# COMPRESSION OF DIFFUSION WEIGHTED MAGNETIC RESONANCE IMAGES USING JPEG2000

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## ABSTRACT

Magnetic Resonance Imaging (MRI) is a medical imaging technique which is routinely used for anatomical and functional imaging. Diffusion Tensor Imaging (DTI) characterizes the diffusion of water molecules using a tensor and allows computation of quantitative metrics, such as fractional anisotropy, which measures anisotropy of the diffusion process for each voxel. The image datasets used in DTI are four-dimensional with three spatial dimensions and a fourth dimension representing the direction of diffusion sensitivity applied during acquisition. In this work,we investigate the use of three-dimensional JPEG2000 for lossy and lossless compression, as well as the trade-off between the number of diffusion directions and bits/pixel used to compress each volume. These experiments show that diffusion weighted MR images can be reliably compressed using JPEG2000.

## INTRODUCTION

Magnetic Resonance Imaging (MRI) is a medical imaging technique which provides excellent soft tissue contrast and is routinely used in clinics for anatomic and functional imaging. Diffusion weighted MRI (DW-MRI) is a method that is used to measure diffusion of water molecules in the body. The most common DW-MRI methodology is Diffusion Tensor Imaging (DTI). DTI characterizes diffusion using a diffusion tensor that describes the magnitude, the degree, and orientation of the diffusion. Figure 1 shows the diffusion random walk for a single water molecule from the green point to the red point with displacement indicated by the yellow arrow on the left. The right describes the displacement probability with time for a group of water molecules over different diffusion times, $\tau$. This molecular water displacement is described by a Gaussian probability density [1]. As $\tau$ increases so does the predicted spread. The diffusion depicted in Figure 1 is spherically symmetric, or isotropic.

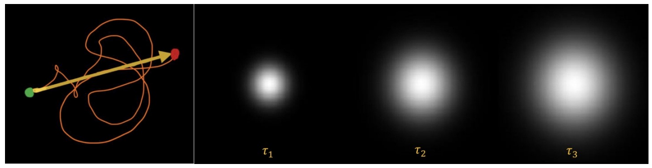


Figure 1: Diffusion change as time increases.

Diffusion weighted MRIs produce multiple image slices at each diffusion level, making it a longer scan that produces more data than traditional brain MRIs. Attempts have been made to reduce the data needed, and therefore time needed, to characterize a sample through deep learning methods [2]. But for these images a lossless image compression method, such as the discrete wavelet transforms used in JPEG2000, would be ideal [3].

### A. *Diffusion Weighted Magnetic Resonance Imaging*

Diffusion tensor imaging is used to characterize and map diffusion of water as a function of spatial location in three dimensions. This diffusion tensor describes the magnitude, the orientation of diffusion anisotropy, and the degree of anisotropy. This method of contrast for magnetic resonance (MR) can estimate the white matter patterns and tractography in the brain using the principle diffusion directions and diffusion anisotropy.

The Einstein diffusion equation, Equation 1, shows that the diffusion coefficient, D, is proportional to the mean squared displacement, $< \Delta r^2 >$, divided by the number of dimensions and the diffusion time and the diffusion coefficient of pure water at 20 degrees Celsius which is approximately $2.0$ x $10^{-3}$ $mm^2/s$ and increases with temperature.

$$D = \frac{< \Delta r^2 >}{2n\Delta t} \tag{1}$$

The water displacement without boundaries can be described by a Gaussian probability density, but diffusion of water in tissues occurs around and through cellular structures. In white matter, water diffusion is mostly unimpeded when parallel to the orientation of the fibers, but water diffusion is highly hindered in directions perpendicular to the orientation of the fibers. This makes diffusion in white matter anisotropic. Equation 2 shows the model introduced by Basser [4] [5] that describes diffusion as a multivariate normal distribution using the diffusion tensor shown as Equation 3 to describe anisotropic diffusion behavior.

$$P(\Delta\overrightarrow{r}, \Delta t) = \frac{1}{\sqrt{(4\pi\Delta t)^3|\mathbf{D}|}} \exp\frac{-\Delta\overrightarrow{r}\mathbf{D}^{-1}\Delta\overrightarrow{r}}{4\Delta t} \tag{2}$$

This diffusion tensor, D, is a 3 x 3 covariance matrix that describes the covariance diffusion displacements in three dimensions normalized by the diffusion time. The diagonal elements are the diffusion variances and the off-diagonal elements are the covariance terms and are symmetric about the diagonal. The eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ and corresponding eigenvectors $(\epsilon_1, \epsilon_2, \epsilon_3)$ of the diffusion tensor are found by diagonalization of the diffusion tensor and describe the directions and diffusivities along the axes of principal diffusion.

$$\mathbf{D} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{bmatrix} \tag{3}$$

A diffusion weighted MRI consists of a four-dimensional set of images. Three dimensions correspond to the usual spatial coordinates, while the fourth dimension corresponding to b-values, or diffusion weights, and b-vectors, or diffusion directions. The data used in this research is from the Human Connectome Project (HCP) [6] and contains one DWI for each of the 25 human subjects, each such DWI is stored as a four-dimensional data set containing 288 three-dimensional MRIs corresponding to 288 diffusion parameter settings for each subject. The HCP utilizes a specific style of imaging protocol that creates 18 images with no diffusion, 90 images that correspond to b = 1000 $s/mm^2$ with varied diffusion directions, 90 images that correspond to b = 2000 $s/mm^2$ with varied diffusion directions, and 90 images that correspond to b = 3000 $s/mm^2$ with varied diffusion directions that make up the 288 3-D images produced. The b-values and b-vectors employed are recorded in a separate file that accompanies the images.

Figure 2 shows how different diffusion weights change the images. As the b-value increases the signal decays and this causes the Signal-to-Noise Ratio of high b-value images to be lower. Figure 3 shows the difference in images taken at the same diffusion weight but different diffusion directions. The difference in intensity throughout the images highlights how the diffusion directions affect the images.

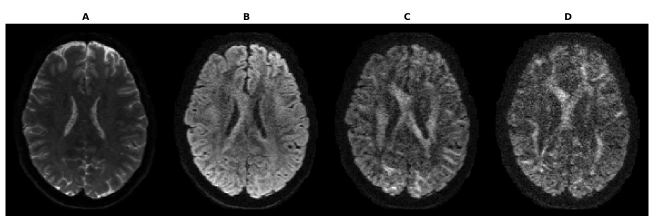


Figure 2: An axial slice of four brain images acquired at (A) b = 0 $s/mm^2$, (B) b = 1000 $s/mm^2$, (C) b = 2000 $s/mm^2$ and (D) b = 3000 $s/mm^2$.

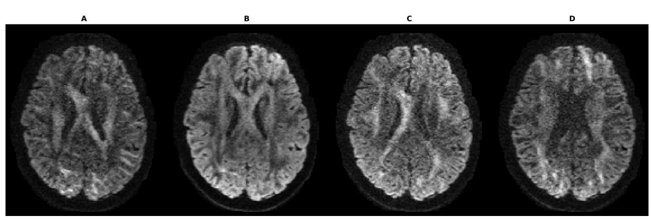


Figure 3: An axial slice of four brain images all acquired at b = 2000 $s/mm^2$ with different diffusion directions. (A) [-0.3829, -0.8437, -0.3762], (B) [0.2044, 0.9072, -0.3676], (C) [0.2589, 0.0774, 0.9628], and (D) [0.9768, 0.2054, -0.0606].

The pulsed-gradient spin echo pulse sequence with a single-shot, echo-planar imaging (EPI) readout is the most common diffusion weighted imaging (DWI) approach [1]. For the diffusion gradient pulses used in this method the signal attenuation for a simple isotropic Gaussian diffusion can be described by Equation 4. S is the DW signal, $S_o$ is the signal without any DW gradients, D is the diffusion coefficient, g is the diffusion direction vector, and b is the diffusion-weighting described by the pulse pair shown in Equation 5. For the pulse pair, $\gamma$ is the gyromagnetic ratio.

$$S_j = S_0 \exp -b_j \mathbf{g_j}^T \mathbf{D} \mathbf{g_j} \tag{4}$$

$$b(\gamma G \delta)^2 (\Delta - \frac{\delta}{3}) \tag{5}$$

The signal attenuation for anisotropic diffusion can be described by Equation 6. $S_i$ in this equation is the DW signal, $\hat{g}_i$ is the unit vector describing the DW encoding direction, supplied by the b-vectors, and $b_i$ is the amount of diffusion weighting. The apparent diffusivity maps can then be estimated using Equation 7 for an image with no diffusion and a single diffusion weight or b-value. Equations 6 and 7 can be expanded in terms of D shown in Equation 8. From this with six different images, all six unknowns can be solved for and the entire diffusion tensor can be filled.

$$S_i = S_o \exp -b \mathbf{\hat{g}_i}^T \mathbf{D} \mathbf{\hat{g}_i} = S_o \exp -b_i D_{i,app} \tag{6}$$

$$D_{i,app} = \frac{ln(S_i) - ln(S_o)}{b_i} \tag{7}$$

$$\frac{-ln(\frac{S_j}{S_o})}{b_i} = (g_{xi}^2 D_{xx} + g_{yi}^2 D_{yy} + g_{zi}^2 D_{zz} + 2g_{xi}g_{yi}D_{xy} + 2g_{xi}g_{zi}D_{xz} + 2g_{yi}g_{zi}D_{yz}) \tag{8}$$

Due to the size of this data it is best to distill this image information into simpler scalar maps. The most widely used rotationally invariant measure of anisotropy is the fractional anisotropy (FA) described in equation 9 by Basser and Pierpaoli [4]. Some other metrics that can be used are the apparent diffusion coefficient ($MD/ADC = \frac{\lambda_1+\lambda_2+\lambda_3}{3}$), the radial diffusivity ($D_r = \frac{\lambda_2+\lambda_3}{2}$), and the axial diffusivity ($D_a = \lambda_1$).

$$FA = \sqrt{\frac{(\lambda_1 - MD)^2 + (\lambda_2 - MD)^2 + (\lambda_3 - MD)^2}{2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}} \tag{9}$$

This degree of anisotropy is shown in the FA maps as a value between one and zero with zero meaning that diffusion is isotropic or equal in all directions and one meaning that diffusion occurs only along one axis. An FA map shows the diffusion pattern of water molecules, allowing the detection of microstructural details of normal or altered anatomy of a given region.

MRtrix3 is a python library that provides a set of tools to perform various types of diffusion MRI analyses, including the creation of FA maps [7]. Two of the provided functions, dwi2tensor and tensor2metric, were used. The function dwi2tensor takes a set of diffusion weighted images and their corresponding b-values and b-vectors to estimate a diffusion tensor. The output of dwi2tensor is then used as the input for tensor2metric to output different maps of tensor-derived parameters, in this case only the FA maps are used.

### *B. JPEG2000 Image Compression*

JPEG2000 is a discrete wavelet transform based compression system and became an International Standard in 2000 [8]. This system is well-suited for image compression that requires high quality needed for a medical image. JPEG2000 compression begins with the discrete wavelet transform made up of a multi-component transform and a spatial wavelet transform [9]. Next, uniform scalar quantization is used. Equations 10 and 11 respectively show the process of quantization and dequantization [10]. A signed integer q is produced from the quantizer from a given wavelet coefficient, x, using a step size, $\delta$. The inverse shown in Equation 11, utilizes a user selectable parameter to control the reconstruction $\gamma$ that is typically 0.5.

$$q = Q(x) = sign(x)\frac{|x|}{\Delta} \tag{10}$$

$$\hat{x} = Q^{-1}(q) = \begin{cases} 0 & q = 0 \\ sign(q)(|q| + \gamma)\Delta & q \neq 0 \end{cases} \tag{11}$$

The image is divided into tiles and the subbands of each tile are divided into precincts and these are divided into code blocks. The code blocks are entropy encoded independently. The process is reversed to expand. The specific application of JPEG2000 used in this research employs the multi-component transforms (MCT) seen in Part 2 of the JPEG2000 standard. The MCT allows the multi-component image data to be decorrelated along the component direction [11]. To compress the three-dimensional data the forward transform goes through the third dimension first, similar to how an RGB image would first go through a color transform, then each component is independently transformed as a two-dimensional image [8].

## METHODS

### *C. Pre-processing of Data*

The data collected from the Human Connectome Project (HCP) [6] contains the DWIs as a four-dimensional data set containing the three-dimensional MRIs corresponding to 288 diffusion weights for each subject. For each subject, b-values and b-vectors files are included as well as the brain mask file that can be used to focus FA maps only on the brain region. Twenty five subjects were used in these experiments.

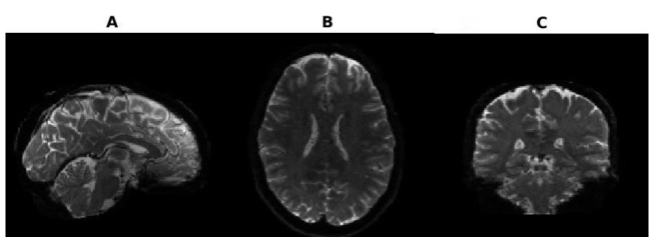


Figure 4: The three different brain planes (A) sagittal, (B) axial, and (C) coronal.

Each 4-D image is separated into 288 different three-dimensional images for compression. To view the different orientations of the brain scans, the three-dimensional images can be sliced in different orders. For example a coronal slice is obtained by fixing the first dimension of the 3-D image to a constant. An axial slice is obtained by fixing the second dimension and sagittal slice by the third. One slice from each of these three orientations can be seen in Figure 4.

*D. Compression of Diffusion Weighted MRI Images*

Kakadu software, a JPEG2000 software development tool, was used to perform lossless compression on each of the 25 DWI images [12]. Each DWI was compressed as 288 separate 3-D MRI images. The 5/3 reversible wavelet was used in each of the 3 dimensions. For the three-dimensional application, a one level discrete wavelet transform is applied across the 2-D slices for a particular orientation. The resulting transformed 2-D slices are then compressed by 2-D JPEG2000. The 288 3-D images (cubes) from each DWI has dimensions 145 x 174 x 145. The cubes are compressed using bitrates ranging from 0.5 bits/pixel up to lossless compression, which was typically around 6.5 bits/pixel, using half bit steps. The images were then expanded using kdu-expand, specifically the skip component and layers command were used to preserve all the different bitrates and output three-dimensional images.

*E. Diffusion Weighted MRI Analysis*

The MRtrix3 python library was then used to create the diffusion tensor, which consist of a 6-D tensor for each location in a 145 x 174 x 145 cube. This tensor can be arranged as a set of six 3-D scalar images (see Figure 5). For a given subject the diffusion tensor matrix and the images with no diffusion can be used to form a predicted DWI via Equation 4.

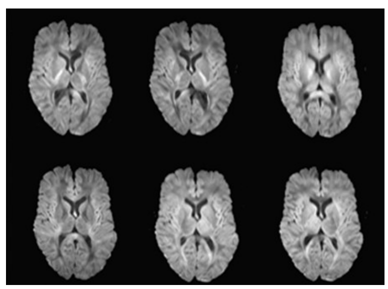

Figure 5: An axial slice of six 3-D images that comprise the diffusion tensor. The top row from right to left would correspond to $D_{xx}, D_{xy}, D_{xz}$ in the diffusion tensor shown in Equation 3 and the bottom row from right to left would correspond to $D_{yy}, D_{yz}, D_{zz}$ in the same tensor.

Figure 5 shows an example of an axial slice of the six images that make up the diffusion tensor. These six images can be produced by the dwi2tensor function. These represent the images with diffusion directions associated with first six values of the diffusion tensor and this tensor is symmetric so only six coefficients need to be found to characterize all nine values. The seventh image is the image taken at b = 0 $s/mm^2$ or no diffusion.

The diffusion tensors were produced using dwi2tensor with the compressed slices at different bitrates used as the input. Varied numbers of diffusion levels were also used to create corresponding tensors. One b = 0 $s/mm^2$ diffusion weighted image and 6, 15, 30 or 45 b = 1000 $s/mm^2$ images were used at the different bitrates. These diffusion tensors were then used in the tensor2metric function to create different FA maps. The FA maps were created by inputting the tensor and the brain mask.

## RESULTS

### *F. JPEG2000 Compression*

The first step was to compress and decompress all of the images within the data set using JPEG2000 multi-component discrete wavelet transforms. Figure 6 shows the MSE across bitrates ranging from a 0.5 to lossless, lossless decompression occurs at no more than 6.5 bits per pixel for each DWI. This figure is grouped by diffusion levels and separated by orientation, the error bars correspond to the standard deviation across all samples used.

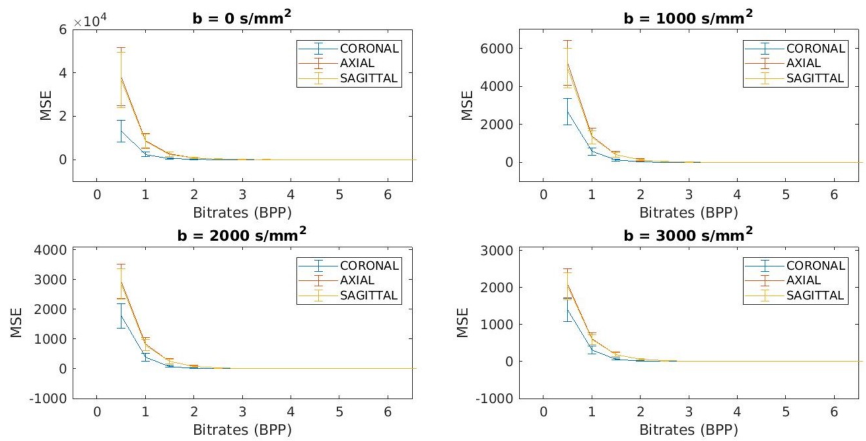


Figure 6: MSE performance of different bitrates in bits per pixel separated by diffusion weights. The error bars correspond to the standard deviation among the data set of 25 subjects.

Across all diffusion weights and samples the coronal orientation performs better by almost a factor of two. As expected the MSE decreases as the bitrate rises. The sagittal and axial orientations are similar in that a given cube consists of 174 slices, each 145 x 145. The coronal orientation consists of 145 slices, each 174 x 145. Of course, the total number of pixels is $174(145)^2$ regardless.

### *G. FA Maps Created Using Different Numbers of Diffusion Weighted Images*

The next experiment was the use of different numbers of diffusion weighted images (DWI) to create FA maps to establish if these metrics can be reliably created with less data. The DWIs are reduced from 18 b = 0 $s/mm^2$ to 1 and the amount of b = 1000 $s/mm^2$ included is varied from the original 90 down to 45, 30, 15 and 6. The b = 2000 $s/mm^2$ and b = 3000 $s/mm^2$ are totally removed. The removal of these images alone already accomplishes compression ratios of approximately 6:1, 9:1, 18:1, and 41:1.

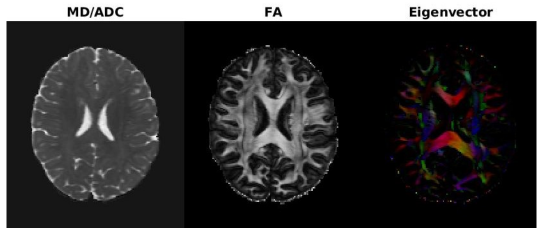


Figure 7: The mean diffusivity(MD/ADC), FA map and Eigenvector

Figure 7 shows the mean diffusivity (MD), FA map, and the major eigenvector direction indicated by color weighted by the FA. All of these were created using all 288 uncompressed diffusion weighted images. Red is right–left, green is anterior–posterior and blue is superior–inferior. Figures 8 and 9 show the FA maps of the varied number of b = 1000 $s/mm^2$ DWIs compressed using 0.5 bits per pixel and the FA maps using the uncompressed images. The reduction of DWIs used makes changes particularly around the boundaries of the skull, but the biggest change is how the maps deteriorate when only six of these images are used.

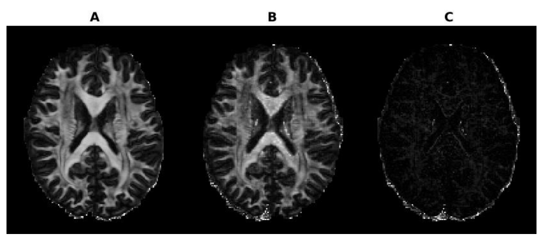


Figure 8: (A) FA map made using all 90 images with diffusion weight b = 1000 $s/mm^2$. (B) FA map made using 45 images of diffusion weight b = 1000 $s/mm^2$ that have been compressed with a bitrate of 0.5 bits per pixel. (C) The difference between A and B multiplied by five for clarity.

Compared to the FA map made using all 90 DWIs (Figure 8B), the FA map created using 45 DWIs shows moderate differences throughout in addition to major differences around the borders. This difference is highlighted in the far right image, (Figure 8C), that is the absolute value of the difference between the two images and multiplied by a constant for clarity.

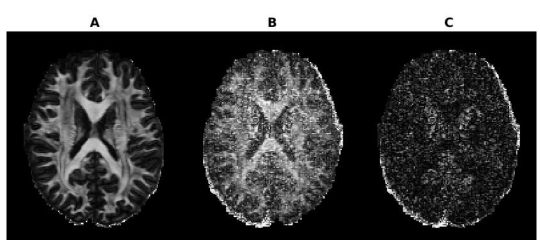


Figure 9: (A) FA map made using all 90 images with diffusion weight b = 1000 $s/mm^2$. (B) FA map made using 6 images of diffusion weight b = 1000 $s/mm^2$ that have been compressed with a bitrate of 0.5 bits per pixel. (C) The difference between A and B multiplied by five for clarity.

The FA map created using only 6 DWIs, labeled B, in Figure 9 shows major differences between the FA map created using 90 DWIs, labeled A. This difference is highlighted in the far right image, labeled C, that is the absolute value of the difference between the two images and multiplied by a constant for clarity.

| | | Number of Diffusion Weighted Images Included | | | |
|---|---|---|---|---|---|
| | | 6 | 15 | 30 | 45 |
| Bitrate (BPP) | 0.5 | 0.0283 | 0.0078 | 0.0062 | 0.0056 |
| | 1 | 0.0308 | 0.0086 | 0.0066 | 0.0060 |
| | 1.5 | 0.0310 | 0.0089 | 0.0069 | 0.0062 |
| | 2 | 0.0308 | 0.0088 | 0.0068 | 0.0061 |
| | 2.5 | 0.0307 | 0.0088 | 0.0068 | 0.0061 |
| | 3 | 0.0307 | 0.0088 | 0.0068 | 0.0061 |
| | 3.5 | 0.0306 | 0.0088 | 0.0068 | 0.0061 |
| | 4 | 0.0306 | 0.0088 | 0.0068 | 0.0060 |
| | 4.5 | 0.0305 | 0.0087 | 0.0067 | 0.0060 |
| | 5 | 0.0305 | 0.0087 | 0.0067 | 0.0060 |
| | 5.5 | 0.0305 | 0.0087 | 0.0067 | 0.0060 |
| | 6 | 0.0305 | 0.0087 | 0.0067 | 0.0060 |

Table 1: Mean squared error of the FA maps created using different amounts of compressed diffusion weighted images over different bitrates.

Table 1 show the effects of varying DWIs used in creating FA maps across different bitrates on the MSE. The biggest effect is seen when the number of DWIs used is lowered and the performance across bitrates changes only modestly as the bitrate varies. This shows that lower bitrates can be used to maintain an FA map, and the DWIs used can be reduced in addition to the compression with consistency until approximately six images used is reached.

These results show that JPEG2000 compression performs very well on the diffusion data. The MSE decreases as the bitrate increases and the slice number increases for the FA maps. The superior performance by the coronal orientation could be because the area of the images that show the brain are smaller than an axial or sagittal which would mean less pixels with values greater than zero.

## CONCLUSION

Diffusion weighted Magnetic Resonance Imaging (MRI) uses measurement of the diffusion properties of water within tissues to provide image contrast, making white matter tractography models possible. But these images are very large and the scans are time consuming and a compression scheme like JPEG2000 is well suited to this medical imaging that has multi-dimensions. The discrete wavelet transform based compression scheme of JPEG2000 was used to compress the diffusion weighted images that were used in MRtrix3 to create different analysis metrics. The compression and reduction of the diffusion slices used in creation of tensor and therefore FA maps shows the ability to reduce the data and maintain the necessary information that a FA map can provide. The performance of JPEG2000 compression applications on the diffusion weighted MRI images explored in this research show consistent results across the multiple samples and the use of compression followed by reduction of slices shows ways to reduce the size of data files needed to create reliable FA maps. As diffusion weighted MRI applications expand, the need for ways to reduce the large amounts of data expand with it and the compression performed in this research are a promising start.

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