

# Superdirective Unidirectional Mixed-Multipole Antenna Designs

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**Abstract**— Highly directive antenna systems are being sought to address the perceived needs of the NextG wireless systems and their applications. However, practical alternatives to complex, power hungry phased arrays are truly desired. Unidirectional mixed-multipole antennas (UMMAs) offer superdirective performance characteristics with high efficiency in a compact format. Superdirective uniform end-fire arrays of active closely-spaced equal-length idealized wire dipoles will be exemplified as reference cases. The Rayleigh quotient (RQ) method typically used to determine their amplitude coefficients and its recent extension to achieve unidirectional outcomes will be discussed. UMMA designs that are approaching those ideal performance characteristics will be presented. These efforts further confirm recent research advances demonstrating practical superdirective systems are achievable.

**Keywords**—Directivity, Huygens dipole antennas (HDAs), Rayleigh quotient, superdirectivity, unidirectional antennas

## I. INTRODUCTION

Superdirective radiating systems, a general definition being an antenna with transverse area  $A$  and excitation wavelength  $\lambda$  that radiates fields with a directivity exceeding the uniformly-excited aperture result [1], [2]:

$$D_{max} = 4\pi \frac{A}{\lambda^2} \quad (1)$$

(note that gain is efficiency times directivity and, hence, this value also represents the maximum possible gain) have been considered at many times over the last century since the concept of "needle radiation" was first introduced [3]. They have met resistance for use in practical applications because of the perceived – and real in many instances – drawbacks, particularly their radiation efficiencies and sensitivities to design parameter tolerances. Nevertheless, there has been a notable increase in both understanding the associated theories and actual realizations of superdirective radiating systems in the last 2 two decades. Most of these efforts have been focused on passive and active, tightly spaced, end-fire arrays based on the seminal work by Uzkov [4]. Examples of recent successful demonstrations include [5], [6].

While a superdirective electrically-small UMMA that is a mixture of electric and magnetic dipoles and an electric quadrupole was reported in [7], its maximum directivity was smaller than the upper bound defined by Harrington [8]:

$$D_{max} = N^2 + 2N \quad (2)$$

where  $N$  is the maximum dominant multipole ( $N = 1$  for a dipole,  $N = 2$  for a quadrupole, ...). Most recently, alternative single-source designs have been investigated based on a mixture of electric dipoles and quadrupoles in an attempt to approach or even surpass it [9]. They will be at the core of my presentation.

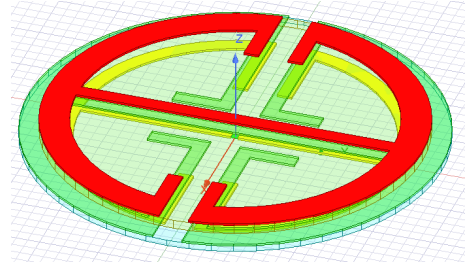


Fig. 1. Superdirective UMMA system whose maximum directivity is broadside directed along the +z-axis.

## II. UMMAS

A UMMA design is illustrated with its HFSS simulation model in Figure 1. Three copper Egyptian axe dipole (EAD) elements are printed on two circular rohacell disks whose dielectric constant and loss tangent are 1.05 and  $4.0 \times 10^{-4}$ . The bottom EAD element (on the bottom surface of the bottom substrate) is the driven EAD, a lumped source being present in a center gap. The other two EADs are near-field resonant parasitic (NFRP) elements (middle EAD is between the two substrates and the top EAD is on the top surface of the top substrate) that form the quadrupole. It is designed for operation in the 28 GHz 5G band. In contrast to the designs reported in [9], this slightly different design is an initial attempt to achieve a self-resonant system.

As shown in Fig. 2(a), the simulated front-to-back ratio (FTBR) reaches its maximum, 20.18 dB, at  $f_{FTBR} = 28.175$  GHz ( $\lambda_0 = 10.64$  mm). Its radiation efficiency (RE) is 88% at this frequency. Electing 10 dB as the FTBR figure-of-merit for unidirectionality, the UMMA exceeds this level over 920 MHz, from 27.61 and 28.53 GHz. The simulated directivity patterns in the two principal vertical planes ( $\phi = 0^\circ$  and  $\phi = 90^\circ$ ) at  $f_{FTBR}$  are displayed in Fig. 2(b). The maximum directivity is 8.98 dB along the broadside direction, i.e., the +z-axis.

The radius of the effective aperture area, i.e., the radius of the largest (middle) EAD, and, hence, approximately that of the minimum enclosing sphere is  $R = 0.329 \lambda_0$  (3.5 mm). This means  $ka = 2.07$  at the maximum FTBR frequency. Moreover, the maximum directivity given by (1) at  $f_{FTBR}$  is then 6.31 dB (4.27). Consequently, this UMMA is superdirective. Furthermore, the upper bound given by (2) with  $N = 2$  for its quadrupole nature is 9.03 dB (8). Thus, its directivity at  $f_{FTBR}$  almost reaches this upper (multipole) bound. In fact, it surpasses it at  $f_{peak \ dir} = 28.5$  GHz where it has its peak broadside directivity, 9.28 dB, also along the +z-axis. Furthermore, it remains significantly higher than the maximum given by (1), i.e., at  $f_{peak \ dir}$  (1) yields 6.41 dB (4.37). Nonetheless, at  $f_{peak \ dir}$  its FTBR decreases to 10.90 dB. There is a trade-off in the design between the peak FTBR and peak directivity performance characteristics. Note that the total

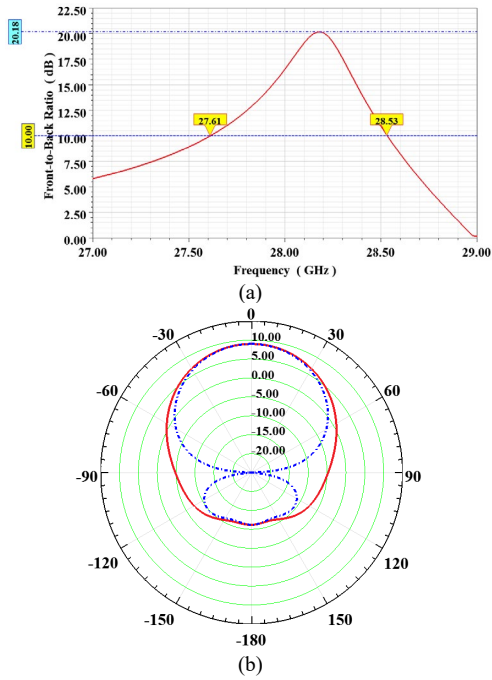


Fig 2. The simulated performance characteristics of the UMMA in Fig. 1. (a) FTBR values as a function of the source frequency. (b) Directivity patterns in the two vertical planes ( $\phi=0^\circ$ , red;  $\phi=90^\circ$ , blue).

thickness of this UMMA is only  $0.033 \lambda_0 \approx \lambda_0 / 30$  (0.355 mm), i.e., it is ultrathin and, hence, low profile.

Reference cases were included in [8] for comparison purposes. One was a uniform linear array of 3 ideal center-fed half-wavelength dipoles separated by  $\lambda_{28 \text{ GHz}} / 20$  that was optimized with the standard unconstrained Rayleigh quotient (RQ) method [10] and the constrained (unidirectional) version developed in [9]. The end-fire peak directivity and FTBR values of the former were, respectively, 10.54 and 10.42 dB, i.e., the backlobe level was quite large being  $-0.12$  dB. With a  $-100$  dB backlobe level specification, the constrained RQ method produced excitation coefficients that yielded an end-fire peak directivity of 9.89 dB and a corresponding FTBR value equal to 111.25 dB, i.e., the backlobe level was  $-101.36$  dB.

While the UMMA is not electrically small, the half-wavelength dipole array is not as well. One finds for this uniform linear array that its  $ka = 1.60$ . While this is 23% smaller than the UMMA's value, the UMMA is a single-source radiator whereas the array contains three active elements, and the RQ methods specify the excitation magnitude and phase of each of them. Moreover, because they were highly optimized, the performance characteristics of these RQ-optimized arrays were quite sensitive to those exact values, the constrained case being even more so than the unconstrained one.

Recall that the maximum directivity of a single half-wavelength dipole is the well-known value: 2.15 dB (1.64) [1] and its  $\text{FTBR}_{\text{hw-dipole}} = 0$  dB. The UMMA's peak directivity value even at  $f_{\text{FTBR}}$  is nearly 6.85 dB larger and its FTBR is 20.18 dB larger. Note, however, that an UMMA cannot reach

the extremely high FTBR values that a RQ-constrained array can. This limitation occurs because its EADs are finite in size and thickness and consist of real, lossy materials. The control over the interactions between them rests solely with the design itself. Nevertheless, as demonstrated in [9], the UMMA design outperforms the RQ-optimized three element array with ideal wires that have the same lengths and spacings as the EADs. The physical sizes of those driven and NFRP elements actually offer more degrees of freedom in tailoring the currents on them and, hence, their overall performance characteristics.

### III. CONCLUSIONS

A UMMA design was presented to introduce some of the basic concepts and considerations associated with mixed-multipole systems and to illustrate their ability to achieve efficient, superdirective performance characteristics in a physically compact structure. In addition to discussing such designs in more depth, my presentation will briefly review the successful prototype Huygens dipole antennas summarized in [11] as the precursors to these more advanced, higher order UMMAs. It will also introduce more of the underlying multipole physics/electromagnetics described in [12] that led to the measured superdirective electrically small UMMA in [7] and to the higher performance superdirective designs reported in [9] and here.

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