector by the following relation:

\[ E_q = P_q E_{q-1} , \quad (A.14) \]
where $q$ is the $q^{th}$ ray intercept. Hence, the net polarization effect of a series of optical elements is represented by a matrix multiplication of each $P$ matrix for each optical element,

$$P_{\text{total}} = P_N P_{N-1} \cdots P_2 P_1 = \prod_{q=1}^{N} P_q.$$  \hfill (A.15)

To calculate the $P$ matrix for each surface, a coordinate transformation is performed to relate the global coordinates of the surface normal to local $s$-$p$ coordinates. The $s$- and $p$-polarization components of a surface along with the propagation vector $\hat{k}$ form a $(\hat{s}, \hat{p}, \hat{k})$ basis that can be used for local coordinates. The local $s$-$p$ coordinates for a ray incident on the $q^{th}$ surface are then given by

$$\hat{s}_q = \frac{\hat{k}_{q-1} \times \hat{n}_q}{|\hat{k}_{q-1} \times \hat{n}_q|} \quad \text{and} \quad \hat{p}_q = \hat{k}_{q-1} \times \hat{s}_q,$$

and the local $s$-$p$ coordinates for a ray exiting the $q^{th}$ surface are given by

$$\hat{s}'_q = \hat{s}_q \quad \text{and} \quad \hat{p}'_q = \hat{k}_q \times \hat{s}_q,$$

where $\hat{n}_q$ is the surface normal of the $q^{th}$ surface and $\hat{k}_{q-1}$ and $\hat{k}_q$ are the propagation vectors for the ray incident on and exiting from the surface. The orthogonal matrices
to transform global coordinates into local coordinates are given by
\[
O_{\text{in},q}^{-1} = \begin{bmatrix}
\hat{s}_x,q & \hat{s}_y,q & \hat{s}_z,q \\
\hat{p}_x,q & \hat{p}_y,q & \hat{p}_z,q \\
\hat{k}_{x,q-1} & \hat{k}_{y,q-1} & \hat{k}_{z,q-1}
\end{bmatrix}
\text{ and } O_{\text{out},q} = \begin{bmatrix}
\hat{s}_x,q & \hat{p}_x,q & \hat{k}_x,q \\
\hat{s}_y,q & \hat{p}_y,q & \hat{k}_y,q \\
\hat{s}_z,q & \hat{p}_z,q & \hat{k}_z,q
\end{bmatrix}. \tag{A.18}
\]

\(O_{\text{in},q}^{-1}\) operates on the incident electric field defined in the global coordinate system and calculates a projection of the global coordinates onto the local \(s\)-\(p\) coordinates. \(O_{\text{out},q}\) rotates the global coordinate system to the exiting \(s\)-\(p\) local coordinates to calculate the exiting electric field in the global coordinate system. The \(P\) matrix for each surface can then be calculated by
\[
P_q = O_{\text{out},q} J_{r,q} O_{\text{in},q}^{-1} \tag{A.19}
\]
where \(J_{r,q}\) is the Jones matrix for reflection which is given by
\[
J_{r,q} = \begin{bmatrix}
r_{s,q} & 0 & 0 \\
0 & r_{p,q} & 0 \\
0 & 0 & 1
\end{bmatrix}, \tag{A.20}
\]
where \(r_{s,q}\) is the Fresnel reflection coefficient for \(s\)-polarization at the \(q\)th surface and \(r_{p,q}\) is the Fresnel reflection coefficient for \(p\)-polarization at the \(q\)th surface. A \(P_q\) matrix can then be calculated for each surface and the total \(P_{\text{total}}\) matrix can be calculated for the system using equation A.15. An incident polarization state represented by a three-dimensional Jones vector can then operate on \(P_{\text{total}}\) to evaluate the exiting electric field, that is,
\[
E_q = P_{\text{total}} E_{q-1} = P_{\text{total}} \begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix}. \tag{A.21}
\]
This procedure was followed for each GMT pupil segment propagating through the parallel DM to calculate the total output electric field for an incident unpolarized
field (two linearly polarized fields $E_x$ and $E_y$). The output electric fields $E'_x$ and $E'_y$ are then given by

$$E'_x = P_{\text{total}} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad E'_y = P_{\text{total}} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}. \quad (A.22)$$

Since the hexpyramid reflects each GMT pupil segment away in six different directions, the incident electric fields $E_x$ and $E_y$ will have different local s- and p-polarization components upon reflection depending on the face of the hexpyramid each GMT segment interacts with. This causes the output electric field $E'_x$ to have a non-zero $y$ component $E'_{xy}$ and the output electric field $E'_y$ to have a non-zero $x$ component $E'_{yx}$ causing a change in the exiting polarization state between each GMT segment that will distort the final PSF.

To calculate the PSF from the polarization aberrations, a Jones pupil is first generated. A Jones pupil is a 2x2 matrix that represents the complex electric field components of the exit pupil for an incident unpolarized electric field represented by two orthogonal linearly polarized states $E_x$ and $E_y$. The Jones pupil maps these complex components for each point in the pupil to evaluate the effects of polarization aberrations. The Jones pupil is defined as

$$J = \begin{bmatrix} E_{xx}'(\xi, \eta) & E_{xy}'(\xi, \eta) \\ E_{yx}'(\xi, \eta) & E_{yy}'(\xi, \eta) \end{bmatrix} = \begin{bmatrix} A_{xx}(\xi, \eta)e^{i\phi_{xx}(\xi, \eta)} & A_{xy}(\xi, \eta)e^{i\phi_{xy}(\xi, \eta)} \\ A_{yx}(\xi, \eta)e^{i\phi_{yx}(\xi, \eta)} & A_{yy}(\xi, \eta)e^{i\phi_{yy}(\xi, \eta)} \end{bmatrix}. \quad (A.23)$$

The $A_{xx}$ term is the X-polarized amplitude of $E_{xx}$ at the exit pupil resulting from an incident X-polarized field $E_x$ in object space. The $\phi_{xx}$ term is the complex argument of $E_{xx}$; the phase shift from the incident X-polarized field to the exiting X-polarized field. This is similar for the $yy$ components. The $yx$ components are the components of the incident X-polarized field that are coupled into the Y-polarized field. Similarly, the $xy$ components are the components of the incident Y-polarized field coupled into the X-polarized field.

Figure A.8 shows the Jones pupil elements for the parallel DM without crossed fold mirrors to show the amplitude and phase variations across the pupil. Ideally,
the Jones pupil would be the identity matrix, for which the phase change would be
independent of the polarization state. However, since the hexpyramid reflects each
GMT pupil segment away in six different directions, incoming unpolarized light from
a star will have different s- and p-polarization components upon reflection depending
on the face of the hexpyramid each GMT segment picks up, causing a change in the
exiting polarization state between each GMT segment.

Figure A.8: The Jones pupil elements at $\lambda = 650$ nm with no crossed fold mirrors
incorporated into the parallel DM.

To evaluate the PSF, the Amplitude Response Matrix (ARM), which is the
Fourier transform of the Jones pupil elements, can be calculated. The ARM is
given by

$$\text{ARM} = \begin{bmatrix} \mathcal{F}[J_{xx}(\xi, \eta)] & \mathcal{F}[J_{xy}(\xi, \eta)] \\ \mathcal{F}[J_{yx}(\xi, \eta)] & \mathcal{F}[J_{yy}(\xi, \eta)] \end{bmatrix},$$

(A.24)

and the intensity of the PSF is given by the sum of the ARM components,

$$I = I_x + I_y = |\text{ARM}_{xx}|^2 + |\text{ARM}_{yx}|^2 + |\text{ARM}_{xy}|^2 + |\text{ARM}_{yy}|^2.$$  

(A.25)

The resulting PSF at $\lambda = 650$ nm for the parallel DM is shown in Figure A.9. The
Strehl ratio is 63%, purely due to the polarization aberrations. The reference PSF
(no polarization) is shown to the right for comparison.
Figure A.9: The resulting PSF at $\lambda = 650$ nm for the parallel DM without crossed fold mirrors. The Strehl ratio is 63%, resulting from the polarization aberrations. The reference PSF with no polarization effects included is shown on the right for comparison.

When crossed fold mirrors are included in the parallel DM, the polarization aberrations cancel. Figure A.10 shows a ray diagram of the parallel DM with crossed fold mirrors. The fold mirror that represents the PI S-325 actuator is rotated to have a plane of incidence that is orthogonal to the plane of incidence of the hexpyramid. Figure A.11 shows the Jones pupil elements for the parallel DM with crossed fold mirrors. Now, the off-diagonal elements have zero amplitude and the result is the identity matrix. Hence, the resulting PSF, shown in Figure A.12, is polarization aberration free. The Strehl ratio is plotted as a function of wavelength in Figure A.13 to show the full effect of polarization aberrations in the parallel DM with and without crossed fold mirrors. Without crossed fold mirrors, the Strehl ratio varies drastically with wavelength, but with crossed fold mirrors, the Strehl ratio stays consistent across all wavelengths, matching the results from the Zemax simulations.

We have found that the crossed fold mirror technique is sufficient for mitigating the polarization aberrations of the parallel DM. However, each mirror surface must have the same exact coating in order for the polarization aberrations to cancel. For this reason, all the fold mirrors for the HCAT parallel DM concept have been coated by the same vendor as the hexpyramid to ensure that they have the same coating.
Figure A.10: A ray diagram of the parallel DM with crossed fold mirrors incorporated. An incoming collimated beam from the GMT intercepts a reflective hexagonal pyramid which reflects each GMT segment in six different directions at 45° AOI, while the central segment propagates through a hole in the center. A fold mirror (PI S-325 actuator) is introduced with a plane of incidence perpendicular to the plane of incidence of the hexpyramid at 45° AOI which folds the light onto a flat mirror (BMC 3k DM). The light then propagates backwards in double pass.
Figure A.11: The Jones pupil elements at $\lambda = 650\text{nm}$ with crossed fold mirrors included in the parallel DM. The amplitude of the off-diagonal components is now zero and the result is the identity matrix.

Figure A.12: The resulting PSF at $\lambda = 650\text{nm}$ for the parallel DM with crossed fold mirrors included. The polarization aberrations cancel and the Strehl ratio is 100%. The reference PSF with no polarization effects included is shown on the right for comparison.
Figure A.13: The Strehl ratio versus wavelength for the Python polarization raytrace simulations of the parallel DM without (left) and with (right) crossed fold mirrors included in the parallel DM.
REFERENCES


